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Long-range quantum gate via Rydberg states of atoms in a thermal microwave cavity

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We propose an implementation of a universal quantum gate between pairs of spatially separated atoms in a microwave cavity at finite temperature. The gate results from reversible laser excitation of Rydberg states of atoms interacting with each other via exchange of virtual photons through a common cavity mode. Quantum interference of different transition paths between the two-atom ground and double-excited Rydberg states makes both the transition amplitude and resonance largely insensitive to the excitations in the microwave cavity quantum bus which can therefore be in any superposition or mixture of photon number states. Our scheme for attaining ultra-long-range interactions and entanglement also applies to mesoscopic atomic ensembles in the Rydberg blockade regime and is scalable to many ensembles trapped within a centimeter-sized microwave resonator.

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Quantum interfaces between solid-state devices and cold atoms are the backbone of a novel class of hybrid quantum systems [1–4] linking fast quantum gates [5,6] with long-lived quantum memories [7,8] and optical quantum communication channels [9–11]. Superconducting coplanar waveguide resonators operating in the microwave regime have been demonstrated to provide strong coupling between solid-state superconducting qubits [12–15] and to mediate quantum state transfer between superconducting qubits and spin ensembles [16–18]. Cold atoms trapped near the surface of an atom chip [19–23] possess good coherence properties and strong optical (Raman) transitions. Therefore, ensembles of trapped atoms or molecules interacting with on-chip microwave resonators were proposed as convenient systems [24–28] for realizing quantum gates and memories as well as optical interfaces. A promising approach to achieve strong coupling of atoms to microwave resonators [27–31] is to employ the atomic Rydberg states having huge electric dipole moments [32].

A common feature of all these schemes is that they require an initially empty microwave cavity which should be kept at very low temperatures of $T \lesssim 100$ mK. This is routinely achieved with solid-state superconducting circuits in dilution refrigerators but is challenging to realize and maintain in combination with ultracold atoms [33]. In turn, atoms are routinely trapped on atom chips at $T \simeq 4$ K [22,31], but then the integrated superconducting cavities have lower quality (Q) factor, and the presence of thermal cavity photons and their fluctuations would preclude high-fidelity quantum operations.

Here we present a scalable scheme for cavity-mediated coherent interactions between Rydberg states of atoms in a thermal microwave cavity. We show that a universal quantum gate between pairs of laser-driven atoms, or atomic ensembles in the Rydberg blockade regime [34,35], can be achieved with current cold-atom experimental technology [21,30,31,36,37].

Our work has been inspired by the seminal proposal of Sørensen and Mølmer [38] to realize quantum computation and entanglement with ions sharing a common vibrational mode subject to thermal fluctuations. Different from the ion trap, in our scheme the cavity mode exchanging photons with the thermal environment simultaneously interacts with atoms on the transitions between neighboring Rydberg states, playing the role of the so-called quantum bus for spatially separated qubits [Fig. 1(a)]. Now the photon-number uncertainty affects not only the amplitude of the laser-mediated transition between the atomic ground and Rydberg states, but also induces fluctuating ac-Stark shifts of atomic levels. By a suitable choice of the system parameters, we ensure that both the multiphoton transition amplitude and its resonant frequency are insensitive to the cavity photon number, making the gate operations immune to the exchange of photons with the thermal environment, which relaxes the need for a low-temperature high-Q cavity.

