Entanglement generation by dissipation in or beyond dark resonances

Xiangming Hu*
Department of Physics, Central China Normal University, Wuhan 430079, China
(Received 28 April 2015; published 12 August 2015)

For dark resonance, one of the most remarkable coherent effects in light-matter interactions, it has commonly been expected that squeezing and entanglement, if existent, are formed via coherent evolutions against dissipation. Contrary to the expectations, here we show that dissipation generates entanglement between two cavity fields and between two dark-state-based spins. The latter correspond also to the atomic ground-state spin squeezing in a limited parameter domain. The dissipation effects, which are hidden deeply behind the coherence-induced nonlinearities, are extracted by probing into the dressed atom-photon interactions, and are widely applicable for the coherently prepared systems in dark resonances or beyond.

DOI: 10.1103/PhysRevA.92.022329 PACS number(s): 03.67.Bg, 42.50.Dv, 32.80.Qk, 42.50.Pq

I. INTRODUCTION

Two optical fields of equal intensities interact resonantly with three-level Λ-type atoms and pump the atoms into a superposition of the ground states, which is a state with maximal coherence and is called "dark state" [1–5]. This coherent phenomenon is usually referred to as "dark resonance." The dark resonance does not mean absence of interactions but existence of resonant transitions back and forth between the two ground states. Once the atoms enter the dark state, they are no longer excited and so become transparent to the applied fields. It is the dark resonance that underlies those well-known coherent effects such as coherent population trapping (CPT) [1] and electromagnetically induced transparency [2–5]. One of the greatest interests is in giant nonlinearities close to the dark resonance. It has been expected that the coherence-induced nonlinearities are used for the creation of squeezed and entangled states [6–10]. It could have been deduced that the coherent evolutions against dissipation are responsible for the squeezed and entangled states, if existent. As a rule, however, coherent evolutions are obtainable only when applied fields are tuned far off resonance with the atoms [11–13]. Otherwise, the desired states are easily destroyed since they are extremely fragile to environmental dissipation or decoherence. Now, a gap still exists between the dark resonance and the squeezing and entanglement.

In this article, we show that dissipation generates squeezing and entanglement in dark resonances or beyond. The dissipation occurs for the Bogoliubov modes of the two cavity fields, and also for the Bogoliubov modes of the dark-state-based spins. Due to the dissipation, squeezing and entanglement are obtainable both for two optical fields and for two dark-state-based spins. The dissipation mechanisms are essentially different from the coherent evolution mechanisms. Squeezing and entanglement by dissipation do not require the preparation of a system in a particular input state. In principle, the squeezing and entanglement last for an arbitrary long time, which is expected to play an important role in quantum information processing. Due to these features, the present dark-resonance system is inherently stable against weak random perturbations, with the dissipative dynamics making the squeezing and entanglement stable. The dissipative effects of the atoms and the fields on each other in the near-resonant or highly excited systems are deeply hidden behind the coherence-induced nonlinearities, which are in divergent series of indefinitely high orders if we expand the dynamical equation in terms of the laser intensities. Our strategy is to merge the high-order nonlinearities into the dressed atomic states and then to reveal the dissipation effects.

The dissipation effects were recently explored to prepare the squeezed and entangled states in different schemes. Pielawa *et al.* [14] showed that Rabi interactions with two beams of two-level atoms can act as dissipative processes and make two optical fields into their squeezed and entangled states. Parkins *et al.* [15] and Torre *et al.* [16] showed that Raman interactions mediated by two vacuum cavity fields and classical fields can used as a dissipative reservoir and drive two atomic ensembles into their squeezed and entangled states. Krauter *et al.* [17] reported on the first experimental demonstration of purely dissipative generation of spin entanglement. Different from the above work, the purpose of this article is to show the dissipation mechanism that hides behind coherent effects. It has been aware that coherence determines the trapping of the atoms in the superposition of the ground or metastable states, the transparency to the optical fields, and the high-order nonlinearities. However, even for the dark-resonance system, it is not necessarily the coherent evolution but it can be the intrinsic dissipation that leads to squeezing and entanglement of the optical fields and/or of the atomic spins. The dissipation hides deeply behind the coherence-induced nonlinearities. In particular, this was demonstrated by the experiment of Sautenkov *et al.* [6], who found that the switching between the photon-photon correlation and anticorrelation at specific two-photon detuning in the Λ system. This switching behavior has not been explained yet in a definite mechanism. We show that the dissipation behind coherence provides a principal explanation of it. The dissipation mechanism can be separated out from the dressed atom-photon interactions. Such mechanisms are most widely existent in coherently prepared systems and practicably applicable for quantum squeezing and entanglement.

The remaining part of this article is organized as follows. In Sec. II, we give the master equation of the atom-field interacting system and present the coherence-induced nonlinearities close to dark resonance. In Sec. III, we merge the

*Corresponding author: xmhu@phy.ccnu.edu.cn

nonlinearities into the dressed atoms and present the dressed atom-photon interactions. In Sec. IV, we show the dissipation mechanisms for entanglement between the two cavity fields and between the two dark-state-based spins. The first two subsections present physical analyses for the Bogoliubov mode dissipations for the cavity fields and for the dark-state-based spins, respectively, under respective adiabatic conditions, the third subsection presents a numerical verification and gives the quantum correlations for general nonadiabatic case, and the fourth subsection shows the ground-state spin squeezing. Finally, discussion and conclusion are given in Sec. V.

II. MASTER EQUATION AND COHERENCE-INDUCED NONLINEARITIES CLOSE TO DARK RESONANCE

An ensemble of N atoms is placed at the intersection of two optical cavities, which are pumped by two external fields, as shown in Fig. 1(a). The two cavity fields are respectively coupled to the two electronic dipole-allowed transitions of the three-level atoms from the ground or metastable states $|1,2\rangle$ to the excited state $|3\rangle$ [Fig. 1(b)]. The master equation for the density operator ρ of the atom-field system is written in the dipole approximation and in an appropriate rotating frame as [18]

$$\dot{\rho} = -\frac{i}{\hbar}[H,\rho] + \mathcal{L}\rho, \tag{1}$$

where the system Hamiltonian reads as

$$H = \sum_{l=1,2} \hbar[-\Delta_l \sigma_{ll} + \Delta_{c_l} a_l^\dagger a_l + g_l(a_l \sigma_{3l} + \sigma_{l3} a_l^\dagger) + i(\varepsilon_l a_l^\dagger - \varepsilon_l^* a_l)], \tag{2}$$

where $\sigma_{kl} = \sum_{\mu=1}^{N} \sigma_{kl}^{\mu}$ ($\sigma_{kl}^{\mu} = |k_\mu\rangle\langle l_\mu|; k,l = 1,2,3$) are the collective projection operators for $k = l$ and the collective spin-flip operators for $k \neq l$, a_l and $a_l^\dagger$ $(l = 1,2)$ are annihilation and creation operators for the cavity fields, $\Delta_l = \omega_{3l} - \omega_l$ and $\Delta_{c_l} = \omega_{c_l} - \omega_l$ are, respectively, the detunings of atomic transition frequencies ω_{3l} and cavity resonance frequencies ω_{c_l} with respect to the driving field frequencies ω_l, g_l are the strengths for the atom-cavity field couplings, and ε_l are the classical driving field amplitudes. The damping term in the master equation takes the form

$$\mathcal{L}\rho = \sum_{l=1}^{2} \left(\kappa_l \mathcal{L}_{a_l}\rho + \frac{\gamma_l}{2} \sum_{\mu=1}^{N} \mathcal{L}_{\sigma_{l3}^{\mu}}\rho \right), \tag{3}$$

where $\mathcal{L}_{a_l}\rho$ and $\mathcal{L}_{\sigma_{l3}^{\mu}}\rho$ describe the cavity and atomic decays with rates $2\kappa_l$ and γ_l, respectively, and $\mathcal{L}_O\rho = 2O\rho O^\dagger - O^\dagger O\rho - \rho O^\dagger O$, $O = a_l, \sigma_{l3}$.

