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Investigating the emergence of time in stationary states with trapped ions

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Even though quantum systems in energy eigenstates do not evolve in time, they can exhibit correlations between internal degrees of freedom in such a way that one of the internal degrees of freedom behaves like a clock variable, and thereby defines an internal time, that parametrizes the evolution of the other degrees of freedom. This situation is of great interest in quantum cosmology where the invariance under reparametrization of time implies that the temporal coordinate dissapears and is replaced by the Wheeler-DeWitt constraint. Here we show that the emergent character of an internal time variable can be investigated experimentally using the exquisite control now available on moderate-size quantum systems. We describe in detail how to implement such an experimental demonstration using the spin and motional degrees of freedom of a single trapped ion.

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Introduction. In classical and quantum mechanics time is an external parameter. However, this external, absolute time is not accessible experimentally. Rather time is measured by clocks, that is dynamical systems whose evolution is related in a simple way to the external absolute time.

In classical mechanics substituting clock time for absolute external time is essentially just a change of variables. The underlying formalism and its interpretation has been recently investigated by Barbour [1–3].

In quantum mechanics novel features appear. First, in quantum theory clocks are affected by quantum fluctuations, have inherent uncertainties, and when a clock and a system interact they necessarily disturb each other [4]. Second, an isolated quantum system in an energy eigenstate is in a stationary state. It does not evolve in terms of external time (except for a physically meaningless overall phase). However, even in this case internal degrees of freedom can be used as clocks and define an internal time [4]. Since the state is stationary, this internal time is totally uncorrelated to the external time. In fact, one can argue that since external absolute time is unobservable, the time-dependent Schrödinger equation is just a mathematical convenience, and that all physical quantities (states, observables) should be time independent [5].

Related to this is the problem of emergence of an effective time-dependent Schrödinger equation from the time-independent Schrödinger equation. This question has a long history going back to Mott's seminal investigation of scattering [6]; see also, e.g., Refs. [7,8]. The case of a radiation field coupled to an atomic system is treated in Ref. [9].

The above issues reappear more forcibly when one considers the quantization of gravity. Indeed, classical general relativity is invariant under reparametrization of time and therefore has no preferred time variable. If one tries to formally quantize gravity, the invariance of the theory under reparametrization of time implies that the temporal coordinate disappears and is replaced by a constraint equation [10]. This constraint equation, the Wheeler-DeWitt equation, is ill defined mathematically because of the appearance of second-order functional derivatives. However, it can be used to study how time emerges in quantum gravity. In particular in quantum cosmology one generally studies minisuperspace models in which only one, or a few, gravitational degrees of freedom are kept. In this context it has been proposed that some internal degrees of freedom can act as clocks and parametrize the evolution of the other degrees of freedom [11–13]. One then recovers an approximate time-dependent Schrödinger equation. The clock variable should be as "heavy," i.e., as classical, as possible, in order that it be affected as little as possible by quantum fluctuations and be as little perturbed as possible by the back action of the other degrees of freedom. For these reasons in quantum cosmology the clock variable is generally taken to be the radius of the universe.

Formally, the problems of defining internal time for stationary solutions of the nonrelativistic Schrödinger equation, and for solutions of the Wheeler-DeWitt constraint in the mini-superspace approximation, are practically identical. (The main difference is that the Wheeler-DeWitt constraint is not positive definite, whereas matter Hamiltonians are.) For this reason the issue of internal versus external time in matter systems can be viewed as a proxy for the more fundamental issue of time in quantum cosmology [4,5,14]. Further works addressing this question can be found in, e.g., Refs. [15–19].

Motivated by the above considerations, in the present work we study how the emergence of time in stationary solutions of the nonrelativistic Schrödinger equation can be studied experimentally. The exquisite control that is now available over moderate-size quantum systems enables an increasing number of foundational questions in quantum mechanics to be investigated experimentally (instead of analytically or numerically). For examples, see the recent review articles, Refs. [20–25].