We first describe the scheme for individual atoms and later adapt it also to atomic ensembles forming Rydberg "superatoms" [34,35,39]. Consider a pair of identical atoms 1 and 2 with the ground state $|g\rangle$ and the highly excited Rydberg state $|r\rangle$ [Fig. 1(b)]. The atoms interact nonresonantly with a common mode of the microwave cavity via the dipole-allowed transitions to the adjacent Rydberg states $|a\rangle$ and $|b\rangle$; the corresponding coupling strengths are denoted by $g_{a,b}$. Two excitation lasers of optical frequencies $\omega_{1,2}$ act on the atomic transitions $|g\rangle_1 \to |a\rangle_1$ and $|g\rangle_2 \to |b\rangle_2$ with the Rabi frequencies $\Omega_{1,2}$. For simplicity we assume for now that each laser interacts only with the corresponding atom (see below for the symmetric coupling of both atoms). The total Hamiltonian of the system in the rotating wave approximation takes the form $H = H_c + \sum_{i=1,2}[H_a^{(i)} + V_{ac}^{(i)} + V_{al}^{(i)}]$. Here $H_c = \hbar\omega_c(\hat{c}^\dagger\hat{c} + \frac{1}{2})$ is the Hamiltonian for cavity field with the photon creation $\hat{c}^\dagger$ and annihilation $\hat{c}$ operators in the mode of frequency ω_c, $H_a^{(i)} = \hbar\sum_\mu \omega_\mu |\mu\rangle_i\langle\mu|$ is the Hamiltonian of the unperturbed atom i with the Bohr (excitation) frequencies ω_μ of the corresponding energy levels $|\mu\rangle$ $(\mu = g,r,a,b)$, $V_{ac}^{(i)} = \hbar(g_a\hat{c}\,|a\rangle_i\langle r| + g_b\hat{c}^\dagger\,|b\rangle_i\langle r| + \text{H.c.})$ describes the atom-cavity interactions, and $V_{al}^{(1)} = \hbar\Omega_1 e^{-i\omega_1 t}\,|a\rangle_1\langle g| + \text{H.c.}$ and $V_{al}^{(2)} = \hbar\Omega_2 e^{-i\omega_2 t}\,|b\rangle_2\langle g| + \text{H.c.}$ describe the laser driving of atoms 1 and 2, respectively.

We assume that the two atoms initially in the ground state $|g\rangle$ are in spatially separated traps at equivalent positions,

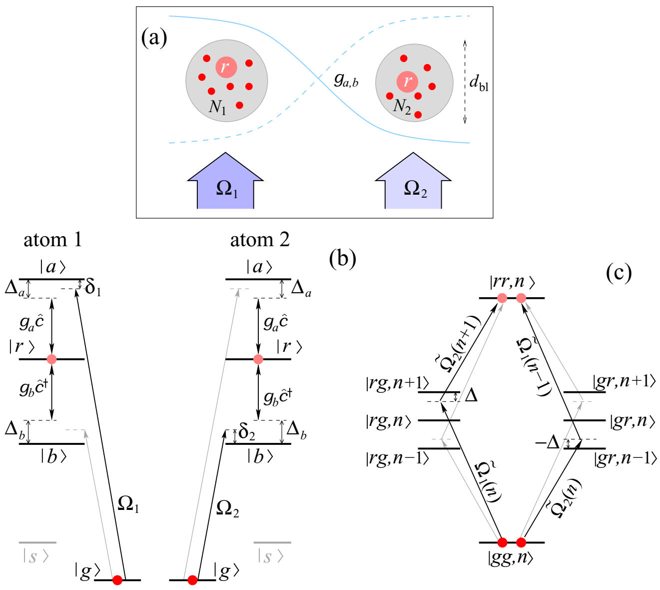

FIG. 1. (Color online) Schematics of the system. (a) Spatially separated atoms, or superatoms composed of $N_{1,2}$ atoms within the Rydberg blockade distance d_{bl}, interact with a pair of optical lasers and a common mode of a microwave cavity. (b) Level scheme of two atoms interacting with the cavity field on the Rydberg transitions $|r\rangle \leftrightarrow |a\rangle, |b\rangle$ with strengths $g_{a,b}$ and driven by the laser fields with Rabi frequencies $\Omega_{1,2}$ on the transitions from the ground state $|g\rangle$ to states $|a\rangle, |b\rangle$. (c) Under appropriate conditions (see text for details), there are two interfering excitation pathways from the two-atom ground state $|gg,n\rangle$ to the double-excited Rydberg state $|rr,n\rangle$, which cancel the dependence of the total transition amplitude $\bar{\Omega}$ on the cavity photon number n.

