All-versus-nothing violation of local realism from the Hardy paradox under no-signaling

Some Sankar Bhattacharya,[1,*] Arup Roy,[1,†] Amit Mukherjee,[1,‡] and Ramij Rahaman[2,§]
[1]*Physics and Applied Mathematics Unit, Indian Statistical Institute, 203 B.T. Road, Kolkata 700108, India*
[2]*Department of Mathematics, University of Allahabad, Allahabad 211002, Uttar Pradesh, India*
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Hardy's is one of the simplest arguments concerning nonlocality. Recently, Chen *et al.* [Phys. Rev. A **88**, 062116 (2013)] have proposed a more generalized Hardy-like argument and have shown that the probability of success increases with the local system's dimension. Here we study the same in a minimally constrained theory, namely, the generalized no-signaling theory(GNST). We find that not only does the probability of success of this argument increase with the local system dimension in GNST, but it also takes a very simple functional form.

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I. INTRODUCTION

In 1964, Bell proved that one can find measurement correlations for a composite quantum system which cannot be described by local-realistic theory (LRT) [1]. Although the work of Bell was pioneering, the approach to the proof was not very impressive due to its statistical nature. Bell's inequalities [2], in fact, are statistical predictions about some sets of measurements which can be made on local subsystems separated far from each other. The violation of such an inequality implies that the statistical description cannot be reproduced by local hidden variables.

Greenberger, Horne, and Zeilinger [3] (GHZ) found a way to show more immediately, without inequalities, that the result or prediction of quantum mechanics is inconsistent with the assumption of Einstein, Podolsky, and Rosen, i.e., locality and reality [4]. Unlike that of Bell, this proof involves only one event and not the statistics of many events. In 1992, Hardy [5] gave a relatively simpler all-or-none-type proof of this no-go theorem for local hidden variables without using any statistical inequalities via some logical contradiction, in the spirit of GHZ [3,6]. Hardy's nonlocality argument deals with two qubits with two dichotomic measurement observables on each qubit. The proof can be extended even for n qubits [7,8]. The argument is also valid for more than two measurements [9] and more than two outcomes [10–13]. The above nonlocality argument can be extended even for generalized no-signaling theory (GNST) [14]. One can also find the opposite approach in literature, where to show that correlations originating from GNST are more nonlocal than quantum correlations, Fritz has considered a "stronger" version of the Hardy paradox in a two-input, two-output scenario [15].

Any physical theory should contain the fact that instantaneous propagation of information is impossible. This is the no-signaling principle. Nonlocality obeying this principle at the operational level is solely responsible for a good number of fascinating phenomena such as secrecy extraction [16], certification of intrinsic randomness [17], and several nonclassical communication tasks.

It has been shown that the maximum success probability for Hardy's argument does not depend on the system's dimension. Recently, Chen *et al.* [18] have formulated a new type of Hardy's nonlocality argument for measurements that have more than two outcomes. Interestingly, the authors have shown numerically that the maximum probability of success for this argument increases with the local system's dimension. Actually, this nonlocality argument is equivalent to a violation of a tight Bell inequality [18]. As one might expect, this argument reduces to *Hardy's original argument* [5] for a special case. So it might be interesting to study this new Hardy-type argument for a minimally constrained theory, namely, GNST. Recently, Mansfield [19] has shown that the probability of witnessing Hardy nonlocality (PN) for two two-level systems can be achieved with certainty under GNST, whereas the paradoxical probability (PP) of two two-level Hardy arguments is bounded by 0.5. PP concerns the quality of a particular Hardy argument, but PN concerns the performance of a correlation regarding the demonstration of nonlocality. Hence, these two concepts are motivated from different perspectives. In this context, PP and PN and their relation under GNST are worth studying. This, in effect, can provide us with an upper bound on the paradoxical probability of this argument that is allowed by relativistic causality alone, in the absence of any further constraints. This study is also important because the optimal success probability of Hardy's argument is deeply connected to the security proof of several information processing tasks [12,20,21].

We also investigate how this maximum probability of success changes with the local system's dimension and provide an analytic functional form for such a feature. We extend our work even for situations where the dimensions of spatially separated parties are not equal. The rest of the article is arranged in the following manner: Sec. II reviews the conventional and new Hardy-type arguments and the results known so far; in Sec. III we set the stage for calculating the maximum success probability of the new Hardy-type paradox for higher-dimensional systems in the no-signaling paradigm. In Sec. IV we present our results, and finally, we conclude in Sec. V.

