

**Repulsive Casimir-Polder potential by a negative reflecting surface**

Qi-Zhang Yuan

*State Key Laboratory of Precision Spectroscopy, Department of Physics, East China Normal University,  
500 Dongchuan Road, Shanghai 200241, People's Republic of China*

(Received 25 March 2015; published 28 July 2015)

We present a scheme to generate an all-range long repulsive Casimir-Polder potential between a perfect negative reflecting surface and a ground-state atom. The repulsive potential is stable and does not decay with time. The Casimir-Polder potential is proportional to  $z^{-2}$  at short atom-surface distances and to  $z^{-4}$  at long atom-surface distances. Because of these advantages, this potential can help in building quantum reflectors, quantum levitating devices, and waveguides for matter waves.

DOI: [10.1103/PhysRevA.92.012522](https://doi.org/10.1103/PhysRevA.92.012522)

PACS number(s): 31.30.jh, 12.20.-m, 34.35.+a, 42.50.Wk

**I. INTRODUCTION**

The quantum force induced by a vacuum electromagnetic field, which is named after Casimir [1,2], is becoming more and more important in developing nanoengineering [3–5]. As machine parts become as small as micro- and nanometer scales, these tiny objects will stick together because of the attractive Casimir force, and that can cause the failure of nanomachines. This negative influence of the Casimir force can be turned into a benefit if the attractive Casimir potential turns into repulsion. Casimir repulsion can keep nano-objects from sticking together and can make nanomachines work without friction. For this reason, there has been increasing interest in finding quantum repulsions.

Some schemes for producing the repulsive Casimir force, both theoretically [6–10] and experimentally [11,12], have been reported. By mediating a dielectric material 3 between material 1 and material 2, if the permittivities of them satisfy  $\epsilon_1 > \epsilon_3 > \epsilon_2$ , the interaction between material 1 and material 2 will be repulsion [11–13]. Quantum repulsion is also found between a perfectly conducting material ( $\epsilon \rightarrow \infty$ ) and an infinitely permeable material ( $\mu \rightarrow \infty$ ) [14,15], as well as between two parallel magnetodielectric plates [16]. Some authors suggest that an atom with anisotropic polarizability may feel repulsive Casimir-Polder (CP) force near conducting ellipsoids [17], edges [18], and dielectric materials having anisotropic permittivities [19].

The idea of using materials with abnormal electromagnetic properties to produce repulsive Casimir forces is attractive [20–22]. It has been published that quantum repulsion can be produced by left-handed material. By sandwiching an infinite perfect lens between two conducting plates, the Casimir potential between the two conducting plates can be turned into repulsion [23]. This scheme works in a certain interplate separation range which is decided by the thickness of the metamaterial; if the plates are moved farther away, the Casimir force becomes attractive again. Similarly, a repulsive resonant CP interaction between an excited atom and a perfect mirror can also be achieved by covering the mirror with a layer of left-handed material [24], but this resonant interaction decays with time because of spontaneous radiation.

Negative reflection, which is very different from negative refraction, is another kind of amazing abnormal phenomenon and has been found in some situations. Negative reflection means the reflected beam and the incident light are at the same

side of the interface normal, and this phenomenon has been reportedly achieved in a uniaxial medium [25], Faraday chiral medium [26], photonic crystals [27], hybrid metamaterial slab [28], and ferrite films [29]. Motivated by the successes in using metamaterials to achieve quantum repulsions and the discovery of negative reflections, we theoretically study the potential for producing a repulsive Casimir force by electromagnetic negative reflecting materials.

In this paper, we present a proposal to investigate the CP interaction between a negative reflecting surface and a two-level atom in the ground state. We assume the negative reflecting surface is “perfect” which means, for an incident beam of plane wave with any frequency and polarization, the reflected light is still plane wave and its wave vector is exactly opposite that of the incident wave. The reported negative reflections are not so “perfect” yet, but we think, with the developments in artificial materials, these good properties will be gradually approached. Quantum electrodynamics is employed in this theoretical research. We use the Wigner-Weisskopf method to deal with the atom-reservoir interaction problem and calculate the interaction energy. The result we get shows that quantum repulsion can be achieved by negative reflecting material. For the atom that is in the ground state, the CP potential does not decay with time and is repulsive in all ranges of atom-plate separation. With these features, negative reflecting materials have the potential for being used as atomic mirrors and atomic waveguides and being employed in levitating, trapping, and even cooling particles.

This paper is organized as follows. The formalism of the interaction between a ground-state atom and an electromagnetic vacuum is reviewed in Sec. II; then we introduce the negative reflection in Sec. III. In Sec. IV, we analyze the dynamical evolution of the atom-vacuum system, followed by Sec. V in which we give out the analytical expression of the CP interaction energy and the conditions to produce a repulsive CP potential. Some properties of the potential are discussed in Sec. VI; then we make conclusions in Sec. VII.

**II. BASIC FORMULAS**

In this section, the basic formulas describing the atom-reservoir interaction are reviewed.

As shown schematically in Fig. 1, a two-level atom is located above the negative reflecting surface. The position of the atom is  $z\hat{e}_z$  and  $\hat{e}_z$  is the unit vector along the  $z$  direction.

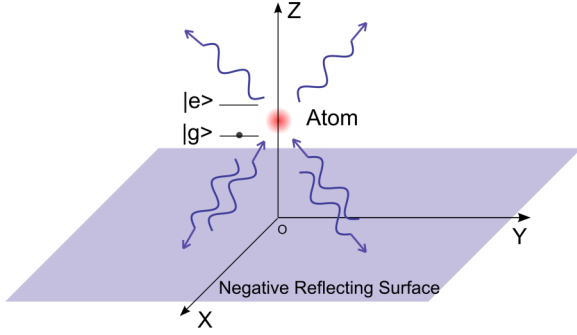


FIG. 1. (Color online) A two-level atom in the ground state is located above the negative reflecting surface, and the atom-surface distance is expressed by  $z$ .

For the surface in the  $x$ - $y$  plane,  $z$  represents the distance between the atom and the surface. To describe the atom-surface interaction in a Schrödinger picture, we use the Schrödinger equation

$$i\hbar \frac{\partial |\psi\rangle}{\partial t} = \hat{H} |\psi\rangle, \quad (1)$$

in which  $|\psi\rangle$  is the quantum state of the atom-reservoir system and  $\hat{H}$  is the total Hamiltonian.