Thus, a first experiment illustrating some aspects of the origin of time in quantum cosmology using two photons was recently reported in Ref. [26]. This experiment, however, used

propagating photons (therefore time evolving) and implements external time evolution by varying the thickness of wave plates (rather than actually waiting for an interval of external time), and the clock variable only had dimension 2. Here, we focus on trapped ion systems and present in detail how such an experiment could be realized in the case where the clock variable is the vibrational degree of freedom of the ion, and the internal state is a spin degree of freedom. Excitation numbers of the order of 10 for the clock variable should be readily attainable.

Model based on spin and vibrational degrees of freedom. We consider the simple model in which the clock is realized by a harmonic oscillator and the other degrees of freedom are realized by an angular momentum degree of freedom (a spin S particle); see Refs. [4,5,9,14]. The Hamiltonian is

$$H = \omega a^\dagger a + \omega \sum_{m=0}^{2S} m|m\rangle\langle m|\,, \tag{1}$$

where $a^\dagger$,a are the creation and destruction operators for the harmonic oscillator. We have adjusted the potential of the oscillator and an external magnetic field that couples to the spin's magnetic moment and thus lifts its degeneracy, such that the frequencies ω of the oscillator and spin are equal.

The harmonic oscillator acts as clock. In classical mechanics time is therefore given by the phase of the oscillator, which can be deduced from the position and momentum through $t = \frac{1}{\omega}\arctan q/p$. However, this procedure cannot be applied to a quantum model because there does not exist a well-defined time operator, equivalent in this case to a well-defined phase operator. But if the total excitation number of the oscillator is less than N, then we can use the Pegg-Barnett phase states [27–29]:

$$|\Theta_k\rangle = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} e^{-i2\pi nk/N}|n\rangle, \quad k = 0,\ldots,N-1, \tag{2}$$

where $|n\rangle = a^{\dagger n}|0\rangle/\sqrt{n!}$ are the number states. The phase states sum to the identity over the subspace $a^\dagger a < N$:

$$\sum_{k=0}^{N-1} |\Theta_k\rangle\langle\Theta_k| = \sum_{n=0}^{N-1} |n\rangle\langle n|. \tag{3}$$

We can therefore define a phase operator,

$$\Theta = \frac{2\pi}{N} \sum_{k=0}^{N-1} k|\Theta_k\rangle\langle\Theta_k|, \tag{4}$$

whose measurement yields a discretized approximation of the phase of the harmonic oscillator, and therefore of time.

Time dependent states. Consider that the spin is initially in the normalised state $|\psi\rangle = \sum_{m=0}^{2S} a_m|m\rangle$ with a_m arbitrary complex amplitudes satisfying $\sum_m |a_m|^2 = 1$. If we evolve this state according to the time-dependent Schrödinger equation we find

$$|\psi(t_{\text{ext}})\rangle = \sum_{m=0}^{2S} a_m e^{-i\omega m t_{\text{ext}}}|m\rangle, \tag{5}$$

where t_{ext} denotes the external time (i.e., the laboratory time). Suppose that the clock is in a phase state $|\Theta_k\rangle$ corresponding to, say, $k = 0$. The overall state at time $t_{\text{ext}} = 0$ is $|\Psi\rangle = |\psi\rangle|\Theta_0\rangle$. If we evolve this state according to the time-dependent Schrödinger equation, the spin and the oscillator evolve independently:

$$|\Psi(t_{\text{ext}})\rangle = \left(\sum_{m=0}^{2S} e^{-i\omega m t_{\text{ext}}} a_m|m\rangle\right)\left(\sum_{n=0}^{N-1} \frac{e^{-i\omega n t_{\text{ext}}}}{\sqrt{N}}|n\rangle\right). \tag{6}$$

