Continuous-variable quantum teleportation with non-Gaussian entangled states generated via multiple-photon subtraction and addition

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We theoretically analyze the Einstein-Podolsky-Rosen (EPR) correlation, the quadrature squeezing, and the continuous-variable quantum teleportation when considering non-Gaussian entangled states generated by applying multiple-photon subtraction and multiple-photon addition to a two-mode squeezed vacuum state (TMSVs). Our results indicate that in the case of the multiple-photon-subtracted TMSVs with symmetric operations, the corresponding EPR correlation, the two-mode squeezing degree, the sum squeezing, and the fidelity of teleporting a coherent state or a squeezed vacuum state can be enhanced for any squeezing parameter r and these enhancements increase with the number of subtracted photons in the low-squeezing regime, while asymmetric multiple-photon subtractions will generally reduce these quantities. For the multiple-photon-added TMSVs, although it holds stronger entanglement, its EPR correlation, two-mode squeezing, sum squeezing, and the fidelity of a coherent state are always smaller than that of the TMSVs. Only when considering the case of teleporting a squeezed vacuum state does the symmetric photon addition make somewhat of an improvement in the fidelity for large-squeezing parameters. In addition, we analytically prove that a one-mode multiple-photon-subtracted TMSVs is equivalent to that of the one-mode multiple-photon-added one. And one-mode multiple-photon operations will diminish the above four quantities for any squeezing parameter r.

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I. INTRODUCTION

In recent years, non-Gaussian entangled states with continuous variables as communication resources have received more attention in quantum information and communication technologies. This is mainly because non-Gaussian states and non-Gaussian operations are indispensable for performing some certain continuous-variable quantum information tasks, such as quantum entanglement distillation [1–13], quantum error correction [14], and universal quantum computation [15].

Photon subtraction and addition are the typical non-Gaussian operations used to generate non-Gaussian states with highly nonclassical properties. Agarwal and Tara [16] first studied the nonclassical properties of a photon-added coherent state, which was implemented [17] via a nondegenerate parametric amplifier with small-coupling strength. The photon-subtracted single-mode squeezed state which can be used to conditionally produce the Schrödinger cat state [18] was implemented by a beam splitter with high transmissivity [19]. Opatrný *et al.* proposed that the entanglement and the fidelity of the quantum teleportation can be enhanced by simultaneously subtracting one photon from both modes of a two-mode squeezed vacuum state (TMSVs) [1], whereas in Ref. [2], Cochrane *et al.* showed that the entanglement and the fidelity of a coherent state with photon subtraction are indeed increased for any nonzero initial squeezing. Considering the transmissivity of a beam splitter and the quantum efficiency of photon detectors, Olivares *et al.* [3] further proved that the inconclusive photon subtraction is an effective method to improve the fidelity of a coherent state when the initial squeezing is below a certain value. Kitagawa *et al.* [5] provided

a detailed numerical analysis of two-mode subtraction by on-off detectors in terms of the explicit changes in entanglement and the fidelity of a coherent state. For a given realistic scenario with lossy transmission channels, Zhang and Loock [11] showed that in order to improve the entanglement and the fidelity, a constraint represented by a lower bound for the beam splitter (used for symmetric photon subtraction) must be satisfied. The more photons are symmetrically subtracted, the higher the entanglement and the fidelity will be. However, this improvement will disappear for large squeezing due to the imperfections in this kind of system. Very recently, Bartley *et al.* [20] extensively investigated different strategies for enhancing quantum entanglement via symmetric multiple-photon subtraction from a TMSVs in a realistic experimental scenario. At present, it has been demonstrated in experiments [6,10] that the TMSVs can be "degaussified" by photon subtraction, resulting in a mixed non-Gaussian state whose entanglement degree and teleportation fidelity are improved. Very recently, Kurochkin *et al.* [21] also experimentally demonstrated that by applying the photon-subtraction operator to both modes of the TMSVs, they raised the fraction of the two-photon component in the state, resulting in an increase of both squeezing and entanglement by about 50%. For a review of quantum-state engineering with photon addition and subtraction, we refer to Refs. [22,23].

For an entangled Gaussian resource, it is known that larger squeezing leads to larger entanglement and higher teleportation fidelity. In addition, Adesso and Illuminati have proven that the fidelity of teleportation and the entanglement of the shared entangled Gaussian resource are in an exact one-to-one correspondence [24]. In experiments, the teleportation of Gaussian states in the standard continuous-variable Vaidman-Braunstein-Kimble (VBK) teleportation protocol [25,26], including the coherent state and the squeezed vacuum state,

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has been reported [27–31]. And the teleportation of a non-Gaussian state was carried out recently [32]. In the ideal and in the realistic VBK protocol, Dell'Anno *et al.* [7,8,33] systematically studied the performance of squeezed Bell states (generalized non-Gaussian entangled states), which coincides with photon-subtracted states and photon-added states, where subtraction and addition operations are referred to the case of a single photon. They found that in the non-Gaussian case, the teleportation fidelity depends not only on the entanglement, but also on the degree of non-Gaussianity and the degree of Gaussian affinity with the two-mode squeezed vacuum. In particular, the Gaussian affinity is crucial. Therefore, stronger entanglement does not mean higher teleportation fidelity when non-Gaussian states are considered as entanglement resources, although the entanglement is indispensable in the quantum teleportation [7–9,33,34]. For implementing symmetric and asymmetric multiple-photon subtraction and addition on both modes of the TMSVs, Navarrete-Benlloch *et al.* [35] demonstrated in a idealistic scenario that the entanglement generally increases with the number of such operations. And the multiple-photon addition typically provides a stronger entanglement enhancement than the multiple-photon subtraction. As mentioned above, stronger entanglement does not mean higher teleportation fidelity when non-Gaussian states are considered as entanglement resources. In this paper, we will extend the work in Refs. [7,35] and theoretically investigate the continuous-variable quantum teleportation in the standard VBK protocol with non-Gaussian states generated by the symmetric and asymmetric multiple-photon subtraction and addition. In a idealistic scenario, we show how symmetric or asymmetric multiple-photon operations affect the EPR correlation and the teleportation fidelity, as well as the relations among the Einstein-Podolsky-Rosen (EPR) correlation, the two-mode squeezing, and the teleportation fidelity.

The paper is organized as follows. In Sec. II, we provide a brief review of the multiple-photon-subtracted TMSVs (PS-TMSVs) and the multiple-photon-added TMSVs (PA-TMSVs), as well as their entanglement entropy. In Sec. III, we extend the work in Refs. [7,35] and further investigate the EPR correlation and the quadrature squeezing of non-Gaussian entangled states. In Sec. IV, considering these non-Gaussian entangled states as entangled resources, we study the teleportation fidelity of a coherent state and a squeezed vacuum state in the standard VBK protocol. Our results indicate that only the symmetric multiple-photon subtraction can effectively improve the EPR correlation, the quadrature squeezing, and the teleportation fidelity, and this improvement is obvious in the low-initial-squeezing regime. Finally, we investigate the performance of the PS-TMSVs for the coherent-state teleportation at a fixed EPR correlation parameter, rather than at a fixed squeezing parameter or the entanglement entropy. Our main results are summarized in Sec. V.

II. MULTIPLE-PHOTON-ADDED AND MULTIPLE-PHOTON-SUBTRACTED TWO-MODE SQUEEZED VACUUM STATES

In this section, we first provide a brief review of the PA-TMSVs and PS-TMSVs, as well as their entanglement entropy. We consider a TMSVs as an input state, which is

a typical Gaussian entangled state produced in experiments. Theoretically, the TMSVs can be obtained by adding the two-mode squeezed operator $S_2(r) = \exp[r(a^\dagger b^\dagger - ab)]$ with squeezing parameter r to a vacuum state with modes A and B, that is, $|r\rangle = S_2(r)|0,0\rangle = \text{sech} r \sum_{n=0}^{\infty} \tanh^n r |n,n\rangle$.

