Deterministic noiseless amplification of coherent states

Meng-Jun Hu and Yong-Sheng Zhang*
Laboratory of Quantum Information, University of Science and Technology of China, Hefei, 230026, China and Synergetic Innovation Center of Quantum Information and Quantum Physics, University of Science and Technology of China, Hefei, 230026, China

(Received 11 March 2015; revised manuscript received 10 June 2015; published 26 August 2015)

A universal deterministic noiseless quantum amplifier has been shown to be impossible. However, probabilistic noiseless amplification of a certain set of states is physically permissible. Regarding quantum state amplification as quantum state transformation, we show that deterministic noiseless amplification of coherent states chosen from a proper set is attainable. The relation between input coherent states and gain of amplification for deterministic noiseless amplification is thus derived. Furthermore, we extend our result to more general situation and show that deterministic noiseless amplification of Gaussian states is also possible. As an example of application, we find that our amplification model can obtain better performance in homodyne detection to measure the phase of state selected from a certain set. Besides, other possible applications are also discussed.

DOI: 10.1103/PhysRevA.92.022352 PACS number(s): 03.67.Ac, 03.65.Ta, 42.50.Dv, 42.50.Ex

I. INTRODUCTION

Quantum amplification plays an essential role in quantum measurement and quantum metrology [1,2]. In order to measure a weak signal, improving the sensitivity of the detector or amplifying the signal are two basic solutions. However, constrained by physical laws, it may be very difficult for detectors to measure a sufficiently weak signal, especially for a quantum signal. What we usually do is amplify the signal first and then measure it with a proper detector. Unfortunately, the noise accompanying the signal is also amplified during the process of signal amplification. Additionally, the added noise may reduce the signal-to-noise ratio (SNR) after amplification. For a linear phase-insensitive quantum amplifier, it has been shown that there is at least $(g^2 - 1/2)\hbar\omega$ total noise (including intrinsic noise and added noise) power per unit bandwidth out of its output port, where g^2 is the power gain [3,4]. The noiseless amplification (without introducing added noise), which is SNR preserving, seems unlikely for a universal linear phase-insensitive quantum amplifier.

In fact, due to the constraint of quantum commutation condition, a universal linear phase-insensitive quantum amplifier which can amplify any coherent states determinately and noiselessly is impossible [5]. However, as the no-cloning theorem [6–9] does not rule out the possibility of probabilistic cloning the state which is randomly chosen from a linear-independent set of states [10], the nonexistence of a universal deterministic noiseless quantum amplifier does not mean the nonexistence of a specific quantum amplifier which can noiselessly amplify a certain input set of states. The noiseless amplification of quantum states essentially is a problem of quantum state transformation. Both deterministic and probabilistic quantum state transformations have already been discussed in detail [11–13]. Using the language of quantum state transformation, the quantum state cloning, the unambiguous discrimination of states, and the quantum state amplification can be demonstrated in a unified framework [14–16]. Recently, there have been some experimental reports on realization of noiseless amplification of quantum light states [17–19]. All these experimental schemes are probabilistic and can only attain unit fidelity asymptotically. The truly probabilistic noiseless amplification of coherent states in a certain set is thus discussed based on quantum state transformation [16].

A natural question arises of whether there exists a specific quantum amplifier which can determinately and noiselessly amplify a coherent state randomly chosen from a definite set. In this paper, we show that it is indeed possible when we regard quantum state amplification as quantum state transformation.

We note here that there is another kind of quantum amplifier which is based on the unambiguous identification of input states and the preparation of desired amplified input state [20,21]. This kind of quantum amplifier is usually called classic-like quantum amplifier analogous to classical amplifier working through measurement and preparation. Limited by success probability of identification of input states (except orthogonal input states), the classic-like quantum amplifier can only probabilistically amplify the input states, though the gain can be arbitrarily high.

