Advantages of nonclassical pointer states in postselected weak measurements

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We investigate, within the weak measurement theory, the advantages of nonclassical pointer states over semiclassical ones for coherent, squeezed vacuum, and Schrödinger cat states. These states are utilized as pointer states for the system operator $\hat{A}$ with property $\hat{A}^2 = \hat{I}$, where $\hat{I}$ represents the identity operator. We calculate the ratio between the signal-to-noise ratio of nonpostselected and postselected weak measurements. The latter is used to find the quantum Fisher information for the above pointer states. The average shifts for those pointer states with arbitrary interaction strength are investigated in detail. One key result is that we find the postselected weak measurement scheme for nonclassical pointer states to be superior to semiclassical ones. This can improve the precision of the measurement process.

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I. INTRODUCTION

The weak measurement, as a generalized von Neumann quantum measurement theory, was proposed by Aharonov, Albert, and Vaidman [1]. In weak measurement, the coupling between pointer and measured systems is sufficiently weak, but its induced weak value of the observable on the measured system can be beyond the usual range of the eigenvalues of that observable [2]. This feature of weak value is usually referred to as an amplification effect for weak signals rather than a conventional quantum measurement that collapses a coherent superposition of quantum states [1,3].

After first optical implementation of weak value [4], it has been applied in different fields to observe very tiny effects, such as beam deflection [5–10], frequency shifts [11], phase shifts [12], angular shifts [13,14], velocity shifts [15], and even temperature shift [16]. Weak value has a nature of being a complex number, which leads the weak measurements to provide an ideal method to examine some fundamentals of quantum physics. Quantum paradoxes (Hardy's paradox [17–19] and the three-box paradox [20]), quantum correlation and quantum dynamics [21–26], quantum state tomography [27–32], violation of the generalized Leggett-Garg inequalities [33–38], and violation of the initial Heisenberg measurement-disturbance relationship [39,40] are just a few examples. In these typical examples, the small effects have been amplified due to the benefit of weak values. This amplifying effect occurs when the preselection and postselection states of the measured system are almost orthogonal. The successful postselection probability tends to decrease in order to have successful amplification effect. For more details about weak measurement and weak value, one can consult the reviews [41–43].

So far, most weak measurement studies focus on using the zero-mean Gaussian state as an initial pointer state. However, recent studies [44,45] have shown that a zero-mean Gaussian pointer state cannot improve the signal-to-noise ratio (SNR) when considering postselection probability. Needless to say, the Gaussian beam is classical and one may naturally ask how about using nonclassical pointer states, and what kind of advantages do they have? This issue has been recently addressed [46], where coherent and coherent squeezed states were utilized as pointers. They showed that the postselected weak measurement improved the SNR compared to the nonpostselected process if the pointer state is nonclassical rather than classical. The focus of the calculation was based on the assumption that the coupling between the measuring device and the measured system is too weak, and hence it was enough to consider the time evolution operator up to its first order. Furthermore, there have been recent studies giving full order effects of the unitary evolution due to the von Neumann interaction, but for classical and semiclassical states [47,48].

In this paper, we address a remaining point of interest constructing a general formula for weak measurement beyond the first order, and utilizing the nonclassical states. We investigate the advantages of nonclassical pointer states over classical (semiclassical) pointer state, within weak values, by considering postselection probability. In order to do so, we use coherent, squeezed vacuum, and Schrödinger cat states as pointer states for system observable $\hat{A}$ with property $\hat{A}^2 = \hat{I}$. We start by presenting an analytical general expression of the shifted values of position and momentum operators for the above-mentioned pointer states with arbitrary measurement strengths. In addition, we present the ratio of SNR between postselected and nonpostselected weak measurement, and also look at quantum Fisher information. Our key results in this

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paper are as follows: (i) Our general expressions of shifted values reduce to Nakamura *et al.*'s [47] main result if we take the zero-mean Gaussian beam as initial pointer state. (ii) As shown in Ref. [46], improving the SNR using postselected weak measurement, one needs the nonclassical pointer states which is better than classical or semiclassical states. (iii) Nonclassical pointer states are much better even when it comes to the parameter estimation process which is characterized by Fisher information.

The rest of the paper is organized as follows. In Sec. II, we give the setup for our system. In Sec. III, we start by giving general expressions for the expectation values of position and momentum operators. After that we discuss the ratio of SNR between postselected and nonpostselected weak measurements of coherent, squeezed vacuum, and Schrödinger cat states. In Sec. IV, we give the Fisher information for those given states in the light of postselection probability. We give a conclusion to our paper in Sec. V. Throughout this paper, we use the unit $\hbar = 1$.

