Frequency shift between coherent superposition states induced by the Berry phase evolving linearly in time

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The frequency shift induced by the Berry phase between two coherent superposition states with $m' = -1$ and $m = 1$ was demonstrated under constant rotation of a magnetic field with frequency f for angle θ from the rotation axis. It was found that the frequency shift is $\nu = 2f - 2f\cos\theta$ for $0 \leqslant \theta \leqslant \pi/3$, $\nu = -2f\cos\theta$ for $\pi/3 < \theta < 2\pi/3$, and $\nu = -2f - 2f\cos\theta$ for $2\pi/3 \leqslant \theta \leqslant \pi$ in the case of the right-handed rotation. For the left-handed rotation, the frequency changes in the opposite sign. The frequency shift is zero at $\theta = 0, \pi/2$, and π, and it jumps by $2f$ in the vicinity of $\theta = \pi/3$ and $2\pi/3$. We confirm that the frequency shift is given by the time derivative of the Berry phase which does not depend on the sign of the g factor.

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I. INTRODUCTION

It is well known that a particle of any spin in an eigenstate of a magnetic field rotating around a circuit C will acquire the Berry phase γ in addition to the dynamical phase. The Berry phase for a whole rotation of the magnetic field is given by $\gamma = -m\Omega(C)$, where m is the spin component along the magnetic field and $\Omega(C)$ is the signed solid angle. If C is a right-handed circuit around a cone of semiangle θ from a rotation axis, the signed solid angle is $\Omega(C) = 2\pi(1 - \cos\theta)$ [1]. Wilczek and Zee [2] and Cina [3] have suggested that a manifestation of the geometric phase should be observed in the evolution of a coherent superposition of states m and m' for a whole rotation. Then, the difference between the Berry phase of the two states is given by $\Delta\gamma = -2\pi(m - m')(1 - \cos\theta)$. The cyclic Berry phase has been demonstrated in several such coherent superposition systems such as linearly polarized light in an optical fiber [4] and neutron spins in a helically wound magnetic field [5]. It was also predicted that such a Berry phase evolving linearly in time will induce a shift of the transition frequency between two states [6]. The frequency shift of the spectrum between two states under the rotation of a magnetic field is thought to be the time derivative of the Berry phase. Under constant rotation with frequency f, the frequency shifts by $\Delta\nu = -\Delta\gamma f/(2\pi)$ from the resonance frequency under a static magnetic field. This frequency shift was verified by adiabatic rotational splitting of the nuclear quadrupole resonance in the free-induction decay signals from a rotating sample by Tycko [7] and by the Fourier transform of the nuclear magnetic resonance spectra under a continuously rotating radio-frequency field by Suter *et al.* [8].

To verify such a phase shift or frequency shift experimentally, the extradynamical phase shift should be removed. For this purpose, a superposition state of the ground hyperfine levels of the $|F' = 1, m' = -1\rangle$ state with a negative Landé g factor to the $|F = 2, m = 1\rangle$ state with a positive g factor of an alkaline atom is suitable because it becomes a magnetic-field-insensitive transition at the magic magnetic field [9]. Previously, we developed an atom interferometer using the two-photon microwave–radio-frequency (MW-rf) transition from the $m' = -1$ to $m = 1$ states of Na atoms at the magic magnetic field of 67.7 μT and measured the Berry phase for a whole rotation of the magnetic field for $\pi/3 < \theta < 2\pi/3$. The obtained phase difference between the two states was $\Delta\gamma = 4\pi\cos\theta$ for the whole rotation of the magnetic field [10]. On the other hand, the frequency shift of the spectrum was $-2f\cos\theta$ under the constant rotation of the magnetic field at frequency f [11]. These results were different from the theoretical values of $-4\pi(1 - \cos\theta)$ and $2f - 2f\cos\theta$, respectively. Therefore, in our previous papers [10–13], we claimed that the signed solid angle for the state with a negative g factor was $\Omega - 4\pi$, when that for the state with a positive g factor was Ω, to explain our experimental results.