II. QUANTUM TRANSFORMATION OF SETS OF PURE STATES

In quantum operation theory, any physically permissible transformation of the state of a quantum system can be represented by a completely positive (CP), linear, trace non-increasing map: $\$: \rho \to \$(\rho)$. If any such map exists, the transformation is realizable in principle [22]. The so-called first representation theorem [23] states that the CP, linear, trace nonincreasing map \$ can be represented as operator-sum form $\$(\rho) = \sum_k A_k^\dagger \rho A_k$, where A_k is the Kraus operator which satisfies $\sum_k A_k^\dagger A_k \leqslant I$ and I is the identity operator. For deterministic transformation $\sum_k A_k^\dagger A_k = I$, while for probabilistic transformation $\sum_k A_k^\dagger A_k < I$. This process has another description that the transformation can be implemented by adding an ancillary system to the quantum system and then a unitary transformation is applied to the composite system. Mathematically, we have $\$(\rho) = \mathrm{tr}_{E'}[U\rho \bigotimes \rho_E U^\dagger I \bigotimes P_{E'}]$, where ρ_E is the initial state of ancillary system and $P_{E'}$ is

*yshzhang@ustc.edu.cn

a projector in transformed ancillary Hilbert space [24]. We begin our formal discussion by first reviewing the well-known quantum transform theorem of sets of pure states.

Lemma 1 [13]. Suppose there is a set of N pure states $A = \{|\psi_i\rangle\}$ which is linear independent and another set of N pure states $B = \{|\phi_i\rangle\}$. A probabilistic transformation $T : A = \{|\psi_i\rangle\} \xrightarrow{\{p_i\}} B = \{|\phi_i\rangle\}$ that transforms state $|\psi_i\rangle$ in set A to the corresponding state $|\phi_i\rangle$ in set B with probability p_i exists if and only if there exists an $N \times N$ matrix Π which satisfies the three conditions: $1 : \Pi \geqslant 0$; $2 : \mathrm{diag}(\Pi) = \vec{p} = (p_1, p_2, \ldots, p_N)$; $3 : G_A - \Pi \circ G_B \geqslant 0$. Here G_A and G_B are Gram matrix of set A and B respectively and $\circ$ denotes Hadamard matrix product.

Proof. If such a transformation exists, there must exist complex coefficients c_{ki} such that

$$A_{ks}|\psi_i\rangle = c_{ki}|\phi_i\rangle, \tag{1}$$

where $A_{ks}(k = 1,2,\ldots,M)$ are the Kraus operators for successful transformation. Consider these coefficients as the elements of a $M \times N$ matrix $C = [c_{ki}]$. We can introduce matrix Π defined by $\Pi = C^\dagger C$. It can be shown that matrix Π satisfies all three conditions [13].

Suppose there is a matrix Π which satisfies all three conditions. Positivity of the matrix enables us to factorize Π as $C^\dagger C$ and then the transformation operators can be constructed as

$$A_{ks} = \sum_i \frac{c_{ki}}{\langle\tilde{\psi}_i|\psi_i\rangle}|\phi_i\rangle\langle\tilde{\psi}_i|, \tag{2}$$

where $\langle\tilde{\psi}_i|\psi_j\rangle = \gamma_i\delta_{ij}$, $\gamma_i \neq 0$ is a constant. State $|\tilde{\psi}_i\rangle$ is orthogonal to any state in set A except for state $|\psi_i\rangle$.

Physically, the above transformation can be implemented by a specific unitary transformation operating on a composite system consists of quantum system and ancillary system [16]:

$$U|\psi_i\rangle = \sqrt{p_i}|\phi_i\rangle|u_i\rangle|0\rangle + \sqrt{1-p_i}|Fail\rangle|v_i\rangle|1\rangle. \tag{3}$$

Taking the inner product of Eq. (3) and its complex conjugate, we have

$$\langle\psi_j|\psi_i\rangle = \sqrt{p_i p_j}\langle\phi_j|\phi_i\rangle\langle u_j|u_i\rangle + \sqrt{(1-p_i)(1-p_j)}\langle v_j|v_i\rangle, \tag{4}$$

which can be recast as

$$G_A = G_B \circ \Pi + K. \tag{5}$$

From the positivity of Gram matrix K, it can be seen that $G_A - G_B \circ \Pi \geqslant 0$ and the Gram matrix Π defined as $\Pi = \sqrt{p_i p_j}\langle u_j|u_i\rangle$ obviously satisfies the conditions 1 and 2.

