# Triplet pair amplitude in a trapped $s$-wave superfluid Fermi gas with broken spin rotation symmetry

Yuki Endo, Daisuke Inotani,* Ryo Hanai, and Yoji Ohashi

*Department of Physics, Faculty of Science and Technology, Keio University, 3-14-1 Hiyoshi, Kohoku-ku, Yokohama 223-8522, Japan*

(Received 21 May 2015; published 10 August 2015)

We investigate the possibility that the broken spatial inversion symmetry caused by a trap potential induces a spin-triplet Cooper pair amplitude in an $s$-wave superfluid Fermi gas. Based on symmetry considerations, we clarify that this phenomenon may occur, when a spin rotation symmetry of the system is also broken. We also numerically confirm that a triplet pair amplitude is actually induced under this condition, using a simple model. Our results imply that this phenomenon is already present in a trapped $s$-wave superfluid Fermi gas with spin imbalance. As an interesting application of this phenomenon, we point out that one may produce a $p$-wave superfluid Fermi gas by suddenly changing the $s$-wave pairing interaction to a $p$-wave one by using the Feshbach resonance technique. Since a Cooper pair is usually classified into a spin-singlet (and even-parity) state and a spin-triplet (and odd-parity) state, our results would be useful in considering how to mix them with each other in a superfluid Fermi gas. Such an admixture has recently attracted much attention in the field of noncentrosymmetric superconductivity, so that our results would also contribute to the further development of this research field, from the viewpoint of cold Fermi gas physics.

DOI: 10.1103/PhysRevA.92.023610 PACS number(s): 03.75.Ss, 67.85.Lm

## I. INTRODUCTION

Since any experiment on a superfluid Fermi gas is done with a trap potential [1–7], it is interesting to explore physical phenomena originating from this spatial inhomogeneity. An example is surface oscillations observed in a $^6$Li superfluid Fermi gas [3,4]. Another example is the phase separation observed in a $^6$Li Fermi gas with spin imbalance [5–7], where the spin-balanced superfluid region in the trap center is spatially surrounded by excess atoms.

In addition to these macroscopic phenomena, spatial inhomogeneity can also affect microscopic superfluid properties. Noting that a trap potential breaks the spatial inversion symmetry when the inversion center is taken to be away from the trap center, we expect that the parity becomes no longer a good quantity for classifying the spatial structure of a Cooper pair, leading to an admixture of even- and odd-parity symmetry. Since a pair wave function is always antisymmetric with respect to the exchange of two fermions, this naturally leads to the mixing of spin-singlet and spin-triplet states. When this phenomenon occurs, the $s$-wave superfluid state is accompanied by a triplet Cooper pair amplitude, in addition to the ordinary singlet component. (The Cooper pair amplitude is symbolically written $\langle c_{\boldsymbol{p},\alpha} c_{-\boldsymbol{p},\alpha'} \rangle$, where $c_{\boldsymbol{p},\alpha}$ is the annihilation operator of a Fermi atom with pseudospin $\alpha = \uparrow,\downarrow$.)

The purpose of this paper is to theoretically explore this possibility in a trapped $s$-wave superfluid Fermi gas. Using symmetry considerations, we prove that this phenomenon may occur when the spin rotation symmetry of this system is also broken, in addition to the broken inversion symmetry caused by a trap potential. In a two-component Fermi gas, this additional condition is realized when two species feel different trap potentials or chemical potentials or when they have different atomic masses. Although this is a necessary condition, we numerically confirm that a triplet pair amplitude is actually induced under this condition, within the mean-field theory for a model two-dimensional lattice Fermi superfluid in a harmonic trap.

In considering a triplet pair amplitude, one should note that the appearance of this quantity does not immediately mean the realization of a triplet superfluid state. Actually, the system is still in the $s$-wave superfluid state, insofar as the system only has an $s$-wave interaction. This is simply because the symmetry of a Fermi superfluid is fully determined by the symmetry of the superfluid order parameter, which is essentially given by the product of a pairing interaction and a pair amplitude. For example, an $s$-wave superfluid Fermi gas with a contact-type $s$-wave pairing interaction $-U_s$ ($<0$) is characterized by the ordinary $s$-wave superfluid order parameter,

$$\Delta_s = U_s \sum_{\boldsymbol{p}} \langle c_{\boldsymbol{p},\uparrow} c_{-\boldsymbol{p},\downarrow} \rangle, \tag{1}$$

which is finite when the pair amplitude $\langle c_{\boldsymbol{p},\uparrow} c_{-\boldsymbol{p},\downarrow} \rangle$ has the $s$-wave component. The odd-parity component does not contribute to $\Delta_s$ in Eq. (1).

However, for an $s$-wave superfluid Fermi gas with both a singlet and a triplet pair amplitude, when one suddenly changes the $s$-wave pairing interaction to a triplet (and odd-parity) one $U(\boldsymbol{p},\boldsymbol{p}')$, while the $s$-wave superfluid order parameter in Eq. (1) immediately vanishes due to the vanishing $s$-wave interaction ($U_s = 0$), the product of the triplet interaction and the triplet component in the pair amplitude $\langle c_{\boldsymbol{p},\uparrow} c_{-\boldsymbol{p},\downarrow} \rangle$ (which is assumed to have existed already in the $s$-wave state) immediately gives a finite triplet superfluid order parameter,

$$\Delta(\boldsymbol{p}) = \sum_{\boldsymbol{p}'} U(\boldsymbol{p},\boldsymbol{p}') \langle c_{\boldsymbol{p}',\uparrow} c_{-\boldsymbol{p}',\downarrow} \rangle, \tag{2}$$

when the triplet interaction $U(\boldsymbol{p},\boldsymbol{p}')$ is chosen so that the momentum summation in Eq. (2) can be finite. In an ultracold Fermi gas, the change of the interaction is possible by using a tunable interaction associated with a Feshbach resonance [8–16]. Then, by definition, one obtains a triplet superfluid Fermi gas characterized by the superfluid order parameter $\Delta(\boldsymbol{p})$ in Eq. (2), at least just after this manipulation. This makes us expect that, when one can induce a $p$-wave pair amplitude

*dinotani@rk.phys.keio.ac.jp

 

in an $s$-wave superfluid Fermi gas, a $p$-wave superfluid Fermi gas may be realized. This possibility has recently been discussed by one of the authors in [17], where a $p$-wave pair amplitude is induced by a synthetic spin-orbit interaction [18–26]. The present paper provides another source of $p$-wave pair amplitude, without using an artificial gauge field.

The admixture of singlet and triplet Cooper pairs has recently attracted much attention in the field of noncentrosymmetric superconductivity [27,28], where a crystal lattice with no inversion center causes this phenomenon. In this field, it has been pointed out that this admixture may be the origin of the anomalous temperature dependence of the penetration depth observed in $Li_2Pt_3B$ [29]. Thus, an $s$-wave superfluid Fermi gas with a triplet pair amplitude would also be helpful in the study of this electron system.

This paper is organized as follows. In Sec. II, we clarify the necessary condition for a triplet Cooper pair amplitude to appear in a trapped $s$-wave superfluid Fermi gas. In Sec. III, we numerically evaluate how high a triplet pair amplitude is induced under the condition obtained in Sec. II. In this section, we treat a superfluid Fermi gas loaded on a two-dimensional square lattice, within the mean-field theory. Throughout this paper, we take $\hbar = k_B = 1$, for simplicity.

## II. CONDITION FOR A TRIPLET PAIR AMPLITUDE TO APPEAR IN A TRAPPED $s$-WAVE SUPERFLUID FERMI GAS

We consider a three-dimensional $s$-wave superfluid Fermi gas, described by the Hamiltonian

$$H = \int d\boldsymbol{r}\Big[\sum_{\alpha,\alpha'}\Psi_\alpha^\dagger(\boldsymbol{r})h_{\alpha,\alpha'}(\boldsymbol{r})\Psi_{\alpha'}(\boldsymbol{r}) - U_s\Psi_\uparrow^\dagger(\boldsymbol{r})\Psi_\downarrow^\dagger(\boldsymbol{r})\Psi_\downarrow(\boldsymbol{r})\Psi_\uparrow(\boldsymbol{r})\Big]. \tag{3}$$

Here, $\Psi_\alpha(\boldsymbol{r})$ is a fermion field operator with pseudospin $\alpha = \uparrow,\downarrow$, describing two atomic hyperfine states. $-U_s$ ($<0$) is a contact-type $s$-wave pairing interaction. $h_{\alpha,\alpha'}(\boldsymbol{r})$ is a one-particle Hamiltonian density, consisting of a kinetic term and a potential term, the detailed expression of which is given later.

We assume that the system is in the ordinary $s$-wave superfluid state with the $s$-wave superfluid order parameter,

$$\Delta_s(\boldsymbol{r}) = U_s\langle\Psi_\uparrow(\boldsymbol{r})\Psi_\downarrow(\boldsymbol{r})\rangle. \tag{4}$$

We also assume that any other spontaneous symmetry breaking is absent (such as the triplet superfluid state).

In this model superfluid, we consider the spin-triplet Cooper pair amplitude, given by

$$\Phi_{\rm t}^{S_z}(\boldsymbol{r},\boldsymbol{r}') = \begin{cases} \langle\Psi_\uparrow(\boldsymbol{r})\Psi_\uparrow(\boldsymbol{r}')\rangle & (S_z=1),\\ \frac{1}{\sqrt{2}}[\langle\Psi_\uparrow(\boldsymbol{r})\Psi_\downarrow(\boldsymbol{r}')\rangle + \langle\Psi_\downarrow(\boldsymbol{r})\Psi_\uparrow(\boldsymbol{r}')\rangle] & (S_z=0),\\ \langle\Psi_\downarrow(\boldsymbol{r})\Psi_\downarrow(\boldsymbol{r}')\rangle & (S_z=-1), \end{cases} \tag{5}$$

where $S_z$ denotes the $z$ component of the total spin of each pair amplitude. The triplet pair amplitude in Eq. (5) does not contribute the $s$-wave superfluid order parameter $\Delta_s(\boldsymbol{r})$ in Eq. (4), because $\Phi_{\rm t}^{S_z}(\boldsymbol{r},\boldsymbol{r}) = 0$. The spin-singlet pair amplitude,

$$\Phi_{\rm s}(\boldsymbol{r},\boldsymbol{r}') = \frac{1}{\sqrt{2}}\big[\langle\Psi_\uparrow(\boldsymbol{r})\Psi_\downarrow(\boldsymbol{r}')\rangle - \langle\Psi_\downarrow(\boldsymbol{r})\Psi_\uparrow(\boldsymbol{r}')\rangle\big], \tag{6}$$

only contributes to Eq. (4).