However, Welte *et al.* recently measured the Berry phase on superposition states of the magnetic quantum numbers $m' = -1$ with a negative g factor and $m = 1$ with a positive g factor of ^{87}Rb using free-induction decay [14] and verified that the Berry phase is independent of the sign of the g factor, as predicted from the equation derived by Berry [1]. They showed that the phase shift varies in accordance with the equation $\Delta\gamma = -4\pi + 4\pi\cos\theta$ for $0 \leqslant \theta \leqslant \pi/2$, where a reasonable boundary condition is $\Delta\gamma(\theta = 0) = 0$. Certainly, their phase shifts and others reported previously [4,5] were not different from those observed by us, except for those of 4π and 2π. Experimentally, an absolute phase difference of multiple of 2π cannot be distinguished. The solid angles that we claimed for a negative g factor were the same as those for a positive g-factor modulo 4π. Therefore, our claim that the Berry phase for a whole turn depended on the sign of the g factor was not reasonable. Thus, the problem is how to explain the frequency shift of $-2f\cos\theta$ under the constant rotation of a magnetic field at frequency f. Up to now, no other experiments have been carried out to investigate the dependence of the frequency shift on θ, although Simon *et al.* observed the frequency shift of a laser by changing the solid angle on the Poincaré sphere [15]. Tycko examined the frequency shift only at an angle of $\cos^2\theta = 1/3(\theta \sim 0.3\pi)$ as a function of the rotation frequency and explained the frequency shift as the time derivative of the phase modulo 2π [7]. Therefore, we aimed to demonstrate the behavior of the frequency shift for the whole angle from the rotation axis.

In this paper, we report the experimental results for the frequency shift for the angle $0 \leqslant \theta \leqslant \pi$ from the rotation axis and explain the frequency shift measured during the evolution of a coherent superposition of two different states evolving linearly in time on the basis of our experimental results.

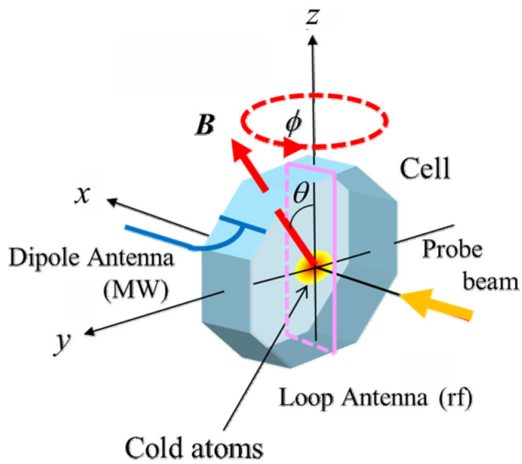

FIG. 1. (Color online) Experimental configuration. Cold atoms trapped in the cell are irradiated by the microwave (MW) and radio-frequency wave (rf) under the rotational magnetic field $\boldsymbol{B}$ with an angle θ from the z axis and a rotation angle ϕ.

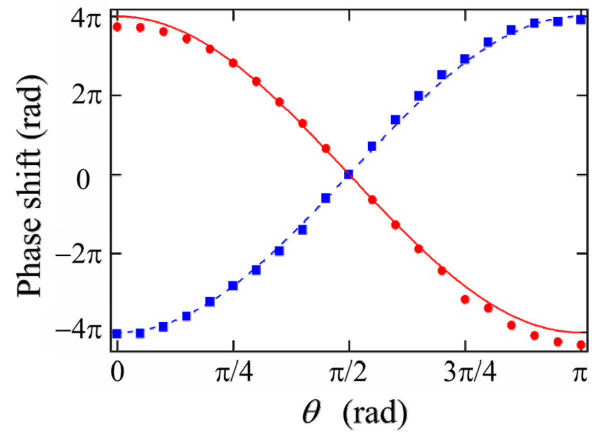

FIG. 2. (Color online) Observed Berry phase of spectrum from $m' = -1$ to $m = 1$ states for a whole rotation of the magnetic field using the atom interferometer as a function of angle θ from the rotation axis. The phase shift is measured on the basis of the phase at $\theta = \pi/2$. (●) Right-handed rotation. (■) Left-handed rotation. Solid curve and dashed curve are $4\pi\cos\theta$ and $-4\pi\cos\theta$, respectively.

II. MEASUREMENT OF FREQUENCY SHIFT

The present measurement method and apparatus were the same as those described in a previous paper [11], except for the direction of the rotation axis of the magnetic field. In the previous experiment, the spectra became smaller as θ decreases and disappeared at $\theta < \pi/3$, as the rotation axis was parallel to the propagation direction of the microwave (MW). Therefore, the rotation axis in this study was set to be the z axis, perpendicular to the propagation directions of MW and rf, as shown in Fig. 1. The dipole antenna produces the microwave with a frequency of hyperfine splitting and the loop antenna produces the radio frequency of around 420 kHz. The two-photon spectra were clearly observed throughout the range $0 \leqslant \theta \leqslant \pi$. The magnetic field in the x-y plane was rotated and the static magnetic field was applied parallel to the z axis. The total strength of the magnetic field was maintained at $67.7 \pm 1.0\,\mu$T by regulating the currents of three Helmholtz coils. The rotation frequency was 400 Hz and the pulse width of the MW-rf was 5 ms. The amplitudes of the MW and rf were regulated to get a good signal-to-noise ratio. Thus, we could obtain the spectrum of the transition from the $m = -1$ to $m = 1$ states (−1-1) with a width of 200 Hz.