Stationary state. Now let us consider a particular energy eigenstate for the system formed by the spin and the clock. We project the state $|\Psi(t_{\text{ext}})\rangle$ onto the subspace of energy $E = \omega M$, for some integer value of M. This yields the stationary entangled state

$$|\Psi_M\rangle = \sum_{m=0}^{\min\{2S,M\}} a_m|m\rangle|M-m\rangle. \tag{7}$$

In order to exhibit the evolution in internal time, we carry out the joint measurement of both the phase operator Θ on the state $|\Psi_M\rangle$ (in order to measure the internal time) and of an operator acting only on the spin degrees of freedom. Suppose the measurement of the phase operator Θ yields the result $\frac{2\pi k}{N}$. The unormalized state of the spin conditional on this measurement outcome is

$$|\psi(k)\rangle = \frac{e^{i2\pi kM/N}}{\sqrt{N}} \sum_{m=0}^{\min\{2S,M\}} a_m e^{-i2\pi km/N}|m\rangle. \tag{8}$$

The interpretation of Eq. (8) is that each value of internal time (i.e., of k) occurs with equal probability (since the norm of $|\psi(k)\rangle$ in Eq. (8) is the probability of finding outcome k, and is independent of k), and that the state of the spin conditional on phase k being measured is identical to the spin having evolved for a time $t_{\text{int}} = 2\pi k/(\omega N)$, where t_{int} is the internal time [compare with Eq. (5)].

(Note that the use of Pegg-Barnett phase states to parametrize time evolution circumvents the problems encountered in other approaches to treat classical turning points of the clock variable; see, e.g., Refs. [9,18].)

Experimental implementation using a trapped ion. The basic requirements for any experiment that wishes to illustrate the emergence of time in stationary quantum states, using the above states are: (1) One needs a system with two degrees of freedom described by the Hamiltonian Eq. (1); this can be realized by trapping a single atomic ion in a harmonic potential where the ion's effective spin (suitable internal ionic states) only interacts with the motional degree of freedom when additional fields are applied. (2) At (external) time $t_{\text{ext}} = 0$ one initializes the system in state $|\Psi_M\rangle$; (3) at a later (external) time t_{ext}, one carries out a measurement of the operator $A_S \otimes |\Theta_k\rangle\langle\Theta_k|$, where A_S is an operator acting on the spin particle degree of freedom. However, such a measurement of correlations between spin and motion of a trapped ion is not always easy to realize experimentally (for instance the proposals of Refs. [30,31] could possibly be used, but to our knowledge have so far not been implemented in the laboratory). Note that the measurement of operator Θ in Eq. (4) will not permit a full tomography of the quantum state of the spin and oscillator, but it is of course enough to check the validity of Eq. (8). We will focus here on measurements that require short total interaction times between electromagnetic

fields and ion and thus are less susceptible to decoherence during the measurement process. They allow one to deduce at least approximately the expectation of $A_S \otimes |\Theta_k\rangle\langle\Theta_k|$. (4) One verifies that the results of the measurement are consistent with the predictions of Eq. (8): namely that all values of internal time $t_{\rm int}$ are equally probable, that the spin state has evolved in internal time, and that the results are independent of the external time $t_{\rm ext}$ at which the system is measured.

In order to realize the Hamiltonian Eq. (1), we consider as spin degree of freedom the hyperfine levels of a trapped ion, and as position degree of freedom the motion along one axis of the trap. By adjusting the magnetic field and/or the trap spring constant, one can adjust the energy hyperfine splitting and/or the trap vibration frequency. In this way one can realize the degenerate Hamiltonian Eq. (1). The hyperfine levels of the ions frequently used in experiments (e.g., $^9Be^+$, $^{43}Ca^+$, $^{171}Yb^+$) could be used to implement moderate values spin $S = 1/2,1,3/2$, etc. In what follows we focus on the situation where we only use two hyperfine states, corresponding to an effective spin $S = 1/2$. We denote by $|\uparrow\rangle$ and $|\downarrow\rangle$ these two hyperfine states.