We first prove that the broken spatial inversion symmetry is necessary for a triplet pair amplitude to appear in an $s$-wave superfluid Fermi gas. For this purpose, we conveniently introduce the inversion operator $\hat{P}(\boldsymbol{R})$ with respect to the inversion center $\boldsymbol{R}$. The field operator is transformed under this operation as

$$\tilde{\Psi}_\alpha(\boldsymbol{r}) \equiv \hat{P}(\boldsymbol{R})\Psi_\alpha(\boldsymbol{r})\hat{P}^{-1}(\boldsymbol{R}) = \Psi_\alpha(\boldsymbol{R}-\boldsymbol{l}), \tag{7}$$

where $\boldsymbol{r} = \boldsymbol{R}+\boldsymbol{l}$. The inverted Hamiltonian $\tilde{H} = \hat{P}H\hat{P}^{-1}$ is then given by

$$\tilde{H} = \int d\boldsymbol{l}\Big[\sum_{\alpha,\alpha'}\Psi_\alpha^\dagger(\boldsymbol{R}-\boldsymbol{l})h_{\alpha,\alpha'}(\boldsymbol{R}+\boldsymbol{l})\Psi_{\alpha'}(\boldsymbol{R}-\boldsymbol{l}) - U_s\Psi_\uparrow^\dagger(\boldsymbol{R}-\boldsymbol{l})\Psi_\downarrow^\dagger(\boldsymbol{R}-\boldsymbol{l})\Psi_\downarrow(\boldsymbol{R}-\boldsymbol{l})\Psi_\uparrow(\boldsymbol{R}-\boldsymbol{l})\Big]. \tag{8}$$

When the one-particle Hamiltonian density $h_{\alpha,\alpha'}(\boldsymbol{r})$ has the symmetry $h_{\alpha,\alpha'}(\boldsymbol{R}+\boldsymbol{l}) = h_{\alpha,\alpha'}(\boldsymbol{R}-\boldsymbol{l})$, this system is invariant ($\tilde{H} = H$) under this symmetry operation. On the other hand, the triplet pair amplitude $\Phi_{\rm t}^{S_z=1}(\boldsymbol{r},\boldsymbol{r}')$ in Eq. (5) with the center-of-mass position $\boldsymbol{R} = [\boldsymbol{r}+\boldsymbol{r}']/2$ is transformed as

$$\begin{aligned}\tilde{\Phi}_{\rm t}^{S_z=1}(\boldsymbol{r},\boldsymbol{r}') &\equiv \langle\tilde{\Psi}_\uparrow(\boldsymbol{r})\tilde{\Psi}_\uparrow(\boldsymbol{r}')\rangle\\ &= \langle\Psi_\uparrow(\boldsymbol{R}-\boldsymbol{r}_{\rm rel}/2)\Psi_\uparrow(\boldsymbol{R}+\boldsymbol{r}_{\rm rel}/2)\rangle\\ &= -\langle\Psi_\uparrow(\boldsymbol{R}+\boldsymbol{r}_{\rm rel}/2)\Psi_\uparrow(\boldsymbol{R}-\boldsymbol{r}_{\rm rel}/2)\rangle\\ &= -\Phi_{\rm t}^{S_z=1}(\boldsymbol{r},\boldsymbol{r}'),\end{aligned} \tag{9}$$

where $\boldsymbol{r}_{\rm rel} = \boldsymbol{r}-\boldsymbol{r}'$ is the relative coordinate. We also find $\tilde{\Phi}_{\rm t}^{S_z=0,-1}(\boldsymbol{r},\boldsymbol{r}') = -\Phi_{\rm t}^{S_z=0,-1}(\boldsymbol{r},\boldsymbol{r}')$. That is, the triplet pair amplitude $\Phi_{\rm t}^{S_z}(\boldsymbol{r},\boldsymbol{r}')$ vanishes when the system has the inversion symmetry ($\tilde{H} = H$) with respect to the inversion center $\boldsymbol{R} = [\boldsymbol{r}+\boldsymbol{r}']/2$. Thus, the broken inversion symmetry is necessary for a triplet pair amplitude to appear.

For the singlet pair amplitude in Eq. (6), this symmetry operation simply gives $\tilde{\Phi}_{\rm s}(\boldsymbol{r},\boldsymbol{r}') = \Phi_{\rm s}(\boldsymbol{r},\boldsymbol{r}')$. As expected, this quantity may be finite.

The one-particle Hamiltonian density $h_{\alpha,\alpha'}(\boldsymbol{r})$ in the ordinary uniform Fermi gas has the form

$$h_{\alpha,\alpha'}(\boldsymbol{r}) = \Big[\frac{\hat{\boldsymbol{p}}^2}{2m} - \mu\Big]\delta_{\alpha,\alpha'}, \tag{10}$$

where $\hat{\boldsymbol{p}} = -i\nabla$, $m$ is the atomic mass, and $\mu$ is the Fermi chemical potential. Equation (10) has the symmetry property, $h_{\alpha,\alpha'}(\boldsymbol{R}+\boldsymbol{l}) = h_{\alpha,\alpha'}(\boldsymbol{R}-\boldsymbol{l})$, with respect to $\boldsymbol{l}$ for an arbitrary $\boldsymbol{R}$. To conclude, any triplet pair amplitude is not induced.

In the presence of a harmonic trap, the one-particle Hamiltonian density becomes inhomogeneous as

$$h_{\alpha,\alpha'}(\boldsymbol{r}) = \Big[\frac{\hat{\boldsymbol{p}}^2}{2m} - \mu + \frac{1}{2}Kr^2\Big]\delta_{\alpha,\alpha'}, \tag{11}$$

so that it does not have the inversion symmetry except at $\boldsymbol{R} = 0$. However, when we consider the $s$-wave superfluid state in this trapped case, any triplet pair amplitude is not actually induced (although we do not explicitly show the result here). Of course, since the condition obtained from the inversion symmetry is a *necessary* condition, the broken inversion symmetry does not guarantee the appearance of a triplet pair amplitude.

In this regard, we point out that the vanishing triplet pair amplitude in the trapped case is due to the fact that this system still has a rotation symmetry in spin space. To see this, we next consider the spin rotation of the field operator, given by

$$\tilde{\Psi}_\alpha(\boldsymbol{r}) = \hat{R}(\boldsymbol{\theta})\Psi_\alpha(\boldsymbol{r})\hat{R}^{-1}(\boldsymbol{\theta}) = \sum_{\alpha'}\left(e^{\frac{i}{2}\boldsymbol{\theta}\cdot\boldsymbol{\sigma}}\right)_{\alpha,\alpha'}\Psi_{\alpha'}(\boldsymbol{r}). \quad (12)$$

Here, $\boldsymbol{\theta} = \theta\boldsymbol{e}_\theta$ describes the spin rotation around the unit vector $\boldsymbol{e}_\theta$ with the angle $\theta$, and $\boldsymbol{\sigma} = (\sigma_x,\sigma_y,\sigma_z)$, where $\sigma_j$ ($j = x,y,z$) are Pauli matrices. (As usual, we take the spin quantization axis in the $\sigma_z$ direction.) For the three "$\pi$ rotations" specified by $\boldsymbol{\theta} = (\pi,0,0)(\equiv \boldsymbol{\theta}_\pi^x)$, $(0,\pi,0)(\equiv \boldsymbol{\theta}_\pi^y)$, and $(0,0,\pi)(\equiv \boldsymbol{\theta}_\pi^z)$, Eq. (12) can be written as

$$\begin{pmatrix}\tilde{\Psi}_\uparrow(\boldsymbol{r})\\ \tilde{\Psi}_\downarrow(\boldsymbol{r})\end{pmatrix}_{\boldsymbol{\theta}=\boldsymbol{\theta}_\pi^j} = i\sigma_j\begin{pmatrix}\Psi_\uparrow(\boldsymbol{r})\\ \Psi_\downarrow(\boldsymbol{r})\end{pmatrix}, \quad (13)$$

where we have used the formula $e^{i\frac{\theta}{2}\sigma_j} = \cos(\theta/2) + i\sigma_j\sin(\theta/2)$. Under the $\pi$ rotation, the Hamiltonian in Eq. (3) is transformed as

$$\begin{aligned}\tilde{H} &= \hat{R}(\boldsymbol{\theta}_\pi^j)H\hat{R}^{-1}(\boldsymbol{\theta}_\pi^j)\\
&= \int d\boldsymbol{r}\Bigg[\sum_{\alpha,\alpha'}\tilde{\Psi}_\alpha^\dagger(\boldsymbol{r})h_{\alpha,\alpha'}(\boldsymbol{r})\tilde{\Psi}_{\alpha'}(\boldsymbol{r})\\
&\quad -U_s\tilde{\Psi}_\uparrow^\dagger(\boldsymbol{r})\tilde{\Psi}_\downarrow^\dagger(\boldsymbol{r})\tilde{\Psi}_\downarrow(\boldsymbol{r})\tilde{\Psi}_\uparrow(\boldsymbol{r})\Bigg]\\
&= \int d\boldsymbol{r}\Bigg[\sum_{\alpha,\alpha'}\Psi_\alpha^\dagger(\boldsymbol{r})(\sigma_j\hat{h}(\boldsymbol{r})\sigma_j)_{\alpha,\alpha'}\Psi_{\alpha'}(\boldsymbol{r})\\
&\quad -U_s\Psi_\uparrow^\dagger(\boldsymbol{r})\Psi_\downarrow^\dagger(\boldsymbol{r})\Psi_\downarrow(\boldsymbol{r})\Psi_\uparrow(\boldsymbol{r})\Bigg]. \end{aligned} \quad (14)$$

Here, $\hat{h}(\boldsymbol{r}) = \{h_{\alpha,\alpha'}(\boldsymbol{r})\}$. Thus, one has $\tilde{H} = H$ when

$$[\hat{h}(\boldsymbol{r}),\sigma_j] = 0 \quad (15)$$

is satisfied.