II. BIPARTITE HARDY PARADOX

Consider a physical system consisting of two subsystems shared between two distant parties, Alice and Bob. The two observers (Alice and Bob) have access to one subsystem each.

*somesankar@gmail.com
†arup145.roy@gmail.com
‡amitisiphys@gmail.com
§ramijrahaman@gmail.com

Assume that Alice (Bob) can measure one of two observables, X_0 and X_1 (Y_0 and Y_1), on her (his) local subsystem. The outcomes a (b) of each such von Neumann measurement can be $1,2,\ldots,d_X^A$ (d_Y^B). Here d_X^A (d_Y^B) is the dimension of Hilbert space associated with Alice's (Bob's) subsystem. The joint probability $P(X=a,Y=b)$ denotes the probability of getting outcome (a,b) for measurement (X,Y).

A. Hardy paradox

The pioneering noninequality paradox regarding incompatibility of any theory with local realism, introduced by Hardy [5] in 1992, is for two two-level systems. A generalized version of this argument for two multilevel systems starts with the following set of joint probability conditions:

$$\begin{aligned} &P\left(X_0=1,Y_0=d_{Y_0}^B\right)=q_H>0,\\ &P\left(X_1=a,Y_0=d_{Y_0}^B\right)=0,\quad \forall a\in\left\{2,3,\ldots,d_{X_1}^A\right\},\\ &P(X_0=1,Y_1=b)=0,\quad \forall b\in\left\{1,2,\ldots,d_{Y_1}^B-1\right\},\\ &P\left(X_1=1,Y_1=d_{Y_1}^B\right)=0.\end{aligned} \tag{1}$$

The logical structure of the argument is as follows: for some ontic variables $\lambda\in\Omega$, the (X_0,Y_0) observables can take the value $(1,d_{Y_0}^B)$, which is the first condition of (1). Let us denote the subspace span by those λ's as $\Omega'(\subset\Omega)$. Now the second condition tells us that for all λ's in Ω' observable X_1 can take only the value 1, as the other possibility of the X_1 observable is excluded. Similarly, the third condition tells us that for the Y_1 observable $d_{Y_1}^B$ is the only possibility for all $\lambda\in\Omega'$. Therefore, the joint possibility for $X_1=1$ and $Y_1=d_{Y_1}^B$ should be nonzero for $\lambda\in\Omega'$. But this contradicts the last condition of (1). Therefore, the four statements of (1) are incompatible with local realism. However, quantum correlations which can reproduce all four conditions of (1) exist [11,22].

At this point one can point out two important quantities: the first is q_H, which is the probability of success of a stand-alone argument (1), i.e., paradoxical probability. On the other hand, given a correlation, one can make use of two or more such arguments to demonstrate nonlocality, which gives rise to PN. Since more than one elementary argument is used, the complementary events (i.e., events which are not considered in the original stand-alone argument) along with the principal events may contribute to PN. One can heuristically write PP + PPC = PN, where PPC is the probability of success contributed by the complementary events. Correlations arising from quantum systems satisfy PP = PN [19]. But the gap between PP and PN becomes visible when one considers postquantum correlations. One such example is the Popescu-Rohrlic (PR) box. For the PR box PP corresponding to (1) is $\frac{1}{2}$, whereas PN = 1. Thus, consideration of general postquantum correlations reveal this curious feature of the nonlocality-without-inequality type of argument.

B. General nonsignaling theory satisfying the Hardy-type argument

In the framework of a general probabilistic theory, consider a system of two separated parties, which together satisfy all the conditions of the Hardy-type argument as given in (1).

1. Positivity conditions

For $P(X=a,Y=b)$ to be a valid probability measure it should satisfy the positivity conditions

$$P(X=a,Y=b)\geqslant 0\,\forall\,X,Y,a,b. \tag{2}$$

2. Normalization conditions

The probability distribution relating the outcomes for a given measurement setting should satisfy the normalization condition.

$$\sum_{a=1}^{d_X^A}\sum_{b=1}^{d_Y^B}P(X=a,Y=b)=1, \tag{3}$$

$\forall\,X\in\{X_0,X_1\}$ and $Y\in\{Y_0,Y_1\}$.