The total Hamiltonian  $\hat{H}$  reads

$$\hat{H} = \hat{H}_A + \hat{H}_{\text{c.m.}} + \hat{H}_F + \hat{H}_{\text{INT}}. \quad (2)$$

In detail, the atomic Hamiltonian is

$$\hat{H}_A = \sum_{i=e,g} \hbar\omega_i \sigma_{ii}, \quad (3)$$

where  $\omega_i$  are the eigenfrequencies of atomic state  $|i\rangle$  and  $\sigma_{ii} = |i\rangle\langle i|$  are the atomic flip operators. The atomic center-of-mass motion Hamiltonian is written

$$\hat{H}_{\text{c.m.}} = \frac{\hat{\mathbf{p}}^2}{2m_a}, \quad (4)$$

in which  $\hat{\mathbf{p}}$  is the atomic center-of-mass momentum operator and  $m_a$  is the mass of the atom. The electromagnetic Hamiltonian can be given as

$$\hat{H}_F = \sum_{k\lambda} \hbar\omega_k \left( \hat{a}_{k\lambda}^\dagger \hat{a}_{k\lambda} + \frac{1}{2} \right), \quad (5)$$

where  $\hat{a}_{k\lambda}^\dagger$  and  $\hat{a}_{k\lambda}$  are respectively the creation and annihilation operators for a photon of mode  $k\lambda$ ;  $\omega_k$  are the light frequencies. And the atom-field interaction Hamiltonian in dipole approximation can be expressed as [30]

$$\hat{H}_{\text{INT}} = -\hat{\mathbf{d}}_a \cdot \left( \hat{\mathbf{E}} + \frac{1}{m_a} \hat{\mathbf{p}} \times \hat{\mathbf{B}} \right), \quad (6)$$

where  $\hat{\mathbf{d}}_a$  is the dipole momentum operator of the atom, while  $\hat{\mathbf{E}}$  and  $\hat{\mathbf{B}}$  are electrical and magnetic operators, respectively. For the gas at room temperature and cold-atom ensemble, the average velocity of atoms is much slower than light speed in vacuum ( $\mathbf{p}/m_a \ll c$ ), so the contribution from  $\hat{\mathbf{B}}$  in Eq. (6) is small enough to be dropped. So we have

$$\hat{H}_{\text{INT}} = -\hat{\mathbf{d}}_a \cdot \hat{\mathbf{E}}(\mathbf{r}_a). \quad (7)$$

For this reason, the atomic internal dynamics and the atomic center-of-mass motion can be decoupled, and the atom-surface interaction is independent of the center-of-mass velocity of the atom. In this paper, we do not discuss the translational motion of the atom for simplicity, and the atomic position  $\mathbf{r}_a$  is treated as a classical variable. According to the analysis above, the total Hamiltonian Eq. (2) reduces to

$$\hat{H} = \hat{H}_A + \hat{H}_F + \hat{H}_{\text{INT}}, \quad (8)$$

and state  $|\psi\rangle$  can be written as

$$|\psi\rangle = C_g |g\rangle |0\rangle + \sum_{k\lambda} D_{e,k\lambda} |e\rangle |k\lambda\rangle, \quad (9)$$

where  $|g\rangle |0\rangle$  indicates the atom is in the ground state with no photon existing, while  $|e\rangle |k\lambda\rangle$  means the atom is excited and a photon with wave vector  $\mathbf{k}$  and polarization  $\lambda$  exists. By solving Eq. (1), the atom-surface interaction energy can be given by

$$V_{\text{int}} = \langle \hat{H}_{\text{INT}} \rangle = \langle \psi | \hat{H}_{\text{INT}} | \psi \rangle. \quad (10)$$

### III. NEGATIVE REFLECTION

In this section, the method we used to describe the negative reflection of electromagnetic waves is introduced. As shown in Fig. 2, the negative reflecting surface is located in the  $x$ - $y$  plane, and the region of  $z > 0$  is a vacuum. A beam of unit plane wave with determined wave vector  $\mathbf{k}$  and polarization  $\lambda$ , which is incident on the negative reflecting surface, can be expressed by

$$\mathbf{W}(\mathbf{k}\lambda, \mathbf{r}) = \hat{\mathbf{e}}_{k\lambda} e^{i\mathbf{k}\cdot\mathbf{r}} \quad (\lambda = \perp, \parallel). \quad (11)$$

We assume that the negative reflective surface is perfect, so the reflected light is still a plane wave, with its wave vector exactly opposite that of the incident wave ( $\mathbf{k} \rightarrow -\mathbf{k}$ ). To express the reflected light, we structure a phenomenological scattering matrix

$$\mathbf{S}(\mathbf{k}) = \begin{bmatrix} g_{\perp \rightarrow \perp}(\mathbf{k}) & g_{\perp \rightarrow \parallel}(\mathbf{k}) \\ g_{\parallel \rightarrow \perp}(\mathbf{k}) & g_{\parallel \rightarrow \parallel}(\mathbf{k}) \end{bmatrix}, \quad (12)$$

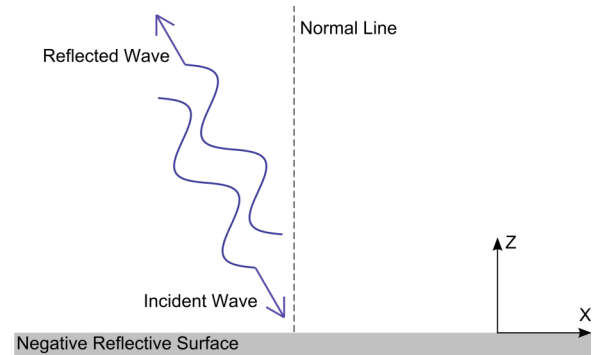


FIG. 2. (Color online) A scheme for the negative reflection of an electromagnetic wave. For a negative reflecting surface, the incident wave and reflected wave are at the same side of the normal line. When the wave vector of the reflected wave is exactly opposite that of the incident wave, the negative reflection is called perfect.

and then the expression of the reflected wave can be written as

$$\mathbf{S}(\mathbf{k}\lambda, \mathbf{r}) = \sum_{\lambda'=\perp, \parallel} g_{\lambda \rightarrow \lambda'}(\mathbf{k}) \hat{\mathbf{e}}_{k\lambda'} e^{-i\mathbf{k}\cdot\mathbf{r}} \quad (\lambda = \perp, \parallel), \quad (13)$$

where  $\hat{\mathbf{e}}_{k\perp}$  and  $\hat{\mathbf{e}}_{k\parallel}$  are respectively unit vectors perpendicular and parallel to the incident plane. We have introduced complex reflection coefficients  $g_{\lambda \rightarrow \lambda'}(\mathbf{k})$  here to describe the amplitude reflection of the electric field, and  $\lambda \rightarrow \lambda'$  implies the polarization change or conservation between the incident and reflected waves. For the conservation of energy, the reflection coefficients must satisfy  $\sum_{\lambda, \lambda'=\perp, \parallel} |g_{\lambda \rightarrow \lambda'}(\mathbf{k})|^2 \leq 1$ . If the absorption of the incident waves is negligible, “=” will stand in the inequality, and otherwise “<” will do. Equations (12) and (13) imply a broadband optical negative reflection which is difficult to achieve nowadays. But, the response of the atom to an electromagnetic field is limited by the lifetime of the atomic excited state, so the negative reflection only needs to be achieved for a wave band that covers the response band of the atom. In this paper, we assume this condition is satisfied and further assume negative reflection is achieved for broad incident angles in this wave band. According to Eqs. (11) and (13), the total electric field in the region of  $z > 0$  can be expressed as

$$\begin{aligned} \mathbf{E}(\mathbf{r}) &= \sum_{k\lambda} E_{k\lambda} \mathbf{M}(\mathbf{k}\lambda, \mathbf{r}) \\ &= \sum_{k\lambda} E_{k\lambda} [\mathbf{W}(\mathbf{k}\lambda, \mathbf{r}) + \mathbf{S}(\mathbf{k}\lambda, \mathbf{r})], \end{aligned} \quad (14)$$

in which  $E_{k\lambda}$  are electric wave amplitudes. To quantize the field, we can replace the amplitudes  $E_{k\lambda}$  in Eq. (14) with annihilation operators and normalization constants. The electric field operator can be written as

$$\begin{aligned} \hat{\mathbf{E}}(\mathbf{r}) &= \sum_{k\lambda} \hat{\mathbf{E}}(\mathbf{k}\lambda, \mathbf{r}) \\ &= \sum_{k\lambda} A_k \hat{a}_{k\lambda} \mathbf{M}(\mathbf{k}\lambda, \mathbf{r}) + \text{H.c.} \end{aligned} \quad (15)$$

We assume the absorption of incident light by the surface is negligible ( $\sum_{\lambda, \lambda'=\perp, \parallel} |g_{\lambda \rightarrow \lambda'}(\mathbf{k})|^2 = 1$ ), so the normalization constant  $A_k = \sqrt{\hbar\omega_k/\epsilon_0 V}/2$ .  $\hbar$  is the modified Plank constant,  $\omega_k$  is the angular frequency of the photon,  $\epsilon_0$  is the permittivity of the vacuum, and  $V$  is the volume of quantization.