By performing the multiple-photon-added operation on the TMSVs, one obtains the normalized output state as [35]

$$
\begin{aligned}
|r\rangle_{\text{pa}} &= C_{k,l}^{-1/2} a^{\dagger k} b^{\dagger l} S_2(r)|0,0\rangle \\
&= \sum_{n=0}^{\infty} \sqrt{\frac{(n+k)!(n+l)!}{(n!)^2 C_{k,l} \cosh^2 r}} \tanh^n r |n+k, n+l\rangle, \quad (1)
\end{aligned}
$$

where $C_{k,l} = \text{Tr}[a^{\dagger k} b^{\dagger l} S_2(r)|0,0\rangle\langle 0,0|S_2(-r)a^k b^l]$ is the normalization factor of the PA-TMSVs. Different from that in Ref. [35], here, for our purpose, we first calculate the expectation value of a general product of operators $a^p a^{\dagger q} b^h b^{\dagger j}$ in the TMSVs. After straightforward calculation, we have (see Appendix)

$$
\begin{aligned}
C_{p,q,h,j} &= \text{Tr}(|r\rangle\langle r|a^p a^{\dagger q} b^h b^{\dagger j}) \\
&= \sum_{m}^{\min[p,h]} \frac{p!q!h!j! \cosh^{2p+2h} r \sinh^{j-h} 2r}{2^{j-h} m!(p-m)!(h-m)!} \\
&\quad \times \frac{\tanh^{2m} r \delta_{p+j,q+h}}{(j-h+m)!}, \quad (2)
\end{aligned}
$$

where the Kronecker δ function $\delta_{p+j,q+h}$ means that all off-antidiagonal elements are zero. Obviously, when $p = q = k$ and $h = j = l$, $C_{p,q,h,j}$ reduces to the normalization factor $C_{k,l}$. Note that the four quantities n, $n + \alpha$, $n + \beta$, and $n + \alpha + \beta$ are non-negative integers, and the Jacobi polynomial can be written as

$$
\begin{aligned}
P_n^{(\alpha,\beta)}(x) &= \frac{1}{2^n} \sum_{k=0}^{n} \binom{n+\alpha}{k} \binom{n+\beta}{n-k} \\
&\quad \times (x-1)^{n-k}(x+1)^k. \quad (3)
\end{aligned}
$$

Thus, the normalization factor $C_{k,l}$ (without loss of generality, assuming $k \geqslant l$) can be written as

$$
C_{k,l} = k!l! \cosh^{2k} r P_l^{(0,k-l)}(\cosh 2r). \quad (4)
$$

When only one of the modes undergoes photon addition while the other is unchanged (for example, $l = 0$), Eq. (4) reduces to

$$
C_{k,0} = k! \cosh^{2k} r. \quad (5)
$$

Then, noting that the relation $S_2(-r)a^\dagger S_2(r) = a^\dagger \cosh r + b \sinh r$, the normalized one-mode-added TMSVs can be written as

$$
\frac{1}{\sqrt{k! \cosh^{2k} r}} a^{\dagger k} S_2(\xi)|0,0\rangle = S_2(\xi)|k,0\rangle, \quad (6)
$$

which is a two-mode squeezed number state.

On the other hand, subtracting multiple photons from two modes of the TMSVs, one obtains the PS-TMSVs. Theoretically, the normalized PS-TMSVs can be written

as [36]

$$|r\rangle_{\mathrm{ps}} = N_{k,l}^{-1/2} a^k b^l S_2(r)|0,0\rangle$$

$$= \sum_{n=\max[k,l]}^{\infty} \sqrt{\frac{(n!)^2 \operatorname{sech}^2 r \tanh^{2n} r}{N_{k,l}(n-k)!(n-l)!}} |n-k,n-l\rangle, \quad (7)$$

where $N_{k,l}$ is the normalization factor of the PS-TMSVs. Similarly, we first derive the expectation value of a general product of operators $a^{\dagger q} a^p b^{\dagger j} b^h$ in the TMSVs (also see Appendix),

$$N_{p,q,h,j} = \operatorname{Tr}(|r\rangle\langle r|a^{\dagger q} a^p b^{\dagger j} b^h)$$

$$= \sum_{m}^{\min[p,h]} \frac{p!q!h!j! \sinh^{2p+2h} r \sinh^{j-h} 2r}{2^{j-h} m!(p-m)!(h-m)!}$$

$$\times \frac{\coth^{2m} r \, \delta_{p+j,q+h}}{(j-h+m)!}. \quad (8)$$

Thus, in the case of $p = q = k$ and $h = j = l$, $N_{p,q,h,j}$ reduces to the normalization factor $N_{k,l}$, i.e.,

$$N_{k,l} = k!l! \sinh^{2k} r \, P_l^{(k-l,0)}(\cosh 2r). \quad (9)$$

For the case $l = 0$, we have

$$N_{k,0} = k! \sinh^{2k} r.$$

Then, the normalized one-mode-subtracted TMSVs is

$$\frac{1}{\sqrt{k! \sinh^{2k} r}} a^k S_2(\xi)|00\rangle = S_2(\xi)|0,k\rangle, \quad (10)$$

which is another two-mode squeezed number state. Due to the symmetry of Eqs. (6) and (10), we can see that adding k photons to the first mode is equivalent to subtracting them from the same mode. In addition, adding k photons to the first mode has the same effect as subtracting them from the second mode [35]. Therefore, both the one-mode photon-subtracted TMSVs and the one-mode photon-added TMSVs have same quantum statistical effects.

In addition, it can be seen that Eq. (2) [or Eq. (4)] and Eq. (8) [or Eq. (9)] substantially differ for the exchange of the hyperbolic coefficients and are important for further studying nonclassical properties of both the PA-TMSVs and PS-TMSVs. Particularly, with the help of Eqs. (2), (4), (8), and (9), it is convenient to explore some quantum optical nonclassicalities that are characterized by the expectation values of field operators, such as sub-Poissonian statistics, the cross correlation, antibunching effects [37], quadrature squeezing properties (including sum squeezing and difference squeezing) [38], as well as the entanglement characterized by some inseparability criteria [39–42].

For our purpose, let us review the von Neumann entropy of both the PA-TMSVs and PS-TMSVs [35]. For a pure state in Schmidt form, $|\psi\rangle_{AB} = \sum_{n=1} c_n |\alpha_n\rangle_A |\beta_n\rangle_B$ (c_n: real positive) with the orthonormal states $|\alpha_n\rangle_A$ and $|\beta_n\rangle_B$, the quantum entanglement is quantified by the partial von Neumann entropy of the reduced density operator [43],

$$E(|\psi\rangle_{AB}) = -\operatorname{Tr}(\rho_A \ln \rho_A) = -\sum_{n=1} c_n^2 \log_2 c_n^2, \quad (11)$$

where the local state is given by $\rho_A = \operatorname{Tr}_B(|\psi\rangle_{AB}\langle\psi|)$. Note that Eqs. (1) and (7) are already in Schmidt form, and thus the entanglement of the PA-TMSVs and the PS-TMSVs, respectively, can be directly obtained [35],

$$E_{\mathrm{pa}}^{k,l} = -\sum_{n=0}^{\infty} \frac{(n+k)!(n+l)!}{(n!)^2 C_{k,l} \cosh^2 r} \tanh^{2n} r$$

$$\times \log_2 \frac{(n+k)!(n+l)!}{(n!)^2 C_{k,l} \cosh^2 r} \tanh^{2n} r \quad (12)$$

and

$$E_{\mathrm{ps}}^{k,l} = -\sum_{n=\max[k,l]}^{\infty} \frac{(n!)^2 N_{k,l}^{-1} \cosh^{-2} r}{(n-k)!(n-l)!} \tanh^{2n} r$$

$$\times \log_2 \frac{(n!)^2 N_{k,l}^{-1} \cosh^{-2} r}{(n-k)!(n-l)!} \tanh^{2n} r. \quad (13)$$