While the singlet pair amplitude in Eq. (6) remains unchanged under these $\pi$ rotations, the triplet component is transformed as

$$\begin{pmatrix}\tilde{\Phi}_t^{S_z=1}(\boldsymbol{r},\boldsymbol{r}')\\ \tilde{\Phi}_t^{S_z=0}(\boldsymbol{r},\boldsymbol{r}')\\ \tilde{\Phi}_t^{S_z=-1}(\boldsymbol{r},\boldsymbol{r}')\end{pmatrix} = \begin{cases}\begin{pmatrix}-\Phi_t^{S_z=-1}(\boldsymbol{r},\boldsymbol{r}')\\ -\Phi_t^{S_z=0}(\boldsymbol{r},\boldsymbol{r}')\\ -\Phi_t^{S_z=1}(\boldsymbol{r},\boldsymbol{r}')\end{pmatrix}(\boldsymbol{\theta} = \boldsymbol{\theta}_\pi^x),\\ \begin{pmatrix}\Phi_t^{S_z=-1}(\boldsymbol{r},\boldsymbol{r}')\\ -\Phi_t^{S_z=0}(\boldsymbol{r},\boldsymbol{r}')\\ \Phi_t^{S_z=1}(\boldsymbol{r},\boldsymbol{r}')\end{pmatrix}(\boldsymbol{\theta} = \boldsymbol{\theta}_\pi^y),\\ \begin{pmatrix}-\Phi_t^{S_z=1}(\boldsymbol{r},\boldsymbol{r}')\\ \Phi_t^{S_z=0}(\boldsymbol{r},\boldsymbol{r}')\\ -\Phi_t^{S_z=-1}(\boldsymbol{r},\boldsymbol{r}')\end{pmatrix}(\boldsymbol{\theta} = \boldsymbol{\theta}_\pi^z).\end{cases} \quad (16)$$

For example, when we set $\boldsymbol{\theta} = \boldsymbol{\theta}_\pi^x$, Eq. (16) means that $\Phi_t^{S_z=0}(\boldsymbol{r},\boldsymbol{r}') = 0$, when $[\hat{h}(\boldsymbol{r}),\sigma_x] = 0$. (The other two components with $S_z = \pm1$ are not excluded in this case.) When the system is invariant under all the $\pi$-rotations ($\boldsymbol{\theta}_\pi^{x,y,z}$), any triplet pair amplitude is not induced.

To conclude, the broken spin rotation symmetry characterized by $\boldsymbol{\theta}_\pi^j$ is necessary for a triplet pair amplitude to be induced in a trapped $s$-wave superfluid Fermi gas. This is the reason why the model case described by Eqs. (3) and (11) is not accompanied by any triplet pair amplitude.

The broken spin rotation symmetry is realized when the strength of the trap potential $K$ in Eq. (11) depends on the spin ($\equiv K_\alpha$). In this case, the one-particle Hamiltonian $\hat{h}(\boldsymbol{r}) = \{h_{\alpha,\alpha'}(\boldsymbol{r})\}$ can be written as

$$\hat{h}(\boldsymbol{r}) = \left[\frac{\hat{\boldsymbol{p}}^2}{2m} - \mu + \frac{K_\uparrow + K_\downarrow}{4}r^2\right] + \frac{K_\uparrow - K_\downarrow}{4}r^2\sigma_z. \quad (17)$$

Equation (15) is then satisfied only when $j = z$. Thus, we find from the last line in Eq. (16) that only the triplet pair amplitude with $S_z = 0$ may be induced. Since the last term in Eq. (17) works as an external magnetic field, this phenomenon is also expected in the presence of spin imbalance [5–7,30,31], where two species feel different Fermi chemical potentials $\mu_\uparrow \neq \mu_\downarrow$. Another possibility is a trapped hetero superfluid Fermi gas [32–40], where two species have different atomic masses $m_\uparrow \neq m_\downarrow$. In Sec. III, we numerically examine these cases.

Before ending this section, we briefly note that the broken inversion symmetry, as well as the broken spin rotation symmetry, is also realized in a spin-orbit-coupled $s$-wave superfluid Fermi gas [18–26]. Indeed, Refs. [17,41,42] predict that a $p$-wave pair amplitude is induced in this case. Although we do not deal with this case in Sec. III, we explain in Appendix A how to apply the present symmetry consideration to this case.

### III. NUMERICAL CONFIRMATION FOR THE INDUCTION OF TRIPLET PAIR AMPLITUDE IN A TRAPPED s-WAVE SUPERFLUID FERMI GAS

To examine whether or not a triplet pair amplitude is induced under the condition obtained in Sec. II, we consider a model $s$-wave Fermi superfluid loaded on an $L \times L$ two-dimensional square lattice, within the mean-field approximation. Although this simple model cannot be directly applied to a real continuum Fermi gas, it is still helpful to grasp basic characters of this phenomenon.

The Hamiltonian is given by

$$\begin{aligned}H_{\rm MF} = &-\sum_{\langle i,j\rangle,\alpha} t_\alpha\left[c_{\boldsymbol{r}_i,\alpha}^\dagger c_{\boldsymbol{r}_j,\alpha} + \text{H.c.}\right]\\
&+\sum_i \Delta_s(\boldsymbol{r}_i)\left[c_{\boldsymbol{r}_i,\uparrow}^\dagger c_{\boldsymbol{r}_i,\downarrow}^\dagger + \text{H.c.}\right]\\
&+\sum_{i,\alpha}[V_\alpha(\boldsymbol{r}_i) - \mu_\alpha - U_s n_{-\alpha}(\boldsymbol{r}_i)]c_{\boldsymbol{r}_i,\alpha}^\dagger c_{\boldsymbol{r}_i,\alpha}.\end{aligned} \quad (18)$$

Here, $c_{\boldsymbol{r}_i,\alpha}^\dagger$ is the creation operator of a Fermi atom at the lattice site $\boldsymbol{r}_i = (r_x^i,r_y^i)$, with pseudospin $\alpha = \uparrow,\downarrow$ and Fermi chemical potential $\mu_\alpha$. $-t_\alpha$ describes the particle hopping between nearest-neighbor sites, and $\langle i,j\rangle$ means the summation over the nearest-neighbor pairs. In Eq. (18), the $s$-wave superfluid order parameter $\Delta_s(\boldsymbol{r}_i) = U_s\langle c_{\boldsymbol{r}_i,\uparrow}c_{\boldsymbol{r}_i,\downarrow}\rangle$, as well as the Hartree potential $-U_s n_{-\alpha}(\boldsymbol{r}_i) = -U_s\langle c^\dagger_{\boldsymbol{r}_i,-\alpha}c_{\boldsymbol{r}_i,-\alpha}\rangle$, is obtained from the mean-field approximation for the on-site Hubbard interaction $-U_s c^\dagger_{\boldsymbol{r}_i,\uparrow}c_{\boldsymbol{r}_i,\uparrow}c^\dagger_{\boldsymbol{r}_i,\downarrow}c_{\boldsymbol{r}_i,\downarrow}$, where $-U_s$ ($<0$) is the interaction strength. $V_\alpha(\boldsymbol{r}_i) = V_0^\alpha(\boldsymbol{r}_i/r_d)^2$ is the harmonic trap potential, where $V_0^\alpha$ is the strength of a trap potential which $\alpha$-spin atoms feel. Here, the spatial position $\boldsymbol{r}_i$ is measured from the center of the $L\times L$ square lattice, and $r_d$ is the distance between the trap center and the edge of the system. For simplicity, we take the lattice constant $a$ to be unity.

In the present model, the spatial inversion symmetry is broken by the trap potential except at the trap center. For spin rotation symmetry, it is broken when one of $t_\alpha$, $\mu_\alpha$, and $V_0^\alpha$ depends on pseudospin $\alpha = \uparrow,\downarrow$. Since all these cases satisfy Eq. (15) only when $j = z$, Eq. (16) indicates that one may only consider the possibility of the triplet pair amplitude with $S_z = 0$. In the present model, this component is given by

$$\Phi_{\rm t}^{S_z=0}(\boldsymbol{r}_i,\boldsymbol{r}_j) = \frac{1}{\sqrt{2}}\left[\langle c_{\boldsymbol{r}_i,\uparrow}c_{\boldsymbol{r}_j,\downarrow}\rangle + \langle c_{\boldsymbol{r}_i,\downarrow}c_{\boldsymbol{r}_j,\uparrow}\rangle\right]. \quad (19)$$

For comparison, we also consider the ordinary singlet component, given by

$$\Phi_{\rm s}(\boldsymbol{r}_i,\boldsymbol{r}_j) = \frac{1}{\sqrt{2}}\left[\langle c_{\boldsymbol{r}_i,\uparrow}c_{\boldsymbol{r}_j,\downarrow}\rangle - \langle c_{\boldsymbol{r}_i,\downarrow}c_{\boldsymbol{r}_j,\uparrow}\rangle\right]. \quad (20)$$

Besides the superfluid order parameter, the condensate fraction is also a fundamental quantity in the superfluid phase [43–46], which physically describes the number of Bose-condensed Cooper pairs. In an ordinary $s$-wave superfluid state, it has the form

$$N_{\rm c}^{\rm s} = \sum_{\boldsymbol{R}=(\boldsymbol{r}_i+\boldsymbol{r}_j)/2} n_{\rm c}^{\rm s}(\boldsymbol{R}), \quad (21)$$

where the local condensate fraction $n_{\rm c}^{\rm s}(\boldsymbol{R})$ is directly related to the singlet pair amplitude in Eq. (20) as

$$n_{\rm c}^{\rm s}(\boldsymbol{R}) = \frac{1}{2N}\sum_{\boldsymbol{r}_{\rm rel}=\boldsymbol{r}_i-\boldsymbol{r}_j} |\Phi_{\rm s}(\boldsymbol{R}+\boldsymbol{r}_{\rm rel}/2,\boldsymbol{R}-\boldsymbol{r}_{\rm rel}/2)|^2. \quad (22)$$

In addition to the singlet component of the condensate fraction $N_{\rm c}^{\rm s}$ in Eq. (21), the present system may also have the spin-triplet component [17], $N_{\rm c}^{\rm t} = \sum_{\boldsymbol{R}=(\boldsymbol{r}_i+\boldsymbol{r}_j)/2} n_{\rm c}^{\rm t}(\boldsymbol{R})$, where

$$n_{\rm c}^{\rm t}(\boldsymbol{R}) = \frac{1}{2N}\sum_{\boldsymbol{r}_{\rm rel}=\boldsymbol{r}_i-\boldsymbol{r}_j} \left|\Phi_{\rm t}^{S_z=0}(\boldsymbol{R}+\boldsymbol{r}_{\rm rel}/2,\boldsymbol{R}-\boldsymbol{r}_{\rm rel}/2)\right|^2. \quad (23)$$

The total condensate fraction is given by $N_{\rm c}^{\rm s} + N_{\rm c}^{\rm t}$. In what follows, we simply call $N_{\rm c}^{\rm s}$ and $N_{\rm c}^{\rm t}$ the singlet and triplet condensate fractions, respectively.

We note that the square lattice in our model does not affect the symmetry consideration in Sec. II. This is because the square lattice is invariant under the spatial inversion with respect to the center-of-mass position $\boldsymbol{R} = [\boldsymbol{r}_i + \boldsymbol{r}_j]/2$ of a triplet pair amplitude $\Phi_{\rm t}^{S_z}(\boldsymbol{r}_i,\boldsymbol{r}_j)$. In addition, since the spin rotation symmetry is also unaffected by the crystal lattice, the necessary condition obtained in Sec. II is still valid for the present case.