#### IV. DYNAMICAL ANALYSIS

By considering electric dipole interaction only, the interaction Hamiltonian should be written from Eq. (7) as

$$\begin{aligned} \hat{H}_{\text{INT}} &= - \sum_{ij} \sum_{k\lambda} A_k \hat{\sigma}_{ij} \hat{a}_{k\lambda} \mathbf{d}_{ij} \cdot \mathbf{M}(\mathbf{k}\lambda, \mathbf{r}) \\ &\quad - \sum_{ij} \sum_{k\lambda} A_k^* \hat{a}_{k\lambda}^\dagger \hat{\sigma}_{ij} \mathbf{d}_{ij} \cdot \mathbf{M}^*(\mathbf{k}\lambda, \mathbf{r}), \end{aligned} \quad (16)$$

where  $A_k = \sqrt{\hbar\omega_k/\epsilon_0 V}/2$  and  $\mathbf{d}_{ij}$  ( $i, j = e, g$ ) are the dipole transition matrix elements between atomic states  $i$  and  $j$ . According to Eq. (13), the expression of mode function  $\mathbf{M}(\mathbf{k}\lambda, \mathbf{r})$  depends on the electromagnetic reflecting properties

of the surface. The atomic flip operators  $\hat{\sigma}_{ij}$  are defined by

$$\hat{\sigma}_{ij} = |i\rangle\langle j| (i, j = e, g). \quad (17)$$

Derived from Eq. (1), the equations of motion for  $C_g(t)$  and  $D_{e, k\lambda}(t)$  can be given as

$$\dot{C}_g(t) = \frac{i}{\hbar} \sum_{k\lambda} D_{e, k\lambda}(t) A_k \mathbf{d}_{ge} \cdot \mathbf{M}(\mathbf{k}\lambda, \mathbf{r}), \quad (18)$$

$$\begin{aligned} \dot{D}_{e, k\lambda}(t) &= -i(\omega_0 + \omega_k) D_{e, k\lambda}(t) \\ &\quad + \frac{i}{\hbar} C_g(t) A_k^* \mathbf{d}_{eg} \cdot \mathbf{M}^*(\mathbf{k}\lambda, \mathbf{r}), \end{aligned} \quad (19)$$

where  $\omega_0 = \omega_e - \omega_g$  is the angular transition frequency between atomic states  $|g\rangle$  and  $|e\rangle$ . To solve these equations, we first derive the formal solution of Eq. (19):

$$\begin{aligned} D_{e, k\lambda}(t) &= D_{e, k\lambda}(0) e^{-i(\omega_k + \omega_0)t} + \frac{i}{\hbar} A_k^* \mathbf{d}_{eg} \cdot \mathbf{M}^*(\mathbf{k}\lambda, \mathbf{r}) \\ &\quad \times \int_0^t d\tau C_g(\tau) e^{-i(\omega_k + \omega_0)(t-\tau)}. \end{aligned} \quad (20)$$

For the atom initially in the ground state, we set  $D_{e, k\lambda}(0)$  to be 0. Furthermore, the integral can be dealt with via the Wigner-Weisskopf method,

$$\int_0^t d\tau C_g(\tau) e^{-i(\omega_k + \omega_0)(t-\tau)} \approx C_g(t) \left[ -\frac{i}{(\omega_k + \omega_0)} \right]. \quad (21)$$

Making use of relationship (21), Eq. (20) can be rewritten as

$$D_{e, k\lambda}(t) = \frac{A_k^* \mathbf{d}_{eg} \cdot \mathbf{M}^*(\mathbf{k}\lambda, \mathbf{r})}{\hbar(\omega_k + \omega_0)} C_g(t), \quad (22)$$

and by substituting Eq. (22) into Eq. (18), we can get the equation about  $C_g(t)$ :

$$\frac{d}{dt} C_g(t) = \frac{i}{\hbar^2} \sum_{k\lambda} |A_k|^2 \frac{\mathbf{d}_{eg} \cdot \mathbf{M}^*(\mathbf{k}\lambda, \mathbf{r}) \mathbf{d}_{ge} \cdot \mathbf{M}(\mathbf{k}\lambda, \mathbf{r})}{\omega_k + \omega_0} C_g(t). \quad (23)$$

It is not difficult to find the solution

$$C_g(t) = C_g(0) e^{i\alpha t}, \quad (24)$$

in which

$$\alpha = \frac{1}{\hbar^2} \sum_{k\lambda} |A_k|^2 \frac{G(\mathbf{k}\lambda, \mathbf{r})}{\omega_k + \omega_0}, \quad (25)$$

and  $G(\mathbf{k}\lambda, \mathbf{r}) = \mathbf{d}_{eg} \cdot \mathbf{M}^*(\mathbf{k}\lambda, \mathbf{r}) \mathbf{M}(\mathbf{k}\lambda, \mathbf{r}) \cdot \mathbf{d}_{ge}$ .

Real coefficient  $\alpha$  is an energy-level shift which decides the interaction between the atom and the surface. According to Eq. (25), the value of  $\alpha$  depends on the properties of the negative reflecting surface, in other words, the complex reflection coefficients  $g_{\lambda \rightarrow \lambda'}$ . For this reason, to produce a repulsive CP force, the values of  $g_{\lambda \rightarrow \lambda'}$  need to be properly chosen. In the next section, we discuss the properties of the atom-surface interaction potential and give the proper choice of  $g_{\lambda \rightarrow \lambda'}$  to produce a repulsive CP force.

### V. GENERATION OF REPULSIVE CASIMIR-POLDER POTENTIAL

Obtained from Eqs. (16) and (24), the atom-surface interaction potential is proportional to level shift  $\alpha$ :

$$V_{\text{int}} = \langle \hat{H}_{\text{INT}} \rangle = -2\hbar\alpha. \quad (26)$$

The atom is in a half-free space, so to calculate Eq. (26) we go to the continuum limit

$$\sum_{\mathbf{k}} \rightarrow \frac{V}{(2\pi)^3} \int d^3k. \quad (27)$$

Under this limit, the expression of level shift  $\alpha$  can be rewritten as

$$\alpha = \frac{1}{4\hbar\epsilon_0(2\pi)^3} \int d^3k \frac{k}{k_0 + k} \sum_{\lambda} G(\mathbf{k}\lambda, \mathbf{r}). \quad (28)$$

To calculate the integral, we introduce a spherical coordinate system in the  $\mathbf{k}$  space for convenience:

$$\alpha = \frac{1}{4\hbar\epsilon_0(2\pi)^3} \int_0^{\infty} dk \frac{k^3}{k_0 + k} \int_{\Omega} d\Omega \sum_{\lambda} G(\mathbf{k}\lambda, \mathbf{r}), \quad (29)$$

where  $\int_{\Omega} d\Omega$  represents an angular integral.