Consider the deterministic transformation which means $p_i = 1$ for all states so that the second term in the right-hand side of Eq. (4) vanishes and Eq. (5) becomes $G_A = G_B \circ \Pi$ with $\Pi = \langle u_j|u_i\rangle$. For any two input states in the set A, we have the following equality:

$$\langle\psi_j|\psi_i\rangle = \langle\phi_j|\phi_i\rangle\langle u_j|u_i\rangle. \tag{6}$$

Equation (6) implies that the overlap between two input states is no more than the overlap between two corresponding output states after deterministic transformation, that is, $|\langle\psi_j|\psi_i| \leqslant |\langle\phi_j|\phi_i\rangle|$. From the point of view of information, the deterministic transformation does not increase the distinguishability of input states.

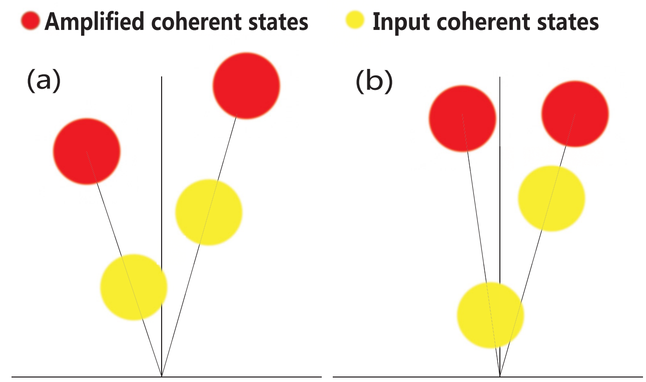

FIG. 1. (Color online) Illustration of Wigner function contours for probabilistic and deterministic noiseless amplification of coherent states. (a) If the distance between two amplified coherent states is longer than the distance between two input coherent states, then the noiseless amplification of coherent states can only be probabilistic. (b) If the distance between two amplified coherent states is shorter than or equal to the distance between two input coherent states, then the noiseless amplification of coherent states can be deterministic as long as the gain of quantum amplifier is state dependent.

III. DETERMINISTIC NOISELESS AMPLIFICATION OF COHERENT STATES

We now focus on the case of coherent states. The distinguishability of two coherent states can be measured by the distance of two coherent states, since $|\langle\alpha_1|\alpha_2\rangle|^2 = \exp(-|\alpha_1 - \alpha_2|^2)$. We can define the distance of two coherent states $|\alpha_1\rangle$ and $|\alpha_2\rangle$ as

$$D(\alpha_1,\alpha_2) = |\alpha_1 - \alpha_2|^2. \tag{7}$$

The distinguishability of any two coherent states is thus proportional to their distance. If there exists a deterministic transformation which transforms the set of coherent states $A = \{|\alpha_i\rangle\}$ to another set of coherent states $B = \{|\beta_i\rangle\}$, there must be $D(\alpha_i,\alpha_j) \geqslant D(\beta_i,\beta_j)$. However, it does not hold for a deterministic noiseless quantum amplifier of coherent states with gain $g > 1$. It is obviously that $D(g\alpha_i,g\alpha_j) = g^2|\alpha_i - \alpha_j|^2 > |\alpha_i - \alpha_j|^2 = D(\alpha_i,\alpha_j)$. For a quantum amplifier of coherent states with fixed gain $g > 1$, deterministic noiseless amplification is thus impossible. However, a fixed gain of amplification is actually not necessary. As shown in Fig. 1, a deterministic noiseless quantum amplifier of coherent states can exist as long as the gain of amplification is state dependent. For the simplicity of discussion, we just consider the case of only two coherent states contained in the input set in the following.

Theorem 1. Suppose there are two sets of two coherent states $A = \{|\alpha_1 e^{i\theta_1}\rangle, |\alpha_2 e^{i\theta_2}\rangle\}$, $B = \{|g_1\alpha_1 e^{i\theta_1}\rangle, |g_2\alpha_2 e^{i\theta_2}\rangle\}$; here we explicitly show the amplitude and phase of the coherent state. The deterministic noiseless quantum amplifier which amplifies the coherent state randomly chosen from set A to the corresponding coherent state in the set B exists if and only if the input coherent states and the gain of amplification satisfy the condition $\cos\eta \geqslant \sqrt{(g_1^2-1)(g_2^2-1)}/(g_1g_2-1)$ with $\eta = |\theta_1-\theta_2|$ being the relative phase between two coherent states.