II. SETUP

For the weak measurement, the coupling interaction between system and measuring device is given by the standard von Neumann Hamiltonian [2]

$$H = g\delta(t - t_0)\hat{A} \otimes \hat{P}. \tag{1}$$

Here, g is a coupling constant and $\hat{P} = \int p|p\rangle\langle p|dp$ is the conjugate momentum operator, while the position operator is $\hat{X} = \int x|x\rangle\langle x|dx$ where $[\hat{X},\hat{P}] = i\hat{I}$. We have taken, for simplicity, the interaction to be impulsive at time $t = t_0$. For this kind of impulsive interaction the time evolution operator becomes as $e^{-ig\hat{A}\otimes\hat{P}}$.

The weak measurement is characterized by the preselection and postselection of the system state. If we prepare the initial state $|\psi_i\rangle$ of the system and the pointer state, and after some interaction time t_0, we then postselect a system state $|\psi_f\rangle$ we obtain the information about a physical quantity $\hat{A}$ from the pointer wave function by the following weak value:

$$\langle A\rangle_w = \frac{\langle\psi_f|\hat{A}|\psi_i\rangle}{\langle\psi_f|\psi_i\rangle}, \tag{2}$$

where the subscript w denotes the weak value. From Eq. (2), we know that when the preselected state $|\psi_i\rangle$ and the postselected state $|\psi_f\rangle$ are almost orthogonal, the absolute value of the weak value can be arbitrarily large. This feature leads to weak value amplification.

We express position operator $\hat{X}$ and momentum operator $\hat{P}$ in terms of the annihilation (creation) operator, $\hat{a}$ ($\hat{a}^\dagger$) in Fock space representation as

$$\hat{X} = \sigma(\hat{a}^\dagger + \hat{a}), \tag{3}$$

$$\hat{P} = \frac{i}{2\sigma}(\hat{a}^\dagger - \hat{a}), \tag{4}$$

where σ is the width of the fundamental Gaussian beam. These annihilation (creation) operators obey the commutation relation $[\hat{a},\hat{a}^\dagger] = \hat{I}$. By substituting Eq. (4) into unitary evolution operator $e^{-ig\hat{A}\otimes\hat{P}}$, bearing in mind that operator $\hat{A}$ satisfies the property $\hat{A}^2 = \hat{I}$, we get

$$e^{-ig\hat{A}\otimes\hat{P}} = \frac{1}{2}(\hat{I} + \hat{A}) \otimes D\left(\frac{s}{2}\right) + \frac{1}{2}(\hat{I} - \hat{A}) \otimes D\left(-\frac{s}{2}\right), \tag{5}$$

where parameter s is defined by $s :\equiv g/\sigma$, and $D(\mu)$ is a displacement operator with complex number μ defined by

$$D(\mu) = e^{\mu\hat{a}^\dagger - \mu^*\hat{a}}. \tag{6}$$

Note that s characterizes the measurement strength. Thus, we can say that the coupling between system and pointer is weak (strong) and so the measurement is a called weak (strong) measurement, if $s \ll 1$ ($s \gg 1$).

III. SHIFTED VALUES AND THE SIGNAL-TO-NOISE RATIO (SNR)

In this section we start by giving general shifted values of semiclassical states (coherent state) and nonclassical states, squeezed vacuum and Schrödinger cat pointer states for arbitrary measurement strength s. To show the advantages of nonclassical pointer states over semiclassical ones, we discuss the ratio of SNR between postselected and nonpostselected weak measurements

$$\chi = \frac{R_X^p}{R_X^n}. \tag{7}$$

Here, R_X^p represents the SNR of postselected weak measurement defined as

$$R_X^p = \frac{\sqrt{NP_s}|\langle X\rangle_{fi}|}{\sqrt{\langle X^2\rangle_f - \langle X\rangle_f^2}}. \tag{8}$$

Here, N is the total number of measurements, P_s is probability of finding the postselected state for a given preselected state, and NP_s is the number of times the system was found in a postselected state. Here, $\langle\rangle_f$ denotes the expectation value of measuring observable under the final state of the pointer.

When dealing with nonpostselected measurement, there is no postselection process after the interaction between system and measuring device due to unitary evolution operator $e^{-ig\hat{A}\otimes\hat{P}}$. Therefore, the definition of R_X^p for nonpostselected weak measurement can be given as

$$R_X^n = \frac{\sqrt{N}|\langle X\rangle_{f'i}|}{\sqrt{\langle X^2\rangle_{f'} - \langle X\rangle_{f'}^2}}. \tag{9}$$

Here, $\langle\rangle_{f'}$ denotes the expectation value of measuring observable under the final state of the pointer without postselection.