A key procedure is measurement of the internal states of the trapped ion, which is usually carried out by registering resonance fluorescence. Depending on the internal state of the ion it will either scatter light or not when subjected to an appropriate light beam [32–34]. Thus, one can detect with near unity efficiency whether the ion is in a specific hyperfine state, say $|m = 0\rangle$. One can also detect with near unit efficiency whether the ion is in the ground vibrational state $|n = 0\rangle$ [35]: by tuning the laser to the red sideband, the ground state will not fluoresce, as opposed to any other excited vibrational state.

Using the above measurement procedure, one can initialize the system (by projection) in the state $|\downarrow\rangle|n = 0\rangle$ [36]. Unitary transformations between the states $|\uparrow\rangle,|\downarrow\rangle$ can readily be carried out. And the vibrational state of motion of the atom can be manipulated with high precision to generate Fock, coherent, or squeezed states [35]. These techniques can be used to produce the state

$$|\Psi_M\rangle = a_\uparrow|\uparrow\rangle|n\rangle + a_\downarrow|\downarrow\rangle|n+1\rangle, \tag{9}$$

for moderate values of n (say $n \simeq 10$), where $a_\uparrow,a_\downarrow$ are arbitrary complex numbers satisfying $|a_\uparrow|^2 + |a_\downarrow|^2 = 1$. This is a particular realization of a state of the form given in Eq. (7).

Following this preparation procedure, one waits a time $t_{\rm ext}$, and then measures the ion. To this end recall that arbitrary unitary transformations can be carried out on the spin degree of freedom, and that displacement and squeezing operations can readily be carried out on the vibrational degree of freedom [35,37]. It is thus possible, using the resonance fluorescence measurement described above, to measure the probability to be in the state

$$(\mu|\uparrow\rangle + \nu|\downarrow\rangle) \otimes D(\alpha)S(z)|n = 0\rangle, \tag{10}$$

where μ,ν are arbitrary complex parameters satisfying $|\mu|^2 + |\nu|^2 = 1$, $D(\alpha) = \exp(\alpha a^\dagger - \alpha^* a)$ is the displacement operator, and $S(z) = \exp((za^{\dagger 2} - z^* a^2)/2)$ the squeezing operator, with arbitrary complex numbers α,z.

By varying the parameters μ,ν,α,z one can determine how the spin is evolving in terms of the internal time. There are different possibilities that we sketch, the first being the projection onto coherent states. To this end, consider the information obtained by measuring the probability to be in the state

$$|\phi_{\mu\nu\alpha}\rangle = (\mu|\uparrow\rangle + \nu|\downarrow\rangle) \otimes |\alpha\rangle, \tag{11}$$

where $|\alpha\rangle = D(\alpha)|n = 0\rangle$ is a coherent state (displaced vacuum state). Projecting onto the coherent state $|\alpha\rangle$ gives us information both on the phase and amplitude of the vibrational degree of freedom. On the other hand, the ideal measurement (of the phase states) gives us only information about the phase. We thus expect that projecting onto a coherent state should be less precise than projecting onto a phase state. This is confirmed by calculation. The probability to find the state $|\phi_{\mu\nu\alpha}\rangle$ is

$$\begin{aligned} P(\mu,\nu,\alpha) &= e^{-|\alpha|^2}\left|\mu^* a_\uparrow \frac{\alpha^{*n}}{\sqrt{n!}} + \nu^* a_\downarrow \frac{\alpha^{*(n+1)}}{\sqrt{(n+1)!}}\right|^2 \\ &= e^{-|\alpha|^2}\frac{|\alpha|^{2n}}{n!}\left|\mu^* a_\uparrow + \nu^* a_\downarrow e^{-i\theta}\frac{|\alpha|}{\sqrt{n+1}}\right|^2, \end{aligned} \tag{12}$$

where $\alpha = e^{i\theta}|\alpha|$. It is thus as if the spin had evolved for internal time $t_{\rm int} = \theta/\omega$. Note that the sum of the probabilities of finding the spin in state $\mu|\uparrow\rangle + \nu|\downarrow\rangle$ and the orthogonal state $\nu^*|\uparrow\rangle - \mu^*|\downarrow\rangle$, $P(\mu,\nu,\alpha) + P(\nu^*, -\mu^*,\alpha)$, is independent of θ, corresponding to the fact that all values of internal time, i.e., of θ, are equally probable.