close to field antinodes of the microwave cavity containing an arbitrary number of photons n. Our aim is to achieve coherent oscillations between the collective states $|gg,n\rangle$ and $|rr,n\rangle$ with maximal amplitude and oscillation frequency which do not depend on n. Denoting the detunings of the laser fields by $\delta_1 = \omega_{ag} - \omega_1$ and $\delta_2 = \omega_{bg} - \omega_2$ and taking Rabi frequencies $\Omega_{1,2} \ll |\delta_{1,2}|$, we adiabatically eliminate the intermediate atomic states $|a\rangle_1$ and $|b\rangle_2$, obtaining two-photon Rabi frequencies $\tilde{\Omega}_1(n) = \frac{\Omega_1 g_a \sqrt{n+1}}{\delta_1}$ and $\tilde{\Omega}_2(n) = \frac{\Omega_2 g_b \sqrt{n}}{\delta_2}$ on the transitions $|gg,n\rangle \to |rg,n+1\rangle$ and $|gg,n\rangle \to |gr,n-1\rangle$ accompanied by addition and subtraction of a cavity photon, respectively [Fig. 1(c)].

We next take large and unequal detunings $\Delta_a = \omega_{ar} - \omega_c$ and $\Delta_b = \omega_{rb} - \omega_c$ of the cavity field from the transition resonances between the atomic Rydberg states, $|\Delta_{a,b}| \gg g_{a,b}$. To avoid cavity-mediated Förster resonances $|rr,n\rangle \to |ab,n\rangle, |ba,n\rangle$ [27], we require that $|\Delta_a - \Delta_b| \gg g_a g_b |\frac{n+1}{\Delta_b} - \frac{n}{\Delta_a}|$ for all $n \lesssim n_{\mathrm{max}}$, where n_{max} is the maximal possible cavity photon number [typically $n_{\mathrm{max}} \approx 2\bar{n}_{\mathrm{th}}$ for a thermal field with the mean number of photons $\bar{n}_{\mathrm{th}} = (e^{\hbar\omega_c/k_B T} - 1)^{-1}$]. If we now choose the two-photon detunings $\Delta_{1,2} \approx \delta_{1,2} \mp \Delta_{a,b}$ of states $|rg,n+1\rangle$ and $|gr,n-1\rangle$ to have equal magnitude but opposite sign, $\Delta_1 = -\Delta_2 = \Delta \gg \tilde{\Omega}_{1,2}$, we can also eliminate these intermediate states and obtain resonant multiphoton transitions between states $|gg,n\rangle$ and $|rr,n\rangle$ involving two laser photons and an exchange of a (virtual) cavity photon between the two atoms. With two equivalent excitation paths from $|gg,n\rangle$ to $|rr,n\rangle$ [Fig. 1(c)], the resulting transition amplitude (effective Rabi frequency) is

$$\bar{\Omega} = \frac{\tilde{\Omega}_1(n)\tilde{\Omega}_2(n+1)}{\Delta_1(n)} + \frac{\tilde{\Omega}_2(n)\tilde{\Omega}_1(n-1)}{\Delta_2(n)} = \frac{\Omega_1\Omega_2 g_a g_b}{\delta_1\delta_2\Delta}. \tag{1}$$

This is the photonic cavity analog of the Sørensen-Mølmer scheme [38] for the ion trap with phonons. The critical question now is how to precisely tune the detunings $\Delta_{1,2}$ and achieve the multiphoton resonance $|gg,n\rangle \leftrightarrow |rr,n\rangle$ for any n.

From the perturbative analysis, we obtain that the detunings $\Delta_{1,2}(n)$ depend on the cavity photon number n through the second-order (ac Stark) shifts of levels $|r\rangle_{1,2}$,

$$\Delta_1(n) \simeq \delta_1 + \frac{\Omega_1^2}{\delta_1} - \Delta_a - \frac{g_a^2(n+1)}{\delta_1} + \frac{g_b^2(n+2)}{\Delta_b},$$

$$\Delta_2(n) \simeq \delta_2 + \frac{\Omega_2^2}{\delta_2} + \Delta_b - \frac{g_a^2(n-1)}{\Delta_a} - \frac{g_b^2 n}{\delta_2}.$$