A. Coherent pointer state

The coherent state is a typical semiclassical state which satisfies the minimum Heisenberg uncertainty relation. Here, we take the coherent state [49] as initial pointer state

$$|\alpha\rangle = D(\alpha)|0\rangle, \tag{10}$$

where $\alpha = re^{i\phi}$ is an arbitrary complex number. After unitary evolution given in Eq. (5), the resultant system state is postselected to $|\psi_f\rangle$. Then, we obtain the following normalized final pointer state:

$$|\Psi_{f_1}\rangle = \frac{\lambda}{2}\bigg[(1+\langle A\rangle_w)e^{-i(s/2)\mathrm{Im}[\alpha]}\bigg|\alpha+\frac{s}{2}\bigg\rangle + (1-\langle A\rangle_w)e^{i(s/2)\mathrm{Im}[\alpha]}\bigg|\alpha-\frac{s}{2}\bigg\rangle\bigg], \tag{11}$$

where the normalization coefficient is given as

$$\lambda = \sqrt{2}\{1+|\langle A\rangle_w|^2+\mathrm{Re}[(1-\langle A\rangle_w^*)(1+\langle A\rangle_w)e^{-2is\mathrm{Im}[\alpha]}] \times e^{-(1/2)s^2}\}^{-1/2}, \tag{12}$$

and Im (Re) represents the imaginary (real) part of a complex number. Using Eqs. (11) and (12) we can calculate general forms of the expectation values of conjugate position operator X and momentum operator P, under the final pointer state $|\Psi_{f_1}\rangle$, to be

$$\langle X\rangle_{f_1} = \sigma|\lambda|^2\{(1+|A_w|^2)\mathrm{Re}[\alpha]+s\mathrm{Re}\langle A\rangle_w + \mathrm{Re}[(1-\langle A\rangle_w^*)(1+\langle A\rangle_w)e^{-2si\mathrm{Im}(\alpha)}]\mathrm{Re}[\alpha]e^{-1/2s^2}\} \tag{13}$$

and

$$\langle P\rangle_{f_1} = \frac{|\lambda|^2}{4\sigma}\big(2(1+|\langle A\rangle_w|^2)\mathrm{Im}[\alpha] - \mathrm{Im}\{(1-\langle A\rangle_w)(1+\langle A\rangle_w^*) \times e^{2is\mathrm{Im}[\alpha]}(s-2i\mathrm{Im}[\alpha])\}e^{-1/2s^2}\big), \tag{14}$$

respectively. Equations (13) and (14) are the general forms of expectation values for system operator $\hat{A}$, with the property $\hat{A}^2=\hat{I}$, and they are valid for any arbitrary value of the measurement strength parameter s.

Here, we assume that the operator to be observed is the spin x component of a spin- 1/2 particle through the von Neuman interaction

$$A = \sigma_x = |\uparrow_z\rangle\langle\downarrow_z| + |\downarrow_z\rangle\langle\uparrow_z|, \tag{15}$$

where $|\uparrow_z\rangle$ and $\langle\downarrow_z|$ are eigenstates of σ_z with corresponding eigenvalues 1 and -1, respectively. When we select the preselected and postselected states as

$$|\psi_i\rangle = \cos\left(\frac{\theta}{2}\right)|\uparrow_z\rangle + e^{i\varphi}\sin\left(\frac{\theta}{2}\right)|\downarrow_z\rangle, \tag{16}$$

and

$$|\psi_f\rangle = |\uparrow_z\rangle, \tag{17}$$

respectively, we can get the weak value by substituting these states to

$$\langle A\rangle_w = \langle\sigma_x\rangle_w = \frac{\langle\psi_f|A|\psi_i\rangle}{\langle\psi_f|\psi_i\rangle}, \tag{18}$$

obtaining

$$\langle A\rangle_w = e^{i\varphi}\tan\left(\frac{\theta}{2}\right), \tag{19}$$

where, $\theta\in[0,\pi]$ and $\varphi\in[0,2\pi)$. Here, the postselection probability is $P_s=\cos^2(\frac{\theta}{2})$. Throughout this paper, we use the above preselected and postselected states and weak value, which are given in Eqs. (16), (17), and (19) for our discussions.