Before the measurement of the frequency shift, we measured the phase shift of the spectrum from $m' = -1$ to $m = 1$ states for the whole rotation of the magnetic field using the atom interferometer comprised of two MW-rf pulses [10] The phase shift was measured on the basis of the phase at $\theta = \pi/2$ as a function of angle θ from the rotation axis, as shown in Fig. 2. Then, the phase shift for the right-handed rotation (or the left-handed rotation) varies monotonically in accordance with the function $4\pi\cos\theta$ (or $-4\pi\cos\theta$) for $0 \leqslant \theta \leqslant \pi$, which confirmed our previous result for $\pi/3 < \theta < 2\pi/3$ [10]. However, if we rewrite it on the basis of the phase at $\theta = 0$, it is in accordance with the function $-4\pi(1 - \cos\theta)$ as in Ref. [14]. Thus, there is no difference between them (modulo 4π).

Typical spectra of the −1-1 transition, together with those of the 0-0 transition as a reference, are shown in Fig. 3 for the case of right-handed rotation with different θ. Figure 3(a) shows the spectrum under the static magnetic field at $\theta = 0$ and other figures show spectra under the rotation of the magnetic field for different θ. From Figs. 3(a)–3(f), we confirm that the peak frequency of the 0-0 spectrum is almost constant within 50 Hz, but the sideband components with the rotation frequency of 400 Hz are generated on both sides as θ increases.

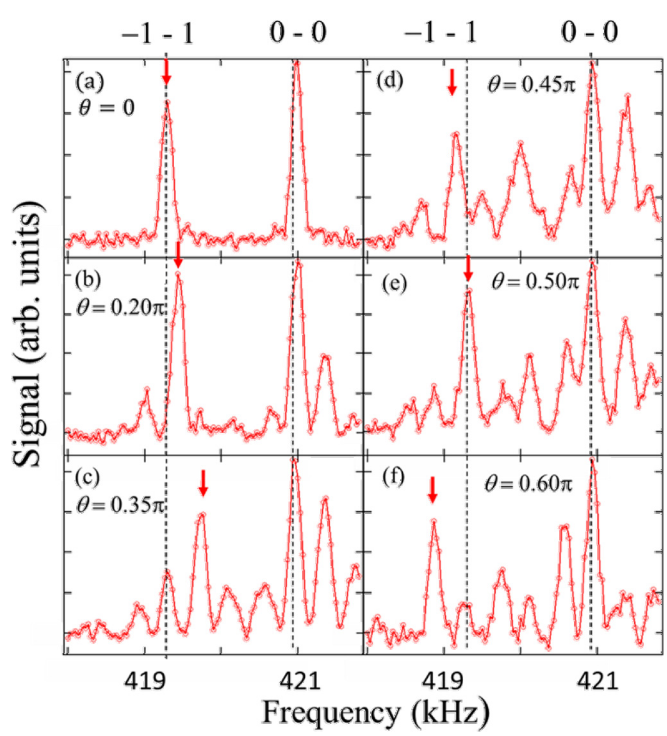

FIG. 3. (Color online) Spectrum from $m' = -1$ to $m = 1$ states (arrow) under the right-handed rotating magnetic field at 400 Hz with various angle θ from the rotation axis, together with that from $m' = 0$ to $m = 0$ states as a reference. The frequency shifts zigzag as θ increases. Dashed lines are their resonance frequencies at static magnetic field.

Thus, the spectrum is composed of primary component f_0 and sideband components f_n with the rotation frequency multiplied by a negative or positive integer n under the rotation of the magnetic field. On the other hand, under the rotation of the magnetic field, the primary component f_0 of the -1-1 line is shifted as θ increases. At $\theta = 0.2\pi$ [Fig. 3(b)], f_0 is shifted to a higher value together with the small sideband f_{-1}. At $\theta = 0.35\pi$ [Fig. 3(c)], the frequency undergoes a higher shift and a second-order sideband f_{-2} with a frequency 800 Hz lower than that of the primary component starts to oscillate. At 0.45π [Fig. 3(d)], the peak spectrum jumps to the second-order sideband f_{-2} from f_0. At 0.50π [Fig. 3(e)], the frequency of the peak sideband f_{-2} is again coincident with that under the static magnetic field and increases as θ increases. This jump of 800 Hz in the frequency of the -1-1 line to a lower sideband f_{-4} occurs again at 0.65π [Fig. 3(f)]. Above 0.75π, other sideband components except for f_{-4} disappeared and the frequency of f_{-4} increased to that under the static magnetic field.