Proof. If the deterministic noiseless amplification exists then we must have

$$D(g_1\alpha_1 e^{i\theta_1}, g_2\alpha_2 e^{i\theta_2}) \leqslant D(\alpha_1 e^{i\theta_1}, \alpha_2 e^{i\theta_2}). \tag{8}$$

According to the definition of the distance, we have

$$|\alpha_1 e^{i\theta_1} - \alpha_2 e^{i\theta_2}|^2 \geqslant |g_1\alpha_1 e^{i\theta_1} - g_2\alpha_2 e^{i\theta_2}|^2. \tag{9}$$

The calculation of Eq. (9) gives $2\alpha_1\alpha_2(g_1g_2-1)\cos\eta \geqslant (g_1^2-1)\alpha_1^2 + (g_2^2-1)\alpha_2^2 \geqslant 2\alpha_1\alpha_2\sqrt{(g_1^2-1)(g_2^2-1)}$. Eliminating the same factor in both sides finally gives

$$\cos\eta \geqslant \frac{\sqrt{(g_1^2-1)(g_2^2-1)}}{g_1g_2-1}. \tag{10}$$

The equality holds only when $\sqrt{g_1^2-1}\alpha_1 = \sqrt{g_2^2-1}\alpha_2$.

If the input states and gain satisfy Eq. (10), we can always construct the Kraus operator of amplification as

$$A_k = \frac{c_{k1}}{\langle\tilde{\psi}_1|\alpha_1 e^{i\theta_1}\rangle}|g_1\alpha_1 e^{i\theta_1}\rangle\langle\tilde{\psi}_1| + \frac{c_{k2}}{\langle\tilde{\psi}_2|\alpha_2 e^{i\theta_2}\rangle}|g_2\alpha_2 e^{i\theta_2}\rangle\langle\tilde{\psi}_2|, \tag{11}$$

where $\sum_k A_k^\dagger A_k = I$ and $\{|\tilde{\psi}_1\rangle, |\tilde{\psi}_1\rangle\}$ are defined as

$$|\tilde{\psi}_1\rangle = \frac{1}{\langle\alpha_2 e^{i\theta_2}|\alpha_1 e^{i\theta_1}\rangle}|\alpha_1 e^{i\theta_1}\rangle - |\alpha_2 e^{i\theta_2}\rangle, \tag{12}$$

$$|\tilde{\psi}_2\rangle = \frac{1}{\langle\alpha_1 e^{i\theta_1}|\alpha_2 e^{i\theta_2}\rangle}|\alpha_2 e^{i\theta_2}\rangle - |\alpha_1 e^{i\theta_1}\rangle. \tag{13}$$

Notice that

$$\langle\tilde{\psi}_s|\alpha_t e^{i\theta_t}\rangle = \frac{1-|\langle\alpha_1 e^{i\theta_1}|\alpha_2 e^{i\theta_2}\rangle|^2}{\langle\alpha_t e^{i\theta_t}|\alpha_s e^{i\theta_s}\rangle}\delta_{s,t}, \tag{14}$$

where $s,t = 1,2$, $\delta_{s,t} = 1$ for $s = t$ and $\delta_{s,t} = 0$ for $s \neq t$. It can be easily verified that

$$A_k|\alpha_1 e^{i\theta_1}\rangle = c_{k1}|g_1\alpha_1 e^{i\theta_1}\rangle, \tag{15}$$

$$A_k|\alpha_2 e^{i\theta_2}\rangle = c_{k2}|g_2\alpha_2 e^{i\theta_2}\rangle. \tag{16}$$

The Kraus operator A_k is the expected operator of amplification.

The above result can be extended into the more general case in which the input set contains more than two coherent states. For that case, a deterministic noiseless quantum amplifier exists if and only if any two coherent states in the set satisfy the relation (10). There is a specific case that all the amplified coherent states have the same amplitude. In two coherent states case, it means $g_1\alpha_1 = g_2\alpha_2$ and the input states and gain have to satisfy more restricted relation. We thus have the following corollary.