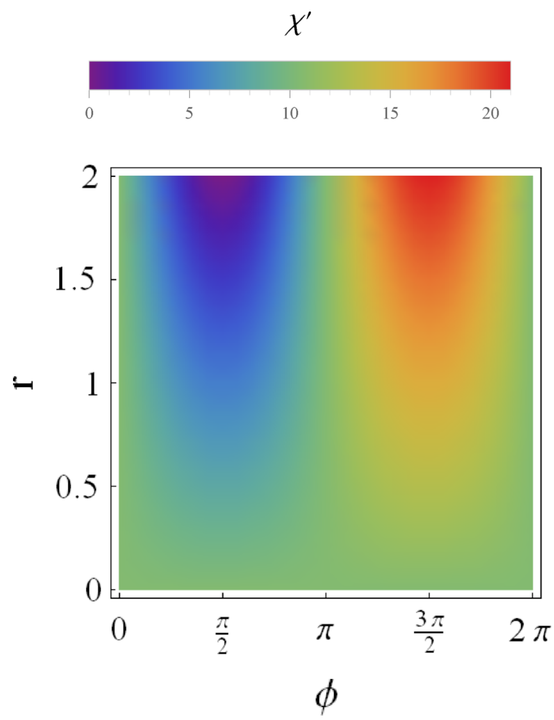

FIG. 1. (Color online) The ratio χ' of SNRs between postselected and nonpostselected weak measurement vs coherent state's parameters ϕ and r. Here we take $\varphi=\pi/4$, $\theta=7\pi/9$, and $s=10^{-5}$.

In the case of the coherent state used as the initial pointer state, we calculate the SNR of postselected and nonpostselected process in a weak measurement regime ($s\ll 1$). In Fig. 1 we plot the ratio $\chi'=(\chi-1.4618)\times 10^5$ against coherent state's parameters r and ϕ, where the ratio χ has the same value 1.4618 in most of the regions. This means that, for the coherent state pointer, the postselected weak measurement is little better than the nonpostselected case which in turn slightly increases the precision of measurement.

B. Squeezed vacuum state

The squeezed vacuum state is a typical quantum state. It has many applications in optical communication, optical measurement, and gravitational wave detection [50]. Here, we assume that the initial pointer is the squeezed vacuum state [49] which is defined by

$$|\xi\rangle = S(\xi)|0\rangle. \tag{20}$$

Here,

$$S(\xi) = \exp\left(\tfrac{1}{2}\xi^* a^2 - \tfrac{1}{2}\xi a^{\dagger 2}\right), \tag{21}$$

where the squeezing parameter $\xi=\eta e^{i\delta}$ is an arbitrary complex number. After the unitary evolution given in Eq. (5), the total system state is postselected to $|\psi_f\rangle$. Then, we obtain the following normalized final pointer state:

$$|\Psi_{f_2}\rangle = \frac{\gamma'}{2}\bigg[(1+\langle A\rangle_w)\bigg|\frac{s}{2},\xi\bigg\rangle + (1-\langle A\rangle_w)\bigg|-\frac{s}{2},\xi\bigg\rangle\bigg], \tag{22}$$

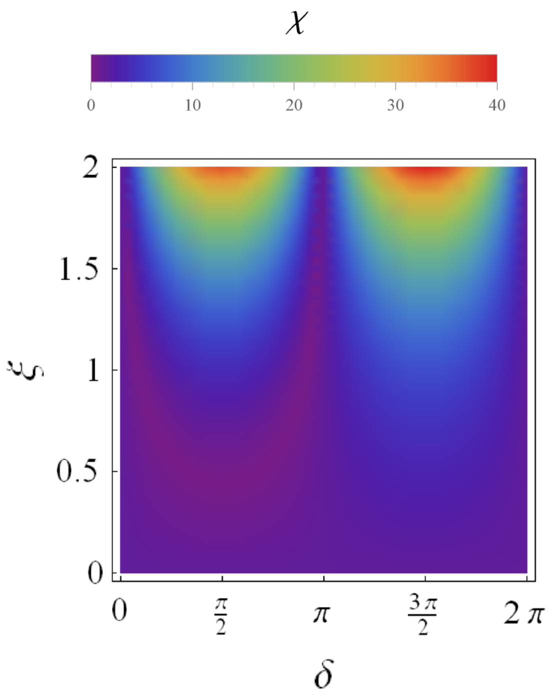

FIG. 2. (Color online) The ratio χ of SNRs between postselected and nonpostselected weak measurements vs squeezed vacuum state's parameters δ and η. Here we take $\varphi = \pi/4, \theta = 7\pi/9$, and $s = 10^{-5}$.