The spectral strengths of the primary and sideband components f_n $(n = 0,-1,-2,-3,-4)$ in the -1-1 spectrum were summarized as shown in Fig. 4. The spectral strength was measured on the basis of the strength of the primary component in the 0-0 spectrum. The strength of f_0 becomes maximum in the -1-1 spectrum for $0 \leqslant \theta < \sim\pi/3$, but decreases as θ increases and disappears at θ larger than $\pi/2$. When the strength of f_0 decreases, the strength of f_{-1} starts increasing at first, but stops increasing at $\theta = \pi/3$. Meanwhile, the strength of f_{-2} increases more rapidly and becomes maximum at around $\theta = \pi/2$. The strength of f_{-4} starts oscillation at around $\theta = \pi/2$ and becomes maximum for $\sim 2\pi/3 < \theta \leqslant \pi$. The sidebands of f_{-1} and f_{-3} did not increase to the dominant component.

The frequency shift of the -1-1 line was summarized as a frequency shift of the dominant component for various θ in $0 \leqslant \theta \leqslant \pi$ at an interval of 0.05π, as shown in Fig. 5. The behavior of the frequency shift can now be clearly observed. For the right-handed rotation, the frequency of primary component f_0 is dominant and increases in accordance with the function $2f - 2f\cos\theta$ (solid line), in $0 \leqslant \theta < \sim\pi/3$. Above $\theta \sim \pi/3$, the sideband f_{-2} becomes the dominant component of the -1-1 line and the frequency shift jumps by $-2f$. It then increases as $-2f\cos\theta$, crossing zero at $\theta = \pi/2$, which confirms our previous result. However, the dominant component is again replaced by the sideband f_{-4} and the frequency shift jumps by $-2f$. Above $\theta = 2\pi/3$, f_{-4} becomes the dominant component and increases as $-2f - 2f\cos\theta$ and reaches 0 at $\theta = \pi$. For the left-handed rotation, the frequency changes in the opposite sign. Thus, the frequency shift induced by the Berry phase evolving linearly in time does not change monotonically. The behavior of the frequency shift can be divided roughly into three ranges of θ, namely $0 \leqslant \theta < \pi/3$, $\pi/3 < \theta < 2\pi/3$, and $2\pi/3 \leqslant \theta \leqslant \pi$.

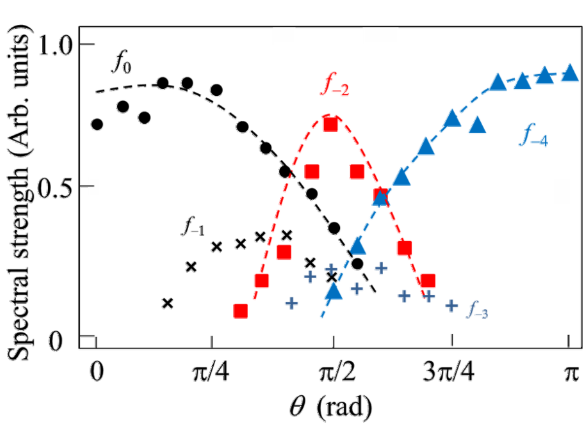

FIG. 4. (Color online) Strength of primary and sideband components in the -1-1 line as a function of θ. The dashed curves are guides for the eyes. f_0 is a carrier component and f_n is the sideband component with the rotation frequency multiplied by integer n.

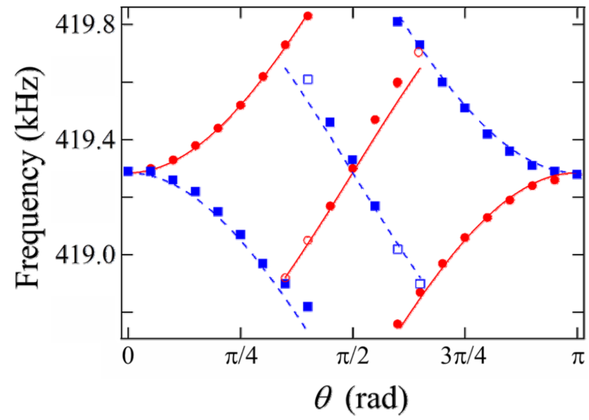

FIG. 5. (Color online) Observed frequency shift of spectrum from $m' = -1$ to $m = 1$ states as a function of angle θ from the rotation axis, which is induced by the Berry phase evolving linearly in time. Rotation frequency of the magnetic field is 400 Hz. (•, ○) Main and subpeaks for right-handed rotation. (■, □) Main and subpeaks for left-handed rotation. Solid lines and dashed lines are theoretical curves (see text).