Note that the measured value of $|\alpha|$ will fluctuate (since in the Fock state $|n\rangle$, $|\alpha|$ has average $\sqrt{n}$ and standard deviation $1/2$). The fluctuations disrupt the measurement through the factor $\frac{|\alpha|}{\sqrt{n+1}}$ on the right-hand side. Indeed only when $|\alpha| = \sqrt{n+1}$ does θ act exactly like the external time, as in Eq. (5). The effect of these fluctuations will decrease as the excitation number n of the harmonic oscillator increases. This shows that as the clock variable becomes more and more classical (large n), the way it is measured becomes less and less relevant, provided some information about the phase of the clock variable is obtained by the measurement.

In the procedure leading to Eq. (8) one determines the spin state conditional on the phase of the oscillator. However, the constraints in ion systems imply that the oscillator state is in fact measured conditional on the measured spin state. By using Bayes theorem, one can recover the time evolution of the spin in terms of the oscillator. In fact, the procedure outlined above can be rephrased as a measurement of the positive Q function of the oscillator conditional on the spin being found in the state $\mu|\uparrow\rangle + \nu|\downarrow\rangle$. By extension one could also measure other quasiprobability distributions of the oscillator conditional on the spin being found in state $\mu|\uparrow\rangle + \nu|\downarrow\rangle$. One possibility would be to determine the Wigner function.

Instead of measuring the whole Wigner function, one could measure the expected value of the generalized quadrature $Y_\theta = (ae^{i\theta} - a^\dagger e^{-i\theta})/2i$ for a set of values of θ. In general, this does not allow full tomography of the vibrational state of the ion but will be enough to verify—with some errors—that the spin and motion are correlated according to Eq. (8). Efficient methods to measure Y_θ for a trapped ion have been proposed, for example, in Refs. [38–40] and applied to detect motional states correlated with internal states of trapped ions, for

example, in Ref. [41]. One could also carry out full tomography of the ion motional state using the procedure of Ref. [42].

Concrete proposal using an Yb ion. As a concrete proposal for such an experiment we outline its implementation using a single Ytterbium ion trapped in an harmonic potential and exposed to a spatially varying magnetic field. First, we define two internal levels, namely two hyperfine states of $^{171}Yb^{+}$: $|\downarrow\rangle \equiv |S_{1/2},F=1,m_F=-1\rangle$ and $|\uparrow\rangle \equiv |S_{1/2},F=1,m_F=0\rangle$. In the presence of a bias magnetic field B, these two levels are no longer degenerate, and their energy-splitting Δ can be controlled by adjusting B. The energy-splitting Δ needs to be chosen as an integer number of the harmonic trap energy-level separation $\hbar\omega$. We choose $\Delta = \hbar\omega$, making the levels $|\downarrow\rangle|n+1\rangle$ and $|\uparrow\rangle|n\rangle$ degenerate.

The states $|\downarrow\rangle$ and $|\uparrow\rangle$ can be individually addressed using an auxiliary internal state $|\text{aux}\rangle \equiv |S_{1/2},F=0\rangle$ [43] separated from levels $|\downarrow\rangle$ and $|\uparrow\rangle$ by about 12.6 GHz. Population transfer between $|\text{aux}\rangle$ and any of the levels $|\downarrow\rangle$ and $|\uparrow\rangle$ is done using microwave pulses at the corresponding frequency and polarization. The qubit $\{|\downarrow\rangle,|\uparrow\rangle\}$ can be rotated using either a resonant radio-frequency field or two microwave fields via state $|\text{aux}\rangle$. A magnetic field gradient allows us to couple vibrational and internal levels using RF or microwave radiation [44–46], and, therefore, resonances between internal states are accompanied by blue and red sideband transitions. Driving these sidebands allows for generating a wide range of effective Hamiltonians coupling internal and vibrational states [38–40,47,48]. Alternatively, laser driven Raman transitions can be employed for this purpose [35].