where the normalization coefficient is given by

$$\gamma' = \sqrt{2}\big(1 + |\langle A\rangle_w|^2 + (1 - |\langle A\rangle_w|^2) \times e^{-(1/2)s^2|\cosh\eta + e^{i\delta}\sinh\eta|^2}\big)^{-1/2} \tag{23}$$

and we note that $|\pm\frac{s}{2},\xi\rangle = D(\pm\frac{s}{2})S(\xi)|0\rangle$ is a squeezed coherent state. Next we will calculate the expectation values of position and momentum operators under the normalized final pointer state $|\Psi_{f_2}\rangle$, and the results read

$$\langle X\rangle_{f_2} = g|\gamma'|^2\mathrm{Re}\langle A\rangle_w - g|\gamma'|^2\mathrm{Im}\langle A\rangle_w e^{-(1/2)s^2|\cosh\eta + e^{i\delta}\sinh\eta|^2}\sinh(2\eta)\sin\delta \tag{24}$$

and

$$\langle P\rangle_{f_2} = \frac{g|\gamma'|^2}{2\sigma^2}\mathrm{Im}\langle A\rangle_w e^{-(1/2)s^2|\cosh\eta + e^{i\delta}\sinh\eta|^2} \times [1 + \sinh(2\eta)\cos\delta], \tag{25}$$

respectively. These formulas are valid not only in the weak measurement regime ($s \ll 1$), but also in the strong measurement regime ($s \gg 1$).

Figure 2 shows the ratio χ of SNR for squeezed pointer state between postselected and nonpostselected weak measurements ($s \ll 1$) plotted as a function of δ and η which are the parameters of the squeezed state. One can see that when η is large and near the points where $\delta = \frac{\pi}{2}, \frac{3\pi}{2}$ the ratio χ is much larger than unity. Evidently, this result indicates that the squeezed pointer state is one of the quantum state candidates that can be utilized to improve the SNR in postselected rather than nonpostselected weak measurement. This result was also confirmed in Ref. [46].

C. Schrödinger cat state

The Schrödinger cat state is another typical quantum state [51] which is a superposition of two coherent correlated states moving in opposite directions. Generally, there are two kinds of Schrödinger cat states [52]; even and odd Schrödinger cat states. Even Schrödinger cat state has very similar properties with squeezed state, since it has superpositions of photon number states with even numbers of quanta. Therefore, we consider the even Schrödinger cat state as the initial pointer state to examine further the advantages of the nonclassical pointer state. The normalized even Schrödinger cat state can be written as

$$|\Theta_+\rangle = K(|\alpha\rangle + |-\alpha\rangle), \tag{26}$$

where $|\pm\alpha\rangle$ are coherent states as defined in Eq. (10) which is characterized by $\alpha = re^{i\phi}$, and the normalization constant is

$$K = \frac{1}{\sqrt{2 + 2e^{-2|\alpha|^2}}}. \tag{27}$$

Following the same procedure as in previous sections, after taking the unitary evolution given in Eq. (5), the outcome will then be projected to postselected state, $|\psi_f\rangle$. Then, we obtain the following normalized final pointer state:

$$|\Psi_{f_3}\rangle = \frac{\kappa'}{2}\left[(1+\langle A\rangle_w)D\left(\frac{s}{2}\right) + (1-\langle A\rangle_w)D\left(-\frac{s}{2}\right)\right]|\Theta_+\rangle, \tag{28}$$

where the normalization coefficient is given by

$$\kappa' = \left[\frac{1}{2}(1 + |\langle A\rangle_w|^2) + K^2(1 - |\langle A\rangle_w|^2)\cos(2s\mathrm{Im}[\alpha])e^{-s^2/2} + \frac{K^2}{2}(1 - |\langle A\rangle_w|^2)\big(e^{-(1/2)|2\alpha+s|^2} + e^{-(1/2)|2\alpha-s|^2}\big)\right]^{-1/2}. \tag{29}$$