Making use of Eqs. (11) and (13), the integrand  $\sum_{\lambda} G(\mathbf{k}\lambda, \mathbf{r})$  can be expressed by

$$\sum_{\lambda} G(\mathbf{k}\lambda, \mathbf{r}) = \mathbf{d}_{eg} \cdot \left[ \begin{array}{c} \mathbf{I} - \hat{\mathbf{k}}\hat{\mathbf{k}} \\ + \sum_{\lambda, \lambda'} (g_{\lambda \rightarrow \lambda'} \hat{\mathbf{e}}_{\mathbf{k}\lambda} \hat{\mathbf{e}}_{\mathbf{k}\lambda'} e^{-i2\mathbf{k}\cdot\mathbf{r}} + g_{\lambda \rightarrow \lambda'}^* \hat{\mathbf{e}}_{\mathbf{k}\lambda'} \hat{\mathbf{e}}_{\mathbf{k}\lambda} e^{i2\mathbf{k}\cdot\mathbf{r}}) \\ + \sum_{\lambda, \lambda'', \lambda'} g_{\lambda \rightarrow \lambda''}^* g_{\lambda \rightarrow \lambda'} \hat{\mathbf{e}}_{\mathbf{k}\lambda''} \hat{\mathbf{e}}_{\mathbf{k}\lambda'} \end{array} \right] \cdot \mathbf{d}_{ge}, \quad (30)$$

in which  $\mathbf{I}$  is a unit matrix and  $\hat{\mathbf{k}} = \mathbf{k}/k$  is the unit vector pointing towards the direction of the wave vector. According to Eq. (30),  $\alpha$  can be separated into two parts,

$$\alpha = \beta_1 + \beta_2, \quad (31)$$

where

$$\beta_1 = \frac{1}{4\hbar\epsilon_0(2\pi)^3} \int_0^{\infty} dk \frac{k^3}{k_0 + k} \int_{\Omega} d\Omega \mathbf{d}_{eg} \cdot \left\{ \sum_{\lambda, \lambda'} [g_{\lambda \rightarrow \lambda'}(\mathbf{k}) \hat{\mathbf{e}}_{\mathbf{k}\lambda} \hat{\mathbf{e}}_{\mathbf{k}\lambda'} e^{-i2\mathbf{k}\cdot\mathbf{r}} + g_{\lambda \rightarrow \lambda'}^*(\mathbf{k}) \hat{\mathbf{e}}_{\mathbf{k}\lambda'} \hat{\mathbf{e}}_{\mathbf{k}\lambda} e^{i2\mathbf{k}\cdot\mathbf{r}}] \right\} \cdot \mathbf{d}_{ge} \quad (32)$$

and

$$\beta_2 = \frac{1}{4\hbar\epsilon_0(2\pi)^3} \int_0^{\infty} dk \frac{k^3}{k_0 + k} \int_{\Omega} d\Omega \mathbf{d}_{eg} \cdot \left[ \mathbf{I} - \hat{\mathbf{k}}\hat{\mathbf{k}} + \sum_{\lambda, \lambda'', \lambda'} g_{\lambda \rightarrow \lambda''}^* g_{\lambda \rightarrow \lambda'}(\mathbf{k}) \hat{\mathbf{e}}_{\mathbf{k}\lambda''} \hat{\mathbf{e}}_{\mathbf{k}\lambda'} \right] \cdot \mathbf{d}_{ge}. \quad (33)$$

Here  $\beta_2$  is the vacuum Lamb shift. The expression of Eq. (33) is obviously divergent but can be dealt with via Bethe's method. As  $\beta_2$  is position independent, it will not contribute any mechanical force. For this reason,  $\beta_2$  can be ignored here, and we do not discuss it below.

The integrand in Eq. (32) oscillates and diverges as  $k \rightarrow \infty$ , which creates difficulties in calculating  $\beta_1$ . To overcome the difficulties, we can renormalize the integral by introducing a convergence factor  $e^{-\gamma|k-k_0|}$  and taking the limit  $\gamma \rightarrow 0^+$  after finishing all calculations [31]. Thus, we get

$$\beta_1 = \frac{1}{4\hbar\epsilon_0(2\pi)^3} \lim_{\gamma \rightarrow 0^+} \int_0^{\infty} dk \frac{k^3}{k_0 + k} e^{-\gamma|k-k_0|} \int_{\Omega} d\Omega \mathbf{d}_{eg} \cdot \left\{ \sum_{\lambda, \lambda'} [g_{\lambda \rightarrow \lambda'}(\mathbf{k}) \hat{\mathbf{e}}_{\mathbf{k}\lambda} \hat{\mathbf{e}}_{\mathbf{k}\lambda'} e^{-i2\mathbf{k}\cdot\mathbf{r}} + g_{\lambda \rightarrow \lambda'}^*(\mathbf{k}) \hat{\mathbf{e}}_{\mathbf{k}\lambda'} \hat{\mathbf{e}}_{\mathbf{k}\lambda} e^{i2\mathbf{k}\cdot\mathbf{r}}] \right\} \cdot \mathbf{d}_{ge}. \quad (34)$$

The physical idea behind Eq. (34) is that the atom can only respond to a limited wave band whose center frequency is  $\omega_0$ . Furthermore, as discussed in Sec. III, we assume that for the very limited atomic response wave band, a wide-angle negative reflection is achieved, and the variations of  $g_{\lambda \rightarrow \lambda'}(\mathbf{k})$  with respect to  $\mathbf{k}$  are much slower than that of the convergence factor  $e^{-\gamma|k-k_0|}$ . For this reason, the reflection coefficients  $g_{\lambda \rightarrow \lambda'}(\mathbf{k})$  can be replaced with  $g_{\lambda \rightarrow \lambda'}(k_0)$  in Eq. (34) as

$$\beta_1 = \frac{1}{4\hbar\epsilon_0(2\pi)^3} \lim_{\gamma \rightarrow 0^+} \int_0^{\infty} dk \frac{k^3}{k_0 + k} e^{-\gamma|k-k_0|} \int_{\Omega} d\Omega \mathbf{d}_{eg} \cdot \left\{ \sum_{\lambda, \lambda'} [g_{\lambda \rightarrow \lambda'}(k_0) \hat{\mathbf{e}}_{\mathbf{k}\lambda} \hat{\mathbf{e}}_{\mathbf{k}\lambda'} e^{-i2\mathbf{k}\cdot\mathbf{r}} + g_{\lambda \rightarrow \lambda'}^*(k_0) \hat{\mathbf{e}}_{\mathbf{k}\lambda'} \hat{\mathbf{e}}_{\mathbf{k}\lambda} e^{i2\mathbf{k}\cdot\mathbf{r}}] \right\} \cdot \mathbf{d}_{ge}. \quad (35)$$

Now the integrand is well behaved.