3. Nonsignaling conditions

For any no-signaling n-partite distribution $P(a,b,c,\ldots|X,Y,Z,\ldots)$, each subset of parties $\{a,b,\ldots\}$ depends on only its corresponding inputs; that is, if we change the input of one party, the marginal probability distribution for the other spatially separated parties is not affected.

For a bipartite generalized probability distribution the no-signaling conditions take the following form:

$$\sum_{b=1}^{d_{Y_0}^B}P(X=a,Y_0=b)=\sum_{b'=1}^{d_{Y_1}^B}P(X=a,Y_1=b')\quad \forall\,X\in\{X_0,X_1\},\,a\in\left\{1,d_X^A\right\} \tag{4}$$

$$\sum_{a=1}^{d_{X_0}^A}P(X_0=a,Y=b)=\sum_{a'=1}^{d_{X_1}^A}P(X_1=a',Y=b)\quad \forall\,Y\in\{Y_0,Y_1\},\,b\in\left\{1,d_Y^B\right\}. \tag{5}$$

What is the maximum probability of success, $P(X_0=1,Y_0=d_{Y_0}^B)$, of the Hardy-type argument (1) under GNST for an arbitrary $d_X^A\times d_Y^B$ system, subject to the constraints given in Eqs. (2)–(5)? We have shown that the maximum probability of success $P(X_0=1,Y_0=d_{Y_0}^B)$ for a two-input, (d_X^A,d_Y^B)-output Hardy's test (1) is $\frac{1}{2}$ under GNST for all d_X^A,d_Y^B (Sec. IV A). Interestingly, the maximum probability of success in the bipartite Hardy-type argument under GNST is dimension independent as in the quantum case. For a two-qubit system the maximum achievable value of Hardy's success is $q_H=\frac{5\sqrt{5}-11}{2}\approx 0.09$ [23,24]. Reference [11] proves that, for two-qutrit systems, the maximum achievable value of Hardy's success probability is the same as that of the two-qubit system and conjectures that it will remain the same for arbitrary dimension n. Recently, Ref. [12] provided a proof of this conjecture. This result tells us that higher-dimensional systems give no advantage in the experimental implementation of such a test, showing the contradiction of quantum mechanics with the *local realism*. Keeping this in mind, the authors of Ref. [18] introduced a Hardy-type argument which applies to measurements with an arbitrarily large number of outcomes. They have also shown that the success probability of this modified Hardy's paradox increases with the increase in the local system's dimension.

C. Relaxed Hardy paradox

The conditions for the new relaxed Hardy-type argument [18] are

$$\begin{aligned} &P(X_0 < Y_0) = q_{RH} > 0,\\ &P(X_1 < Y_0) = 0,\\ &P(Y_1 < X_1) = 0,\\ &P(X_0 < Y_1) = 0, \end{aligned} \tag{6}$$

where $P(X_i < Y_j) = \sum_{a<b} P(X_i = a, Y_j = b)$. Therefore, if events $X_1 < Y_0$, $Y_1 < X_1$, and $X_0 < Y_1$ never happen, then, in any local theory, event $X_0 < Y_0$ must never happen either. However, this is not the case with quantum correlations. The sets of conditions (1) and (6) cannot be satisfied by any LRT [5,18]. One can generalize the above conditions (6) by replacing the last zero condition with a nonzero condition $P(X_0 < Y_1) = p < q_{RH}$. For $d_X^A = d_Y^B = 2$, the above two sets of conditions, (1) and (6), give us the conventional two-level Hardy paradox [5]:

$$\begin{aligned} &P(X_0 = 1, Y_0 = 2) = q_{RH} = q_H > 0,\\ &P(X_1 = 1, Y_0 = 2) = 0,\\ &P(Y_1 = 2, X_1 = 1) = 0,\\ &P(X_0 = 1, Y_1 = 2) = 0. \end{aligned} \tag{7}$$

In [18] the authors have shown that in quantum theory the success probability of the relaxed Hardy test (6) surpasses that of the conventional Hardy's test. It has also been shown that the probability of nonlocal events increases with the local system dimension, and for high enough dimensions q_{RH} is almost four times higher than q_H. They have also claimed that the nonlocality argument proposed by them is the most natural and powerful generalization of Hardy's argument concerning higher-dimensional local systems. To test such a proposal one might wonder how useful this generalized Hardy argument is in a theory which contains the minimum number of features or restrictions. In the following sections we have studied this relaxed Hardy argument in GNST, where the only restriction on theory is that it does not violate relativistic causality.