It is well known that the nonlinearities that the fields and the atoms experience depend remarkably on the atom-field detunings $\Delta_{1,2}$ and the pump-cavity detunings $\Delta_{c_{1,2}}$. When we choose the same atom-field detunings $\Delta_1 = \Delta_2$, CPT occurs and the atoms decouple from the fields. This does not mean absence of the atom-field interactions. Instead, resonant transitions exist between the two ground states and exert their effects on the quantum correlations at the sidebands [19]. For arbitrary detunings, the nonlinearities are generally so complicated that they conceal the interaction mechanisms for the quantum correlations. Fortunately, there is a symmetric case for the atom-field detunings $\Delta_2 = -\Delta_1 = \Delta$, in which we can derive explicit expressions for the dressed atomic states. Particularly, these dressed sublevels are equally spaced and most suitable for studying the interaction mechanisms for the quantum correlations. On the other hand, the dressed interactions are most effective when the cavity fields are resonant with the corresponding dressed transitions. This is guaranteed by choosing the symmetric pump-cavity detunings $\Delta_{c_1} = -\Delta_{c_2} = \Delta_c$.

For the above reasons, we focus on the choice of the above symmetrical detunings. At the same time, for the sake of clarity we also assume the other parameters to be equal: $g_{1,2} = g$, $\gamma_{1,2} = \gamma$, and $\kappa_{1,2} = \kappa$. Then, the high-order nonlinearities can be seen from the field equations after adiabatic elimination of the atomic variables. The input intensity $I_{\rm in} = \frac{|g\varepsilon_l|^2}{\kappa^2}$ $(l = 1,2)$ and the cavity field intensity $I = \frac{|g\langle a_l\rangle|^2}{\gamma^2}$ satisfy the equation

$$I_{\rm in} = I[(1 + A)^2 + (\bar{\Delta}_c + D)^2], \tag{4}$$

where A and D represent, respectively, the absorption and the dispersion due to the atoms and depend strongly on the cavity field intensity I through the relations

$$A = \frac{C\bar{\Delta}^2}{\bar{\Delta}^4 + \bar{\Delta}^2 + \bar{\Delta}^2 I + I^2}, \qquad D = \frac{C\bar{\Delta}(\bar{\Delta}^2 - I)}{\bar{\Delta}^4 + \bar{\Delta}^2 + \bar{\Delta}^2 I + I^2}. \tag{5}$$

Here, we have defined $C = \frac{g^2 N}{2\kappa\gamma}$, $\bar{\Delta} = \frac{\Delta}{\gamma}$, and $\bar{\Delta}_c = \frac{\Delta_c}{\kappa}$. Once $\Delta = 0$, we have $A = D = 0$, which means that the atoms are transparent to the fields. For $\Delta \neq 0$, expansion of Eqs. (4) and (5) gives a divergent series of infinitely high orders when the intensity is of considerable value. Bistability or multistability is obtainable due to the nonlinearities. The stability is determined by the negative eigenvalues of the drift matrix of the linearized atom-field system. It is invalid to treat the nonlinearities in a perturbative way. Close to the dark resonance ($|\Delta| \ll |g\langle a_l\rangle|$),

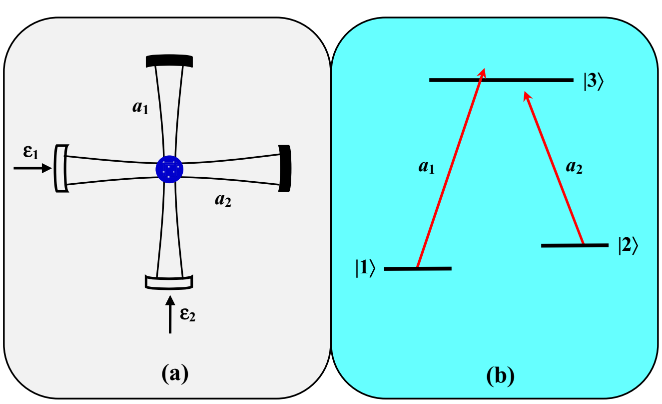

FIG. 1. (Color online) The atom-field interacting system. (a) Two cavity fields $a_{1,2}$, pumped by two external fields $\varepsilon_{1,2}$, are coupled to an ensemble of atoms at the intersection of the two cavities. (b) The atomic transitions in Λ configuration.

the nonlinearity to absorption ratio is much larger than unity

$$\frac{|D|}{A} \approx \frac{I}{|\bar{\Delta}|} \gg 1. \tag{6}$$

It has usually been expected that the coherence-induced nonlinearities lead to the nonclassical correlations. Dantan *et al.* [7] predicted the existence of the ground-state spin squeezing and the light entanglement in the CPT system. They interpreted the mechanism as the Faraday rotations [20,21] plus the cavity feedback. In principle, this interpretation means the coherent evolutions against dissipation. In contrast, however, we will show that Bogoliubov mode dissipation, which hides behind the nonlinearities or the nonlinear Faraday effect, is a direct mechanism to generate the squeezing and entanglement of the cavity fields and the atomic spins. For this purpose, we merge the nonlinearities into the dressed atomic states and analyze the dressed atom-photon interactions as follows.

III. DRESSED ATOM-PHOTON INTERACTIONS

It is convenient to make a unitary transformation [22] via $U_1 = \Pi_{l=1,2} \exp(\lambda_l a_l^\dagger - \lambda_l^* a_l)$ with $\lambda_l = \frac{\varepsilon_l}{\kappa + i\Delta_{c_l}}$, and to linearize the cavity fields $a_l = \langle a_l \rangle + \delta a_l$ $(l = 1,2)$. After doing so, we decompose the Hamiltonian into three parts:

$$H = H_a + H_c + H_I, \tag{7}$$

where the first part

$$H_a = \hbar\Delta(\sigma_{22} - \sigma_{11}) + \sum_{l=1,2} \hbar(\Omega_l \sigma_{3l} + \Omega_l^* \sigma_{l3}) \tag{8}$$

represents the interaction of the atoms with the classical pump fields ε_l and the semiclassical part of the cavity fields $\langle a_l \rangle$ (with total Rabi frequency $\Omega_l = \lambda_l + g\langle a_l \rangle$), the second part

$$H_c = \hbar\Delta_c(\delta a_1^\dagger \delta a_1 - \delta a_2^\dagger \delta a_2) \tag{9}$$

denotes the free part for the fluctuating parts of the cavity fields δa_l, and the last part H_I describes the interaction of the atoms with the field fluctuations δa_l and will be given later. By transferring the phase of Ω_l to the atomic operators, we have real values for $\Omega_l = |\Omega_l|$. We assume that the Rabi frequencies are equal and much stronger than the atomic and cavity decay rates $\Omega_l = \Omega \gg (\gamma,\kappa)$. After diagonalization of H_a we obtain the dressed states that are expressed in terms of the bare atomic states as [23]

$$\begin{aligned}
|+\rangle &= \frac{1+\sin\theta}{2}|1\rangle + \frac{1-\sin\theta}{2}|2\rangle + \frac{\cos\theta}{\sqrt{2}}|3\rangle, \\
|0\rangle &= -\frac{\cos\theta}{\sqrt{2}}|1\rangle + \frac{\cos\theta}{\sqrt{2}}|2\rangle + \sin\theta|3\rangle, \\
|-\rangle &= \frac{1-\sin\theta}{2}|1\rangle + \frac{1+\sin\theta}{2}|2\rangle - \frac{\cos\theta}{\sqrt{2}}|3\rangle,
\end{aligned} \tag{10}$$