By using Eq. (28) we calculate, in a straightforward manner, the general forms of the expectation values for both conjugate position and momentum operators as

$$\langle X\rangle_{f_3} = 2\sigma|\kappa'|^2K^2 \times \big(s\mathrm{Re}\langle A\rangle_w(1 + e^{-2|\alpha|^2}) + 2\mathrm{Im}\langle A\rangle_w\mathrm{Re}[\alpha]\sin\{2s\mathrm{Im}[\alpha]e^{-(1/2)s^2} - \mathrm{Im}\langle A\rangle_w\mathrm{Im}[\alpha]\big(e^{-(1/2)|2\alpha+s|^2} - e^{-(1/2)|2\alpha-s|^2}\big)\}\big) \tag{30}$$

and

$$\langle P\rangle_{f_3} = \frac{|\kappa|^2K^2\mathrm{Im}\langle A\rangle_w}{2\sigma} \times \{(2\mathrm{Re}[\alpha] + s)e^{-(1/2)|2\alpha+s|^2} + 4\sin(2s\mathrm{Im}[\alpha])\mathrm{Im}[\alpha]e^{-(1/2)s^2} + 2s\cos(2s\mathrm{Im}[\alpha]) \times e^{-(1/2)s^2} - (2\mathrm{Re}[\alpha] - s)e^{-(1/2)|2\alpha-s|^2}\}, \tag{31}$$

respectively.

In Fig. 3, we plot the ratio χ of SNRs between postselected and nonpostselected weak measurements for the Schrödinger

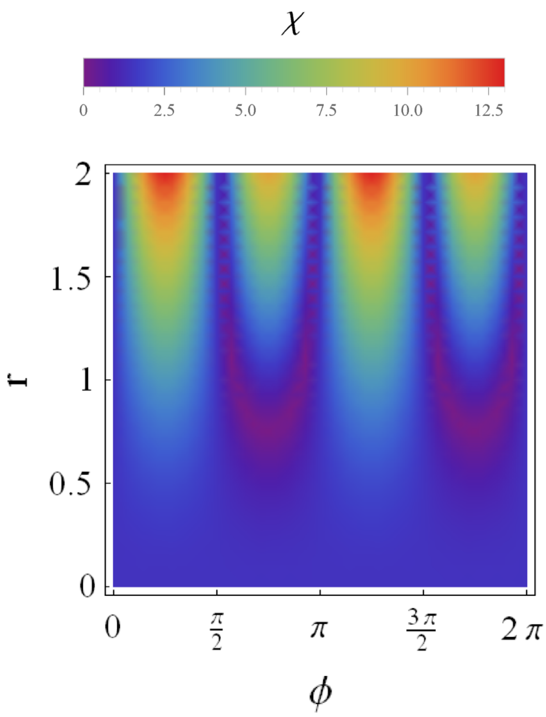

FIG. 3. (Color online) The ratio χ of SNRs between postselected and nonpostselected measurement vs Schrödinger cat state's parameters ϕ and r. Here we take $\varphi = \pi/4$, $\theta = 7\pi/9$, and $s = 10^{-5}$.

cat pointer state. It is, clearly, indicating that when r is increased and passed near $\phi = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$, the ratio of SNRs is much larger than unity. Furthermore, when comparing Fig. 3 to Fig. 1, we find that the ratio χ of the nonclassical Schrödinger cat pointer state is higher than the semiclassical coherent pointer state for the same parameters. This, evidently, leads to the improvement of SNR. However, when comparing between the two nonclassical states in Figs. 2 and 3, one can see that these two figures have some similarity, where both of them have the ratio χ larger than unity, while a much stronger value is obtained for the case of the squeezed state.

We have to emphasize at this point that we have also calculated the odd Schrödinger cat pointer states but found that they have similar properties and results like the even Schrödinger cat pointer states. And in order to avoid repetition, therefore, we just report the results of the even Schrödinger cat states.

For the ratio of SNRs between postselected and nonpostselected weak measurements, we can conclude that nonclassical pointer states (squeezed vacuum, and Schrödinger cat state) are better than the semiclassical one (coherent sate) in order to improve the SNR in postselected weak measurements ($s \ll 1$) for complex weak values. This conclusion can be seen clearly from Figs. 1–3.

The general expectation values of position and momentum operators for the above three pointer states—coherent, squeezed, and Schrödinger cat states—can have the same property. This can be achieved if we assume the initial pointer state to be a zero-mean Gaussian beam (this corresponds to $r = 0$ for coherent states and the Schrödinger cat state, and $\eta = 0$ for the squeezed vacuum state, respectively), then all expressions reduced to

$$\langle X\rangle_f = \frac{g\mathrm{Re}\langle A\rangle_w}{\mathcal{Z}}, \tag{32}$$

and

$$\langle P\rangle_f = \frac{g\mathrm{Im}\langle A\rangle_w}{2\sigma^2\mathcal{Z}} e^{-(1/2)s^2}. \tag{33}$$

Here,

$$\mathcal{Z} = 1 + \tfrac{1}{2}(1 - |\langle A\rangle_w|^2)\left(e^{-(1/2)s^2} - 1\right). \tag{34}$$

This result is given in Nakamura's work [47].