To calculate the angular integrals in Eq. (35), we express the unit vectors by

$$\begin{aligned} \hat{\mathbf{k}} &= \sin\theta \cos\phi \hat{\mathbf{e}}_x + \sin\theta \sin\phi \hat{\mathbf{e}}_y + \cos\theta \hat{\mathbf{e}}_z, \\ \hat{\mathbf{e}}_{\mathbf{k}\parallel} &= -\cos\theta \cos\phi \hat{\mathbf{e}}_x - \cos\theta \sin\phi \hat{\mathbf{e}}_y + \sin\theta \hat{\mathbf{e}}_z, \\ \hat{\mathbf{e}}_{\mathbf{k}\perp} &= \sin\phi \hat{\mathbf{e}}_x - \cos\phi \hat{\mathbf{e}}_y, \end{aligned} \quad (36)$$

and, as shown in Fig. 1, we set the position of the atom to be  $\mathbf{r} = z \hat{\mathbf{e}}_z$ ;  $\hat{\mathbf{e}}_x$ ,  $\hat{\mathbf{e}}_y$ , and  $\hat{\mathbf{e}}_z$  are unit vectors along the  $x$ ,  $y$ , and  $z$  directions, respectively. Thus, we have

$$\beta_1 = \frac{1}{4\hbar\epsilon_0(2\pi)^2} \lim_{\gamma \rightarrow 0^+} \int_0^\infty k^3 dk e^{-\gamma|k-k_0|} \left\{ \begin{aligned} & \left[ \frac{\sin(2kr)}{2kr} + \frac{2\cos(2kr)}{(2kr)^2} - \frac{2\sin(2kr)}{(2kr)^3} \right] \frac{d_x^2 + d_y^2}{k_0 + k} \\ & - \left[ \frac{4\cos(2kr)}{(2kr)^2} - \frac{4\sin(2kr)}{(2kr)^3} \right] \frac{d_z^2}{k_0 + k} \end{aligned} \right\} (g_{\parallel \rightarrow \parallel} + g_{\parallel \rightarrow \parallel}^*) \\ + \frac{1}{4\hbar\epsilon_0(2\pi)^2} \lim_{\gamma \rightarrow 0^+} \int_0^\infty k^3 dk e^{-\gamma|k-k_0|} \frac{\sin(2kr)}{2kr} \frac{d_x^2 + d_y^2}{k_0 + k} (g_{\perp \rightarrow \perp} + g_{\perp \rightarrow \perp}^*), \quad (37)$$

in which  $d_x$ ,  $d_y$ , and  $d_z$  are respectively the  $x$ -,  $y$ -, and  $z$ -direction components of the dipolar transition momentum  $\mathbf{d} = \mathbf{d}_{eg} = \mathbf{d}_{ge}$ ;  $g_{\lambda \rightarrow \lambda}(k_0)$  are written as  $g_{\lambda \rightarrow \lambda}$  for simplicity.

Equation (37) can be calculated by the residue theorem and becomes

$$\beta_1 = -\frac{k_0}{4\hbar\epsilon_0(2\pi)^2} \int_0^\infty du \frac{u^3 e^{-2uz}}{k_0^2 + u^2} \times \left[ \begin{aligned} & f_1(uz, \mathbf{d})(g_{\parallel \rightarrow \parallel} + g_{\parallel \rightarrow \parallel}^*) \\ & + f_2(uz, \mathbf{d})(g_{\perp \rightarrow \perp} + g_{\perp \rightarrow \perp}^*) \end{aligned} \right], \quad (38)$$

in which

$$f_1(uz, \mathbf{d}) = \left[ \frac{1}{2uz} + \frac{2}{(2uz)^2} + \frac{2}{(2uz)^3} \right] (d_x^2 + d_y^2) - \left[ \frac{4}{(2uz)^2} + \frac{4}{(2uz)^3} \right] d_z^2, \\ f_2(uz, \mathbf{d}) = \frac{1}{2uz} (d_x^2 + d_y^2).$$

This result is very different from that in the standard CP scenario (the interaction between a two-level atom and a perfect mirror) [2]. By interacting with a perfect mirror, the level shift of the atomic ground state is

$$\alpha_{\text{conducting}} = \frac{k_0}{\hbar\epsilon_0(2\pi)^2} \int_0^\infty du \frac{u^3 e^{-2uz}}{k_0^2 + u^2} \times \left\{ \begin{aligned} & \left[ \frac{1}{2uz} + \frac{1}{(2uz)^2} + \frac{1}{(2uz)^3} \right] (d_x^2 + d_y^2) \\ & + 2 \left[ \frac{1}{(2uz)^2} + \frac{1}{(2uz)^3} \right] d_z^2 \end{aligned} \right\}, \quad (39)$$

which is positive definite. Associated with the minus sign in Eq. (26), we can see the standard CP potential is always attractive. But in the negative reflection case discussed in this paper, the sign of the level shift depends on the signs of the reflection coefficients and the polarization of the atom. This feature makes it possible to tune the CP potential into repulsion.

According to the integral in Eq. (32), the integrand contains the product of the polarization unit vectors of the incident waves and the scattered waves. That means it is the interferences of the incident waves and scattered waves that do contribute to the CP potential. We find that, in the perfect negative reflection case, the polarization directions of incident waves and scattered waves are always parallel to each other, whereas in the standard CP case, they are not. Because of these different reflecting properties, the angular integrals give different results in the negative reflection case and the standard

CP case. This is the origin of the difference between Eqs. (38) and (39).

Now we consider a case in which the atom is isotropically polarized. For the atom in the ground state, this choice is reasonable. Denoting  $d = |\mathbf{d}|$  and setting  $d_x^2 = d_y^2 = d_z^2 = d^2/3$ , then Eq. (38) can be calculated,

$$\beta_1 = -\frac{d^2 k_0^3}{\hbar\epsilon_0 24\pi^2} \frac{\text{Re}(g_{\perp \rightarrow \perp} + g_{\parallel \rightarrow \parallel})}{\bar{z}} \int_0^\infty d\xi \frac{\xi^2 e^{-2\xi\bar{z}}}{1 + \xi^2}, \quad (40)$$

in which  $k_0 \equiv \omega_0/c$ , and  $c$  is light speed in vacuum. The position of the atom is expressed by dimensionless values  $\bar{z} \equiv k_0 z$ .

By checking the expression of term  $\beta_1$ , we find it is  $z$  dependent, and decided by reflection coefficients  $g_{\perp \rightarrow \perp}$  and  $g_{\parallel \rightarrow \parallel}$ . This term represents that the interaction between the atom and the vacuum electromagnetic field is modified by the negative reflecting surface. When the relative distance between the atom and the surface changes, the ground-state level of the atom-surface-vacuum interaction system will be shifted, which leads to a CP force between the atom and the surface. The vacuum electromagnetic reservoir acts as a medium to produce this force.

Up to here, we can give out the analytic expression of the CP potential energy. By defining

$$I(\bar{z}) = \int_0^\infty d\xi \frac{\xi^2 e^{-2\xi\bar{z}}}{1 + \xi^2} \quad (41)$$

and

$$V(\bar{z}) \equiv \frac{I(\bar{z})}{\bar{z}}, \quad (42)$$

the atom-surface CP potential can be expressed as

$$V_{\text{int}} = \frac{d^2 k_0^3}{12\pi^2 \epsilon_0} \text{Re}(g_{\perp \rightarrow \perp} + g_{\parallel \rightarrow \parallel}) V(\bar{z}). \quad (43)$$

### The condition to produce a CP repulsion

As mentioned above, whether the CP force is repulsive or attractive is decided by the sign of the real part of  $(g_{\perp \rightarrow \perp} + g_{\parallel \rightarrow \parallel})$ . According to Eq. (43) and Fig. 3, we find that in the condition of

$$\text{Re}(g_{\perp \rightarrow \perp} + g_{\parallel \rightarrow \parallel}) > 0, \quad (44)$$

the atom-surface CP interaction is repulsive.