where we have defined $\cos\theta = \frac{\sqrt{2}\Omega}{\bar{\Omega}}$, $\sin\theta = \frac{\Delta}{\bar{\Omega}}$, and $\bar{\Omega} = \sqrt{\Delta^2 + 2\Omega^2}$. These dressed states $|0\rangle$ and $|\pm\rangle$ have their eigenvalues $E_{0,\pm} = 0, \pm\hbar\bar{\Omega}$, which are equally spaced. The free Hamiltonian H_a for the dressed atoms now becomes

$$H_a = \hbar\bar{\Omega}(\sigma_{++} - \sigma_{--}). \tag{11}$$

Transforming the relaxation terms of the atoms to the dressed states representation, we obtain the steady-state populations $N_l = \langle \sigma_{ll} \rangle$ $(l = 0,\pm)$ as

$$\begin{aligned}
N_0 &= \frac{N\cos^4\theta}{1 + 3\sin^4\theta}, \\
N_+ &= N_- = \frac{1}{2}(N - N_0).
\end{aligned} \tag{12}$$

It should be noted that when $\Delta = 0$ $(\sin\theta = 0, \cos\theta = 1)$, the atoms are trapped in the dark state $|0\rangle = \frac{1}{\sqrt{2}}(-|1\rangle + |2\rangle)$, i.e., $N_0 = N$ and $N_\pm = 0$. For the CPT case, one has the maximal coherence $\langle \sigma_{12} \rangle = -\frac{N}{2}$ but no entanglement.

Here, we focus on the case of nonvanishing detunings $\Delta \neq 0$. In terms of the dressed atomic states, H_a and H_c constitute the total free Hamiltonian for the dressed atom-field system

$$\begin{aligned}
H_0 &= H_a + H_c \\
&= \hbar\bar{\Omega}(\sigma_{++} - \sigma_{--}) + \hbar\Delta_c(a_1^\dagger a_1 - a_2^\dagger a_2),
\end{aligned} \tag{13}$$

where we have dropped the symbol δ and do so from now on (i.e., by a_l we mean δa_l). We tune the cavity fields resonant with the Rabi sidebands $\Delta_c = \bar{\Omega}$. The case of $\Delta_c = -\bar{\Omega}$ is treated in the same way. The dressed states are well separated from each other since $\bar{\Omega} \gg (\gamma_{1,2},\kappa_{1,2})$. We can make a further unitary transformation with $U_2 = \exp(-iH_0 t/\hbar)$ and a rotating-wave approximation. After doing so, we obtain the interaction Hamiltonian

$$\begin{aligned}
H_I = {} & \tfrac{1}{2}\hbar g[-a_1\cos^2\theta + a_2^\dagger \sin\theta(1 - \sin\theta)]\sigma_{+0} \\
& + \tfrac{1}{2}\hbar g[-a_2\cos^2\theta + a_1^\dagger \sin\theta(1 - \sin\theta)]\sigma_{-0} \\
& + \text{H.c.},
\end{aligned} \tag{14}$$

where we have used H.c. for Hermitian conjugate of the terms before it. It is easy to see from the Hamiltonian that each field is in resonant interaction with two cascaded dressed transitions, as shown in Fig. 2(a). The entire system is now looked upon as four interacting parts, two of which are the cavity fields $a_{1,2}$ and the other two are the dark-state-based spins $\sigma_{0\pm}$. It is necessary to describe the atomic modes before we analyze the effects of the interactions. They have their commutation relations $[\sigma_{0\pm},\sigma_{\pm 0}] = \sigma_{00} - \sigma_{\pm\pm}$. We keep remembering that $a_{1,2}$ represent the fluctuations of the cavity fields. We are looking for the conditions under which the fluctuations of the collective modes of $a_{1,2}$ are absorbed [24]. Such conditions are existent in some regions, as will be shown in the following. In this case, we have vanishing spin modes at steady state $\langle \sigma_{0\pm} \rangle = 0$, and then $\delta\sigma_{0\pm} = \sigma_{0\pm}$. Thus, by $\sigma_{0\pm}$ we denote the fluctuations of the dark-state-based spins. The z quadratures of the two spin modes have the mean values $N_0 - N_\pm$. Using the Hamiltonian (14), we can analyze the collective dissipations of the cavity fields and the dark-state-based spins.

The dressed atom-photon interactions described in Hamiltonian (14) are both in the parametric amplifier types and in the beam-splitter types. These interactions are alternately cascaded in a closed quadrilateral contour, as shown in Fig. 2(b). It is known that the parametric amplifier interactions

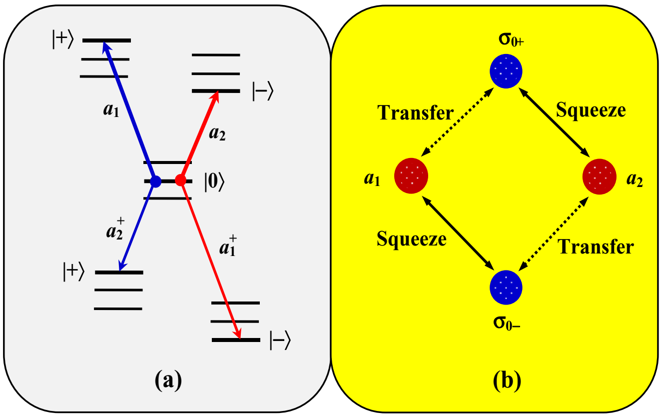

FIG. 2. (Color online) Dressed atom-photon interactions. (a) Resonant transitions between adjacent triplets of the dressed atoms. Each field is resonant with two different dressed transitions. (b) An alternately cascaded quadrilateral loop of beam-splitter interactions (for state transfer) and parameter amplifier interactions (for squeezed states) of the cavity fields $a_{1,2}$ with the dark-state-based spins $\sigma_{0\pm}$.

are responsible for squeezing and entanglement while the beam-splitter interactions lead to the quantum state transfer. Two limiting cases clearly show that the dissipative interactions are responsible for the light entanglement and the spin entanglement. When $\gamma \gg \kappa$, the adiabatically evolving atomic spin σ_{0-} (σ_{0+}) entangles itself with a_1 (a_2), and transfers immediately its state to a_2 (a_1). As a consequence, the cavity modes $a_{1,2}$ are prepared in the two-mode squeezed and entangled state. In the same way, when the conditions are changed to $\kappa \gg \gamma$, the adiabatically evolving cavity fields drive dark-state-based spins into the two-mode squeezed and entangled states. The above mechanisms are more clearly described as follows by using the Bogoliubov modes [14,15].

IV. ENTANGLEMENT GENERATION BY DISSIPATION

Here, we first present a physical analysis of the effects of dressed atom-photon interactions on quantum correlations, and then give the numerical results.