A remaining issue is to examine the connection between weak and strong postselected measurement. Thus, we plot the R_X^p, which is defined in Eq. (8), as function of arbitrary measurement strength parameter s and preselection angle θ. From Fig. 4, particularly for the squeezed vacuum pointer state, we can see that at $\theta = \pi/2$ the R_X^p increase with the increase of s; this is the strong measurement result. The reason is that at $\theta = \pi/2$ the preselected state Eq. (16) is the eigenstate of operator σ_x which has the eigenvalue $+1$. This figure does not only make the connection between weak and strong postselected measurements, but also indicates that nonclassical pointer states are also good enough compared with semiclassical ones in generalized von Neumann measurements [48].

IV. QUANTUM FISHER INFORMATION

Fisher information is the maximum amount of information about the parameter that we can extract from the system. For a pure quantum state $|\psi_s\rangle$, the quantum Fisher information estimating s is

$$F^{(Q)} = 4[\langle\partial_s\psi_s|\partial_s\psi_s\rangle - |\langle\psi_s|\partial_s\psi_s\rangle|^2], \tag{35}$$

where the state $|\psi_s\rangle$ represents the final pointer states of the system. Here, this can be used for coherent, squeezed vacuum, or Schrödinger cat states when only dealing with the postselected weak measurement in the first-order evolution of unitary operator $e^{-ig\hat{A}\otimes\hat{P}}$. Here, $s \equiv g/\sigma$ is the measurement strength parameter which is directly related to the coupling constant g in our Hamiltonian of Eq. (1).

The variance of unknown parameter Δs is bounded by the Cramér-Rao bound

$$\Delta s \geqslant \frac{1}{NF^{(Q)}}, \tag{36}$$

where N is the total number of measurements. Thus, the Fisher information set the minimal possible estimate for parameter s, while higher Fisher information means a better estimation. In weak measurement, if we consider the successful postselection probability, then Fisher information would be $F_p^{(Q)} = P_s F^{(Q)}$. In Ref. [46], one can find general proof showing that quantum Fisher information is higher in postselected rather than nonpostselected weak measurement. Thus, we just focus on the postselected weak measurement process and look into Fisher information for semiclassical and nonclassical pointer states.

We proceed investigating the variation of Fisher information in the weak measurement regime for different weak values. Our numerical results in Fig. 5 show that the quantum Fisher information is higher in the weak measurement regime ($s \ll 1$) when the preselection and postselection states are almost