Corollary 1. Suppose there are two sets of two coherent states $A = \{|\alpha_1 e^{i\theta_1}\rangle, |\alpha_2 e^{i\theta_2}\rangle\}$, $B = \{|g_1\alpha_1 e^{i\theta_1}\rangle, |g_2\alpha_2 e^{i\theta_2}\rangle\}$ with $g_1\alpha_1 = g_2\alpha_2$. The deterministic noiseless quantum amplifier which amplifies the coherent state randomly chosen from set A to the corresponding coherent state in the set B exists if and only if the input states and the gain satisfy the condition $\cos\eta \geqslant \frac{(2g_1^2-1)\alpha_1^2-\alpha_2^2}{2g_1^2\alpha_1^2-2\alpha_1\alpha_2}$.

Proof. The proof is the same as in Theorem 1. The only difference is that the input states and gain must satisfy more restricted relations due to the requirement of $g_1\alpha_1 = g_2\alpha_2$. Substituting $g_1\alpha_1 = g_2\alpha_2$ into the Eq. (9), we thus get

$$\cos\eta \geqslant \frac{(2g_1^2-1)\alpha_1^2-\alpha_2^2}{2g_1^2\alpha_1^2-2\alpha_1\alpha_2}. \tag{17}$$

Among deterministic noiseless quantum amplifiers which amplify coherent states to the same final amplitude, the best amplifier for a definite input set can be defined as g_1 has the maximum value. The maximum value of g_1 can be calculated from Eq. (17):

$$g_{1\max} = \sqrt{\frac{\alpha_1^2+\alpha_2^2-2\alpha_1\alpha_2\cos\eta}{2\alpha_1^2(1-\cos\eta)}}. \tag{18}$$

IV. DETERMINISTIC AMPLIFICATION OF GENERAL STATES

We now extend our results of coherent states to more general states. As coherent states are Gaussian states in phase space, it is natural to consider Gaussian states first. A Gaussian state is usually defined as such a state that its characteristic function is Gaussian function and is fully characterized by its first and second moments [25,26]. The first moments $\vec{d} = (d_1,d_2)$ of a single-mode Gaussian state $|\psi\rangle$ are defined as $d_i = \langle\psi|\hat{X}_i|\psi\rangle$, where $\hat{X}_i$ represent quadrature operators. The second moments $\vec{\sigma}$ which form the so-called covariance matrix $\vec{\sigma} = (\sigma_{ij})(i,j = 1,2)$ are given by $\sigma_{ij} = \langle\psi|\hat{X}_i\hat{X}_j + \hat{X}_j\hat{X}_i|\psi\rangle - 2d_id_j$. The Gaussian state $|\phi\rangle$ is the amplified state of the Gaussian state $|\psi\rangle$ with gain of amplification g means $\vec{d}_{|\phi\rangle} = g\vec{d}_{|\psi\rangle}$. For noiseless amplification, the second moments should stay unchanged, which gives $\vec{\sigma}_{|\phi\rangle} = \vec{\sigma}_{|\psi\rangle}$. The distance between two Gaussian states can be defined as $D(|\psi_1\rangle,|\psi_2\rangle) = (\vec{d}_{|\psi_1\rangle} - \vec{d}_{|\psi_2\rangle})^2$. It is obvious that the distance between two coherent states is a specific case if we rewrite it with the real and imaginary part of α. Suppose Gaussian states $|\phi_1\rangle$ and $|\phi_2\rangle$ are amplified states corresponding to Gaussian states $|\psi_1\rangle$ and $|\psi_2\rangle$ with $\vec{d}_{|\phi_1\rangle} = g_1\vec{d}_{|\psi_1\rangle}$, $\vec{d}_{|\phi_2\rangle} = g_2\vec{d}_{|\psi_2\rangle}$ and $\vec{\sigma}_{|\phi_1\rangle} = \vec{\sigma}_{|\psi_1\rangle}$, $\vec{\sigma}_{|\phi_2\rangle} = \vec{\sigma}_{|\psi_2\rangle}$. According to the previous discussions, the distance between amplified states should be no longer than the distance between input states after deterministic noiseless amplification which means $D(|\phi_1\rangle,|\phi_2\rangle) \leqslant D(|\psi_1\rangle,|\psi_2\rangle)$. A simple calculation gives $\cos\gamma \geqslant \sqrt{(g_1^2-1)(g_2^2-1)}/(g_1g_2-1)$, which coincides with Eq. (10), where γ denotes the relative phase of two Gaussian states in phase space. On the other hand, if inequality $\cos\gamma \geqslant \sqrt{(g_1^2-1)(g_2^2-1)}/(g_1g_2-1)$ is satisfied, we can always construct a deterministic noiseless amplifier which amplifies $|\psi\rangle$ to the corresponding state $|\phi\rangle$. We thus obtain the theorem about deterministic noiseless amplification of Gaussian states similar to that of coherent states.