With an appropriate choice of $\delta_{1,2}$ and $\Omega_{1,2}$, and requiring that $\frac{g_a^2}{\Delta_a} = \frac{g_b^2}{\Delta_b}$, the n dependence of the detunings is greatly suppressed, $\Delta_{1,2}(n) \simeq \Delta_{1,2}(0)(1 + \frac{g_{a,b}^2}{\Delta_{a,b}^2} n)$, and we can satisfy the condition $|\Delta_1(n) + \Delta_2(n)| \ll \bar{\Omega}$ for any n. This leads to $\bar{\Omega}$ that only weakly depends on n, $\bar{\Omega}(n) \simeq \bar{\Omega}(0)(1 - \frac{g_a^2}{\Delta_a^2} n)$. In order to ensure the multiphoton resonance on the transition $|gg,n\rangle \to |rr,n\rangle$, we also need to consider higher-order level-shifts of $|gg,n\rangle$ and $|rr,n\rangle$. To fourth order in the laser and cavity field couplings, the largest contribution to the level shift of $|gg,n\rangle$ is given by $S_{gg}(n) = \frac{\Omega_1^2 g_a^2(n+1)}{\delta_1^2 \Delta_1(n)} + \frac{\Omega_2^2 g_b^2 n}{\delta_2^2 \Delta_2(n)}$, which, remarkably, has the same structure as $\bar{\Omega}$. Since $\Delta_1(n) \simeq -\Delta_2(n)$, we can choose $\frac{\Omega_1 g_a}{|\delta_1|} = \frac{\Omega_2 g_b}{|\delta_2|}$ to make S_{gg} nearly independent on n and absorb it into $\Delta_{1,2}(n)$. Finally, the fourth-order shifts of $|rr,n\rangle$, $S_{rr}(n) \propto \frac{g_{a,b}^4 n^2}{\Delta_{a,b}^3}$ [27] are small in comparison and can therefore be neglected, which we verify below through exact numerical simulations for the complete system.

In general, the atom-cavity couplings g_a and g_b are not equal, since they are proportional to the electric dipole matrix elements on different transitions $|r\rangle \to |a\rangle$ and $|r\rangle \to |b\rangle$, while the corresponding detunings Δ_a and Δ_b can be tuned by static electric (Stark) or magnetic (Zeeman) fields [32]. These, together with the flexibility to choose the laser detunings $\delta_{1,2}$ and Rabi frequencies $\Omega_{1,2}$, allows us to satisfy all of the above conditions for n-independent resonant Rabi oscillations between states $|gg\rangle$ and $|rr\rangle$. We can estimate the maximal attainable oscillation frequency $\bar{\Omega}$, assuming that the main limiting factor is the atom-cavity coupling strengths $g_{a,b}$ [28,30], since the laser Rabi frequencies can be collectively enhanced in the superatom regime [34,35,39]. Recall that we require the intermediate-state detunings $|\Delta_{1,2}(n)| \gg \tilde{\Omega}_{1,2}(n)$ for each $n \lesssim n_{\mathrm{max}}$. Then, with $\eta = |\delta_2|/\Omega_2$ and $\tilde{\eta} = |\Delta_1|/\tilde{\Omega}_1$ and all of the above conditions satisfied, we obtain that $\bar{\Omega}_{\mathrm{max}} \leqslant \frac{g_b}{\eta\tilde{\eta}\sqrt{n_{\mathrm{max}}+1}}$. As an estimate, assuming $g_{a,b} \simeq 2\pi \times 10$ MHz, $\eta, \tilde{\eta} \simeq 10$ and a cavity mode with $\omega_c \simeq 2\pi \times 15$ GHz at $T = 4$ K, leading to $\bar{n}_{\mathrm{th}} \approx 5$ ($n_{\mathrm{max}} \approx 2\bar{n}_{\mathrm{th}}$), we have $\tilde{\Omega}_{\mathrm{max}} \approx 2\pi \times 30$ kHz.