A. Bogoliubov field dissipation by atoms under adiabatic condition ($\gamma \gg \kappa$)

If the dark-state-based spins undergo adiabatic evolutions ($\gamma \gg \kappa$), that will lead to dissipation common for the two cavity fields. To describe the collective dissipation, we introduce the Bogoliubov modes for the cavity fields [25]

$$\begin{aligned} b_1 &= a_1 \cosh r + a_2^\dagger \sinh r, \\ b_2 &= a_2 \cosh r + a_1^\dagger \sinh r, \end{aligned} \tag{15}$$

where we have defined the squeezing parameter $r = \arctan\left(\frac{-\sin\theta}{1+\sin\theta}\right)$ for $\frac{\Delta}{\Omega} > -\sqrt{\frac{2}{3}}$ and $r = \arctan\left(\frac{1+\sin\theta}{-\sin\theta}\right)$ for $\frac{\Delta}{\Omega} < -\sqrt{\frac{2}{3}}$. The interaction Hamiltonian (14) is rewritten as

$$H_I = \sum_{l=1,2} \hbar g_b (b_l \sigma_l^+ + \sigma_l b_l^\dagger), \tag{16}$$

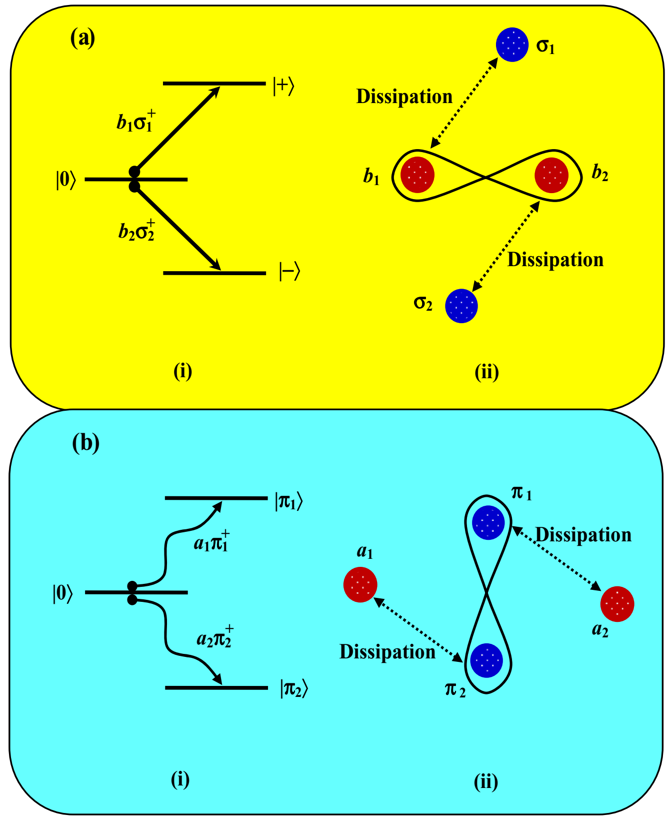

FIG. 3. (Color online) Diagrammatic sketch for the entanglement generation by dissipation. (a) The upper half part shows (i) the dressed transitions for absorption of the Bogoliubov fields $b_{1,2}$, and (ii) the dissipations of Bogoliubov field modes $b_{1,2}$ by the dark-state-based spin $\sigma_{1,2}$, respectively. (b) The lower half part represents (i) the interactions of the Bogoliubov spins $\pi_{1,2}$ with the individual fields $a_{1,2}$ and (ii) the dissipations of the Bogoliubov spin modes $\pi_{1,2}$ by the cavity fields $a_{1,2}$, respectively.

where we have defined the dark-state-based spins $\sigma_{1,2} = \sigma_{0\pm}$ and the corresponding coupling strength $g_b = \frac{1}{2} g(-1+\sin\theta)\sqrt{1+2\sin\theta}$ for $\frac{\Delta}{\Omega} > -\sqrt{\frac{2}{3}}$, and $\sigma_{1,2} = \sigma_{\pm 0}$ and $g_b = \frac{1}{2} g(1-\sin\theta)\sqrt{-1-2\sin\theta}$ for $\frac{\Delta}{\Omega} < -\sqrt{\frac{2}{3}}$.

Pictorially, we show in Fig. 3(a-i) the interactions of the Bogoliubov fields $b_{1,2}$ with the dressed atoms, and in Fig. 3(a-ii) the dissipations of the Bogoliubov fields $b_{1,2}$ by the spins $\sigma_{1,2}$. The dressed atom-field interactions are strongly dependent on the atom-field normalized detuning $\frac{\Delta}{\Omega}$. When $\frac{\Delta}{\Omega} > -\sqrt{\frac{2}{3}}$, annihilation (creation) of the new modes $b_{1,2}$ is accompanied with the dressed transitions from $|0\rangle$ to $|\pm\rangle$ (from $|\pm\rangle$ to $|0\rangle$), respectively. On the contrary, when $\frac{\Delta}{\Omega} < -\sqrt{\frac{2}{3}}$, annihilation (creation) of the new modes $b_{1,2}$ is caused by the dressed transitions from $|\pm\rangle$ to $|0\rangle$ (from $|0\rangle$ to $|\pm\rangle$), respectively. Whether the field fluctuations are suppressed or amplified depends on the dressed state population differences $N_0 - N_\pm$. The populations follow the relations $N_+ = N_-$, $N_0 > N_\pm$ for $|\frac{\Delta}{\Omega}| < 1$, and $N_0 < N_\pm$ for $|\frac{\Delta}{\Omega}| > 1$. It is seen from the above that the absorption of the $b_{1,2}$ modes is dominant over the amplification in the regions of

$\frac{\Delta}{\Omega} = (-\sqrt{\frac{2}{3}},0), (0,1), (-\infty,-1)$, while the amplification dominates over the absorption in the ranges of $\frac{\Delta}{\Omega} = (-1,-\sqrt{\frac{2}{3}})$, $(1,\infty)$. The squeezing and entanglement of the fields are possible only when the absorption (dissipation) of the Bogoliubov modes dominates. The domains for squeezing and entanglement are confined to $\frac{\Delta}{\Omega} = (-\sqrt{\frac{2}{3}},0), (0,1), (-\infty,-1)$. As the dark-state-based spins $\sigma_{1,2}$ evolve adiabatically ($\Gamma \gg \kappa$) into the thermal vacuum states, the Bogoliubov modes $b_{1,2}$ will dissipate into the thermal vacuum states. This corresponds to the squeezing and entanglement of the original cavity fields $a_{1,2}$. On the other hand, once we tune to the other regions $\frac{\Delta}{\Omega} = (-1,-\sqrt{\frac{2}{3}}), (1,\infty)$, the fluctuations are amplified and the squeezing and entanglement do not occur. Even in the presence of the large nonlinearities in these regions, the squeezing and entanglement are possible only when the dissipation is dominant over amplification. This perhaps explains why the photon-photon correlation and anticorrelation switching occurs at a specific two-photon detuning, as demonstrated in a recent experiment [6].

B. Bogoliubov spin dissipation by the fields under adiabatic condition ($\kappa \gg \gamma$)

Similarly, if the cavity fields evolve adiabatically ($\kappa \gg \gamma$), they will induce dissipation common for the two dark-state-based spins. To show this clearly, we use the Bogoliubov modes for the spins

$$\begin{aligned} \pi_1 &= \sigma_1 \cosh r + \sigma_2^+ \sinh r, \\ \pi_2 &= \sigma_2 \cosh r + \sigma_1^+ \sinh r, \end{aligned} \tag{17}$$

and rewrite the interaction Hamiltonian (14) in the form

$$H_I = \sum_{l=1,2} \hbar g_b (a_l \pi_l^+ + \pi_l a_l^\dagger). \tag{18}$$

Figure 3(b-i) shows the interactions of the Bogoliubov spins $\pi_{1,2}$ with the individual fields $a_{1,2}$, and Fig. 3(b-ii) pictorially describes the dissipations of the Bogoliubov spins $\pi_{1,2}$ by the individual fields $a_{1,2}$. Here, by $|\pi_{1,2}\rangle$ we have denoted the excited states for the Bogoliubov spins $\pi_{1,2}$. For the same ranges of the atom-field detunings as above, $\frac{\Delta}{\Omega} = (-\sqrt{\frac{2}{3}},0),(0,1),(-\infty,-1)$, the absorption (amplification) of the modes $a_{1,2}$ is accompanied with the excitation (deexcitation) of the new atomic modes $\pi_{1,2}$, respectively. As the cavity fields $a_{1,2}$ evolve adiabatically ($\kappa \gg \gamma$) into the vacuum state, the Bogoliubov spin modes $\pi_{1,2}$ will experience dissipations and arrive at the thermal vacuum state. This corresponds to the squeezing and entanglement of the individual spin modes $\sigma_{1,2}$ [26]. On the other hand, the spin squeezing and entanglement are not existent for $\frac{\Delta}{\Omega} = (-1,-\sqrt{\frac{2}{3}})$, $(1,\infty)$ because of the amplification of the fluctuations. It is clear that the Bogoliubov mode dissipation determines the squeezing and entanglement of the dark-state-based spins even for the large nonlinearities.