Theorem 2. Suppose there are two sets of two Gaussian states $A = \{|\psi_1\rangle,|\psi_2\rangle\}, B = \{|\phi_1\rangle,|\phi_2\rangle\}$ with $\vec{d}_{|\phi_1\rangle} = g_1\vec{d}_{|\psi_1\rangle}$, $\vec{d}_{|\phi_2\rangle} = g_2\vec{d}_{|\psi_2\rangle}$ and $\vec{\sigma}_{|\phi_1\rangle} = \vec{\sigma}_{|\psi_1\rangle}$, $\vec{\sigma}_{|\phi_2\rangle} =$

$\overrightarrow{\sigma}_{|\psi_2\rangle}$. The deterministic noiseless quantum amplifier which amplifies the Gaussian state randomly chosen from set A to the corresponding Gaussian state in the set B exists if and only if the input Gaussian states and the gain of amplification satisfy the condition $\cos\gamma \geqslant \sqrt{(g_1^2-1)(g_2^2-1)}/(g_1g_2-1)$ with γ is the relative phase between two Gaussian states in phase space.

For non-Gaussian states, the deterministic quantum amplifier is also possible according to the theory of quantum state transformation. However, since the distance between non-Gaussian states cannot de defined definitely as Gaussian states, the condition for deterministic noiseless quantum amplifier is more complicated.

V. EXAMPLE OF APPLICATION

While the amplitude is increased after amplification, the phase of coherent states is unchanged and we can exploit this property to improve the precision of phase measurement using a balanced homodyne detector. In balanced homodyne detection, the difference of two photodetector measurements is obtained and thus the output signal is determined by the number difference operator $\hat{n}_d = -i(\hat{a}^\dagger\hat{b} - \hat{b}^\dagger\hat{a})$ [27]. The local oscillator as a reference is excited into a large amplitude coherent state $|\beta\rangle$ with fixed phase. When the input is coherent state $|\alpha\rangle$, the mean measured signal is $\langle\hat{n}_d\rangle = 2|\alpha||\beta|\sin\delta$, where δ is the relative phase between coherent states $|\alpha\rangle$ and $|\beta\rangle$. The variance of the output signal is calculated as $\Delta\hat{n}_d = \sqrt{|\alpha|^2+|\beta|^2}$. According to error transfer formula, the sensitivity of measured phase is $\Delta\delta = \Delta\hat{n}_d/(|\partial\langle\hat{n}_d\rangle/\partial\delta|) = \sqrt{1+(|\beta|/|\alpha|)^2}/(2|\beta||\cos\delta|)$. Suppose the input coherent state is randomly chosen from a definite set of coherent states which satisfies the condition of deterministic noiseless amplification; then we can determinately and noiselessly amplify it before detection. In this case, according to our calculations, not only the mean output signal is enhanced but also the precision of measured phase is improved. This kind of measurement is particularly useful for the phase measurement of weak coherent state.

VI. DISCUSSION AND SUMMARY

Besides the application of phase measurement with homodyne detection, we can also conjecture some other possible applications. For instance, in the task of continuous variable quantum key distribution using coherent light pulses [28], the noiseless amplification of coherent light pulses cannot only increase the transmission distance but also may be beneficial to the improvement of signal transmission rate [29–31]. Other applications including loss suppression [32,33], entanglement distillation [34,35], or quantum cloning [36] may also be possible using our protocol of deterministic noiseless amplification.