The amount of the entanglement of the TMSVs (in the case of $k = l = 0$) is analytically given by $E = \cosh^2 r \log_2(\cosh^2 r) - \sinh^2 r \log_2(\sinh^2 r)$, and for other states it can be evaluated numerically by their Schmidt coefficients. When $k = l$ (symmetric operation), one can analytically prove that $E_{\mathrm{pa}} = E_{\mathrm{ps}}$. Actually, Eq. (7) can be
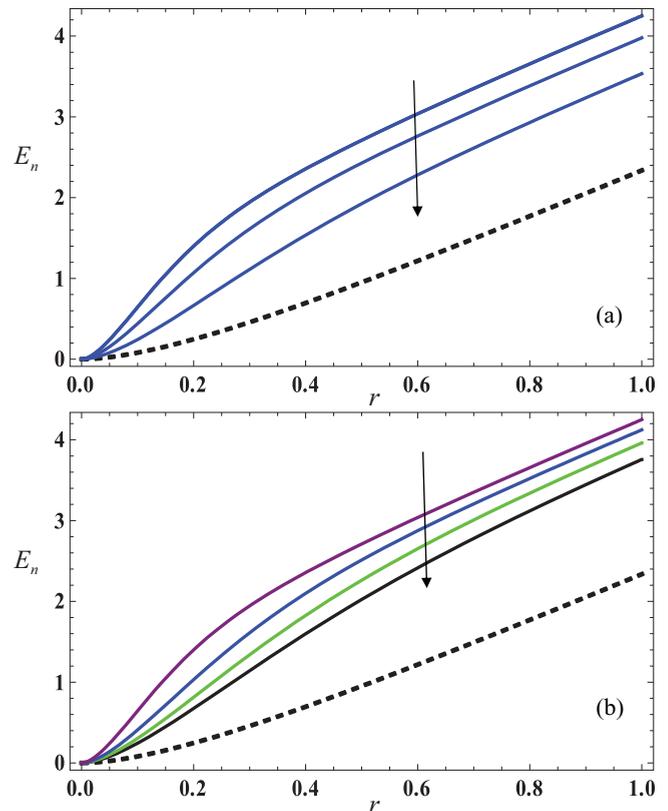
FIG. 1. (Color online) Entanglement entropy as a function of the squeezing parameter r for a PS-TMSVs (or PA-TMSVs) with different values of (k,l). (a) PS-TMSVs or PA-TMSVs: from top to bottom, lines correspond to $(3,3)$, $(2,2)$, and $(1,1)$. (b) PS-TMSVs: from top to bottom, lines correspond to $(3,3)$, $(3,2)$, $(3,1)$, and $(3,0)$. The black dashed curve corresponds to the TMSVs.

rewritten as

$$|\xi\rangle_{\text{ps}} = \sum_{n=0}^{\infty} \sqrt{\frac{(n+k)!(n+k)!}{(n!)^2 N_{k,k} \tanh^{-2k} r \cosh^2 r}} \tanh^{n+k} r |n,n\rangle. \tag{14}$$

From Eq. (8), we can derive

$$N_{k,k} \tanh^{-2k} r = \sum_{m=0}^{k} \frac{(k!)^4 (\cosh r \sinh r)^{2k}}{[m!(k-m)!]^2} \coth^{2m} r. \tag{15}$$

If setting $k - m = m'$, one can immediately obtain

$$N_{k,k} \tanh^{-2k} r = C_{k,k} = \sum_{m=0}^{k} \frac{(k!)^4 \cosh^{4k} r \tanh^{2m} r}{[m!(k-m)!]^2}. \tag{16}$$

Therefore, for a symmetric operation, both the PA-TMSVs and PS-TMSVs hold the same set of Schmidt coefficients which leads to the exact same quantum entanglement, a result noted in Ref. [7] ($k = l = 1$). For multiple-photon-added and multiple-photon-subtracted operations, as pointed out in Ref. [35], the optimal entanglement enhancement is obtained when the same number of operations is applied to both modes, as shown in Fig. 1, where addition and subtraction give the same entanglement enhancement. And the entanglement increases with the number of operations. For an asymmetric operation, their numerical analysis shows that it is always better to perform addition rather than subtraction in order to increase the entanglement, i.e., $E_{\text{pa}}^{k,l} > E_{\text{ps}}^{k,l}$. For a detailed discussion of the entanglement of these non-Gaussian states, please see Ref. [35].

III. EPR CORRELATION AND SQUEEZING PROPERTIES

In this section, we will further investigate the EPR correlation and the quadrature squeezing effects (including the two-mode squeezing and the sum squeezing) of non-Gaussian entangled states generated by multiple-photon addition and multiple-photon subtraction.

A. Einstein-Podolsky-Rosen correlation

Besides the degree of entanglement, non-Gaussian states expressed by Eqs. (1) and (7) can be characterized by the second-order EPR correlations between quadrature-phase components of the two modes. As counterparts of position and momentum operators of a massive particle, the quadrature-phase operators of each mode are defined as $X_j = (a_j + a_j^{\dagger})/\sqrt{2}$ and $P_j = (a_j - a_j^{\dagger})/(i\sqrt{2})$ $(j = A, B)$. Historically, continuous-variable entanglement originated in the paper of Einstein *et al.*, arguing on the incompleteness of quantum mechanics [44]. They proposed an ideal state which is the common eigenstate of a pair of EPR-like operators, $X_A - X_B$ (the relative position) and $P_A + P_B$ (the total momentum). Its explicit form is given by [45,46]

$$|\eta\rangle = \exp\left[-\tfrac{1}{2}|\eta|^2 + \eta a^{\dagger} - \eta^* b^{\dagger} + a^{\dagger} b^{\dagger}\right]|00\rangle, \tag{17}$$

where $\eta = \eta_1 + i\eta_2$ is a complex number. In order to quantify how well two-mode states approximate the EPR state of Eq. (17), one can define the EPR correlation parameter for a generic state ρ as [47,48]

$$\begin{aligned} \Upsilon(\rho) &= \Delta^2 (X_A - X_B) + \Delta^2 (P_A + P_B) \\ &= 2(\langle a^{\dagger} a \rangle + \langle b^{\dagger} b \rangle - \langle ab \rangle - \langle a^{\dagger} b^{\dagger} \rangle + 1) \\ &\quad - 2(\langle a \rangle - \langle b^{\dagger} \rangle)(\langle a^{\dagger} \rangle - \langle b \rangle), \end{aligned} \tag{18}$$

which is the total variance of EPR-like operations $X_A - X_B$ and $P_A + P_B$. In the EPR state [44–46] expressed by Eq. (17), one can easily prove that the EPR correlation $\Upsilon(\rho)$ equals zero. For separable two-mode states or any classical two-mode states, the total variance is larger than or equal to 2. The condition

$$\Upsilon(\rho) < 2 \tag{19}$$

indicates quantum entanglement, which is a crucial resource for quantum protocols using continuous variables [25,26].