C. Field and spin dissipation by each other for nonadiabatic case

So far, we have presented a physical analysis of the Bogoliubov mode dissipations for the cavity fields $a_{1,2}$ and for the dark-state-based spins $\sigma_{1,2}$ under respective adiabatic conditions. In what follows, we present a numerical verification for a general case. We exemplify the case for $\frac{\Delta}{\Omega} > -\sqrt{\frac{2}{3}}$, and the case for $\frac{\Delta}{\Omega} < -\sqrt{\frac{2}{3}}$ is treated in a similar way. Following the standard technique [27], working in the dressed states representation, and defining the c-number and operator correspondences $\alpha_{1,2} \leftrightarrow a_{1,2}$ and $\beta_{1,2} \leftrightarrow \frac{i\sigma_{0\pm}}{\sqrt{N_0 - N_\pm}}$ we derive the Langevin equations as follows:

$$\begin{aligned} \dot{\alpha}_1 &= -\kappa\alpha_1 + \bar{g}_b\beta_1 \cosh r - \bar{g}_b\beta_2^\dagger \sinh r + F_{\alpha_1}, \\ \dot{\alpha}_2 &= -\kappa\alpha_2 + \bar{g}_b\beta_2 \cosh r - \bar{g}_b\beta_1^\dagger \sinh r + F_{\alpha_2}, \\ \dot{\beta}_1 &= -\Gamma\beta_1 - \gamma_c\beta_2^\dagger - \bar{g}_b\alpha_1 \cosh r - \bar{g}_b\alpha_2^\dagger \sinh r + F_{\beta_1}, \\ \dot{\beta}_2 &= -\Gamma\beta_2 - \gamma_c\beta_1^\dagger - \bar{g}_b\alpha_2 \cosh r - \bar{g}_b\alpha_1^\dagger \sinh r + F_{\beta_2}, \end{aligned} \tag{19}$$

where we have defined $\bar{g}_b = g_b\sqrt{N_0 - N_+}$, $\Gamma = \gamma(1 - \frac{1}{2}\cos^4\theta)$, and $\gamma_c = \frac{\gamma}{8}\sin^2(2\theta)$. The F's are noise terms with zero means and correlations $\langle F_O(t)F_{O'}(t')\rangle = D_{OO'}\delta(t-t')$, where the nonzero diffusion coefficients are $2D_{\beta_l^\dagger\beta_l} = 2\Gamma\Pi$ ($\Pi = \frac{N_+}{N_0 - N_+}$) and $D_{\alpha_k\beta_l} = D_{\alpha_k^\dagger\beta_l^\dagger} = -\bar{g}_b \sinh r$ $(k,l = 1,2; k \neq l)$. It is seen clearly from Eq. (19) that the $\cosh r$ terms describe the beam-splitter interactions, while the $\sinh r$ terms correspond to the parametric amplifier interactions. In fact, the nonvanishing diffusion coefficients $D_{\alpha_k\beta_l}$ and $D_{\alpha_k^\dagger\beta_l^\dagger}$ are the very consequence of the parametric amplifier interactions. The γ_c terms are due to the coherence transfer between the degenerate transitions of the dressed atoms. The stability analysis shows that the coupled system is always stable, and the solutions at steady state are $\langle\alpha_{1,2}\rangle = \langle\beta_{1,2}\rangle = 0$.

In order to describe the quantum correlations of the cavity fields and the dark-state-based spins, we write the field and atomic fluctuation variables $\delta\alpha_{1,2} = \alpha_{1,2}$ and $\delta\beta_{1,2} = \beta_{1,2}$ in terms of the position and moment quadratures as $\delta\alpha_l = \frac{1}{\sqrt{2}}(\delta x_{\alpha_l} + i\delta p_{\alpha_l})$ and $\delta\beta_l = \frac{1}{\sqrt{2}}(\delta x_{\beta_l} + i\delta p_{\beta_l})$. For the collective quadratures $X_{O_\pm} = x_{O_1} \pm x_{O_2}$ and $P_{O_\pm} = p_{O_1} \pm p_{O_2}$ $(O = \alpha_{1,2},\beta_{1,2})$, squeezing occurs when the variance of any quadrature is less than unity [18,25,28]:

$$\langle(\delta Z)^2\rangle < 1, \tag{20}$$

where $Z = X_{\alpha_\pm}, P_{\alpha_\pm}, X_{\beta_\pm}, P_{\beta_\pm}$. The cavity fields are entangled with each other if [29]

$$\langle(\delta X_{\alpha_\pm})^2\rangle + \langle(\delta P_{\alpha_\mp})^2\rangle < 2, \tag{21}$$

and the dark-state-based spins are entangled with each other if [30]

$$\langle(\delta X_{\beta_\pm})^2\rangle + \langle(\delta P_{\beta_\mp})^2\rangle < 2. \tag{22}$$

In Eqs. (21) and (22), the X variance with plus (minus) sign subscript matches the P variance with minus (plus) sign subscript. Which inequality is met depends on the parameter regime. It should be noted that although the squeezing or entanglement conditions for the fields have the same forms as for the dark-state-based spins, the essential difference exists between them. This difference lies in the presence of the expectation values $N_0 - N_\pm$ of the dark-state-based spins in the redefined spin variables $\beta_{1,2}$. The origin is the difference of the commutation relations of the spins from those of the bosonic fields [28,30] .

The steady-state variances are calculated from Eq. (19) for $\Delta < 0$:

$$\begin{aligned}\langle(\delta X_{\alpha_+})^2\rangle &= \langle(\delta P_{\alpha_-})^2\rangle \\ &= 1 - \frac{\Gamma_1(1-e^{-2r}) - 2\Gamma\Pi e^{-2r}}{(\kappa+\Gamma_1)\left(1+C_1^{-1}\right)}\end{aligned} \tag{23}$$

and

$$\begin{aligned}\langle(\delta X_{\beta_-})^2\rangle &= \langle(\delta P_{\beta_+})^2\rangle \\ &= 1 - \frac{\kappa(1-e^{-2r}) - \Gamma\Pi\left(1+\frac{\kappa+\Gamma_2}{C_2\Gamma_2}\right)}{(\kappa+\Gamma_2)\left(1+C_2^{-1}\right)},\end{aligned} \tag{24}$$

where we have defined the cooperativity parameters $C_{1,2} = g_b^2(N_0 - N_+)/\kappa\Gamma_{1,2}$ and two different decay rates $\Gamma_{1,2} = \Gamma \pm \gamma_c$. The unity next to the equality signs represents the standard quantum limit, the $(1-e^{-2r})$ terms are due to the dressed atom-photon interactions, and the Π terms come from the atomic spontaneous emission. For $\Delta > 0$, we substitute $\langle(\delta X_{\alpha_-})^2\rangle = \langle(\delta P_{\alpha_+})^2\rangle$ for the first equality in Eq. (23), and $\langle(\delta X_{\beta_+})^2\rangle = \langle(\delta P_{\beta_-})^2\rangle$ for the first equality in Eq. (24). Since we have equal variances $\langle(\delta X_{\alpha_\pm})^2\rangle = \langle(\delta P_{\alpha_\mp})^2\rangle$ and $\langle(\delta X_{\beta_\pm})^2\rangle = \langle(\delta P_{\beta_\mp})^2\rangle$, the conditions for the squeezing are the same as for the entanglement. This is due to the symmetry between the two dressed interaction pathways $|0\rangle \xleftrightarrow{b_1} |+\rangle$ and $|0\rangle \xleftrightarrow{b_2} |-\rangle$. That is not necessarily the case for the other atom-field interacting systems. Generally, X_{α_+} and P_{α_-} (X_{α_-} and P_{α_+}) are not necessarily squeezed simultaneously, and nor are X_{β_-} and P_{β_+} (X_{β_+} and P_{β_-}). Once either quantity of any pair has enough variance above the standard quantum limit 1 to counteract the squeezing effect of the other, entanglement is no longer existent even though the squeezing is still existent.