Though the distinguishability of coherent states does not increase after deterministic noiseless amplification, the situation may be different in a noisy channel. Consider two coherent states $|\alpha_1e^{i\theta_1}\rangle$ and $|\alpha_2e^{i\theta_2}\rangle$ which satisfy Eq. (10) such that they can be deterministically noiselessly amplified to $|g_1\alpha_1e^{i\theta_1}\rangle$ and $|g_2\alpha_2e^{i\theta_2}\rangle$ respectively. When the amplified coherent states are sent through a noisy channel, then the distinguishability of two amplified states through the noisy channel may be larger than the two coherent states through the same noisy channel without amplification. To see this more explicitly, use a superoperator $V(t)$ to describe the noisy channel such that the state evolution of quantum system in noisy channel is $\rho(t) = V(t)[\rho(0)]$. For two coherent states $\rho_1(0) = |\alpha_1e^{i\theta_1}\rangle\langle\alpha_1e^{i\theta_1}|$ and $\rho_2(0) = |\alpha_2e^{i\theta_2}\rangle\langle\alpha_2e^{i\theta_2}|$, the distance of two states in noisy channel is in general decreased monotonously $D(\rho_1(t),\rho_2(t)) \leqslant D(\rho_1(0),\rho_2(0))$. Define the decay rate of distance as $\sigma(\rho_1(t),\rho_2(t)) = \frac{d}{dt}D(\rho_1(t),\rho_2(t))$ [37]. Obviously, $\sigma(\rho_1(t),\rho_2(t)) \leqslant 0$ means the distinguishability of two states decreases with time in noisy channel. Similarly, we can obtain the decay rate of two amplified coherent states in the same noisy channel $\sigma(\rho_1^{g_1}(t),\rho_2^{g_2}(t)) = \frac{d}{dt}D(\rho_1^{g_1}(t),\rho_2^{g_2}(t))$, where $\rho_1^{g_1}(t) = |g_1\alpha_1e^{i\theta_1}\rangle\langle g_1\alpha_1e^{i\theta_1}|$ and $\rho_2^{g_2}(t) = |g_2\alpha_2e^{i\theta_2}\rangle\langle g_2\alpha_2e^{i\theta_2}|$ are amplified states. In general, the decay rate of distance depends on the initial states which means the decay rate may be different for the initial states and the amplified coherent states. It thus possible that the distinguishability of two amplified coherent states through the noisy channel may be larger than the two coherent states through the same noisy channel without amplification. Besides, the detectors we use to distinguish coherent states are not truly ideal in practice. The unavoidable dark noise will cause dark counting in the detector which lowers our precision of distinguishing coherent states. For a nonideal detector, the amplified coherent states may be more distinguishable than the coherent states without amplification. As an exactly solvable noisy model has not been found for deeper investigation, we leave it as an open question whether amplified coherent states can perform better in a noisy environment with the above protocols.

In conclusion, we have shown that the deterministic noiseless amplification of coherent states is physically attainable when we regard the quantum state amplification as quantum state transformation. Our results are based on two facts: the process of deterministic noiseless amplification does not increase the distinguishability of any two amplified states and the gain of amplification can be state dependent. The relation between input coherent states and gain of amplification for deterministic noiseless amplification is thus derived. Furthermore, we extend our results to more general states and give an explicit formalism about deterministic noiseless amplification of Gaussian states. We also proposed the application of phase measurement with balanced homodyne detection. The possible cases with noisy channel and nonideal detection are also discussed. Our results about deterministic noiseless amplification of coherent states not only enrich the research of quantum amplification but also may be helpful in quantum metrology, quantum communication, and quantum information processing.

ACKNOWLEDGMENTS

We thank X. F. Zhou and G. Y. Xiang for helpful discussions. This work is supported by National Natural Science Foundation of China (No. 61275122) and National Fundamental Research Program of China and Strategic Priority Research Program (B) of CAS (No. XDB01030200).