Based on Eqs. (2) and (8), we can prove that $\langle a \rangle = \langle a^{\dagger} \rangle = \langle b \rangle = \langle b^{\dagger} \rangle = 0$ and $\langle \hat{a}\hat{b} \rangle = \langle \hat{a}^{\dagger}\hat{b}^{\dagger} \rangle$, as well as $\langle a^2 b^2 \rangle = \langle a^{\dagger 2} b^{\dagger 2} \rangle$. As a matter of convenience, we derive the expectation values of operators $a^{\dagger} a, b^{\dagger} b, ab, a^2 b^2$ in the PS-TMSVs as follows:

$$\langle a^{\dagger} a \rangle_{\text{ps}} = \frac{N_{k+1,l}}{N_{k,l}}, \quad \langle b^{\dagger} b \rangle_{\text{ps}} = \frac{N_{k,l+1}}{N_{k,l}},$$

$$\langle ab \rangle_{\text{ps}} = \frac{N_{k+1,k,l+1,l}}{N_{k,l}}, \quad \langle a^2 b^2 \rangle_{\text{ps}} = \frac{N_{k+2,k,l+2,l}}{N_{k,l}}, \tag{20}$$

and

$$\langle a^{\dagger} a b^{\dagger} b \rangle_{\text{ps}} = \frac{N_{k+1,l+1}}{N_{k,l}}. \tag{21}$$

For the PA-TMSVs, we have

$$\langle a^{\dagger} a \rangle_{\text{pa}} = \frac{C_{k+1,l}}{C_{k,l}} - 1, \quad \langle b^{\dagger} b \rangle_{\text{pa}} = \frac{C_{k,l+1}}{C_{k,l}} - 1,$$

$$\langle ab \rangle_{\text{pa}} = \frac{C_{k+1,k,l+1,l}}{C_{k,l}}, \quad \langle a^2 b^2 \rangle_{\text{pa}} = \frac{C_{k+2,k,l+2,l}}{C_{k,l}}, \tag{22}$$

and

$$\langle a^{\dagger} a b^{\dagger} b \rangle_{\text{pa}} = \frac{C_{k+1,l+1} - C_{k+1,l} - C_{k,l+1}}{C_{k,l}} + 1. \tag{23}$$

Thus, the EPR correlation of the PS-TMSVs reads

$$\Upsilon(\rho_{\text{ps}}) = 2 \frac{N_{k+1,l} + N_{k,l+1} - 2N_{k+1,k,l+1,l} + N_{k,l}}{N_{k,l}}. \tag{24}$$

In the case of the PA-TMSVs, the EPR correlation is described in the following form:

$$\Upsilon(\rho_{\text{pa}}) = 2 \frac{C_{k+1,l} + C_{k,l+1} - 2C_{k+1,k,l+1,l} - C_{k,l}}{C_{k,l}}. \tag{25}$$

Particularly, when $l = 0$ (or $k = 0$), the PS-TMSVs and the PA-TMSVs have the same EPR correlation; then, Eqs. (24) and (25) reduce to a simple form,

$$\Upsilon(\rho_{\text{ps}})|_{l=0} = \Upsilon(\rho_{\text{pa}})|_{l=0} = (2k+2)e^{-2r}. \tag{26}$$

This is not surprising, since adding k photons to the first mode has the same effect as subtracting them from the same mode. In the case of $k = l = 0$, Eq. (26) reduces to the EPR correlation of the TMSVs,

$$\Upsilon(\rho_0) = 2e^{-2r}, \tag{27}$$
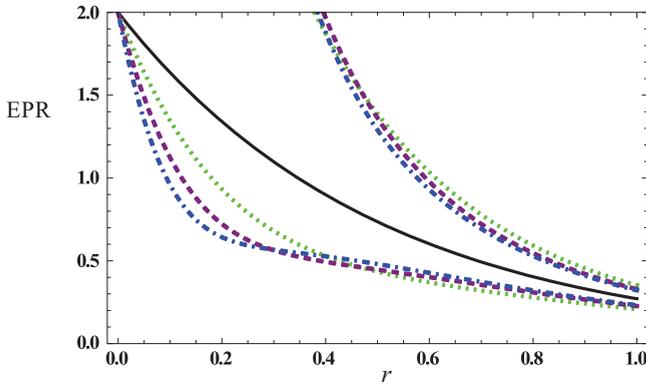
FIG. 2. (Color online) EPR correlation as a function of the squeezing parameter r for different non-Gaussian entangled states with $k = l$. The three upper lines correspond to the PA-TMSVs with $(1,1)$ (green dotted line), $(2,2)$ (purple dashed line), and $(3,3)$ (blue dot-dashed line). The three lower lines correspond to the PS-TMSVs with $(1,1)$ (green dotted line), $(2,2)$ (purple dashed line), and $(3,3)$ (blue dot-dashed line). The intermediate black curve corresponds to the TMSVs.

which tends to zero (the ideal EPR value) asymptotically for the squeezing parameter $r \to \infty$. From Eq. (26), it clearly shows that non-Gaussian entangled states generated by one-mode operations have a lower EPR correlation than that of the TMSVs. Therefore, one-mode photon-addition and photon-subtraction operations will diminish the EPR correlation. In the experiment, Takahashi *et al.* [10] have shown that subtracting one photon simultaneously from both modes of the TMSVs can improve the entanglement and the EPR correlation. And subtracting a single photon from one mode of the TMSVs enhances the entanglement, while it diminishes the EPR correlation.

In order to clearly see the EPR correlation of both the PS-TMSVs and PA-TMSVs with different values of (k,l), Fig. 2 shows the symmetric operations $(k = l)$. From Fig. 2, it can be seen that the EPR correlation of the PS-TMSVs is always lower than that of the TMSVs in the whole range of the initial squeezing. And the EPR correlation decreases with the number of subtracted photons in the regime of low-squeezing parameter r. For larger squeezing, this decrease becomes negligible. However, the EPR correlation of the PA-TMSVs is always larger than that of the input state for any values of (k,l). For asymmetric operations, both photon addition and subtraction generally reduce the EPR correlation, as shown in Fig. 3, which is different from that of the entanglement entropy (asymmetric subtraction and addition can also be used to improve the entanglement). In addition, our results indicate that the EPR correlation of both the PS-TMSVs and PA-TMSVs is optimized at symmetric operations $(k = l)$. For large-squeezing parameter r, the EPR correlation approaches zero, and photon addition or subtraction cannot improve much on this. This is because when $r \to \infty$, the EPR correlation is already closely approaching the ideal EPR value. In a realistic scenario, the photon subtraction will indeed degrade the EPR correlation for large initial squeezing due to the imperfections in the systems [11].
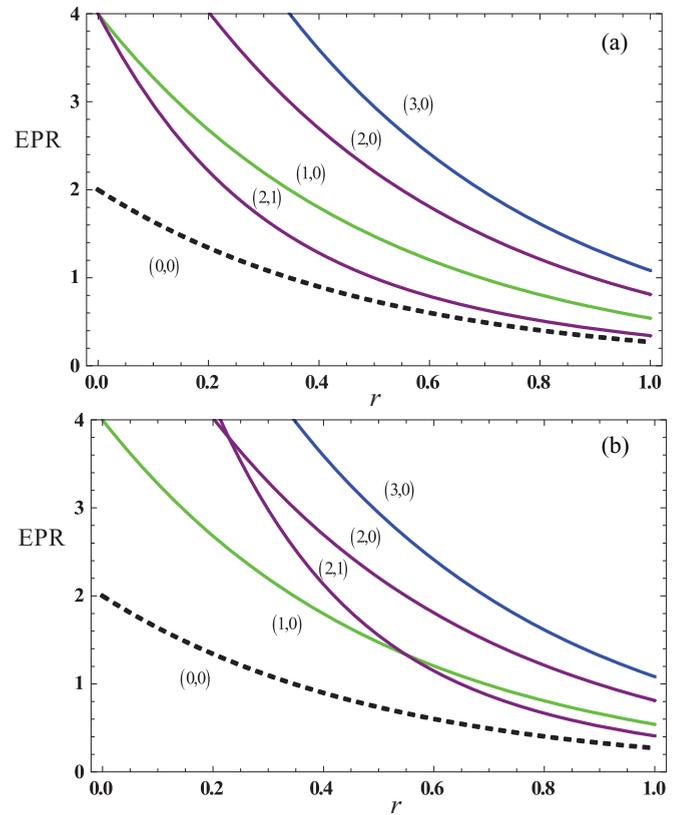
FIG. 3. (Color online) EPR correlation as a function of the squeezing parameter for the states with different values of (k,l): (a) PS-TMSVs, (b) PA-TMSVs. The black dashed line corresponds to the TMSVs.