Plotted in Figs. 4 and 5, respectively, are the variances $\langle(\delta X_{\alpha_\pm})^2\rangle$ ($= \langle(\delta P_{\alpha_\mp})^2\rangle$) and $\langle(\delta X_{\beta_\pm})^2\rangle$ ($= \langle(\delta P_{\beta_\mp})^2\rangle$) below the standard quantum limit for different decay rates. Our numerical results are presented as follows.

(1) Almost ideal two-mode squeezing is achievable for the cavity fields or for the dark-state-based spins under respective adiabatic conditions. From Figs. 4 and 5, we see that the variances for the cavity fields at $\frac{\Delta}{\Omega} = -\sqrt{\frac{2}{3}} \approx -0.82$ tend to vanish,

$$\langle(\delta X_{\alpha_+})^2\rangle = \langle(\delta P_{\alpha_-})^2\rangle \to 0, \tag{25}$$

when $\gamma \gg \kappa$, and so do the variances for the dark-state-based spins,

$$\langle(\delta X_{\beta_-})^2\rangle = \langle(\delta P_{\beta_+})^2\rangle \to 0, \tag{26}$$

when $\kappa \gg \gamma$. This clearly shows that almost ideal squeezed states and Einstein-Podolsky-Rosen entangled states [31] are obtained for cavity fields and for the dark-state-based spins. Generally, in order to have good squeezing we need the three conditions: relatively large squeezing parameter ($r \gtrsim 1$), not too deep saturation ($\Pi \lesssim 1$), and remarkably large cooperativity parameters ($C_{1,2} \gg 1$). Because of the strong dependence of the quantities $(r,\Pi,C_{1,2})$ on the common parameter $\frac{\Delta}{\Omega}$, the above conditions are well satisfied for $-\sqrt{\frac{2}{3}} \lesssim \frac{\Delta}{\Omega} \ll 0$. In this regime, strong dissipation occurs for the Bogoliubov field or spin modes with a large value of squeezing parameter r. In addition, the cooperativity parameters $C_{1,2}$ depend on the atomic and cavity decay rates. For the other given parameters, a decrease in γ or κ corresponds to an increase in the cooperativity parameter $C_{1,2}$. As $C_{1,2}$ becomes large, the engineered dissipation is enhanced, and so are the squeezing and entanglement.

(2) Squeezing and entanglement are confined within the ranges of $\frac{\Delta}{\Omega} = (-\sqrt{\frac{2}{3}},0)$, $(0,1)$, $(-\infty,-1)$. Figures 4 and 5 show an agreement with the above qualitative analysis. Squeezing and entanglement happen within the predicted ranges. We note $N_+ > N_0$ in the range of $\frac{\Delta}{\Omega} = (-\infty,-1)$. In

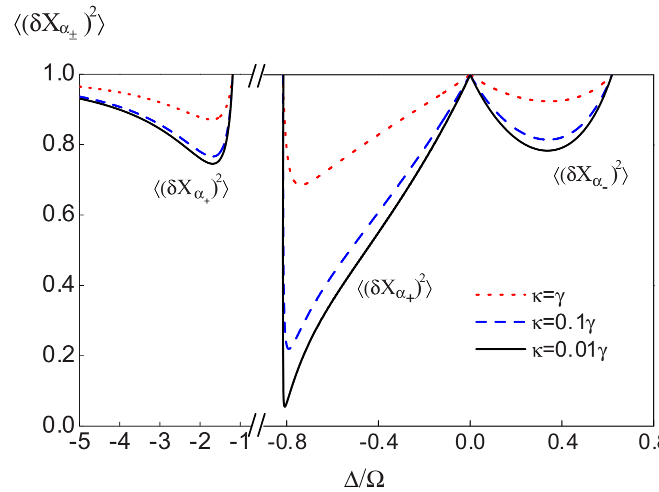

FIG. 4. (Color online) Two-mode field variances $\langle(\delta X_{\alpha_\pm})^2\rangle$ versus the normalized detuning $\frac{\Delta}{\Omega}$ for $g\sqrt{N} = 20\gamma$ and $\kappa = \gamma$ (dotted line), 0.1γ (dashed line), 0.01γ (solid line).

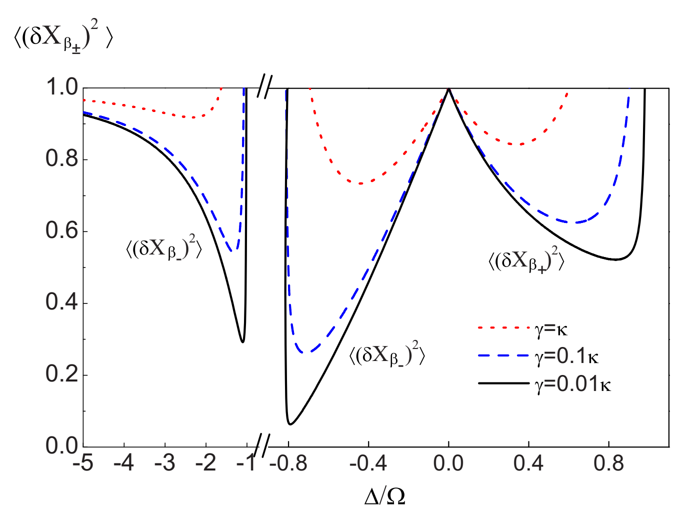

FIG. 5. (Color online) Two-mode spin variances $\langle(\delta X_{\beta_\pm})^2\rangle$ versus the normalized detuning $\frac{\Delta}{\Omega}$ for $g\sqrt{N} = 20\kappa$ and $\gamma = \kappa$ (dotted line), 0.1κ (dashed line), 0.01κ (solid line).

this case, the system is far beyond the dark state. However, squeezing and entanglement are still achievable. This is because the Bogoliubov mode dissipation dominates over the individual mode dissipation due to the vacuum environment. Beyond the above regions, i.e., $(-1,-\sqrt{\frac{2}{3}})$ and $(1,\infty)$, no squeezing occurs even for large nonlinearity or the large ratio of the nonlinearity to absorption. This is because the fluctuations are amplified in these regions. This also explains why the field correlation changes from positive to negative values at a special two-photon detuning, as demonstrated in Ref. [6]. It is clear that the coherence-induced nonlinearities are not sufficient to describe the squeezing and entanglement generation. In addition, we should note that if we tune the cavity fields such $\Delta_c = -\bar{\Omega}$ that the dependence of the quantum correlations on the atom-field detuning $\frac{\Delta}{\Omega}$ is symmetrically exchanged with respect to $\Delta = 0$. The best squeezing will happen at $\frac{\Delta}{\Omega} \to \sqrt{\frac{2}{3}}$.