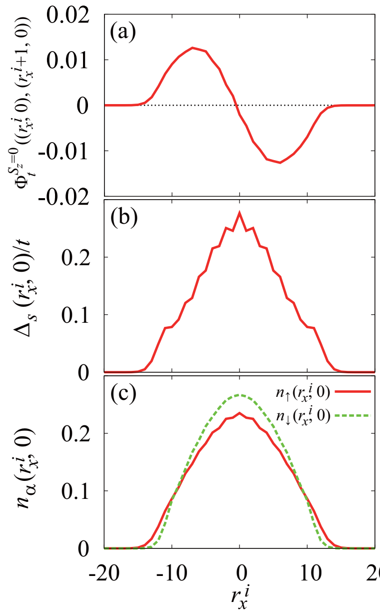


FIG. 1. (Color online) (a) Calculated triplet pair amplitude $\Phi_{\rm t}^{S_z=0}((r_x^i,0),(r_x^i+1,0))$ along the $x$ axis ($r_y^i = 0$) in an $s$-wave superfluid Fermi gas with a trap-potential difference. (b) $s$-wave superfluid order parameter $\Delta_s(r_x^i,0)$. (c) Density profile $n_\alpha(r_x^i,0)$. We take $V_0^\uparrow/V_0^\downarrow = 0.5$, $t_\uparrow/t_\downarrow = 1$, and $U_s/t = 2.5$. This parameter set is also used in Figs. 2–4.

As usual, we diagonalize the mean-field Hamiltonian $H_{\rm MF}$ in Eq. (18) by the Bogoliubov transformation [47]. Since this is a standard procedure [48], we do not explain the details here but summarize the outline in Appendix B. We numerically carry out the Bogoliubov transformation to self-consistently determine $\Delta_s(\boldsymbol{r}_i)$, $n_\alpha(\boldsymbol{r}_i)$, and $\mu_\alpha$. We then evaluate the triplet pair amplitude $\Phi_{\rm t}^{S_z=0}(\boldsymbol{r}_i,\boldsymbol{r}_j)$ in Eq. (19).

In numerical calculations, we take the lattice size $L = 41$, and $V_0^\uparrow = 2t$, where $t = [t_\uparrow + t_\downarrow]/2$. To avoid lattice effects, we consider the low-density region, by setting $N_\uparrow = N_\downarrow = 59$ in the absence of spin imbalance (where $N_\alpha$ is the number of Fermi atoms in the $\alpha$-spin component). The total number $N = N_\uparrow + N_\downarrow$ of Fermi atoms then equals $N = 118$. In this case, the particle density is at most $n_\alpha(\boldsymbol{r}_i) \lesssim 0.3 \ll 1$ even in the trap center. We take a low but finite temperature $T/t = 0.01$, in order to suppress the effects of discrete energy levels associated with the finite system size.

Figure 1(a) shows the evidence that a triplet pair amplitude with $S_z = 0$ is induced in the $s$-wave superfluid state when both the spatial inversion symmetry and the spin rotation symmetry are broken by the trap potential $V_\alpha(\boldsymbol{r}_i)$. From comparison of this figure with Figs. 1(b) and 1(c), one finds that $\Phi_{\rm t}^{S_z=0}((r_x^i+1,0),(r_x^i,0))$ appears everywhere in the gas cloud where the $s$-wave superfluid order parameter $\Delta_s(r_x^i,0)$, as well as the atom density $n_\alpha(r_x^i,0)$, is finite, except at the trap center. Since the system still has spatial inversion symmetry at the trap center, the node structure shown in Fig. 1(a) agrees with the symmetry consideration in Sec. II. We emphasize that the triplet pair amplitude is not induced when $V_0^\uparrow = V_0^\downarrow$, although we do not explicitly show the result here.

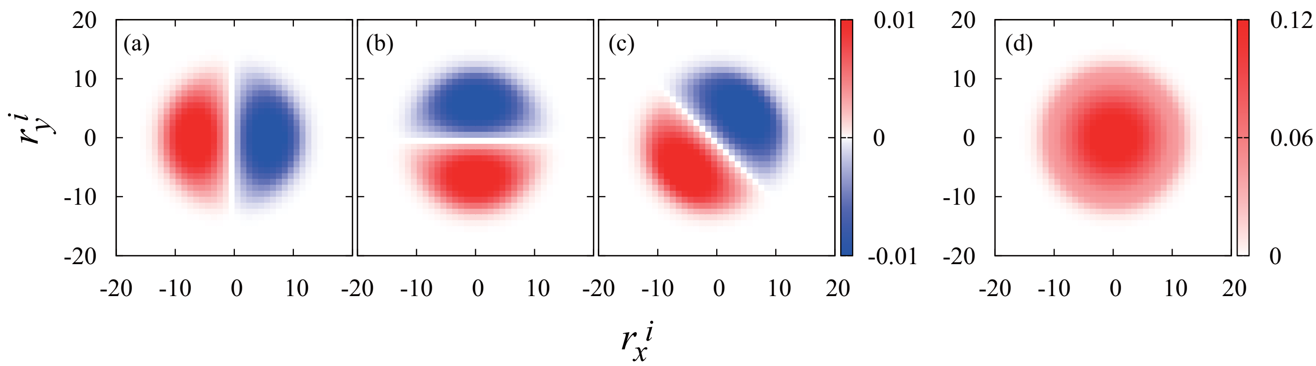


FIG. 2. (Color online) Calculated triplet pair amplitude $\Phi_t^{S_z=0}(\boldsymbol{r}_i,\boldsymbol{r}_j)$ in an $s$-wave superfluid Fermi gas with a trap-potential difference. (a) $\Phi_t^{S_z=0}((r_x^i,r_y^i),(r_x^i+1,r_y^i))$. (b) $\Phi_t^{S_z=0}((r_x^i,r_y^i),(r_x^i,r_y^i+1))$. (c) $\Phi_t^{S_z=0}((r_x^i,r_y^i),(r_x^i+1,r_y^i+1))$. (d) Singlet pair amplitude $\Phi_s(\boldsymbol{r}_i,\boldsymbol{r}_i)$.

Figure 2(a) shows that the point node shown in Fig. 1(a) is actually a line node along the $y$ axis. This node structure comes from the symmetry property that, while the present square-lattice model has reflection symmetry with respect to the $y$ axis, the triplet pair amplitude $\Phi_t^{S_z}(\boldsymbol{r}_i,\boldsymbol{r}_j)$ behaves as

$$\Phi_t^{S_z}(\boldsymbol{R}+\boldsymbol{r}_{\rm rel}/2,\boldsymbol{R}-\boldsymbol{r}_{\rm rel}/2) = -\Phi_t^{S_z}(\boldsymbol{R}-\boldsymbol{r}_{\rm rel}/2,\boldsymbol{R}+\boldsymbol{r}_{\rm rel}/2) \tag{24}$$

when $\boldsymbol{R}=[\boldsymbol{r}_i+\boldsymbol{r}_j]/2=(0,R_y)$ and $\boldsymbol{r}_{\rm rel}=\boldsymbol{r}_i-\boldsymbol{r}_j=(r_{\rm rel}^x,0)$. Since the present lattice model is also invariant under the reflections with respect to the $x$ axis, as well as the lines along $y=\pm x$, the triplet pair amplitude $\Phi_t^{S_z}(\boldsymbol{r}_i,\boldsymbol{r}_j)$, with the relative vector $\boldsymbol{r}_{\rm rel}=\boldsymbol{r}_i-\boldsymbol{r}_j$ being perpendicular to one of them, has the line node along the reflection line. [See Figs. 2(b) and 2(c).] In a continuum system with no lattice, the triplet pair amplitude is expected always to have the line node, which is perpendicular to the relative vector of the pair amplitude. We briefly note that such a node is not obtained in the singlet component, as shown in Fig. 2(d).

Figure 3(a) shows the spatial structure of the triplet pair amplitude $\Phi_t^{S_z=0}(\boldsymbol{r}_i,\boldsymbol{r}_j)$ with respect to the relative coordinate $\boldsymbol{r}_{\rm rel}=\boldsymbol{r}_i-\boldsymbol{r}_j$. Noting that the pairing symmetry is determined by the angular dependence in relative-momentum space, we find that the induced pair amplitude has $p$-wave symmetry. That is, the pair amplitude has $p_x$-wave ($p_y$-wave) symmetry, when the center-of-mass position is on the $x$ axis ($y$ axis).

An advantage of the cold Fermi gas system is that one can tune the pairing interaction by adjusting the threshold energy of a Feshbach resonance. Although this technique is usually used to adjust the interaction strength for a fixed interaction channel, one may also use this technique to change the interaction channel from an $s$-wave one to a $p$-wave one. For example, an ultracold $^6$Li Fermi gas consisting of two atomic hyperfine states $|F,m_F\rangle=|1/2,\pm1/2\rangle$ is known to have both an $s$-wave and a $p$-wave Feshbach resonance at $B_s\simeq 822$ G [2] and at $B_p\simeq 186$ G [11,12], respectively, so that one may change the $s$-wave interaction to a $p$-wave one by rapidly decreasing the external magnetic field from $B_s$ to $B_p$. In the $s$-wave superfluid Fermi gas with triplet pair amplitude shown in Fig. 3(a), when one suddenly changes the $s$-wave interaction to a $p$-wave one [49,50],

$$H_{p\text{-wave}} = -U_p\sum_{\boldsymbol{p},\boldsymbol{p}',\boldsymbol{q}}\boldsymbol{p}\cdot\boldsymbol{p}'c^\dagger_{\boldsymbol{p}+\boldsymbol{q}/2,\uparrow}c^\dagger_{-\boldsymbol{p}+\boldsymbol{q}/2,\downarrow} \times c_{-\boldsymbol{p}'+\boldsymbol{q}/2,\downarrow}c_{\boldsymbol{p}'+\boldsymbol{q}/2,\uparrow}, \tag{25}$$

the $p$-wave superfluid order parameter,

$$\Delta_p(\boldsymbol{p},\boldsymbol{R}) = U_p\sum_{\boldsymbol{p}'}\boldsymbol{p}\cdot\boldsymbol{p}'\Phi_t^{S_z=0}(\boldsymbol{p}',\boldsymbol{R}), \tag{26}$$

immediately becomes finite. Here, $\Phi_t^{S_z=0}(\boldsymbol{p}',\boldsymbol{R})$ is the Fourier-transformed triplet pair amplitude with respect to the relative coordinate $\boldsymbol{r}_{\rm rel}$. We emphasize that this triplet pair amplitude