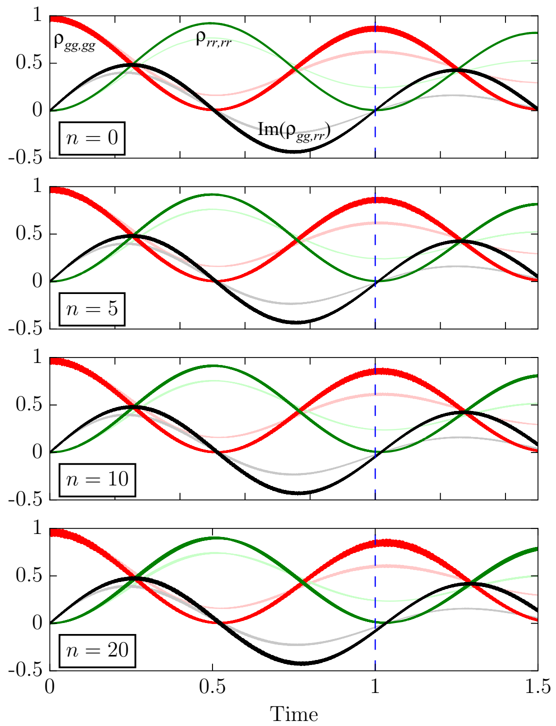

FIG. 2. (Color online) Rabi oscillation between states $|gg,n\rangle$ and $|rr,n\rangle$ for the cavity photon numbers $n = 0,5,10,20$. Thick (full) lines show coherent oscillations of populations $\rho_{gg,gg} = \langle gg|\,\hat{\rho}\,|gg\rangle$, $\rho_{rr,rr} = \langle rr|\,\hat{\rho}\,|rr\rangle$ and coherence $\rho_{gg,rr} = \langle gg|\,\hat{\rho}\,|rr\rangle$ for the Rydberg state decay $\Gamma = 0.142\,[\bar{\Omega}(0)/2\pi]$ and no dephasing, $\gamma = 0$, while thinner (shaded) lines show damped oscillations of the same quantities for large dephasing $\gamma = 0.4\,[\bar{\Omega}(0)/2\pi]$. Parameters are $\Omega_1 = 56.50, \Omega_2 = 60.00, \delta_1 = 663.8, \delta_2 = -742.0, g_a = 9.5, g_b = 10.0, \Delta_a = 722.0, \Delta_b = 800.0$ ($\times 2\pi$ MHz), leading to $\bar{\Omega}(0) \approx 2\pi \times 21.1$ kHz. Time is in units of $[2\pi/\bar{\Omega}(0)]$.

Note that, similarly to the ion trap scheme [38], our scheme would work in exactly the same way for the symmetric coupling of both atoms to both excitation lasers [Fig. 1(b)]. This opens two new excitation paths from $|gg,n\rangle$ to $|rr,n\rangle$ via states $|rg,n-1\rangle$ and $|gr,n+1\rangle$ [Fig. 1(c)], which enhances the transition amplitude $\bar{\Omega}$ by a factor of two. Care should be taken only to properly account for, and compensate for, the additional second- and fourth-order level shifts of the atomic ground states.

To validate our perturbative calculations, we numerically solve the master equation $\frac{\partial}{\partial t}\hat{\rho} = -\frac{i}{\hbar}[H,\hat{\rho}]$ for the density operator $\hat{\rho}$ using the exact Hamiltonian H for the pair of atoms initially in the ground state $|g\rangle$ and the cavity field with n photons. Results for different n are shown in Fig. 2, which verifies that with a proper choice of parameters, the transition resonance $|gg,n\rangle \leftrightarrow |rr,n\rangle$ and the effective Rabi frequency $\bar{\Omega}$ can simultaneously be made nearly independent of the number of photons in the cavity. We have examined various Fock, coherent and thermal states as the initial states of the cavity field, all yielding very similar results.

We include the realistic relaxation processes affecting the Rydberg states of atoms using the standard Liouvillians [40] with the Lindblad generators $\hat{L}_{\nu}^{(i)} = \sqrt{\Gamma}\,|g\rangle_i\langle\nu|$ for the decay with rate Γ (assumed the same for all $\nu = r,a,b$), and $\hat{L}_{g}^{(i)} = \sqrt{\gamma/2}(\,|g\rangle_i\langle g| - \sum_{\nu}\,|\nu\rangle_i\langle\nu|\,)$ for the dephasing (with respect to the ground state) with rate γ. In Fig. 2 we show strongly damped Rabi oscillations caused by the single-atom decay Γ and relatively large dephasing γ comparable to $\bar{\Omega}$.