Some works have pointed out that the larger amount of entanglement does not always means stronger EPR correlations [7–9,33]. And those states holding large entanglement may not be useful to improve quantum information processing. In the VBK protocol of quantum teleportation [25,26], the quantum channel is based on the EPR correlations and the fidelity of teleported states depends on the EPR correlations. Hence, we expect that the quality of quantum teleportation of continuous variables can be improved by the symmetric multiple-photon subtraction, and the fidelity can increase with the number of subtracted photons.

B. Two-mode squeezing

In quantum optics, squeezing is one of the earliest studied nonclassical phenomena. There are various types of squeezing, including single-mode squeezing, two-mode squeezing, sum squeezing and difference squeezing [49,50]. Here, we study the two-mode squeezing level in which the correlation between modes starts to play a role. For a two-mode system, the quadrature-phase amplitudes can be expressed by $X = (X_A + X_B)/\sqrt{2}$ and $P = (P_A + P_B)/\sqrt{2}$ ($[X,P] = 1$), respectively. Based on Eqs. (2) and (8), it is easy to see that $\langle a \rangle = \langle a^\dagger \rangle = \langle b \rangle = \langle b^\dagger \rangle = 0$ and $\langle a^2 \rangle = \langle a^{\dagger 2} \rangle = \langle b^2 \rangle = \langle b^{\dagger 2} \rangle = 0$ as well as $\langle ab^\dagger \rangle = \langle a^\dagger b \rangle = 0$, which lead to $\langle X \rangle = 0$ and $\langle P \rangle = 0$. Thus, the covariances of operators X and P in the PA-TMSVs

or PS-TMSVs can be written, respectively, as

$$(\Delta X)^2 = \frac{\langle a^\dagger a\rangle + \langle b^\dagger b\rangle + 2\langle ab\rangle + 1}{2} \qquad (28)$$

and

$$(\Delta P)^2 = \frac{\langle a^\dagger a\rangle + \langle b^\dagger b\rangle - 2\langle ab\rangle + 1}{2}. \qquad (29)$$

According to quantum mechanics, operators X and P satisfy the uncertainty relation $(\Delta X)^2(\Delta P)^2 \geq 1/4$. When $(\Delta X)^2 < 1/2$ or $(\Delta P)^2 < 1/2$, we can say there exists squeezing in the X or P direction. Compared with Eqs. (18), (24), and (25), we see that the EPR correlation of non-Gaussian entangled states is four times as much as that of the corresponding covariance of operator P, i.e.,

$$\Upsilon(\rho) = 4(\Delta P)^2, \qquad (30)$$

which indicates that the conditions of squeezing and entanglement become identical, which is an interesting result. Therefore, the variations of the two-mode squeezing level for both non-Gaussian states with different values of (k,l) yield the same law, as shown in Figs. 2 and 3. The subtraction operation can enhance the degree of the two-mode squeezing, particularly in the case of the symmetric operation. However, photon additions always weaken the squeezing level of the TMSVs. According to Eqs. (19) and (26), one can see that one-mode photon-addition and photon-subtraction operations will diminish the two-mode squeezing level and the EPR correlation of the TMSVs.

C. Sum squeezing

Sum and difference squeezing are both higher-order, two-mode squeezing effects [38,51]. For two arbitrary modes A and B, the sum squeezing is associated with a so-called two-mode quadrature operator V_φ of the form [38]

$$V_\varphi = \tfrac{1}{2}(e^{i\varphi}a^\dagger b^\dagger + e^{-i\varphi}ab), \qquad (31)$$

where φ is an angle made by V_φ with the real axis in the complex plane. A state is said to be sum squeezed for a φ if

$$\langle(\Delta V_\varphi)^2\rangle < \tfrac{1}{4}\langle a^\dagger a + b^\dagger b + 1\rangle. \qquad (32)$$

From Eq. (32), one can define the degree of sum squeezing S in the form of normally ordered operators as follows:

$$S = \frac{4\langle(\Delta V_\varphi)^2\rangle - \langle a^\dagger a + b^\dagger b + 1\rangle}{\langle a^\dagger a + b^\dagger b + 1\rangle}. \qquad (33)$$

Substituting Eq. (31) into Eq. (33), we obtain S as

$$S = \frac{2\langle a^\dagger ab^\dagger b\rangle + 2\mathrm{Re}(e^{-2i\varphi}\langle a^2 b^2\rangle) - 4\mathrm{Re}^2(e^{-i\varphi}\langle ab\rangle)}{\langle a^\dagger a + b^\dagger b + 1\rangle}. \qquad (34)$$

Then its negative value in the range $[-1,0]$ indicates sum squeezing (or higher-order nonclassicality). It is clear that S has a lower bound equal to -1. Hence, the closer the value of S to -1, the higher the degree of sum squeezing is. When $l = 0$ (or $k = 0$), the optimal degree of the sum squeezing of both the PS-TMSVs and PA-TMSVs reduces to that of the TMSVs
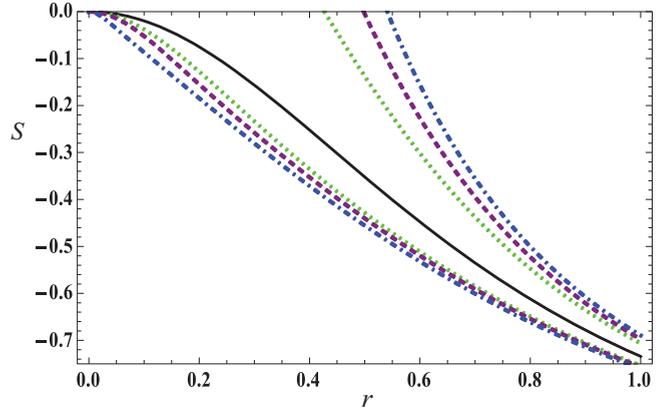
FIG. 4. (Color online) The sum squeezing degree S as a function of the squeezing parameter r for different states. The three upper lines correspond to the PA-TMSVs with $(1,1)$ (green dotted line), $(2,2)$ (purple dashed line), and $(5,5)$ (blue dot-dashed line). The three lower lines correspond to the PS-TMSVs with $(1,1)$ (green dotted line), $(2,2)$ (purple dashed line), and $(5,5)$ (blue dot-dashed line). The intermediate black curve corresponds to the TMSVs.

at $\varphi = \pi/2$,

$$S_{\mathrm{opt}} = -\frac{(e^{2r} - 1)^2}{(e^{4r} + 1)}, \qquad (35)$$

which is another interesting result. Thus, one-mode photon subtraction or addition does not change the sum squeezing of the TMSVs.

The sum squeezing of the PS-TMSVs is also optimized at $\varphi = \pi/2$. Our numerical analysis shows that the sum squeezing of the PS-TMSVs is always larger than that of the TMSVs, and the optimal sum squeezing is obtained for symmetric operations, as shown in Fig. 4. Thus, the degree of the sum squeezing can be improved by the photon subtraction, particularly in the case of symmetric operations. On the other hand, the photon addition weakens the sum squeezing.