This result can be understood as follows. The atom immersed in an electromagnetic vacuum may jump to the excited state and go back to the ground state by emitting and reabsorbing a photon, respectively. And the atom can get

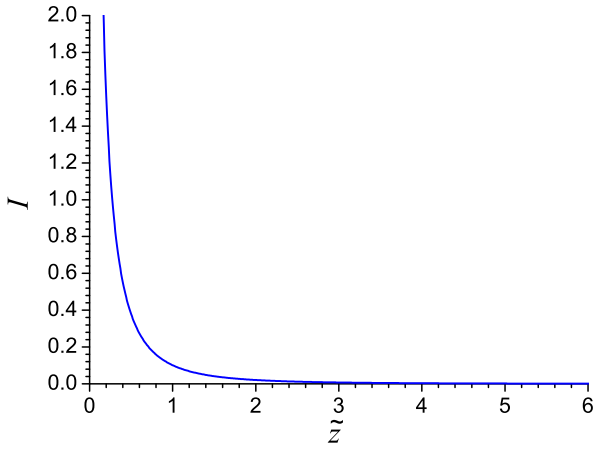


FIG. 3. (Color online) The monotonic, positive-definite function  $I(\bar{z})$  diverges for  $\bar{z} \rightarrow 0$  and converges towards zero for  $\bar{z} \rightarrow \infty$ . These features of function  $I(\bar{z})$  make it possible to produce an all-range long repulsive CP force.

recoil momentum when it emits and absorbs the photons. No mechanical force will press on one atom in free space because the average of these recoil momenta is zero. But, when the atom is located near a negative surface, as shown in Fig. 4, the photon emitted from the atom may be backward reflected by the negative reflecting surface before it is reabsorbed, and the atom will get recoil momenta twice which push the atom away from the surface. As the average of these recoil momenta, the atom will feel a repulsion from the surface. Furthermore, according to Eq. (30), it is the interferences of incident waves and scattered waves which contribute to the atom-surface interaction. If the signs of reflection coefficients are positive, the vacuum electromagnetic field near the surface is “enhanced” and leads to a stronger atom-vacuum interaction, so the atom tends to leave far away from the surface to get a lower potential energy. On the contrary, if the signs of the reflection coefficients are negative, the vacuum field near the surface is “decreased” and the atom tends to get closer to

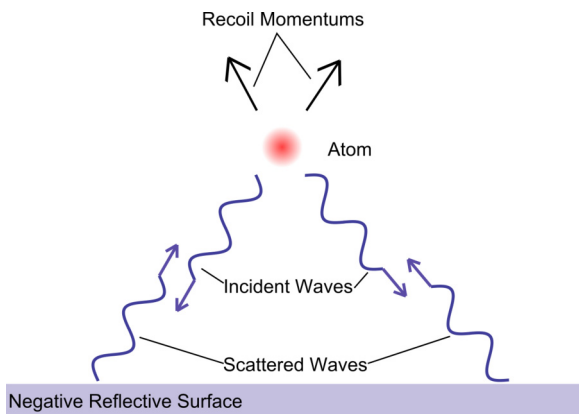


FIG. 4. (Color online) If a photon is emitted by the atom, negatively reflected by the surface, and reabsorbed by the atom again, the atom can get recoil momentum from the photon twice and be pushed away from the surface. On average, a repulsion normal to the surface can be produced between the surface and the atom.

the surface. The signs of the reflection coefficients are decided by the phase changes of the incident waves when they are reflected on the surface, so these phase changes play important roles in producing CP repulsion.

### VI. ANALYSIS OF THE RESULTS

We have proved that when Eq. (44) is satisfied, a repulsive CP potential can be produced by a perfect negative reflecting surface. In the following, we check the properties of this potential.

According to Eq. (43), the CP potential depends only on the reflection coefficients  $g_{\perp \rightarrow \perp}$  and  $g_{\parallel \rightarrow \parallel}$ , which imply polarization conservations, while the other two reflection coefficients  $g_{\perp \rightarrow \parallel}$  and  $g_{\parallel \rightarrow \perp}$ , which imply polarization changes, do not contribute. Figure 5 can help in explaining the result. Figure 5 shows a typical one-loop process of a ground-state atom interacting with a vacuum field. An atom in the ground state may jump to the excited state by emitting a virtual photon, and go back to ground state by absorbing the photon. Enlightened by the graph, we can just imagine that the atom is excited by a virtual light beam incident on the surface. Obviously, the atom should have a polarization identical to the virtual light beam. When the incident beam is reflected by the surface, only the polarization-conservative part can be felt by the atom, while the polarization-changing part cannot. For this reason, the reflection coefficients  $g_{\perp \rightarrow \parallel}$  and  $g_{\parallel \rightarrow \perp}$  cannot contribute to the atom-surface interaction, and they do not appear in potential Eq. (43).

Now we discuss the space translation symmetry of the atom-surface system. If we assume the surface is infinite, when the atom moves a relative displacement parallel to the surface, the situations before and after the motion are physically equivalent. So the atom-surface interaction will not result in a force parallel to the surface. This analysis is in agreement with Eq. (43). The atom-surface potential only depends on  $z$ , which means the force repressing the atom is along the direction normal to the surface. When the potential is repulsive, atoms will always be pushed away from the surface. This characteristic of the CP potential is suitable for building quantum levitating and reflecting devices.

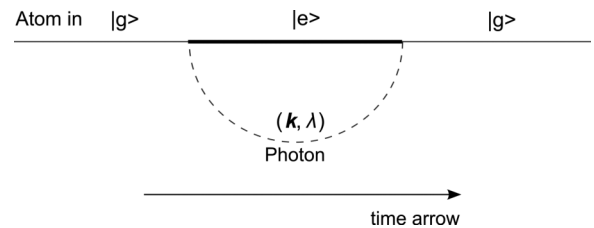


FIG. 5. Time-ordered graph for interaction of one atom with virtual photons. The thin straight line represents the atom in the ground state, while the thick straight line represents the atom in the excited state. The dashed semicircle represents a photon in mode  $(\mathbf{k}, \lambda)$ . The arrow points out the direction of time evolution. This graph can help in understanding Eq. (43). Once the atom jumps to the excited state and emits a photon, the directions of the polarizations of the excited atom and the photon will be parallel. If the polarization of the photon was changed when it was reflected on the surface, the photon would not interact with the atom again. So only the reflection coefficients  $g_{\perp \rightarrow \perp}$  and  $g_{\parallel \rightarrow \parallel}$  appear in Eq. (43).

In the following, we discuss the properties of potential Eq. (43) in the near-field case ( $z \rightarrow 0$ ) and the far-field case ( $z \rightarrow \infty$ ).