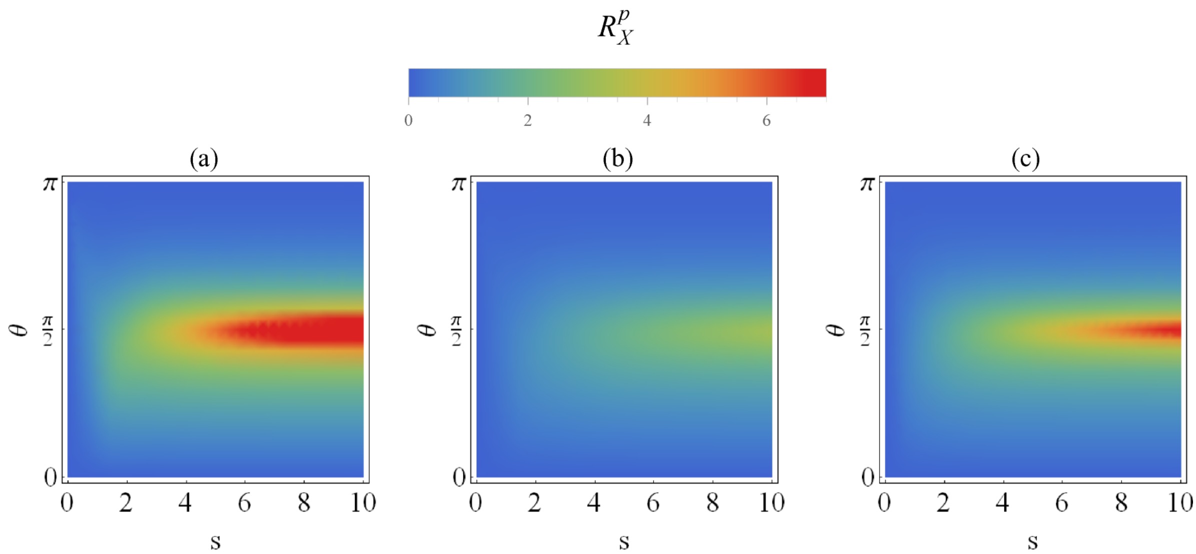

FIG. 4. (Color online) The R_X^p for arbitrary measurement strength parameter s, and θ for different weak values. Here, we take $\varphi = 0$ and $N = 1$. (a) For coherent pointer state, $r = 1$, $\phi = \pi/4$. (b) For Schrödinger cat pointer sates, $r = 1$,$\phi = \pi/4$. (c) For squeezed vacuum pointer state, $\eta = 1$, $\delta = \pi/4$.

orthogonal. The other important result is that the nonclassical pointer states have more advantages over the semiclassical ones, which in turn leads to a better estimation process.

V. CONCLUSION

In summary, we give general expressions for the shifted values of position and momentum operators for different pointer states (coherent, squeezed vacuum, and Schrödinger cat states); these expressions are valid in weak and strong measurement regimes. In the next step, we investigate the SNR and the quantum Fisher information only in the weak measurement regime. We find that if we take the initial state as a zero mean Gaussian state, our general expressions of shifted values would be reduced to Eqs. (32) and (33), which are given in Ref. [47]. By giving the ratio of SNR between postselected and nonpostselected weak measurement, we find that the postselected weak measurement process for nonclassical pointer states gives more information about the system compared to the nonpostselected process. This result is consistent with Pang *et al*'s work [46]. If one wants to quantify the quantum Fisher information in order to improve the precision of unknown parameter estimation, then he can consider using the nonclassical pointer state and avoiding the semiclassical one.

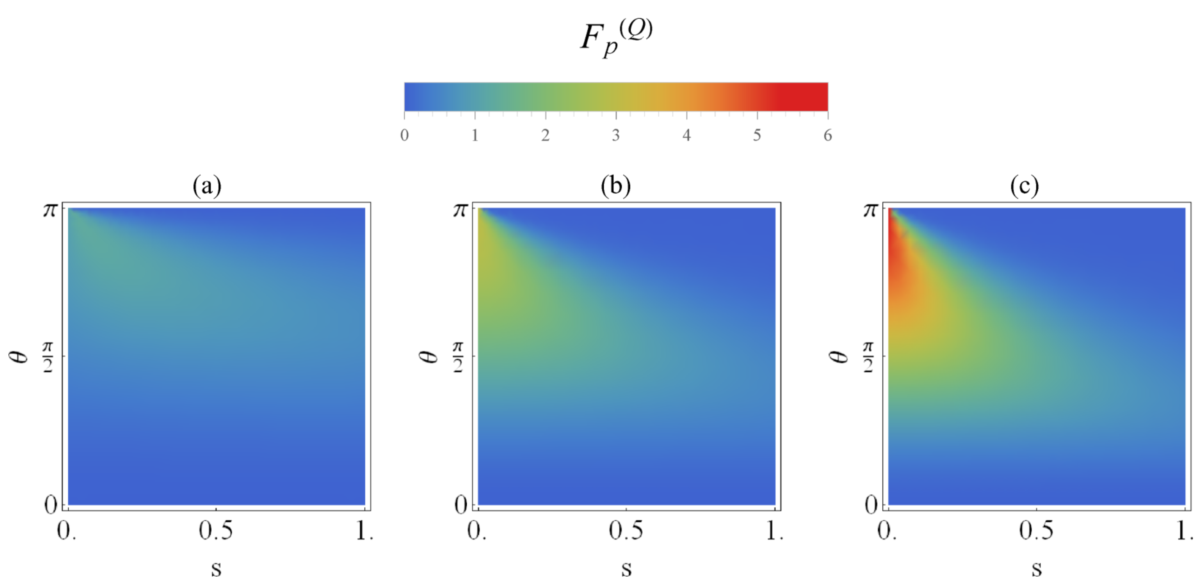

FIG. 5. (Color online) The quantum Fisher information in weak measurement regime vs measurement strength parameter s and θ for different weak values. Here, we take $\varphi = \pi/4$ and $N = 1$. (a) For coherent pointer state, $r = 1$, $\phi = \pi/4$. (b) Schrödinger cat pointer sates, $r = 1$, $\phi = \pi/4$. (c) For squeezed vacuum pointer state, $\eta = 1$, $\delta = \pi/4$.

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