In this section, we demonstrate that symmetric photon subtractions can enhance EPR correlation, the two-mode squeezing, and the sum squeezing of the TMSVs. And these quantities can be better improved with a large number of symmetric photon subtractions in the low-initial-squeezing regime. However, photon addition does not enhance these quantities at all, excluding the entanglement entropy.

IV. QUANTUM TELEPORTATION USING NON-GAUSSIAN ENTANGLED STATES

Bennet *et al.* [52] first proposed the idea of quantum teleportation in the discrete variable regime. After that, Vaidman [25] put forward the idea of continuous-variable quantum teleportation. Later, the quantum-optical protocol for the continuous-variable teleportation of the phase-quadrature components of a light was proposed by Braunstein and Kimble [26]. The best possible fidelity in teleporting a coherent state without entangled resources is $1/2$ [53], so the fidelity over the classical bound $1/2$ may be considered as a success for continuous-variable quantum teleportation. Based on the VBK protocol, the perfect teleportation can occur with an infinitely

entangled resource that exhibits an ideal EPR correlation [i.e., $\Delta^2(X_A - X_B) + \Delta^2(P_A + P_B) \to 0$].

The success probability to teleport a pure quantum state can be described through the teleportation fidelity, $F = \mathrm{Tr}[\rho_{\mathrm{in}}\rho_{\mathrm{out}}]$, which is a measure of how close it is between the initial input state and the final (mixed) output quantum state. In the formalism of the characteristic functions of the continuous variable, the fidelity can be written as [54]

$$F = \int \frac{d^2\alpha}{\pi} \chi_{\mathrm{in}}(-\alpha)\chi_{\mathrm{out}}(\alpha), \tag{36}$$

where $\chi_{\mathrm{out}}(\alpha) = \chi_{\mathrm{in}}(\alpha)\chi_{\mathrm{E}}(\alpha^*,\alpha)$ [55] is the characteristic function of the output teleported state. Here, $\chi_{\mathrm{E}}(\alpha^*,\alpha) = \mathrm{Tr}[D_1(\alpha)D_2(\beta)\rho_{\mathrm{E}}]$ is the characteristic function of an entangled resource, and $\chi_{\mathrm{in}}(\alpha) = \mathrm{Tr}[D(\alpha)\rho_{\mathrm{in}}]$ is that of the input state. In the following, we consider the non-Gaussian entangled states as entangled resources to teleport a coherent state and a squeezed vacuum state in the standard VBK teleportation protocol, respectively.

A. Teleporting a coherent state

Let us first consider the behavior of the fidelity for input of a coherent state $|\beta\rangle$, whose characteristic function is

$$\chi_{\mathrm{coh}}(\alpha) = \exp\left[-\tfrac{1}{2}|\alpha|^2 + 2i\,\mathrm{Im}[\alpha\beta^*]\right]. \tag{37}$$

Using the same approach as that used to derive Eq. (2), the characteristic functions of the PS-TMSVs and the PA-TMSVs read, respectively,

$$\chi_{\mathrm{ps}}(\alpha,\beta) = \frac{\chi(\alpha,\beta)\partial^{2k+2l}}{N_{k,l}\partial f^k \partial s^k \partial t^l \partial \tau^l} e^{(fs+t\tau)\sinh^2 r + (ft+s\tau)\frac{\sinh 2r}{2}}$$
$$\times e^{f(\alpha\sinh^2 r - \beta^*\frac{\sinh 2r}{2}) - s(\alpha^*\sinh^2 r - \beta\frac{\sinh 2r}{2})}$$
$$\times e^{t(\beta\sinh^2 r - \alpha^*\frac{\sinh 2r}{2})}$$
$$\times e^{-\tau(\beta^*\sinh^2 r - \alpha\frac{\sinh 2r}{2})}\big|_{f,s,t,\tau=0} \tag{38}$$

and

$$\chi_{\mathrm{pa}}(\alpha,\beta) = \frac{\chi(\alpha,\beta)\partial^{2k+2l}}{C_{k,l}\partial f^k \partial s^k \partial t^l \partial \tau^l} e^{(fs+t\tau)\cosh^2 r + (ft+s\tau)\frac{\sinh 2r}{2}}$$
$$\times e^{f(\alpha\cosh^2 r - \beta^*\frac{\sinh 2r}{2}) - s(\alpha^*\cosh^2 r - \beta\frac{\sinh 2r}{2})}$$
$$\times e^{t(\beta\cosh^2 r - \alpha^*\frac{\sinh 2r}{2})}$$
$$\times e^{-\tau(\beta^*\cosh^2 r - \alpha\frac{\sinh 2r}{2})}\big|_{f,s,t,\tau=0}, \tag{39}$$

where $\chi(\alpha,\beta)$ is the characteristic function of the TMSVs,

$$\chi(\alpha,\beta) = e^{-\frac{\cosh 2r}{2}(|\alpha|^2+|\beta|^2)+(\alpha\beta+\alpha^*\beta^*)\frac{\sinh 2r}{2}}. \tag{40}$$

It can be seen that Eqs. (38) and (39) substantially differ for the exchange of the hyperbolic coefficients. Upon substituting these characteristic functions into Eq. (36), we can work out the fidelities for teleporting a coherent state. For the PS-TMSVs, we have the teleportation fidelity of a coherent state,

$$F_{\mathrm{ps}} = \frac{N_{k,l}^{-1}2^k k! l!}{e^{-2r}+1}\left[\frac{(e^{2r}-1)^2}{4e^{2r}+4}\right]^l P_k^{l-k,0}\left(\frac{e^{4r}+2e^{2r}+5}{4e^{2r}+4}\right), \tag{41}$$

where $P_n^{(\alpha,\beta)}(x)$ is the Jacobi polynomial. Different from that work of Ref. [56], here we obtain the general expression of the

fidelity for teleporting a coherent state. For the PA-TMSVs, the teleportation fidelity of a coherent state can be written in a simple form,

$$F_{\mathrm{pa}} = \frac{(k+l)!}{4^{k+l}C_{k,l}}\frac{(e^{2r}+1)^{k+l}}{e^{-2r}+1}, \tag{42}$$

which is a special case of that in Ref. [57]. It can be seen that the fidelities depend on the squeezing parameter r and the number of operations (k,l). Note that the fidelity is independent of the amplitudes of the coherent state; thus Eqs. (41) and (42) are just the fidelity for teleporting a vacuum state. In the case of one-mode operations (for example $l = 0$), Eqs. (41) and (42) reduce to

$$F' = \left(\frac{1}{e^{-2r}+1}\right)^{k+1}, \tag{43}$$

which is always smaller than that of the TMSVs, for any values of the squeezing parameter r. Therefore, one-mode operations will also diminish the fidelity of teleporting a coherent state.