### The near-field and far-field behaviors of the CP potential

Rewriting  $I(\tilde{z})$  as

$$I(\tilde{z}) = \frac{1}{2\tilde{z}} - \int_0^\infty d\xi \frac{e^{-2\xi\tilde{z}}}{1+\xi^2}, \quad (45)$$

if  $\tilde{z} \rightarrow 0$ ,

$$\int_0^\infty d\xi \frac{e^{-2\xi\tilde{z}}}{1+\xi^2} \rightarrow \int_0^\infty d\xi \frac{1}{1+\xi^2} = \frac{\pi}{2}, \quad (46)$$

and the CP potential

$$V_{\text{int}}(\tilde{z} \rightarrow 0) \propto \frac{1}{2} \left( \frac{1}{\tilde{z}^2} - \frac{\pi}{\tilde{z}} \right). \quad (47)$$

Equation (47) shows that when the atom-surface separation tends to zero, the CP potential is approximately inversely proportional to  $\tilde{z}^2$ . This behavior is very different from that of the standard CP potential [2], which is proportional to  $z^{-3}$ . According to Eq. (39), when  $z \rightarrow 0$ , the  $1/(2uz)^3$  terms become dominant, so the level shift of the atomic ground state in a classical CP scenario is given by

$$\alpha_{\text{conducting}}(z \rightarrow 0) = \frac{k_0(d_x^2 + d_y^2 + 2d_z^2)}{\hbar\epsilon_0(2\pi)^2} \times \int_0^\infty du \frac{e^{-2uz}}{k_0^2 + u^2} \frac{1}{(2z)^3}, \quad (48)$$

Making use of Eq. (46), we have

$$\alpha_{\text{conducting}}(z \rightarrow 0) \propto \frac{1}{(2z)^3}. \quad (49)$$

But in the negative reflection case, as the atom is isotropically polarized ( $d_x^2 = d_y^2 = d_z^2 = d^2/3$ ), the terms containing  $(2uz)^{-2}$  and  $(2uz)^{-3}$  in Eq. (38) just cancel each other. According to Eq. (38), the level shift  $\alpha$  is

$$\alpha_{\text{negative}} \propto \frac{1}{2z} \int_0^\infty du \frac{u^2 e^{-2uz}}{k_0^2 + u^2}, \quad (50)$$

and according to Eqs. (45) and (46), we get the  $z^{-2}$  behavior of the level shift,

$$\alpha_{\text{negative}}(z \rightarrow 0) \propto \frac{1}{2} \left( \frac{1}{z^2} - \frac{k_0\pi}{z} \right), \quad (51)$$

which agrees with Eq. (47). Because the terms containing  $(2uz)^{-2}$  and  $(2uz)^{-3}$  in Eq. (38) vanish, the  $(2uz)^{-1}$  terms become dominant and lead to a  $z^{-2}$  behavior of the CP potential in near-field limit.

To check the far-field properties of the CP potential, we express the function  $I(\tilde{z})$  as

$$I(\tilde{z}) = \frac{1}{4} \frac{\partial^2}{\partial \tilde{z}^2} \int_0^\infty d\xi \frac{e^{-2\xi\tilde{z}}}{1+\xi^2}. \quad (52)$$

Making use of function  $\delta(\xi)$ ,

$$\delta(\xi) = \lim_{\tilde{z} \rightarrow +\infty} \tilde{z} e^{-2|\xi|\tilde{z}}, \quad (53)$$

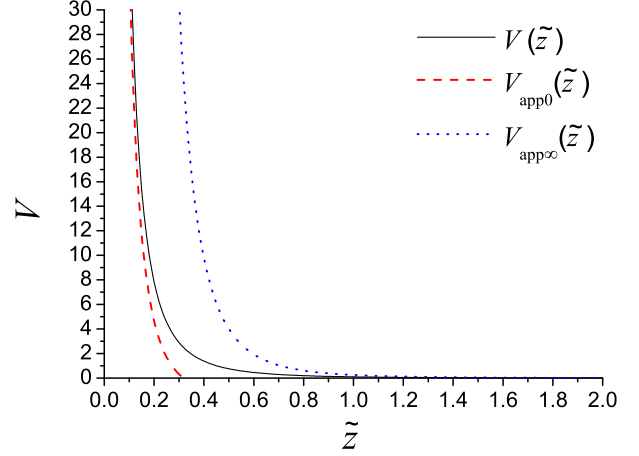


FIG. 6. (Color online) The functions  $V(\tilde{z})$ ,  $V_{\text{app}0}(\tilde{z})$ , and  $V_{\text{app}\infty}(\tilde{z})$  are plotted. It can be seen that  $V(\tilde{z})$  converges towards  $V_{\text{app}0}(\tilde{z})$  for  $\tilde{z} \rightarrow 0$  and fits  $V_{\text{app}\infty}(\tilde{z})$  when  $\tilde{z} \rightarrow \infty$ . This result shows the CP potential decays very fast for large atom-surface separation.

Eq. (52) can be written as

$$I(\tilde{z} \rightarrow +\infty) = \frac{1}{4} \frac{\partial^2}{\partial \tilde{z}^2} \left[ \frac{1}{\tilde{z}} \int_0^\infty d\xi \frac{\delta(\xi)}{1+\xi^2} \right], \quad (54)$$

which is easy to calculate,

$$I(\tilde{z} \rightarrow +\infty) = \frac{1}{4\tilde{z}^3}, \quad (55)$$

and the CP potential is inversely proportional to  $\tilde{z}^4$ :

$$V_{\text{int}}(\tilde{z} \rightarrow +\infty) \propto \frac{1}{4\tilde{z}^4}. \quad (56)$$

We define

$$V_{\text{app}0}(\tilde{z}) = \frac{1}{2} \left( \frac{1}{\tilde{z}^2} - \frac{\pi}{\tilde{z}} \right) \quad (57)$$

and

$$V_{\text{app}\infty}(\tilde{z}) = \frac{1}{4\tilde{z}^4}. \quad (58)$$

The comparison of function  $V(\tilde{z})$  with  $V_{\text{app}0}(\tilde{z})$  and  $V_{\text{app}\infty}(\tilde{z})$  is shown in Fig. 6.

This analysis shows that the CP potential decreases very fast ( $\propto z^{-4}$ ) for large atom-surface separation. This feature is very useful in building good quantum reflectors. According to Fig. 6, when  $\tilde{z} < 1$ , the atom will be strongly pushed away from the surface; but in the region of  $\tilde{z} > 1$ , the CP potential is so small that the atom can be thought free.

## VII. CONCLUSION

In summary, we have studied a perfect negative reflecting surface and investigated the CP interaction between the surface and a ground-state atom. Quantum electrodynamics is employed in analyzing the atom-reservoir interaction. We estimate the level shift of the ground state of the atom-vacuum-surface interacting system and give the analytic expression of the atom-surface interaction energy. The result proves the possibility of producing an all-range long repulsive CP

potential between an atom and one negative reflecting surface. This potential is proportional to  $z^{-2}$  at short atom-surface distances and to  $z^{-4}$  at long atom-surface distances. For an atom in the ground state, this repulsive potential does not decay with time. With these advantages, the scheme advised in this paper has the potential for building quantum reflectors, levitating devices, and trapping devices and can help in building frictionless nanomachines.

#### ACKNOWLEDGMENTS

I thank Dr. Ke-Ye Zhang, Dr. Chun-Hua Yuan, and Dr. L. Zhou for constructive discussions and great help. I thank the referees for their insightful comments and criticisms as well as the great suggestions which helped me in improving this paper.