III. RELAXED HARDY PARADOX IN NO-SIGNALING THEORIES

Here we study Hardy's paradox for higher-dimensional systems within the framework of generalized probabilistic theories. The only condition that we impose on the generalized probability distribution is the no-signaling condition, which all known physical theories respect.

The set of boxes which satisfy Eqs. (2)–(5) can be divided into two types: local and nonlocal. A local box can be simulated using only shared randomness, whereas to simulate a nonlocal box with shared randomness, the observers must communicate. Due to the linearity of the constraints [Eqs. (2)–(5)], the set of all nonsignaling boxes with a finite number of inputs and outputs forms a polytope $\boldsymbol{P}$ with finite vertices. The convex property of such a polytope follows from the argument that a probabilistic mixture of any two boxes satisfying the linear constraints will also be a member of the polytope $\boldsymbol{P}$. Here we consider the case of two inputs and d outputs.

A. No-signaling polytope $\boldsymbol{P(2,d)}$

We have two parties, Alice and Bob, who choose from two inputs X and $Y \in \{0,1\}$ and receive outputs a and b with a joint probability $P(X = a, Y = b)$. We denote the number of distinct outputs associated with inputs X and Y by d_X^A and d_Y^B. Any event in this scenario is described as a point in the polytope $\boldsymbol{P(2,d)}$, i.e., the polytope consisting of all no-signaling boxes with two inputs and an arbitrary large number of outputs. A vertex of $\boldsymbol{P(2,d)}$ must satisfy (2), (3), (4), and (5) and dim[$\boldsymbol{P(2,d)}$] of the positivity inequalities (2) replaced with equalities where

$$\dim[\boldsymbol{P(2,d)}] = \sum_{X,Y=0}^{1} d_X^A d_Y^B - \sum_{X=0}^{1} d_X^A - \sum_{Y=0}^{1} d_Y^B. \tag{8}$$

The extremal points of $\boldsymbol{P(2,d)}$ are of two kinds: partial-output vertices [at least one of the conditions $P(X = a) = 0$ and $P(Y = b) = 0$ holds] and full-output vertices [all $P(X = a) \neq 0$ and $P(Y = b) \neq 0$] [25]. Partial-output vertices correspond to the vertices of some other polytope $\tilde{\boldsymbol{P}}$ with fewer local dimensions (i.e., $d_X^{'A} < d_X^A$ or $d_Y^{'B} < d_Y^B$). On the other hand, the vertices of a polytope $\tilde{\boldsymbol{P}}$ can be extended to vertices of $\boldsymbol{P}$ by assigning a zero probability $P(X = a) = 0$ and $P(Y = b) = 0$ to extra outcomes. From this mapping it is quite evident that for full-output vertices all outcomes contribute to the no-signaling box. So we need to construct only the full-output vertices for a polytope characterized by d_X^A and d_Y^B. The extremal points of the dimension-asymmetric cases, where $d_X^A \neq d_Y^B$, will be the full-output extremal points of d-outcome polytopes for $d \in \{2, \dots, \min(d_X^A, d_Y^B)\}$.

1. Local vertices

Local vertices of polytope $\boldsymbol{P(2,d)}$ correspond to the extremal boxes which realize deterministic strategies using only shared randomness. Allowing for reversible relabeling of the observers' outputs by the local vertices takes the following form [25]:

$$P_L^{\alpha\beta\gamma\delta} = \begin{cases} 1 & \text{if } a = \alpha X \oplus \beta, \quad b = \gamma Y \oplus \delta,\\ 0 & \text{otherwise,} \end{cases}$$

where the indices $\alpha, \beta, \gamma, \delta \in \{0, \dots, \min(d_X^A, d_Y^B) - 1\}$ correspond to the reversible relabeling and $\oplus$ denotes sum modulo d, where $d = \min(d_X^A, d_Y^B)$.