Now, we can numerically study the behavior of the fidelity for teleporting a coherent state by making use of the PS-TMSVs and the PA-TMSVs. In Fig. 5(a), we plot the fidelity
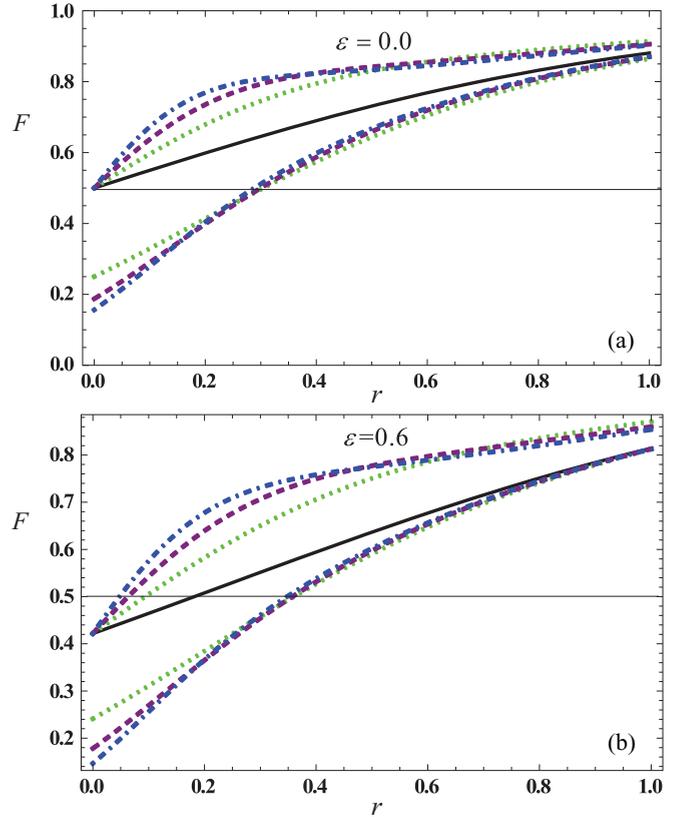
FIG. 5. (Color online) Fidelity as a function of the squeezing parameter for different states. The three upper lines correspond to the PS-TMSVs with $(1,1)$ (green dotted line), $(2,2)$ (purple dashed line), and $(3,3)$ (blue dot-dashed line). The three lower lines correspond to the PA-TMSVs with $(1,1)$ (green dotted line), $(2,2)$ (purple dashed line), and $(3,3)$ (blue dot-dashed line). The intermediate solid curve corresponds to the TMSVs. (a) The single-mode squeezing parameter $\varepsilon = 0.0$; (b) $\varepsilon = 0.6$.
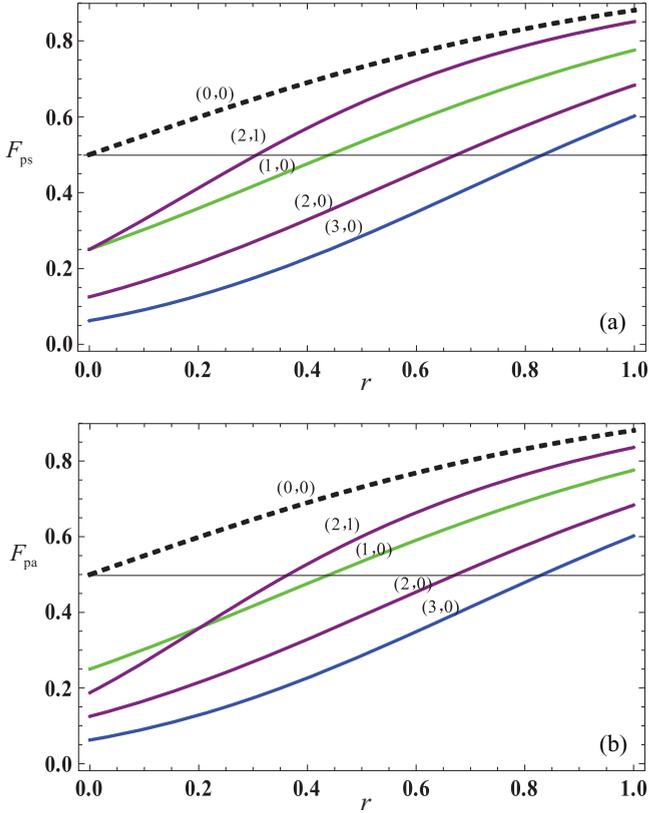
FIG. 6. (Color online) Fidelity as a function of the squeezing parameter for different states: (a) PS-TMSVs and (b) PA-TMSVs.

for input of a coherent state in the case of $k = l$, i.e., the symmetric operation. From Fig. 5(a), we can see that the fidelity for the PS-TMSVs is always larger than that of the TMSVs, while the fidelity for the PA-TMSVs is always smaller than that of the TMSVs, even smaller than $1/2$ in the low-squeezing regime. For the asymmetric operation ($k \neq l$), our numerical analysis shows that both photon subtraction and addition generally weaken the fidelity, as shown in Fig. 6. Thus, the optimal fidelity is arrived at for symmetric operations.

In the VBK protocol of quantum teleportation of continuous variables, the fidelity of teleported states depends on the EPR correlations. Thus, for teleporting a coherent state, the higher EPR correlation means the higher fidelity. Comparing the teleportation fidelity with the EPR correlation in Figs. 2 and 4, one can observe that the EPR correlation and the fidelity can be enhanced by the symmetric photon-subtraction operation in the whole region of the squeezing parameter r. In addition, the teleportation fidelity for both the PS-TMSVs and PA-TMSVs could be beyond the classical limit of $1/2$ without the EPR correlation, as shown in Figs. 2 and 4. Thus, the teleportation fidelity, which is larger than $1/2$, does not guarantee the existence of the EPR correlation.

B. Teleporting a squeezed vacuum state

The characteristic function of the squeezed Gaussian input state, $|\varepsilon\rangle = S(\varepsilon)|0\rangle$ with the single-mode squeezing operator $S(\varepsilon) = \exp[\varepsilon(a^{\dagger 2} - a^2)/2]$ (ε is the single-mode squeezing

parameter), reads

$$\chi_{\rm in}(\alpha) = \exp\left[-\frac{\cosh 2\varepsilon}{2}|\alpha|^2 + (\alpha^2 + \alpha^{*2})\frac{\sinh 2\varepsilon}{4}\right]. \quad (44)$$

With the help of the integral formula

$$\int \frac{d^2 z}{\pi} e^{\zeta|z|^2 + \xi z + \eta z^* + f z^2 + g z^{*2}} = \frac{1}{\sqrt{\zeta^2 - 4fg}} e^{\frac{-\zeta\xi\eta + \xi^2 g + \eta^2 f}{\zeta^2 - 4fg}}, \quad (45)$$

whose convergent condition is $\mathrm{Re}(\zeta \pm f \pm g) < 0$ and $\mathrm{Re}(\frac{\zeta^2 - 4fg}{\zeta \pm f \pm g}) < 0$, we can work out the fidelity for teleporting a squeezed state by using the PS-TMSVs

$$
\begin{aligned}
F_{\rm ps} = {} & \frac{F_0}{N_{k,l}} \frac{\partial^{2k+2l}}{\partial f^k \partial s^k \partial t^l \partial \tau^l} \\
& \times \exp\left\{(fs + t\tau)\sinh^2 r + (ft + s\tau)\frac{\sinh 2r}{2}\right. \\
& \left. + \frac{(f - \tau)(t - s)(e^{-2r} - 1)^2(e^{-2r} + \cosh 2\varepsilon)}{4(2e^{-2r}\cosh 2\varepsilon + e^{-4r} + 1)} \right. \\
& \left. + \frac{[(f - \tau)^2 + (t - s)^2](e^{-2r} - 1)^2 \sinh 2\varepsilon}{8(2e^{-2r}\cosh 2\varepsilon + e^{-4r} + 1)}\right\}\Bigg|_{f,s,t,\tau=0}, \\
& \hspace{8cm} (46)
\end{aligned}
$$

where F_0 is the fidelity for the TMSVs,

$$F_0 = \sqrt{\frac{1}{2e^{-2r}\cosh 2\varepsilon + e^{-4r} + 1}}. \quad (47)$$

It can be seen that the fidelity is not only dependent on the squeezing parameter r and the number of subtracted photons (k,l), but also on the single-mode squeezing parameter ε. Because of the arbitrary order partial derivatives in Eq. (46), finding a general expression presents challenges. When the squeezing parameter $\varepsilon = 0$, Eq. (46) reduces to Eq. (41), i.e., the fidelity of a coherent state. For the PA-TMSVs, we obtain the fidelity of teleportation of a squeezed vacuum state as

$$
\begin{aligned}
F_{\rm pa} = {} & \frac{F_0}{C_{k,l}} \frac{\partial^{2k+2l}}{\partial f^k \partial s^k \partial t^l \partial \tau^l} \\
& \times \exp\left\{(fs + t\tau)\cosh^2 r + (ft + s\tau)\frac{\sinh 2r}{2}\right. \\
& \left. + \frac{(f - \tau)(t - s)(e^{-2r} + 1)^2(e^{-2r} + \cosh 2\varepsilon)}{4(2e^{-2r}\cosh 2\varepsilon + e^{-4r} + 1)} \right. \\
& \left. + \frac{[(f - \tau)^2 + (t - s)^2](e^{-2r} + 1)^2 \sinh 2\varepsilon}{8(2e^{-2r}\cosh 2\varepsilon + e^{-4r} + 1)}\right\}\Bigg|_{f,s,t,\tau=0}. \\
& \hspace{8cm} (48)
\end{aligned}
$$

In the case of $\varepsilon = 0$, Eq. (48) reduces to a simple form expressed by Eq. (42).