[1] C. M. Caves, Phys. Rev. D **26**, 1817 (1982).
[2] B. Yurke and J. S. Denker, Phys. Rev. A **29**, 1419 (1984).
[3] C. M. Caves, J. Combes, Z. Jiang, and S. Pandey, Phys. Rev. A **86**, 063802 (2012).
[4] A. A. Clerk, M. H. Devoret, S. M. Girvin, F. Marquardt, and R. J. Schoelkopf, Rev. Mod. Phys. **82**, 1155 (2010).
[5] U. Gavish, B. Yurke, and Y. Imry, Phys. Rev. Lett. **93**, 250601 (2004).
[6] W. K. Wootters and W. H. Zurek, Nature (London) **299**, 802 (1982).
[7] H. P. Yuen, Phys. Lett. A **113**, 405 (1986).
[8] G. M. D'Ariano and H. P. Yuen, Phys. Rev. Lett. **76**, 2832 (1996).
[9] H. Barnum, C. M. Caves, C. A. Fuchs, R. Jozsa, and B. Schumacher, Phys. Rev. Lett. **76**, 2818 (1996).
[10] L. M. Duan and G. C. Guo, Phys. Rev. Lett. **80**, 4999 (1998).
[11] A. Chefles, Phys. Lett. A **239**, 339 (1998).
[12] A. Chefles, Phys. Lett. A **270**, 14 (2000).
[13] A. Chefles, Phys. Rev. A **65**, 052314 (2002).
[14] X. F. Zhou, Q. Lin, Y. S. Zhang, and G. C. Guo, Phys. Rev. A **75**, 012321 (2007).
[15] V. Dunjko and E. Andersson, J. Phys. A: Math. Theor. **45**, 365304 (2012).
[16] V. Dunjko and E. Andersson, Phys. Rev. A **86**, 042322 (2012).
[17] G. Y. Xiang, T. C. Ralph, A. P. Lund, N. Walk, and G. J. Pryde, Nat. Photon. **4**, 316 (2010).
[18] A. Zavatta, J. Fiurášek, and M. Bellini, Nat. Photon. **5**, 52 (2011).
[19] S. Kocsis, G. Y. Xiang, T. C. Ralph, and G. J. Pryde, Nat. Phys. **9**, 23 (2013).
[20] K. Hammerer, M. M. Wolf, E. S. Polzik, and J. I. Cirac, Phys. Rev. Lett. **94**, 150503 (2005).
[21] G. Adesso and G. Chiribella, Phys. Rev. Lett. **100**, 170503 (2008).
[22] M. A. Nielsen and I. L. Chuang, *Quantum Computation and Quantum Information* (Cambridge University Press, Cambridge, 2000).
[23] K. Kraus, *States, Effects, and Operations* (Springer-Verlag, Berlin, 1983).
[24] J. Priskill, http://www.theory.caltech.edu/people/preskill/ph229/.
[25] X. B. Wang, T. Hiroshima, A. Tomita, and M. Hayashi, Phys. Rep. **448**, 1 (2007).
[26] G. Adesso, S. Ragy, and A. R. Lee, Open Syst. Inf. Dyn. **21**, 1440001 (2014).
[27] M. O. Scully and M. S. Zubairy, *Quantum Optics* (Cambridge University Press, Cambridge, 1997).
[28] F. Grosshans and P. Grangier, Phys. Rev. Lett. **88**, 057902 (2002).
[29] R. Blandino, A. Leverrier, M. Barbieri, J. Etesse, P. Grangier, and R. Tualle-Brouri, Phys. Rev. A **86**, 012327 (2012).
[30] N. Walk, T. C. Ralph, T. Symul, and P. K. Lam, Phys. Rev. A **87**, 020303(R) (2013).
[31] J. Fiurášek and N. J. Cerf, Phys. Rev. A **86**, 060302(R) (2012).
[32] M. Mičuda, I. Straka, M. Miková, M. Dušek, N. J. Cerf, J. Fiurášek, and M. Ježek, Phys. Rev. Lett. **109**, 180503 (2012).
[33] G. S. Meng, S. Yang, X. B. Zou, S. L. Zhang, B. S. Shi, and G. C. Guo, Phys. Rev. A **86**, 042305 (2012).
[34] T. C. Ralph and A. P. Lund, *Quantum Communication Measurement and Computing Proceedings of the Ninth International Conference*, edited by A. Lvovsky (AIP, New York, 2009), pp. 155–160.
[35] S. Yang, S. L. Zhang, X. B. Zou, S. W. Bi, and X. L. Lin, Phys. Rev. A **86**, 062321 (2012).
[36] C. R. Müller, C. Wittmann, P. Marek, R. Filip, C. Marquardt, G. Leuchs, and U. L. Andersen, Phys. Rev. A **86**, 010305(R) (2012).
[37] H. P. Breuer, E. M. Laine, and J. Piilo, Phys. Rev. Lett. **103**, 210401 (2009).