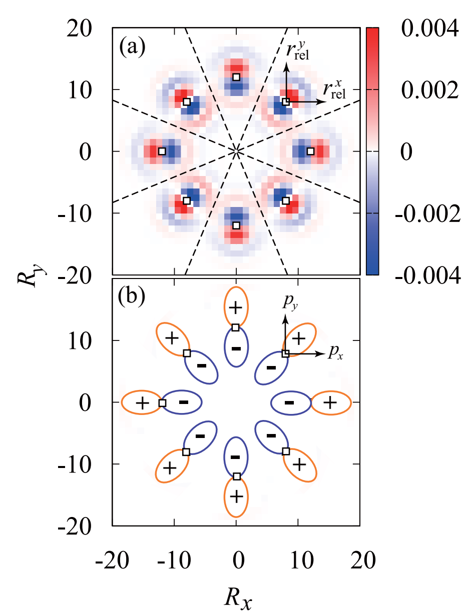


FIG. 3. (Color online) (a) Triplet pair amplitude $\Phi_t^{S_z=0}(\boldsymbol{r}_i,\boldsymbol{r}_j)$ as a function of the relative coordinate $\boldsymbol{r}_{\rm rel}=\boldsymbol{r}_i-\boldsymbol{r}_j$. We take $V_0^\uparrow/V_0^\downarrow=0.5$. For each center-of-mass position $\boldsymbol{R}=[\boldsymbol{r}_i+\boldsymbol{r}_j]/2$ (small open squares), the pair amplitude is plotted inside the region between two dashed lines, by taking $\boldsymbol{R}$ as the origin. (b) Spatial variation of synthesized $p$-wave superfluid order parameter $\Delta_p(\boldsymbol{p},\boldsymbol{R})$. The $\boldsymbol{p}$ dependence of $\Delta_p(\boldsymbol{p},\boldsymbol{R})$ is shown schematically, centered at $\boldsymbol{R}$ (small open squares).

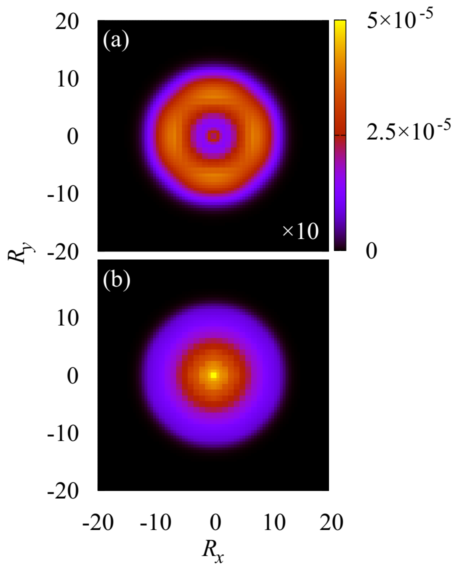


FIG. 4. (Color online) Local condensate fraction in an $s$-wave superfluid Fermi gas with a trap-potential difference. (a) Triplet component $n_c^t(\boldsymbol{R})$. The intensity is magnified to 10 times. (b) Singlet component $n_c^s(\boldsymbol{R})$.

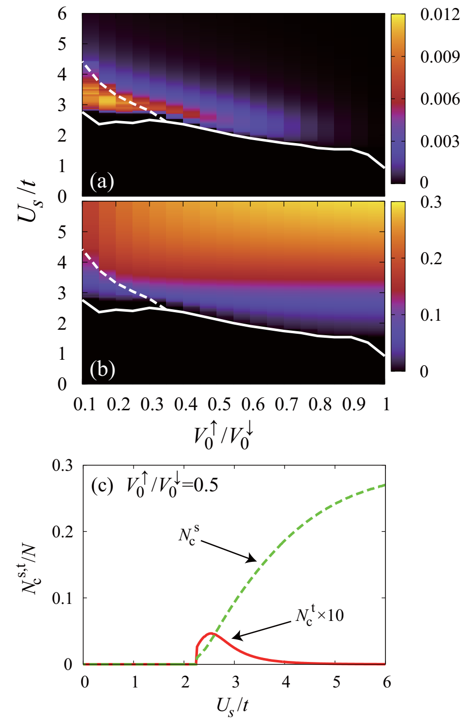


FIG. 5. (Color online) Condensate fraction $N_c^{s,t}=\sum_{\boldsymbol{R}} n_c^{s,t}(\boldsymbol{R})$ in an $s$-wave superfluid Fermi gas with a trap-potential difference. (a) Triplet component $N_c^t$. (b) Singlet component $N_c^s$. (c) $N_c^t$ and $N_c^s$, as functions of the interaction strength $U_s$, when $V_0^\uparrow/V_0^\downarrow=0.5$. In (a) and (b), the region above the solid line is in the superfluid state within our numerical accuracy. (Note that we take $T/t=0.01>0$ in our numerical calculations.) The region between the solid line and the dashed line is in the FFLO phase, characterized by a spatially oscillating superfluid order parameter $\Delta_s(\boldsymbol{r}_i)$. These lines are also drawn in Fig. 6.

has already existed before the change of the interaction. Thus, just after this manipulation, we expect the spatial structure of the induced $p$-wave superfluid order parameter schematically shown in Fig. 3(b). The $s$-wave superfluid order parameter immediately disappears because of the vanishing $s$-wave interaction ($U_s=0$), and the $s$-wave pair amplitude $\Phi_s(\boldsymbol{r}_i,\boldsymbol{r}_j)$ only remains. Thus, at least immediately after this manipulation, by definition, the system is in the $p$-wave superfluid state with the synthesized $p$-wave superfluid order parameter in Eq. (26). This unconventional superfluid phase would be in the nonequilibrium state, so that we need further analyses on the time evolution of this state. However, the combined Feshbach technique with the induced triplet pair amplitude is an interesting idea for realizing a $p$-wave superfluid Fermi gas.

Figure 4 shows the local condensate fraction $n_c^{s,t}(\boldsymbol{R})$ in an $s$-wave superfluid Fermi gas with trap-potential imbalance. In Fig. 4(a), the triplet component $n_c^t(\boldsymbol{R})$ is enhanced around $|\boldsymbol{R}|=6$, as well as the region near the trap center (except at $\boldsymbol{R}=0$, where the triplet condensate fraction vanishes). On the other hand, Fig. 4(b) shows that the singlet component $n_c^s(\boldsymbol{R})$ monotonically decreases as one goes away from the trap center. The latter behavior is consistent with the spatial variation of the $s$-wave superfluid order parameter $\Delta_s(\boldsymbol{r}_i)$ shown in Fig. 1(b).

The large triplet condensate fraction $n_c^t(\boldsymbol{R})$ near the trap center shown in Fig. 4(a) is due to the spin imbalance $[n_\uparrow(r_x^i,0)>n_\downarrow(r_x^i,0)]$ in the trap center. [See Fig. 1(c).] This naturally leads to broken spin rotation symmetry through the Fermi chemical potential $\mu_\alpha$, as well as the Hartree potential $-U_s n_{-\alpha}(\boldsymbol{r}_i)$ in Eq. (18). Thus, although two spin components feel almost the same trap potential $[V_\uparrow(\boldsymbol{r}_i)\simeq V_\downarrow(\boldsymbol{r}_i)]$ around the trap center, the triplet condensate fraction is enhanced there (except at $\boldsymbol{R}=0$).

While the difference $V_\uparrow(\boldsymbol{r}_i)-V_\downarrow(\boldsymbol{r}_i)$ becomes remarkable as one goes away from the trap center, the spin imbalance $[n_\uparrow(r_x^i,0)-n_\downarrow(r_x^i,0)]$ becomes small and eventually vanishes at $|\boldsymbol{R}|\simeq 9$. [See Fig. 1(c).] In the outer region, the spin imbalance again occurs as $n_\uparrow(r_x^i,0)<n_\downarrow(r_x^i,0)$. These enhance $n_c^{S_z=0}(|\boldsymbol{R}|\sim 6)$, as shown in Fig. 4(a).

Summing up the local condensate fraction $n_c^{s,t}(\boldsymbol{R})$ in the gas cloud, one obtains the condensate fraction $N_c^{s,t}$ in Fig. 5. As expected, Fig. 5(a) shows that the triplet component $N_c^t$ is enhanced when $V_0^\uparrow/V_0^\downarrow \ll 1$. We also find that $N_c^t$ becomes large in the intermediate-coupling regime but becomes small when $U_s/t\gg 1$. In the strong-coupling regime, most Fermi atoms form singlet molecules, which suppresses the effects of broken inversion and spin rotation symmetry. Indeed, Fig. 5(b) shows that the singlet component $N_c^s$ monotonically increases with increasing interaction strength $U_s$. To clearly see the difference between $N_c^t$ and $N_c^s$, Fig. 5(c) shows these quantities as functions of the interaction strength $U_s$.

In Figs. 5(a) and 5(b), one sees the FFLO (Fulde-Ferrell-Larkin-Ovchinnikov) phase [51–54]. In this regard, since we are dealing with a two-dimensional lattice model within the simple mean-field theory, it is unclear whether or not the FFLO phase still remains in a realistic three-dimensional continuum Fermi gas [55]. However, since Fig. 5(a) indicates that the triplet condensate fraction is also induced in the ordinary BCS region, we find that the FFLO state is not necessary for the triplet pair amplitude to appear.

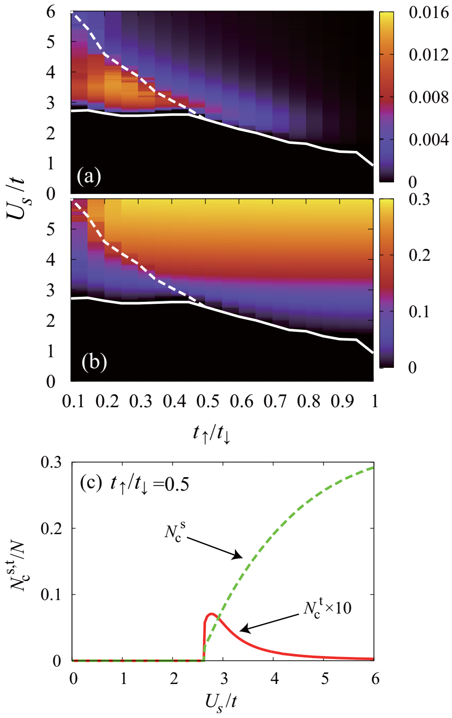


FIG. 6. (Color online) Condensate fraction $N_c^{s,t}$ in a trapped $s$-wave superfluid Fermi gas in the presence of a mass imbalance $t_\uparrow/t_\downarrow \neq 1$. (a) Triplet component $N_c^t$. (b) Singlet component $N_c^s$. (c) $N_c^t$ and $N_c^s$, as functions of the interaction strength $U_s/t$, when $t_\uparrow/t_\downarrow = 0.5$.