(3) The best squeezing approaches 50% for the nonadiabatic conditions ($\kappa \sim \gamma$) and large cooperativity parameter $C_{1,2} \gg 1$. For $C_{1,2} \gg 1$, $e^{-2r} \ll 1$, and $\Pi \lesssim 1$, we easily obtain from Eqs. (21) and (22)

$$\begin{aligned}\langle(\delta X_{\alpha_+})^2\rangle &= \langle(\delta P_{\alpha_-})^2\rangle \approx \frac{\kappa}{\kappa+\Gamma_1},\\ \langle(\delta X_{\beta_-})^2\rangle &= \langle(\delta P_{\beta_+})^2\rangle \approx \frac{\Gamma_2}{\kappa+\Gamma_2},\end{aligned} \tag{27}$$

which approach $\frac{1}{2}$ for $\kappa \approx \Gamma_{1,2} \approx \gamma$. This shows that the best squeezing of 50% is obtainable. Note that what is shown in Figs. 4 and 5 (for $\kappa = \gamma$) does not reach 50%. This is because the conditions $C_{1,2} \gg 1$ are not well satisfied for a large value of the squeezing parameter r. If we increase the number of atoms N and/or decrease both of the cavity and atomic decay rates κ and γ ($\kappa \sim \gamma$), the cooperativity parameters $C_{1,2}$ are increased and then the squeezing is enhanced. We also should note that the present case is in sharp contrast to the quantum memory case [32,33], in which the nonadiabatic dissipation can enhance quantum noise reduction. In that case, the coherent evolutions were employed against the dissipation due to the environment. Naturally, an incomplete dissipation of the atoms introduces less noise into the coupled fields. However, the present case differs. While the two cavity fields (the two dark-state-based spins) are considered as two system modes, the two dark-state-based spins (the two cavity fields) serve as two engineered reservoir components. The reservoir exerts its dissipative effects on the Bogoliubov modes of the system. Each system mode entangles with one different reservoir component, and the other system mode receives the state of the entangled reservoir component. The state transfer processes are fast because of large dissipation rates $C_1\kappa$ or $C_2\Gamma_2$. When the engineered reservoir arrives at their steady states much sooner than the system, the state of one reservoir component entangled with one system mode is completely transferred to the other system mode. Therefore, the faster the dissipation induced by the engineered reservoir, the better the squeezing and entanglement of the system. This is the essential difference between the coherent evolution mechanisms and the Bogoliubov mode dissipation mechanisms.

D. Ground-state spin squeezing

We should point out that the two-mode squeezing of the dark-state-based spins also corresponds to the ground-state spin squeezing in some regions. In order to show this, let us define the ground- or metastable-state spin quadratures

$$\begin{aligned}J_x &= \sigma_{12} + \sigma_{21},\\ J_y &= -i(\sigma_{12} - \sigma_{21}),\\ J_z &= \sigma_{11} - \sigma_{22},\end{aligned} \tag{28}$$

which follow the commutation relation $[J_y, J_z] = 2iJ_x$. At steady state, we have the mean values of the spin components $\langle J_x\rangle = -(N_0 - N_+)\cos^2\theta$ and $\langle J_y\rangle = \langle J_z\rangle = 0$. The spin squeezing occurs when either of the two inequalities $\langle(\delta J_{y,z})^2\rangle < |\langle J_x\rangle|$ is satisfied. In terms of the dark-state-based spins we write the ground-state spin as

$$\begin{aligned}J_z = &\sin\theta(\sigma_{++} - \sigma_{--})\\ &-\frac{\cos\theta}{\sqrt{2}}(\sigma_{+0} + \sigma_{0-} + \sigma_{0+} + \sigma_{-0}).\end{aligned} \tag{29}$$

The ground-state spin variance is obtained for $\frac{\Delta}{\Omega} = (-\sqrt{\frac{2}{3}},0)$ as

$$\langle(\delta J_z)^2\rangle = \langle(\delta X_{\beta_+})^2\rangle|\langle J_x\rangle| + 2N_+\sin^2\theta, \tag{30}$$

where the first term is the contribution of the interaction determined correlation, and the second term is due to the atomic spontaneous emission. We plot in Fig. 6 the variance $\langle(\delta J_z)^2\rangle/|\langle J_x\rangle|$ for different atomic decay rates. It is shown that the ground-state spin squeezing is confined to the ranges of detuning $(-\sqrt{\frac{2}{3}},0)$ for $\Delta_c = \bar{\Omega}$ and to $(0,\sqrt{\frac{2}{3}})$ for $\Delta_c = -\bar{\Omega}$, where the atoms populate dominantly in the dark state $N_0 > N_+$. The best squeezing appears roughly when $\frac{\Delta}{\Omega} \to \mp 0.6$ for $\Delta_c = \pm\bar{\Omega}$, respectively.

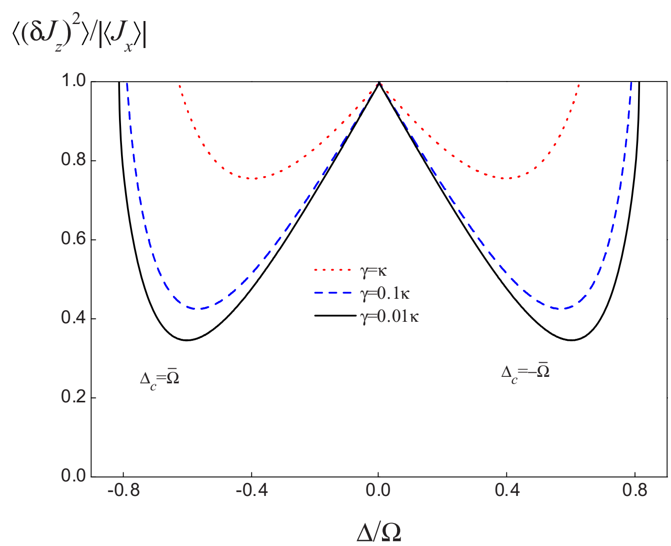

FIG. 6. (Color online) Ground-state spin variance $\langle(\delta J_z)^2\rangle/|\langle J_x\rangle|$ versus the normalized detuning $\frac{\Delta}{\Omega}$ for $g\sqrt{N} = 20\kappa$ and $\gamma = \kappa$ (dotted line), 0.1κ (dashed line), 0.01κ (solid line). The $\Delta < 0$ part is for $\Delta_c = \bar{\Omega}$ while the $\Delta > 0$ part is for $\Delta_c = -\bar{\Omega}$.