2. Nonlocal vertices

The nonlocal vertices of polytope $\boldsymbol{P(2,d)}$ correspond to the strategies which cannot be realized without the observers communicating. Under local reversible relabeling all such nonlocal vertices take the form [25]

$$P_{NL}^{\alpha\beta\gamma} = \begin{cases} \frac{1}{d} & \text{if } (b \ominus a) = XY \oplus \alpha X \oplus \beta Y \oplus \gamma,\\ & a,b \in \{1, \dots, d\},\\ 0 & \text{otherwise,} \end{cases}$$

where the indices $\alpha,\beta,\gamma \in \{0,\ldots,\min(d_X^A,d_Y^B)-1\}$ correspond to the local reversible relabeling and $\oplus$ and $\ominus$ denote sum modulo d and subtraction modulo d, respectively, where $d=\min(d_X^A,d_Y^B)$.

IV. RESULTS FOR HIGHER-DIMENSIONAL SYSTEMS

A. Results for the bipartite (2,*d*) scenario

At this point we are ready to present the main results of this work. Let q_H be the probability of success of a bipartite two-input, (d_X^A,d_Y^B)-output conventional Hardy paradox (1) for any nonlocal vertex of $\boldsymbol{P(2,d)}$. The structure of the conventional Hardy argument (1) suggests that we assign zero probability to all outcomes other than $(1,d_X^A)$ on Alice's side and $(1,d_Y^B)$ on Bob's side, which essentially corresponds to a partial-output vertex of $\boldsymbol{P(2,d)}$. This situation can be mapped to a full-output vertex of $\boldsymbol{P(2,2)}$. Thus, the value for q_H achieved by any nonlocal full-output vertex of $\boldsymbol{P(2,2)}$ is

$$q_H^{\text{full}} = \tfrac{1}{2}, \tag{9}$$

and it becomes independent of local dimension. Now moving to the relaxed Hardy argument, let q_{RH} be the probability of success of a bipartite two-input, (d_X^A,d_Y^B)-output relaxed Hardy paradox (6) for any nonlocal vertex of $\boldsymbol{P(2,d)}$. It can easily be shown that for a full-output vertex of $\boldsymbol{P(2,d)}$, the maximum number of nonzero elements contributing to the success probability of the relaxed Hardy test is $(d-1)$, where $d=\min(d_X^A,d_Y^B)$, since these are the only possible events satisfying the following two conditions with the input being (X_1,Y_1):

$$(b-a)\,\text{mod}\,d = 1;\quad a,b\in\{1,\ldots,d\}, \tag{10}$$

$$a<b. \tag{11}$$

While Eq. (10) refers to the condition for nonzero value of events for a nonlocal full-output vertex, Eq. (11) denotes the condition for nonzero probability of success of the relaxed Hardy's test (6). Thus, it can easily be shown that the maximum value of q_{RH} that can be achieved by a full-output vertex of $\boldsymbol{P(2,d)}$ takes the following form:

$$q_{RH}^{\text{full}} = \frac{d-1}{d}. \tag{12}$$

Note that this success probability of the relaxed Hardy's test (12) increases with the local dimensions. In the asymptotic limit, i.e., for $d=\min(d_X^A,d_Y^B)\to\infty$, q_{RH}^{full} tends to 1, which is optimal. Here a natural question is whether these values $(q_H^{\text{full}}, q_{RH}^{\text{full}})$ are optimal for any finite-dimensional correlation in GNST. In the following section we address this question.

B. q^{opt} in GNST

Let us define the no-signaling limit of the probability of success of Hardy's test as q_H^{opt} and the relaxed Hardy's test as q_{RH}^{opt} and let the correlations that achieve these optimal values be P_H^{opt} and P_{RH}^{opt}, respectively. Due to the convexity of the no-signaling polytope $\boldsymbol{P(2,2)}$ and $\boldsymbol{P(2,d)}$, it readily follows that P_H^{opt} and P_{RH}^{opt} can be written as a probabilistic mixture of local and nonlocal full-output vertices of $\boldsymbol{P(2,2)}$ and $\boldsymbol{P(2,d)}$, respectively. Thus, we can conclude that in any GNST