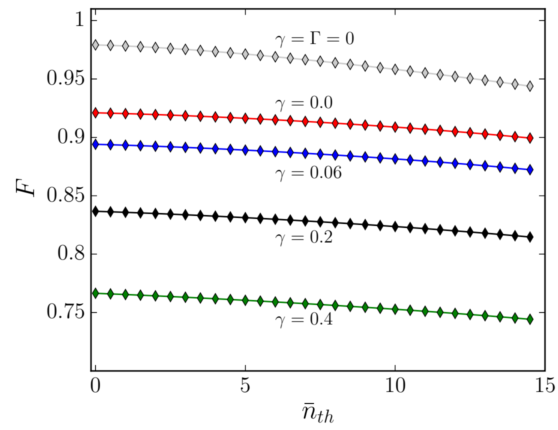

FIG. 3. (Color online) Transfer fidelity F at time $t_\pi = \pi/\bar{\Omega}(0)$ vs the mean number $\bar{n}_{\rm th}$ of thermal photons in the cavity, for the dephasing rates $\gamma = 0,0.06,0.2,0.4$ and the decay rates $\Gamma = 0$ (uppermost reference curve) and $\Gamma = 0.142$ (all other curves), in units of $[\bar{\Omega}(0)/2\pi]$. Other parameters are as in Fig. 2.

In Fig. 3 we plot the fidelity $F = \sum_n p(n|\bar{n}_{\rm th})\langle rr,n|\,\hat{\rho}(t_\pi)\,|rr,n\rangle$ of transfer $|gg\rangle \to |rr\rangle$ at time $t_\pi = \pi/\bar{\Omega}(0)$ (effective π pulse for $n = 0$) as a function of the dephasing rate γ and the mean number of thermal photons $\bar{n}_{\rm th}$ which determines the photon number probability distribution $p(n|\bar{n}_{\rm th}) = \bar{n}_{\rm th}^n/(\bar{n}_{\rm th}+1)^{n+1}$. We observe that large dephasing detrimentally affects the transfer fidelity by damping the amplitude of Rabi osculations between $|gg\rangle$ and $|rr\rangle$. On the other hand, the fidelity only weakly and nearly linearly decreases with increasing the cavity photon number, due to the slight decrease of the effective Rabi frequency $\bar{\Omega}(n)$ discussed above. This can be compensated for by appropriately correcting the transfer time t_π if $\bar{n}_{\rm th}$ is approximately known.

We now discuss the implications of our results for quantum information applications. Each atom can encode a qubit as a coherent superposition of long-lived states $|g\rangle$ and $|s\rangle$ [Fig. 1(b)]. Gate operation can be performed on any pair of atoms within the cavity by addressing the desired atoms with focused laser pulses $\Omega_{1,2}$. Assuming that state $|s\rangle$ is decoupled from the laser field(s), the two-atom state $|ss\rangle$ is immune to the lasers. If only one of the atoms is initially in state $|g\rangle$ and the other atom is in $|s\rangle$, both atoms remain in their initial states due to the absence of multiphoton resonances to any Rydberg state. Finally, if both atoms are in state $|g\rangle$, the application of lasers for time $t_{2\pi} = 2\pi/\bar{\Omega}$ will drive a complete Rabi cycle on the transition $|gg\rangle \leftrightarrow |rr\rangle$, resulting in the sign change of $|gg\rangle$. Since the other initial states remain unaltered, this transformation corresponds to the universal two-qubit CPHASE logic gate [40,41].