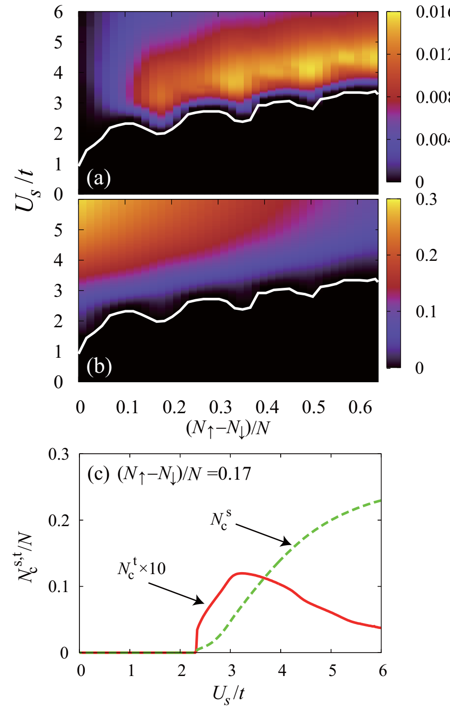


FIG. 7. (Color online) Condensate fraction $N_c^{s,t}$ in a trapped $s$-wave superfluid Fermi gas with spin imbalance $N_\uparrow/N_\downarrow \neq 1$. (a) Triplet component $N_c^t$. (b) Singlet component $N_c^s$. (c) $N_c^t$ and $N_c^s$, as functions of the interaction strength $U_s/t$, when $[N_\uparrow - N_\downarrow]/N = 0.17$ ($N_\uparrow - N_\downarrow = 20$). In (a) and (b), the region above the solid line is in the superfluid phase. In the superfluid region shown here, the superfluid order parameter in the outer region of the gas cloud always exhibits an FFLO-type oscillation in the radial direction.

Figure 6 confirms that the triplet pair amplitude $\Phi_t^{S_z=0}(\boldsymbol{r}_i,\boldsymbol{r}_j)$ is also induced when the spin rotation symmetry is broken by mass imbalance ($t_\uparrow/t_\downarrow \neq 1$). In addition, Fig. 7 shows that this phenomenon also occurs in a trapped $s$-wave superfluid Fermi gas with spin imbalance ($N_\uparrow/N_\downarrow \neq 1$) [56]. In the latter case, the Fermi chemical potential $\mu_\alpha$ depends on the pseudospin $\alpha = \uparrow,\downarrow$, which breaks the spin rotation symmetry [57].

In the presence of spin imbalance, phase separation is known to occur [5–7], where the superfluid region in the trap center is surrounded by excess atoms. Figure 8(a) shows this case. In this figure, since the spin imbalance is almost absent around the trap center, the triplet condensate fraction is suppressed there, compared to the case where phase separation does not occur [Fig. 8(b)]. In addition, the region around the edge of the gas cloud is highly spin polarized, so that the triplet pair amplitude is also suppressed there. As a result, when the phase separation occurs, the triplet pair amplitude is localized around the edge of the gas cloud of the minority component ($\alpha = \downarrow$), as shown in Fig. 8(a).

## IV. SUMMARY

To summarize, we have discussed the possibility of inducing a triplet pair amplitude in a trapped $s$-wave superfluid Fermi gas. Using symmetry considerations, we have clarified that both broken spatial inversion symmetry and broken spin rotation symmetry are necessary for this phenomenon to occur. We have numerically confirmed that a triplet pair amplitude is induced when this condition is satisfied, within the mean-field theory for a two-dimensional lattice model. In this confirmation, we considered the three cases of (i) trap-potential difference, (ii) mass imbalance, and (iii) spin imbalance. In the

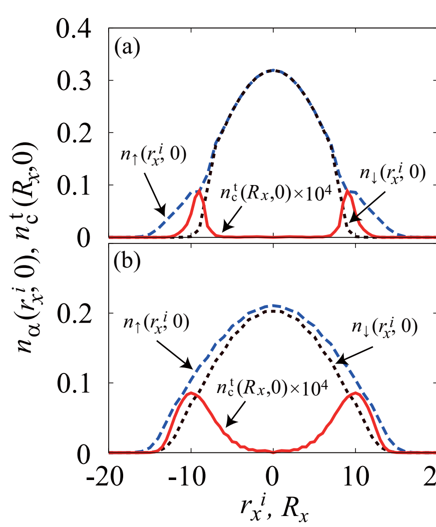


FIG. 8. (Color online) Calculated density profile $n_\alpha(r_x^i,0)$ and triplet condensate fraction $n_c^t(R_x,0)$ in a trapped $s$-wave superfluid Fermi gas with spin imbalance. (a) $U_s/t = 6$. (b) $U_s/t = 3$. We set $(N_\uparrow - N_\downarrow)/N = 0.17$.

first case, we showed that the induced triplet pair amplitude is dominated by a $p$-wave symmetry. Among these three cases, the trapped $s$-wave superfluid Fermi gas with spin imbalance has been realized [5–7]. Thus, our results imply that a triplet pair amplitude is already present in this system, although there is no experimental evidence yet.

Since the symmetry of a Fermi superfluid is fully determined by the symmetry of the superfluid order parameter, the induction of a triplet pair amplitude does not immediately mean the realization of a triplet Fermi superfluid. In the present case, the system is still in the $s$-wave superfluid state, which is characterized by the $s$-wave superfluid order parameter, even in the presence of a triplet pair amplitude. In this situation, however, when one suddenly changes the $s$-wave pairing interaction to an appropriate $p$-wave one, the product of the $p$-wave interaction and the triplet pair amplitude that has been induced before this manipulation may immediately give a finite $p$-wave superfluid order parameter. Since the $s$-wave superfluid order parameter vanishes, by definition, we have a $p$-wave superfluid state, characterized by this $p$-wave superfluid parameter. Change of the interaction would be possible using the Feshbach resonance technique.

In cold Fermi gas physics, although the realization of a $p$-wave superfluid state is a crucial challenge, current experiments are facing various difficulties originating from $p$-wave interaction, such as three-particle loss [58–60], as well as dipolar relaxation [14]. In this regard, the above idea may avoid these difficulties to some extent, because the triplet pair amplitude is prepared in an $s$-wave superfluid Fermi gas with no $p$-wave interaction. In addition, since we can start from a finite value of the $p$-wave superfluid order parameter, the system would be in the $p$-wave superfluid state for a while, until it is strongly damaged by the particle loss and dipolar relaxation after the $p$-wave interaction is introduced. In this sense, the induction of a triplet pair amplitude discussed in this paper is important not only as a fundamental physical phenomenon, but also from the viewpoint of the challenge of the realization of a $p$-wave superfluid Fermi gas.

In this paper, we have treated a lattice model to simply confirm the induction of a triplet pair amplitude. To quantitatively evaluate this quantity, we need to extend the present analyses to a realistic continuum Fermi superfluid. To assess the idea that one produces a $p$-wave superfluid Fermi gas from the induced triplet pair amplitude, it is also important to clarify the time evolution of the $p$-wave superfluid order parameter after the $s$-wave interaction is replaced by a $p$-wave one. These problems remain for the future. Since a pair amplitude always exists in a Fermi superfluid, our results will be useful for the study of this fundamental quantity in cold Fermi gas physics.

### ACKNOWLEDGMENTS

Y.O. thanks N. Yamamoto for useful comments. This work was supported by the KiPAS project at Keio University. Y.E. and R.H. were supported by a grant-in-aid for JSPS fellows. Y.O. was supported by Grants-in-Aid for Scientific Research from MEXT, Japan (No. 25105511, No. 25400418, and No. 15H00840).

### APPENDIX A: TRIPLET PAIR AMPLITUDE IN A SPIN-ORBIT-COUPLED UNIFORM $s$-WAVE SUPERFLUID FERMI GAS

We consider a uniform $s$-wave superfluid Fermi gas with a spin-orbit interaction. The model Hamiltonian is given by

$$H = \sum_{\boldsymbol{p},\alpha} \xi_{\boldsymbol{p}} c^\dagger_{\boldsymbol{p},\alpha} c_{\boldsymbol{p},\alpha} + H_{\rm so} - U_s \sum_{\boldsymbol{p},\boldsymbol{p}',\boldsymbol{q}} c^\dagger_{\boldsymbol{p}+\boldsymbol{q}/2,\uparrow} c^\dagger_{-\boldsymbol{p}+\boldsymbol{q}/2,\downarrow} c_{-\boldsymbol{p}'+\boldsymbol{q}/2,\downarrow} c_{\boldsymbol{p}'+\boldsymbol{q}/2,\uparrow}. \tag{A1}$$

Here, $c^\dagger_{\boldsymbol{p},\alpha}$ is the creation operator of a Fermi atom with the kinetic energy $\xi_{\boldsymbol{p}} = p^2/(2m) - \mu$, measured from the chemical potential $\mu$. The antisymmetric spin-orbit interaction $H_{\rm so}$ has the form [17]

$$H_{\rm so} = \sum_{\boldsymbol{p},\alpha,\alpha'} c^\dagger_{\boldsymbol{p},\alpha} h^{\alpha,\alpha'}_{\rm so}(\boldsymbol{p}) c_{\boldsymbol{p},\alpha'}, \tag{A2}$$

where $\hat{h}_{\rm so}(\boldsymbol{p}) = \{h^{\alpha,\alpha'}_{\rm so}\}$ is assumed as

$$\hat{h}_{\rm so}(\boldsymbol{p}) = \lambda_\perp[p_x\sigma_x + p_y\sigma_y] + \lambda_z\sigma_z. \tag{A3}$$

Here, $\lambda_\perp$ and $\lambda_z$ are spin-orbit couplings.

We assume that the system is in the $s$-wave superfluid state with the superfluid order parameter $\Delta_s = U_s \sum_{\boldsymbol{p}} \langle c_{\boldsymbol{p}\uparrow} c_{-\boldsymbol{p}\downarrow}\rangle$. We also assume that any other spontaneous symmetry breaking is absent.