The proposed experiment consist of three concrete steps. Step 1: prepare the ion state given in Eq. (9). First, using sideband cooling the ion is brought to the state $|\text{aux}\rangle|n=0\rangle$. Then, a circularly polarized microwave π pulse on the blue sideband of the $|\text{aux}\rangle - |\downarrow\rangle$ resonance prepares state $|\downarrow\rangle|k=1\rangle$. Then, a subsequent π pulse applied on the red sideband of the $|\text{aux}\rangle - |\downarrow\rangle$ resonance (or on the carrier of this resonance) yields state $|\text{aux}\rangle|k=2\rangle$ (or $|\text{aux}\rangle|k=1\rangle$). This procedure can be repeated until the desired motional state $|\downarrow\rangle|k=n\rangle$ is reached. Finally, an effective $\pi/2$ pulse on the red sideband of the $|\downarrow\rangle - |\uparrow\rangle$ resonance (microwave Raman transition via state $|\text{aux}\rangle$) generates the superposition state $\frac{1}{\sqrt{2}}|\uparrow\rangle|n\rangle + \frac{1}{\sqrt{2}}|\downarrow\rangle|n+1\rangle$ (where for definiteness we have chosen $a_\uparrow = a_\downarrow = \frac{1}{\sqrt{2}}$). Using this procedure it should be possible to prepare the desired superposition for moderate values of n, say $n=10$. By way of example: For currently used experimental parameters [49], the preparation procedure should take external time $\approx n\times 3\times 10^{-4}$ s. Step 2: After having prepared the initial state one waits for an interval of external time t_{ext}. The duration of the waiting time will be limited by the coherence time of the internal and motional states that can be of order 10^{-1} s in a well-shielded macroscopic ion trap. Internal states insensitive to magnetic field fluctuations to first order could also be used with coherence times on the order of minutes. In that case Raman transitions would be used for state preparation.

In step 3 the internal state and the motional state of the ion are measured. The measurement scheme is based on collecting state-selectively scattered resonance fluorescence on the $|S_{1/2},F=1\rangle \leftrightarrow |P_{1/2},F=0\rangle$ resonance near 369 nm [50]. By preceding the scattering of resonance fluorescence by a microwave π pulse transferring the population from state $|\downarrow\rangle$ (or $|\uparrow\rangle$) to $|\text{aux}\rangle$, one can measure the internal state of the qubit $\{|\downarrow\rangle,|\uparrow\rangle\}$ in the σ_z basis. The absence of resonance fluorescence in the subsequent measurement indicates initial population of $|\downarrow\rangle$ (or $|\uparrow\rangle$). A detection of the qubit state in an arbitrary basis is attained by an appropriate rotation of the qubit $\{|\downarrow\rangle,|\uparrow\rangle\}$ preceding the detection process described above [51]. Importantly, if no light is scattered, the motional state is not altered during the measurement. Therefore, after a null detection event, the motional state correlated to $|\downarrow\rangle$ (or $|\uparrow\rangle$) is measured by mapping the motional states onto two internal states of the ion [38–41,47,48], such as $|S_{1/2},F=0\rangle$ and $|S_{1/2},F=1,m_F=+1\rangle$), and then measuring the internal states as described above. This mapping again is achieved by using red and blue sideband transitions that accompany the resonance between the ion's internal states.

In future work it would be interesting to consider clock variables that are even closer to those encountered in quantum cosmology. The difficulty is that the clock variable in minisuperspace would have to be modeled as a system with negative kinetic energy. At first sight this seems impossible. It could, however, possibly be realized in atomic lattices, using ideas borrowed from a recent experiment that demonstrated negative temperatures [52]. This would allow the investigation of many additional phenomena, including the back reaction of the clock on the matter degrees of freedom, and possible violations of unitarity in the evolution of the matter degrees of freedom when the clock is not classical enough.

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