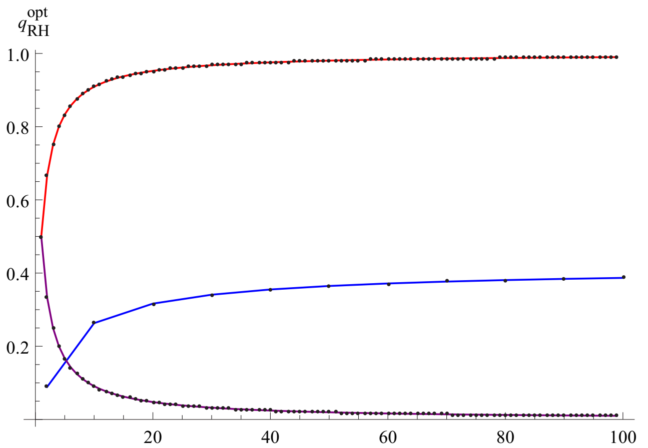

FIG. 1. (Color online) The line in blue (dark gray) shows the increase of q_{RH}^{opt} with increasing system dimension for quantum systems [18]. The red (light gray) line shows the optimal paradoxical probability q_{RH}^{opt} for generalized no-signaling correlations. The purple (medium gray) line shows the decrease of the contribution of complementary events to PN with increasing local dimension for generalized no-signaling correlations.

$$q_H^{\text{opt}} = \frac{1}{2}, \tag{13}$$

$$q_{RH}^{\text{opt}} = \frac{\min\{d_X^A,d_Y^B\}-1}{\min\{d_X^A,d_Y^B\}}. \tag{14}$$

Here we see that for an arbitrarily large system dimension the success probability of the relaxed Hardy's test (6) tends to its possible maximum [from Eqs. (3) and (6)] value, i.e., 1 in the no-signaling paradigm. This is an interesting feature in line with the quantum case where the maximum probability of success increases with local dimension [18]. Unlike the quantum case considered in [18], here we consider even the dimension-asymmetric scenario $(d_X^A \neq d_Y^B)$. Figure 1 shows the success probability of the relaxed Hardy's test against the dimension of the subsystems in a generalized no-signaling theory and in quantum theory [18].

For the Hardy paradox with many outcomes we have seen that the paradoxical probability reaches very close to 1 under a generalized no-signaling theory. This result has great significance, which indicates the fact that relativistic causality alone does not stop one from demonstrating a contradiction with local realism using an argument like (6) with almost 100% success, in contrast to the partial success in the quantum case [5,18].

One can interpret this phenomenon by making two consecutive observations.

Observation 1. For the generalized Hardy-type argument (6), the optimal PPC corresponding to the optimal PP decreases with increasing minimum dimension d in GNST.

Observation 2. PP $\approx$ PN for extremal nonlocal correlations with high minimal dimension d in GNST.

Whereas the first observation indicates that the optimal contribution of the paradoxical probability connected to complementary events to the total probability of witnessing the Hardy nonlocality decreases with increasing local minimum dimension [purple (medium gray) line in Fig. 1], the second observation tells us that in the asymptotic limit the paradoxical probability corresponding to argument (6) is equal to PN, which is 1.

V. CONCLUSION

In comparison to Bell's statistical argument Hardy's paradox more easily demonstrates the fact that quantum mechanics contains correlations which cannot be simulated with shared randomness alone. Chen *et al.* [18] provided the natural generalized version of Hardy's nonlocality argument for higher-dimensional systems. They showed that for $d = 2$ it is exactly Hardy's original nonlocality argument, whereas in the quantum domain for $d > 2$, the paradoxical probability of the relaxed Hardy's argument increases with d. Here we have generalized Chen *et al.*'s conclusion in the no-signaling paradigm. We observe that in any theory that respects relativistic causality, the maximum paradoxical probability of the nonlocality argument increases with the local dimensions of the two subsystems in the bipartite scenario. Finally, we conclude our work by providing a proof which emphasizes a simple functional dependence of the paradoxical probability of the generalized Hardy's nonlocality argument on local dimensions. This fact indicates that with increasing local minimum dimension, the paradoxical probability corresponding to the relaxed Hardy-type argument approaches the probability of witnessing Hardy nonlocality in GNST. This interpretation of our result also suggests that the nonlocality argument proposed in [18] is the most natural higher-dimensional generalization of Hardy's argument [5] in a two-input scenario.

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