#### APPENDIX: OPPOSITE SIGNS ON THE RIGHT-HAND SIDES OF EQS. (38) AND (39)

We think it is necessary to discuss the opposite signs on the right-hand sides of Eqs. (38) and (39). Using the method introduced in Sec. III, we separate the total electric field near the perfect conducting surface into incident field and reflected field. For the incident field, the unit vectors have been defined in Eq. (36). And for the reflected field, we consider

the properties of specular reflection ( $k_x \hat{e}_x + k_y \hat{e}_y + k_z \hat{e}_z \rightarrow k_x \hat{e}_x + k_y \hat{e}_y - k_z \hat{e}_z$ ) and then define

$$\begin{aligned} \hat{k}' &= \sin \theta \cos \phi \hat{e}_x + \sin \theta \sin \phi \hat{e}_y - \cos \theta \hat{e}_z, \\ \hat{e}_{k'\parallel} &= -\cos \theta \cos \phi \hat{e}_x - \cos \theta \sin \phi \hat{e}_y - \sin \theta \hat{e}_z, \\ \hat{e}_{k'\perp} &= \sin \phi \hat{e}_x - \cos \phi \hat{e}_y, \end{aligned} \quad (\text{A1})$$

in which  $k'$  represents the wave vector of the reflected wave. According to Eqs. (36) and (A1), the level shift  $\alpha$  in the standard CP scenario can be calculated as

$$\begin{aligned} \alpha_{\text{conducting}} &= -\frac{k_0}{\hbar \epsilon_0 (2\pi)^2} \int_0^\infty du \frac{u^3 e^{-2uz}}{k_0^2 + u^2} \\ &\times \left\{ \begin{aligned} &\frac{1}{2uz} (d_x^2 + d_y^2) \frac{\text{Re}(g_{\parallel \rightarrow \parallel}) + \text{Re}(g_{\perp \rightarrow \perp})}{2} \\ &+ \left[ \frac{1}{(2uz)^2} + \frac{1}{(2uz)^3} \right] (d_x^2 + d_y^2) \text{Re}(g_{\parallel \rightarrow \parallel}) \\ &+ 2 \left[ \frac{1}{(2uz)^2} + \frac{1}{(2uz)^3} \right] d_z^2 \text{Re}(g_{\parallel \rightarrow \parallel}) \end{aligned} \right\}. \end{aligned} \quad (\text{A2})$$

Note that the sign on the right-hand side of Eq. (A2) is a minus, which agrees with that in Eq. (38). But for specular reflection, the reflection coefficients  $g_{\parallel \rightarrow \parallel} = g_{\perp \rightarrow \perp} = -1$ . Substituting the values of the reflection coefficients into Eqs. (A2) and (39) can be done. We can see that the difference in the signs on the right-hand sides of Eqs. (38) and (39) is because of the minus values of the reflection coefficients in normal specular reflection.

- 
- [1] H. B. G. Casimir, Proc. K. Ned. Akad. Wet. **51**, 793 (1948).
  - [2] H. B. G. Casimir and D. Polder, *Phys. Rev.* **73**, 360 (1948).
  - [3] K. A. Milton, *J. Phys.: Conf. Ser.* **161**, 012001 (2009).
  - [4] B.-S. Skagerstam, P. K. Rekdal, and A. H. Vaskinn, *Phys. Rev. A* **80**, 022902 (2009).
  - [5] M. Boström, S. Å. Ellingsen, I. Brevik, M. F. Dou, C. Persson, and B. E. Sernelius, *Eur. Phys. J. B* **85**, 377 (2012).
  - [6] M. S. Tomaš, *J. Phys. A* **39**, 6785 (2006).
  - [7] R. Zhao, T. Koschny, E. N. Economou, and C. M. Soukoulis, *Phys. Rev. B* **83**, 075108 (2011).
  - [8] K. A. Milton, E. K. Abalo, P. Parashar, N. Pourtolami, I. Brevik, and S. Å. Ellingsen, *Int. J. Mod Phys: Conf. Ser.* **14**, 181 (2012).
  - [9] K. A. Milton, E. K. Abalo, P. Parashar, N. Pourtolami, I. Brevik, and S. Å. Ellingsen, *J. Phys. A* **45**, 374006 (2012).
  - [10] M. Al-Amri and M. Babiker, *Eur. Phys. J. D* **48**, 417 (2008).
  - [11] J. N. Munday, F. Capasso, and V. A. Parsegian, *Nature (London)* **457**, 170 (2009).
  - [12] A. Milling, P. Mulvaney, and I. Larsonj, *J. Colloid Interface Sci.* **180**, 460 (1996).
  - [13] I. E. Dzyaloshinskii, E. M. Lifshitz, and L. P. Pitaevskii, *Sov. Phys. Usp.* **4**, 153 (1961).
  - [14] T. H. Boyer, *Phys. Rev. A* **9**, 2078 (1974).
  - [15] S. C. Lim and L. P. Teo, *New J. Phys.* **11**, 013055 (2009).
  - [16] V. K. Pappakrishnan, P. C. Mundru, and D. A. Genov, *Phys. Rev. B* **89**, 045430 (2014).
  - [17] K. A. Milton, P. Parashar, N. Pourtolami, and I. Brevik, *Phys. Rev. D* **85**, 025008 (2012).
  - [18] K. A. Milton, E. K. Abalo, P. Parashar, N. Pourtolami, I. Brevik, and S. A. Ellingsen, *Phys. Rev. A* **83**, 062507 (2011).
  - [19] K. V. Shajesh and M. Schaden, *Phys. Rev. A* **85**, 012523 (2012).
  - [20] F. S. S. Rosa, D. A. R. Dalvit, and P. W. Milonni, *Phys. Rev. Lett.* **100**, 183602 (2008).
  - [21] R. Zhao, J. Zhou, Th. Koschny, E. N. Economou, and C. M. Soukoulis, *Phys. Rev. Lett.* **103**, 103602 (2009).
  - [22] V. Yannopoulos and N. V. Vitanov, *Phys. Rev. Lett.* **103**, 120401 (2009).
  - [23] U. Leonhardt and T. G. Philbin, *New J. Phys.* **9**, 254 (2007).
  - [24] A. Sambale, D.-G. Welsch, H. T. Dung, and S. Y. Buhmann, *Phys. Rev. A* **78**, 053828 (2008).
  - [25] H. Chen, S. Xu, and J. Li, *Opt. Lett.* **34**, 3283 (2009).
  - [26] T. G. Mackay and A. Lakhtakia, *Microwave Opt. Tech. Lett.* **50**, 1368 (2008).
  - [27] V. Mocella, P. Dardano, L. Moretti, and I. Rendina, *Opt. Express* **15**, 6605 (2007).
  - [28] Z. Y. Wei, H. Q. Li, C. Wu, Y. Cao, J. Z. Ren, Z. H. Hang, H. Chen, D. Z. Zhang, and C. T. Chan, *Opt. Express* **18**, 12119 (2010).
  - [29] A. V. Vashkovsky and E. H. Lock, *Phys. Usp.* **49**, 389 (2006).
  - [30] S. Scheel and S. Y. Buhmann, *Phys. Rev. A* **80**, 042902 (2009).
  - [31] D. P. Craig and T. Thirunamachandran, in *Molecular Quantum Electrodynamics* (Academic Press, London, 1984), Chap. 7, p. 148.