III. DISCUSSION

The present results in the range of $0 \leqslant \theta < \pi/3$ confirmed the reasonable result that the frequency shift is zero at $\theta = 0$ and verified that the frequency shift of $\nu = 2f - 2f\cos\theta$ for the right-handed rotation is just the time derivative of the Berry phase for a whole rotation $-4\pi(1 - \cos\theta)$ during a period T. Then, how can we explain the frequency shifts observed in other ranges? These results will be explained as follows. Under the rotation of the magnetic field, the sideband components f_n with the rotation frequency multiplied by integer n are generated. The frequency of each sideband mode f_n changes according to $\nu = (2+n)f - 2f\cos\theta$. Namely, it is given by a time derivative of the Berry phase plus each sideband frequency of nf. In the sideband component with even n, the sideband component whose frequency is the nearest one to the resonance frequency under the static magnetic field becomes dominant. Therefore, in $\pi/3 < \theta < 2\pi/3$ the sideband component f_{-2} becomes dominant and the frequency shift changes according to $-2f\cos\theta$ and in $2\pi/3 \leqslant \theta \leqslant \pi$ the sideband component f_{-4} becomes dominant and frequency shift changes according to $-2f - 2f\cos\theta$. The boundary conditions are $\theta = \pi/3$ and $2\pi/3$.

For the left-handed rotation, the frequency of f_n changes according to $\nu = (-2+n)f + 2f\cos\theta$. Therefore, in the range of $0 \leqslant \theta \leqslant \pi/3$ the frequency of f_0 changes according to $\nu = -2f + 2f\cos\theta$, in $\pi/3 < \theta < 2\pi/3$ the sideband component f_2 becomes dominant and the frequency shift changes according to $2f\cos\theta$, and in $2\pi/3 \leqslant \theta \leqslant \pi$ the sideband component f_4 becomes dominant and the frequency shift changes according to $2f + 2f\cos\theta$. On the other hand, the strength of the sideband components of f_{-1}, f_{-3}, f_1, and f_3 does not become dominant, because the present case is $m - m' = 2(\text{modulo } 4\pi)$. The fact that the frequency shift is zero for the rotation at $\theta = \pi/2$ shows that the explanation of the experimental results in Ref. [16] is not reasonable. Thus, the frequency shift could be explained by the time derivative of the Berry phase, not depending on the sign of the g factor [10,11].

On the other hand, in the third range of $2\pi/3 \leqslant \theta \leqslant \pi$, the frequency shift $-2f - 2f\cos\theta$ can be also explained by the time derivative of the phase shift measured by the Berry phase for a whole rotation, if we define an axis with a semiangle of cone smaller than $\pi/2$ as the rotation axis. Then, the rotation axis is reversed to the $-z$ axis, the angle from the $-z$ axis becomes $\pi - \theta$, and the sense of the rotation of the magnetic field is reversed. The Berry phase becomes $4\pi(1+\cos\theta)$; thus the frequency shift is given by $\nu = -2f - 2f\cos\theta$.

IV. CONCLUSIONS

In conclusion, we measured the frequency shift between coherent superposition states induced by the Berry phase evolving linearly in time for a rotation of the magnetic field with angle θ from the rotation axis. The behavior of the frequency shift can be divided roughly into three ranges of the angle θ. In the case of the right-handed rotation, the frequency shift is $\nu = 2f - 2f\cos\theta$ for $0 \leqslant \theta < \sim\pi/3$, $\nu = -2f\cos\theta$ for about $\theta = \pi/2$, and $\nu = -2f - 2f\cos\theta$ for $\sim 2\pi/3 \leqslant \theta \leqslant \pi$. The frequency of all sideband components changes according to $(2+n)f - 2f\cos\theta$, which is a time derivative of the Berry phase plus each sideband frequency of nf. The sideband component whose frequency is the nearest to the resonance frequency under the static magnetic field becomes dominant in the sideband components with even n. The boundary conditions are $\theta = \pi/3$ and $2\pi/3$. As a result, the frequency shift of the spectrum under the rotation of a magnetic field at a constant rate is zero at $\theta = 0, \pi/2$, and π, and it jumps by $2f$ in the vicinity of $\theta = \pi/3$ and $2\pi/3$. These results can be explained by the time derivative of the Berry phase and we confirm independence of the sign of the g factor [1,14].

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