In momentum space, the spatial inversion $\hat{P}$ is described as $\tilde{c}_{\boldsymbol{p},\alpha} = \hat{P} c_{\boldsymbol{p},\alpha} \hat{P}^{-1} = c_{-\boldsymbol{p},\alpha}$. Under this operation, each term in Eq. (A1) is invariant except for the spin-orbit interaction, which is transformed as

$$\tilde{H}_{\rm so} = \hat{P} H_{\rm so} \hat{P}^{-1} = \sum_{\boldsymbol{p},\alpha,\alpha'} c^\dagger_{-\boldsymbol{p},\alpha} h^{\alpha,\alpha'}_{\rm so}(\boldsymbol{p}) c_{-\boldsymbol{p},\alpha'} = \sum_{\boldsymbol{p},\alpha,\alpha'} c^\dagger_{\boldsymbol{p},\alpha} h^{\alpha,\alpha'}_{\rm so}(-\boldsymbol{p}) c_{\boldsymbol{p},\alpha'} = -H_{\rm so}. \tag{A4}$$

Thus, the spin-orbit interaction $H_{\rm so}$ in Eq. (A2) breaks the inversion symmetry.

For the spin rotation $\hat{R}(\boldsymbol{\theta})$, the three $\pi$ rotations ($\boldsymbol{\theta} = \boldsymbol{\theta}^j_\pi$, $j = x,y,z$) corresponding to Eq. (13) are given by

$$\begin{pmatrix} \tilde{c}_{\boldsymbol{p}\uparrow} \\ \tilde{c}_{\boldsymbol{p}\downarrow} \end{pmatrix}_{\boldsymbol{\theta}=\boldsymbol{\theta}^j_\pi} = i\sigma_j \begin{pmatrix} c_{\boldsymbol{p}\uparrow} \\ c_{\boldsymbol{p}\downarrow} \end{pmatrix} \ (j = x,y,z). \tag{A5}$$

When $\lambda_\perp = 0$ and $\lambda_z \neq 0$ (single-component spin-orbit interaction), Eq. (A1) is not invariant under the spin rotations $\hat{R}(\boldsymbol{\theta}^x_\pi)$ and $\hat{R}(\boldsymbol{\theta}^y_\pi)$, because the spin-orbit interaction $H_{\rm so}$ is transformed as

$$\tilde{H}_{\rm so} = \hat{R}\big(\boldsymbol{\theta}^{x,y}_\pi\big) H_{\rm so} \hat{R}^{-1}\big(\boldsymbol{\theta}^{x,y}_\pi\big) = \lambda_z \sum_{\boldsymbol{p},\alpha,\alpha'} \tilde{c}^\dagger_{\boldsymbol{p},\alpha} \sigma^{\alpha,\alpha'}_z \tilde{c}_{\boldsymbol{p},\alpha'} = -\lambda_z \sum_{\boldsymbol{p},\alpha,\alpha'} c^\dagger_{\boldsymbol{p},\alpha} \sigma^{\alpha,\alpha'}_z c_{\boldsymbol{p},\alpha'} = -H_{\rm so}. \tag{A6}$$

Thus, Eq. (A1) is invariant only under the $\pi$ rotation with $\boldsymbol{\theta} = \boldsymbol{\theta}_\pi^z$. Noting that the triplet pair amplitude,

$$\Phi_{\rm t}^{S_z}(\boldsymbol{p}) = \begin{cases} \langle c_{\boldsymbol{p}\uparrow} c_{-\boldsymbol{p}\uparrow}\rangle & (S_z = 1), \\ \frac{1}{\sqrt{2}}[\langle c_{\boldsymbol{p}\uparrow} c_{-\boldsymbol{p}\downarrow}\rangle + \langle c_{\boldsymbol{p}\downarrow} c_{-\boldsymbol{p}\uparrow}\rangle] & (S_z = 0), \\ \langle c_{\boldsymbol{p}\downarrow} c_{-\boldsymbol{p}\downarrow}\rangle & (S_z = -1), \end{cases} \tag{A7}$$

is transformed under the three $\pi$ rotations as

$$\begin{pmatrix} \tilde{\Phi}_{\rm t}^{S_z=1}(\boldsymbol{p}) \\ \tilde{\Phi}_{\rm t}^{S_z=0}(\boldsymbol{p}) \\ \tilde{\Phi}_{\rm t}^{S_z=-1}(\boldsymbol{p}) \end{pmatrix} = \begin{cases} \begin{pmatrix} -\Phi_{\rm t}^{S_z=-1}(\boldsymbol{p}) \\ -\Phi_{\rm t}^{S_z=0}(\boldsymbol{p}) \\ -\Phi_{\rm t}^{S_z=1}(\boldsymbol{p}) \end{pmatrix} (\boldsymbol{\theta} = \boldsymbol{\theta}_\pi^x), \\ \begin{pmatrix} \Phi_{\rm t}^{S_z=-1}(\boldsymbol{p}) \\ -\Phi_{\rm t}^{S_z=0}(\boldsymbol{p}) \\ \Phi_{\rm t}^{S_z=1}(\boldsymbol{p}) \end{pmatrix} (\boldsymbol{\theta} = \boldsymbol{\theta}_\pi^y), \\ \begin{pmatrix} -\Phi_{\rm t}^{S_z=1}(\boldsymbol{p}) \\ \Phi_{\rm t}^{S_z=0}(\boldsymbol{p}) \\ -\Phi_{\rm t}^{S_z=-1}(\boldsymbol{p}) \end{pmatrix} (\boldsymbol{\theta} = \boldsymbol{\theta}_\pi^z), \end{cases} \tag{A8}$$

we find that only $\Phi_{\rm t}^{S_z=0}(\boldsymbol{p})$ may be induced. Indeed, Ref. [17] shows that it is induced in this case.

When $\lambda_\perp \neq 0$, the spin-orbit interaction $H_{\rm so}$ is not invariant under any $\pi$ rotations $\hat{R}(\boldsymbol{\theta}_\pi^{x,y,z})$. Within this analysis, one concludes that all the triplet pair amplitudes in Eq. (A7) may be induced. However, within the mean-field theory, Ref. [17] shows that the component with $S_z = 0$ is not induced when $\lambda_\perp \neq 0$ and $\lambda_z = 0$. This is because, in this two-component case, the mean-field BCS Hamiltonian,

$$H_{\rm BCS} = \sum_{\boldsymbol{p},\alpha} \xi_{\boldsymbol{p}} c_{\boldsymbol{p},\alpha}^\dagger c_{\boldsymbol{p},\alpha} + H_{\rm so} + \Delta_s \sum_{\boldsymbol{p}} [c_{\boldsymbol{p},\uparrow}^\dagger c_{-\boldsymbol{p},\downarrow}^\dagger + {\rm H.c}], \tag{A9}$$

is invariant under the momentum-dependent $\pi$-spin rotation, which is followed by the $U(1)$ gauge transformation, given by

$$\begin{pmatrix} \tilde{c}_{\boldsymbol{p}\uparrow} \\ \tilde{c}_{\boldsymbol{p}\downarrow} \end{pmatrix} = e^{-i\frac{\pi}{2}} \times e^{i\frac{\pi}{2}\hat{\boldsymbol{p}}_\perp \cdot \boldsymbol{\sigma}} \begin{pmatrix} c_{\boldsymbol{p}\uparrow} \\ c_{\boldsymbol{p}\downarrow} \end{pmatrix}, \tag{A10}$$

where $\hat{\boldsymbol{p}}_\perp = (p_x, p_y)/\sqrt{p_x^2 + p_y^2}$. In this case, the triplet pair amplitude with $S_z = 0$ is transformed as $\Phi_{\rm t}^{S_z=0}(\boldsymbol{p}) \to -\Phi_{\rm t}^{S_z=0}(\boldsymbol{p})$, so that one finds $\Phi_{\rm t}^{S_z=0}(\boldsymbol{p}) = 0$, as obtained in Ref. [17].

## APPENDIX B: DIAGONALIZATION OF THE BCS HAMILTONIAN IN EQ. (18)

The mean-field BCS Hamiltonian in Eq. (18) can be diagonalized by the Bogoliubov transformation in real space, given by

$$\begin{pmatrix} c_{\boldsymbol{r}_1,\uparrow} \\ \vdots \\ c_{\boldsymbol{r}_{L^2},\uparrow} \\ c_{\boldsymbol{r}_1,\downarrow}^\dagger \\ \vdots \\ c_{\boldsymbol{r}_{L^2},\downarrow}^\dagger \end{pmatrix} = \hat{W} \begin{pmatrix} \gamma_1 \\ \vdots \\ \gamma_{L^2} \\ \gamma_{L^2+1} \\ \vdots \\ \gamma_{2L^2} \end{pmatrix}. \tag{B1}$$

Here, $\hat{W}$ is a $2L^2 \times 2L^2$ orthogonal matrix, which is chosen so that $H_{\rm MF}$ in Eq. (18) can be diagonalized as

$$H_{\rm MF} = \sum_{j=1}^{2L^2} E_j \gamma_j^\dagger \gamma_j, \tag{B2}$$

where $E_j$ is a Bogoliubov single-particle excitation energy. After the diagonalization, the superfluid order parameter $\Delta_s(\boldsymbol{r}_i)$, as well as the number density $n_\alpha(\boldsymbol{r}_i) = \langle c_{\boldsymbol{r}_i,\alpha}^\dagger c_{\boldsymbol{r}_i,\alpha}\rangle$, is evaluated as, respectively,

$$\Delta_s(\boldsymbol{r}_i) = U_s \sum_{j=1}^{2L^2} W_{i,j} W_{i+L^2,j} f(-E_j) \tag{B3}$$

$$n_\uparrow(\boldsymbol{r}_i) = \sum_{j=1}^{2L^2} W_{i,j}^2 f(E_j), \tag{B4}$$

$$n_\downarrow(\boldsymbol{r}_i) = \sum_{j=1}^{2L^2} W_{i+L^2,j}^2 f(-E_j), \tag{B5}$$

where $f(E) = 1/[e^{\beta E_j} + 1]$ is the Fermi distribution function. The number $N_\alpha$ of Fermi atoms in the $\alpha$-spin component is given by

$$N_\alpha = \sum_{i=1}^{L^2} n_\alpha(\boldsymbol{r}_i). \tag{B6}$$

We numerically calculate Eqs. (B1) and (B3)–(B6), to self-consistently determine $\Delta_s(\boldsymbol{r}_i)$, $n_\alpha(\boldsymbol{r}_i)$, and $\mu_\alpha$. The triplet pair amplitude $\Phi_{\rm t}^{S_z=0}(\boldsymbol{r}_i,\boldsymbol{r}_j)$ in Eq. (19), as well as the singlet pair amplitude $\Phi_{\rm s}(\boldsymbol{r}_i,\boldsymbol{r}_j)$ in Eq. (20), is then calculated as, respectively,

$$\Phi_{\rm t}^{S_z=0}(\boldsymbol{r}_i,\boldsymbol{r}_j) = \sum_{k=1}^{2L^2} [W_{i,k} W_{j+L^2,k} f(-E_k) + W_{i+L^2,k} W_{j,k} f(E_k)], \tag{B7}$$

$$\Phi_{\rm s}(\boldsymbol{r}_i,\boldsymbol{r}_j) = \sum_{k=1}^{2L^2} [W_{i,k} W_{j+L^2,k} f(-E_k) - W_{i+L^2,k} W_{j,k} f(E_k)]. \tag{B8}$$


[1] C. A. Regal, M. Greiner, and D. S. Jin, Phys. Rev. Lett. **92**, 040403 (2004).