Our scheme is also applicable to ensembles of trapped atoms in the Rydberg blockade regime [34,35]. Individual ensembles, each containing $N_i \gg 1$ atoms, can encode qubits in the collective ground $|G\rangle \equiv$

$|g_1,g_2,\ldots,g_{N_i}\rangle$ and symmetric single spin-flip (hyperfine) $|S\rangle \equiv \frac{1}{\sqrt{N_i}}\sum_{k=1}^{N_i}|g_1,\ldots,s_k,\ldots,g_{N_i}\rangle$ states. For optically dense ensembles, the qubit encoding superposition of states $|G\rangle$ and $|S\rangle$ can be reversibly mapped onto photonic qubits via stimulated Raman techniques [8]. We assume that each ensemble of linear dimension smaller than a certain blockade distance $d_{\rm bl}$ [Fig. 1(a)] behaves as a Rydberg superatom [34,35,39], wherein multiple excitations are suppressed by the strong Rydberg-state interactions. This allows implementation of arbitrary single-qubit and universal two-qubit quantum gates as follows. In each ensemble, before and after the gate execution, we apply a single-atom π pulse on the transition $|s\rangle \to |r'\rangle$, where $|r'\rangle$ is a Rydberg state which can block excitation of any other atom from state $|g\rangle$ to states $|r'\rangle$, $|r\rangle$ (and, possibly, to $|a\rangle$, $|b\rangle$) due to the strong dipole-dipole or van der Waals interactions [35]. This operation reversibly maps the qubit state $|S\rangle$ onto the symmetric single Rydberg excitation state $|R'\rangle \equiv \frac{1}{\sqrt{N_i}}\sum_{k=1}^{N_i}|g_1,\ldots,r'_k,\ldots,g_{N_i}\rangle$. Single-qubit transformations are then performed in the two-state subspace of $|G\rangle$ and $|R'\rangle$ by resonant lasers with collectively enhanced Rabi frequencies $\Omega = \sqrt{N_i}\Omega^{(1)}$, where $\Omega^{(1)}$ is the single-atom Rabi frequency on the transition $|g\rangle \leftrightarrow |r'\rangle$ [34]. For the two-qubit operations, any pair of atomic ensembles i,j trapped within the cavity can be addressed by appropriate lasers with collectively enhanced Rabi frequencies $\Omega_{1,2} = \sqrt{N_{i,j}}\Omega^{(1)}_{1,2}$. Then, during time $t_{2\pi}$, the initial state $|G\rangle_i|G\rangle_j$ will undergo a complete Rabi cycle to $|R\rangle_i|R\rangle_j$ and back, acquiring a π phase shift (sign change), assuming that in each ensemble multiple excitations of $|r\rangle$ are blocked by strong Rydberg-state interactions. If any, or both, of the ensembles were initially in state $|S\rangle$ mapped onto $|R'\rangle$, the atom in $|r'\rangle$ would preclude the transfer of ground-state atoms $|g\rangle$ to $|r\rangle$, and therefore such initial states will remain unaltered. This completes the implementation of the CPHASE logic gate with Rydberg superatoms.

To conclude, our scheme to implement long-range quantum gates is feasible with present-day experimental technologies [21,22,30,31,33] involving optical excitation of Rydberg states of trapped atoms and their interactions with microwave resonators. This gate is largely insensitive to the number of cavity photons and it can therefore operate in a finite-temperature microwave cavity with modest photon lifetimes $1/\kappa \gtrsim 1\,\mu$s. The main decoherence mechanisms reducing the achievable gate fidelity $F \simeq 1-(\Gamma+\gamma)\tau$ during time $\tau = 2\pi/\bar{\Omega}(0) \sim 50\,\mu$s are the decay $\Gamma \simeq 3$ kHz of Rydberg states and dephasing $\gamma \lesssim 5$ kHz of the nonresonant multiphoton transitions [with $\Delta \gg \tilde{\Omega}_{1,2}(n)$], which are slow by construction [38]. It would thus be interesting to explore the near-resonant excitations [with $\Delta/\tilde{\Omega}_1(0) = 2\sqrt{m}$ $(m = 1,2,\ldots)$] analogous to the fast-gate regime of the ion traps [42]. Unlike our present gate, however, such a fast-gate scheme will be sensitive to the change of cavity photon number [43] during the gate time $\tau = 2\pi\sqrt{m}/\tilde{\Omega}_1(0) \sim 10\,\mu$s requiring cavities with longer photon lifetimes $1/\kappa \gg \tau$.

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