V. DISCUSSION AND CONCLUSION

Finally, it is interesting to recall and compare with the role of the dissipation in the formation of CPT. The atomic part in the damping term (3) for the resonant and symmetrical case is rewritten in terms of the dressed states as

$$\mathcal{L}_{\text{atom}}\rho = \frac{\gamma}{2}\sum_{\mu=1}^{N}\left(\mathcal{L}_{\sigma^{\mu}_{0+}}\rho + \mathcal{L}_{\sigma^{\mu}_{0-}}\rho + \mathcal{L}_{\sigma^{\mu}_{p}}\rho\right) + \frac{\gamma}{4}\sum_{\mu=1}^{N}\left(\mathcal{L}_{\sigma^{\mu}_{-+}}\rho + \mathcal{L}_{\sigma^{\mu}_{+-}}\rho\right), \tag{31}$$

where $\sigma^{\mu}_{p} = \sigma^{\mu}_{++} - \sigma^{\mu}_{--}$. $\mathcal{L}_{\sigma_{0\pm}}\rho$ represent the decay $|\pm\rangle \rightsquigarrow |0\rangle$, $\mathcal{L}_{\sigma^{\mu}_{p}}\rho$ describes the dephasing behavior between the $|+\rangle$ and $|-\rangle$ states, and $\mathcal{L}_{\sigma^{\mu}_{-+}}\rho$ and $\mathcal{L}_{\sigma^{\mu}_{+-}}$ stand for the bidirectional population transfer between the $|+\rangle$ and $|-\rangle$ states [18]. We note that there are no terms like $\mathcal{L}_{\sigma_{\pm 0}}\rho$ in Eq. (31). That means absence of the $|0\rangle \rightsquigarrow |\pm\rangle$ channels away from the dark state. For this reason, the atoms, once pumped into the dark state $|0\rangle$, are no longer excited. Essentially, the atoms are deposited in the dark state with the help of the incoherent channels $|\pm\rangle \rightsquigarrow |0\rangle$ [3]. In other words, the atomic coherence between the ground states is established via dissipation processes. Off resonance, there remain such dissipation processes. Moreover, there appear the dissipation channels $|0\rangle \rightsquigarrow |\pm\rangle$. Since the dark state of the atoms is established via the dissipation, it is easy to understand that the quantum coherence properties of the atoms and the fields are determined by the dissipation processes.

In conclusion, by probing into the dressed atom-photon interactions, we have separated out the Bogoliubov mode dissipation mechanisms from the coherence-induced nonlinearities of infinitely high orders in the laser intensities. It has been shown that the Bogoliubov mode dissipation by the atoms leads to two-mode squeezing and entanglement of the cavity fields, and the Bogoliubov mode dissipation by the cavity fields results in two-mode squeezing and entanglement of the dark-state-based spins. The latter case corresponds to the ground-state squeezing in a limited range of the normalized detuning. The squeezing and entanglement by dissipation are robust to the environmental fluctuations and are generally utilizable for coherently prepared systems.

ACKNOWLEDGMENTS

This work was supported by the National Natural Science Foundation of China (Grants No. 11474118 and No. 61178021) and the National Basic Research Program of China (Grant No. 2012CB921604).

[1] E. Arimondo, in *Progress in Optics*, edited by E. Wolf (Elsevier Science, Amsterdam, 1996), Vol. 35, p. 257.
[2] S. E. Harris, Phys. Today **50**(7), 36 (1997).
[3] J. P. Marangos, J. Mod. Opt. **45**, 471 (1998).
[4] M. D. Lukin, Rev. Mod. Phys. **75**, 457 (2003).
[5] M. Fleischhauer, A. Imamoglu, and J. P. Marangos, Rev. Mod. Phys. **77**, 633 (2005).
[6] V. A. Sautenkov, Y. V. Rostovtsev, and M. O. Scully, Phys. Rev. A **72**, 065801 (2005).
[7] A. Dantan, J. Cviklinski, E. Giacobino, and M. Pinard, Phys. Rev. Lett. **97**, 023605 (2006).
[8] A. Sinatra, Phys. Rev. Lett. **97**, 253601 (2006); J. F. Roch, K. Vigneron, Ph. Grelu, A. Sinatra, J. Ph. Poizat, and Ph. Grangier, *ibid.* **78**, 634 (1997); A. Sinatra, J. F. Roch, K. Vigneron, Ph. Grelu, J. Ph. Poizat, K. Wang, and P. Grangier, Phys. Rev. A **57**, 2980 (1998).
[9] P. Barberis-Blostein and N. Zagury, Phys. Rev. A **70**, 053827 (2004).
[10] G. L. Cheng, X. M. Hu, W. X. Zhong, and Q. Li, Phys. Rev. A **78**, 033811 (2008); F. Wang, X. M. Hu, W. X. Shi, and Y. Z. Zhu, *ibid.* **81**, 033836 (2010).
[11] M. D. Reid, D. F. Walls, and B. J. Dalton, Phys. Rev. Lett. **55**, 1288 (1985).
[12] R. Guzman, J. C. Retamal, E. Solano, and N. Zagury, Phys. Rev. Lett. **96**, 010502 (2006).
[13] Y. Wu, M. G. Payne, E. W. Hagley, and L. Deng, Phys. Rev. A **69**, 063803 (2004).
[14] S. Pielawa, G. Morigi, D. Vitali, and L. Davidovich, Phys. Rev. Lett. **98**, 240401 (2007).
[15] A. S. Parkins, E. Solano, and J. I. Cirac, Phys. Rev. Lett. **96**, 053602 (2006).
[16] E. G. Dalla Torre, J. Otterbach, E. Demler, V. Vuletic, and M. D. Lukin, Phys. Rev. Lett. **110**, 120402 (2013).
[17] H. Krauter, C. A. Muschik, K. Jensen, W. Wasilewski, J. M. Petersen, J. I. Cirac, and E. S. Polzik, Phys. Rev. Lett. **107**, 080503 (2011).
[18] M. O. Scully and M. S. Zubairy, *Quantum Optics* (Cambridge University Press, Cambridge, UK, 1997).
[19] M. Fleischhauer, Phys. Rev. Lett. **72**, 989 (1994); M. Fleischhauer and T. Richter, Phys. Rev. A **51**, 2430 (1995); X. M. Hu and C. H. Oh, *ibid.* **83**, 053842 (2011).
[20] S. M. Rochester, D. S. Hsiung, D. Budker, R. Y. Chiao, D. F. Kimball, and V. V. Yashchuk, Phys. Rev. A **63**, 043814 (2001); A. B. Matsko, I. Novikova, M. S. Zubairy, and G. R. Welch, *ibid.* **67**, 043805 (2003); S. Pustelny, D. F. Jackson Kimball, S. M. Rochester, V. V. Yashchuk, W. Gawlik, and D. Budker, *ibid.* **73**, 023817 (2006).
[21] H. Wang, D. J. Goorskey, and M. Xiao, Phys. Rev. A **65**, 011801 (2001); A. Joshi and M. Xiao, Phys. Rev. Lett. **91**, 143904 (2003).
[22] G. S. Agarwal, W. Lange, and H. Walther, Phys. Rev. A **48**, 4555 (1993).
[23] C. Cohen-Tannoudji, J. Dupont-Roc, and G. Grynberg, *Atom-Photon Interactions* (Wiley, New York, 1992).
[24] M. Sargent III, M. O. Scully, and W. E. Lamb, Jr., *Laser Physics* (Addison-Wesley, Boston, MA, 1974), Chap. 16.
[25] D. F. Walls and G. J. Milburn, *Quantum Optics* (Springer, Berlin, 1994).
[26] J. Ma, X. G. Wang, C. P. Sun, and F. Nori, Phys. Rep. **509**, 89 (2011) and references therein.
[27] C. W. Gardiner and P. Zoller, *Quantum Noise*, 2nd ed. (Springer, Berlin, 2000).

[28] D. W. Berry and B. C. Sanders, Phys. Rev. A **66**, 012313 (2002); X. G. Wang and B. C. Sanders, *ibid.* **68**, 033821 (2003).

[29] L. M. Duan, G. Giedke, J. I. Cirac, and P. Zoller, Phys. Rev. Lett. **84**, 2722 (2000).

[30] M. G. Raymer, A. C. Funk, B. C. Sanders, and H. de Guise, Phys. Rev. A **67**, 052104 (2003).

[31] A. Einstein, B. Podolsky, and N. Rosen, Phys. Rev. **47**, 777 (1935).

[32] M. O. Scully, G. Süssmann, and C. Benkert, Phys. Rev. Lett. **60**, 1014 (1988); C. Benkert, M. O. Scully, and G. Süssmann, Phys. Rev. A **41**, 6119 (1990).

[33] M. D. Reid, Phys. Rev. A **37**, 4792 (1988).