[2] M. W. Zwierlein, C. A. Stan, C. H. Schunck, S. M. F. Raupach, A. J. Kerman, and W. Ketterle, Phys. Rev. Lett. **92**, 120403 (2004).

[3] J. Kinast, S. L. Hemmer, M. E. Gehm, A. Turlapov, and J. E. Thomas, Phys. Rev. Lett. **92**, 150402 (2004).

[4] M. Bartenstein, A. Altmeyer, S. Riedl, S. Jochim, C. Chin, J. H. Denschlag, and R. Grimm, Phys. Rev. Lett. **92**, 203201 (2004).

[5] M. W. Zwierlein, A. Schirotzek, C. H. Schunck, and W. Ketterle, Science **311**, 492 (2006).

[6] G. B. Partridge, W. Li, R. I. Kamar, Y.-A. Liao, and R. G. Hulet, Science **311**, 503 (2006).

[7] Y. Shin, C. H. Schunck, A. Schirotzek, and W. Ketterle, Nature (London) **451**, 689 (2008).

[8] For a review, see C. Chin, R. Grimm, P. Julienne, and E. Tiesinga, Rev. Mod. Phys. **82**, 1225 (2010).

[9] C. A. Regal, C. Ticknor, J. L. Bohn, and D. S. Jin, Phys. Rev. Lett. **90**, 053201 (2003).

[10] C. Ticknor, C. A. Regal, D. S. Jin, and J. L. Bohn, Phys. Rev. A **69**, 042712 (2004).

[11] J. Zhang, E. G. M. van Kempen, T. Bourdel, L. Khaykovich, J. Cubizolles, F. Chevy, M. Teichmann, L. Tarruell, S. J. J. M. F. Kokkelmans, and C. Salomon, Phys. Rev. A **70**, 030702(R) (2004).

[12] C. H. Schunck, M. W. Zwierlein, C. A. Stan, S. M. F. Raupach, W. Ketterle, A. Simoni, E. Tiesinga, C. J. Williams, and P. S. Julienne, Phys. Rev. A **71**, 045601 (2005).

[13] T. Nakasuji, J. Yoshida, and T. Mukaiyama, Phys. Rev. A **88**, 012710 (2013).

[14] J. P. Gaebler, J. T. Stewart, J. L. Bohn, and D. S. Jin, Phys. Rev. Lett. **98**, 200403 (2007).

[15] Y. Inada, M. Horikoshi, S. Nakajima, M. Kuwata-Gonokami, M. Ueda, and T. Mukaiyama, Phys. Rev. Lett. **101**, 100401 (2008).

[16] J. Fuchs, C. Ticknor, P. Dyke, G. Veeravalli, E. Kuhnle, W. Rowlands, P. Hannaford, and C. J. Vale, Phys. Rev. A **77**, 053616 (2008).

[17] T. Yamaguchi and Y. Ohashi, arXiv:1504.03835.

[18] For a review, see J. Dalibard, F. Gerbier, G. Juzeliūnas, and P. Öhberg, Rev. Mod. Phys. **83**, 1523 (2011).

[19] Y.-J. Lin, R. L. Compton, A. R. Perry, W. D. Phillips, J. V. Porto, and I. B. Spielman, Phys. Rev. Lett. **102**, 130401 (2009).

[20] Y.-J. Lin, R. L. Compton, K. Jiménez-García, J. V. Porto, and I. B. Spielman, Nature **462**, 628 (2009).

[21] Y.-J. Lin, R. L. Compton, K. Jiménez-García, W. D. Phillips, J. V. Porto, and I. B. Spielman, Nat. Phys. **7**, 531 (2011).

[22] Y.-J. Lin, K. Jiménez-García, and I. B. Spielman, Nature **471**, 83 (2011).

[23] L. W. Cheuk, A. T. Sommer, Z. Hadzibabic, T. Yefsah, W. S. Bakr, and M. W. Zwierlein, Phys. Rev. Lett. **109**, 095302 (2012).

[24] P. Wang, Z.-Q. Yu, Z. Fu, J. Miao, L. Huang, S. Chai, H. Zhai, and J. Zhang, Phys. Rev. Lett. **109**, 095301 (2012).

[25] J. D. Sau, R. Sensarma, S. Powell, I. B. Spielman, and S. Das Sarma, Phys. Rev. B **83**, 140510(R) (2011).

[26] B. M. Anderson, I. B. Spielman, and G. Juzeliūnas, Phys. Rev. Lett. **111**, 125301 (2013).

[27] E. Bauer, G. Hilscher, H. Michor, Ch. Paul, E. W. Scheidt, A. Gribanov, Yu. Seropegin, H. Noël, M. Sigrist, and P. Rogl, Phys. Rev. Lett. **92**, 027003 (2004).

[28] V. P. Mineev and M. Sigrist, in *Non-centrosymmetric Superconductors*, edited by E. Bauer and M. Sigrist (Springer-Verlag, Berlin, 2012), Chap. 4.

[29] H. Q. Yuan, D. F. Agterberg, N. Hayashi, P. Badica, D. Vandervelde, K. Togano, M. Sigrist, and M. B. Salamon, Phys. Rev. Lett. **97**, 017006 (2006).

[30] S. Nascimbène, N. Navon, S. Pilati, F. Chevy, S. Giorgini, A. Georges, and C. Salomon, Phys. Rev. Lett. **106**, 215303 (2011).

[31] T. Kashimura, R. Watanabe, and Y. Ohashi, Phys. Rev. A **86**, 043622 (2012); **89**, 013618 (2014).

[32] M. Taglieber, A.-C. Voigt, T. Aoki, T. W. Hänsch, and K. Dieckmann, Phys. Rev. Lett. **100**, 010401 (2008).

[33] E. Wille, F. M. Spiegelhalder, G. Kerner, D. Naik, A. Trenkwalder, G. Hendl, F. Schreck, R. Grimm, T. G. Tiecke, J. T. M. Walraven, S. J. J. M. F. Kokkelmans, E. Tiesinga, and P. S. Julienne, Phys. Rev. Lett. **100**, 053201 (2008).

[34] F. M. Spiegelhalder, A. Trenkwalder, D. Naik, G. Hendl, F. Schreck, and R. Grimm, Phys. Rev. Lett. **103**, 223203 (2009).

[35] N. Takemori and A. KogaJ. Phys. Soc. Jpn. **81**, 063002 (2012).

[36] G.-D. Lin, W. Yi, and L.-M. Duan, Phys. Rev. A **74**, 031604(R) (2006).

[37] T.-L. Dao, A. Georges, and M. Capone, Phys. Rev. B **76**, 104517 (2007).

[38] M. A. Cazalilla, A. F. Ho, and T. Giamarchi, Phys. Rev. Lett. **95**, 226402 (2005).

[39] R. Hanai, T. Kashimura, R. Watanabe, D. Inotani, and Y. Ohashi, Phys. Rev. A **88**, 053621 (2013).

[40] R. Hanai and Y. Ohashi, Phys. Rev. A **90**, 043622 (2014).

[41] L. Jiang, X. J. Liu, H. Hu, and H. Pu, Phys. Rev. A **84**, 063618 (2011).

[42] H. Hu, L. Jiang, X.-J. Liu, and H. Pu, Phys. Rev. Lett. **107**, 195304 (2011).

[43] C. N. Yang, Rev. Mod. Phys. **34**, 694 (1962).

[44] L. Salasnich, N. Manini, and A. Parola, Phys. Rev. A **72**, 023621 (2005).

[45] G. E. Astrakharchik, J. Boronat, J. Casulleras, and S. Giorgini, Phys. Rev. Lett. **95**, 230405 (2005).

[46] N. Fukushima, Y. Ohashi, E. Taylor, and A. Griffin, Phys. Rev. A **75**, 033609 (2007).

[47] P. G. de Gennes, *Superconductivity of Metals and Alloys* (Addison-Wesley, New York, 1989).

[48] Y. Ohashi, Phys. Rev. A **83**, 063611 (2011).

[49] Y. Ohashi, Phys. Rev. Lett. **94**, 050403 (2005).

[50] T.-L. Ho and R. B. Diener, Phys. Rev. Lett. **94**, 090402 (2005).

[51] P. Fulde and R. A. Ferrell, Phys. Rev. **135**, A550 (1964).

[52] A. I. Larkin, and Yu. N. Ovchinnikov, Sov. Phys. JETP **20**, 762 (1965).

[53] S. Takada and T. Izuyama, Prog. Theor. Phys. **41**, 635 (1969).

[54] T. Mizushima, M. Ichioka, and K. Machida, J. Phys. Soc. Jpn. **76**, 104006 (2007).

[55] Y. Ohashi, J. Phys. Soc. Jpn. **71**, 2625 (2002).

[56] The oscillating behavior of the phase boundary (solid line) in Figs. 7(a) and 7(b) is due to the discrete energy levels in a harmonic trap potential. To explain this in a simple manner, when we consider a two-dimensional harmonic potential $V(\boldsymbol{r}) = m\omega^2\boldsymbol{r}^2/2$, one particle energy is given by $E(n_x,n_y) = \omega[n_x + n_y + 1]$ ($n_x,n_y = 0,1,2,\dots$). In this case, the degeneracy of an eigenenergy $E = \omega[N_{xy} + 1]$ equals $N_{xy} + 1$. When degenerate energy levels below $E$ are fully occupied by $\uparrow$-spin atoms or $\downarrow$-spin atoms, the energy gap $\omega$ suppresses the superfluid phase transition to some extent, leading to the oscillation of the phase boundary in Figs. 7(a) and 7(b).

[57] The situation, $\mu_\uparrow \neq \mu_\downarrow$, actually occurs also in the case with a trap-potential difference, as well as the case with a mass imbalance. Figure 7 indicates that a triplet pair amplitude is

induced even when only a difference in the chemical potentials exists.

[58] J. Levinsen, N. R. Cooper, and V. Gurarie, Phys. Rev. Lett. **99**, 210402 (2007).

[59] M. Jona-Lasinio, L. Pricoupenko, and Y. Castin, Phys. Rev. A **77**, 043611 (2008).

[60] J. Levinsen, N. R. Cooper, and V. Gurarie, Phys. Rev. A **78**, 063616 (2008).