For the one-mode operation, Eqs. (46) and (48) reduce to

$$F' = \frac{F_0^{2k+1}}{(e^{-2r}\cosh 2\varepsilon + 1)^{-k}} \sum_m^{[k/2]} \frac{k!\left[\frac{\sinh 2\varepsilon}{(\cosh 2\varepsilon + e^{2r})}\right]^{2m}}{2^{2m}(m!)^2(k - 2m)!}. \quad (49)$$
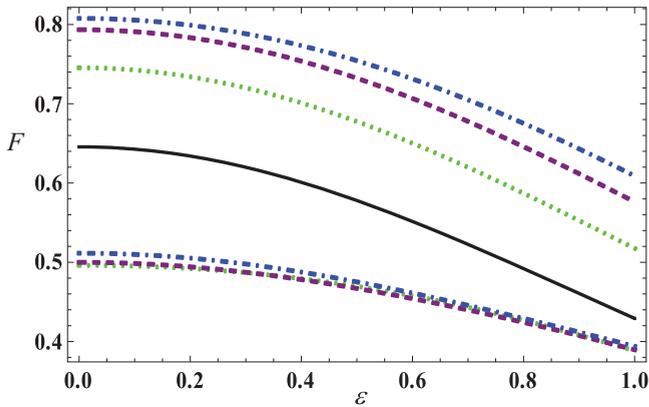
FIG. 7. (Color online) Fidelity as a function of the single-mode squeezing parameter ε with the two-mode squeezing $r = 0.3$ for different states. The three upper lines correspond to the PS-TMSVs with $(1,1)$ (green dotted line), $(2,2)$ (purple dashed line), and $(3,3)$ (blue dot-dashed line). The three lower lines correspond to the PA-TMSVs with $(1,1)$ (green dotted line), $(2,2)$ purple dashed line), and $(3,3)$ (blue dot-dashed line). The intermediate black solid curve corresponds to the TMSVs.

Noting that the new expression of Legendre polynomials [58] is

$$P_k(x) = x^k \sum_{m=0}^{[k/2]} \frac{k! \left(1 - \frac{1}{x^2}\right)^m}{2^{2m}(m!)^2(k-2m)!}, \quad (50)$$

Eq. (49) can be written as

$$F' = F_0^{k+1} P_k \left[\frac{(e^{-2r} \cosh 2\varepsilon + 1)}{\sqrt{2e^{-2r} \cosh 2\varepsilon + e^{-4r} + 1}} \right]. \quad (51)$$

Because the factor $F_0^k P_k(x)$ in Eq. (51) is always smaller than 1 for any values of both squeezing parameter r and ε, one-mode photon subtraction and photon addition also diminish the teleportation fidelity of a squeezed vacuum state. Obviously, when $\varepsilon = 0$, Eq. (51) reduces to Eq. (43), which is the fidelity of teleporting a coherent state.

Compared with the coherent state, the fidelity of teleporting a squeezed vacuum state decreases with its squeezing parameter ε, as shown in Figs. 5(b) and 7. For symmetric operation $(k = l)$, the fidelity for the PS-TMSVs is always larger than that of the TMSVs when teleporting a squeezed vacuum state, while the fidelity for the PA-TMSVs is generally smaller than that of the TMSVs, even smaller than $1/2$ in the low-squeezing regime. Only in the large-squeezing regime can the fidelity for the PA-TMSVs be larger than that of the TMSVs, as shown in Fig. 5(b). For the asymmetric operation $(k \neq l)$, both photon addition and subtraction generally weaken the fidelity, which is similar to that behavior of teleporting a coherent state as shown in Fig. 6. Thus, the optimal fidelity of teleporting a squeezed vacuum state is also arrived at for symmetric operations.

Although the numerical analysis shows that it is always better to perform addition rather than subtraction in order to increase the entanglement, only non-Gaussian entangled states generated by symmetric photon subtraction can result in the advantage in the teleportation fidelity of a coherent or a squeezed vacuum state, compared to just using the

TABLE I. The fidelity varies for some different PS-TMSVs given the EPR correlation parameter $\Upsilon(\rho) = 1.0$. The required initial-squeezing parameter r and the corresponding von Neumann entropies are also presented.

(k,l)	$(2,2)$	$(1,1)$	$(0,0)$	$(1,0)$	$(2,1)$	$(2,0)$
r	0.1226	0.1798	0.3462	0.6931	0.5000	0.8959
F_{ps}	0.6632	0.6637	0.6665	0.6400	0.6379	0.6300
$E_{\mathrm{ps}}^{k,l}$	0.5841	0.5755	0.5662	2.094	2.0925	3.1624

corresponding Gaussian TMSVs with the same initial-squeezing parameter r. It has been known that in the non-Gaussian case, the teleportation fidelity becomes a highly complicated function of three variables: the entanglement, the degree of non-Gaussianity, and the degree of Gaussian affinity [7,8,33]. Thus the optimal teleportation fidelity does not, in general, correspond to the maximal entanglement. In the following, we consider the PS-TMSVs as an entangled resource to teleport a coherent state. In Table I, we fix the EPR correlation parameter $\Upsilon(\rho) = 1.0$, and then we can obtain the corresponding initial-squeezing parameter and the teleportation fidelity for different (k,l), as well as the von Neumann entropy. From Table I, we can see that the non-Gaussian entangled states generated by the symmetric subtraction almost hold the same fidelity as the fixed EPR correlation. The fidelity for the TMSVs is just a little higher than that for the PS-TMSVs. Although the optimal entangled resource for the teleportation of a coherent state via the ideal VBK scheme actually reduces to the TMSVs, it is at a price, i.e., the need of the higher-initial-squeezing parameter. In addition, Table I clearly shows again that stronger entanglement does not mean higher teleportation fidelity even in the multiple-photon-subtraction scheme, which is an important illustration of the general results for the squeezed Bell states that coincides with photon-subtracted states [7]. In the limit of infinite squeezing, since the TMSVs tends to become the ideal EPR state with perfect EPR correlations, the fidelity of teleportation also approaches unity. Up to the present, the largest achievable two-mode squeezing in a stable optical configuration is about $r \approx 1.15$ (i.e., about 10 dB) [59]. Hence, techniques that improve the performance of the teleportation without demanding higher initial squeezing are still useful in quantum information. In this regard, for a given two-mode squeezing parameter r, the TMSVs which is engineered by non-Gaussian operation, such as symmetric multiple-photon subtraction, contains more of the two-mode squeezing or holds a higher EPR correlation. Thus non-Gaussian entangled states may still be advantageous for the quantum teleportation.

On the other hand, it may be interesting to investigate the performance of different non-Gaussian entangled states for teleporting a Gaussian state at the fixed entanglement entropy, rather than at the fixed squeezing parameter. When making the comparison at the fixed entanglement entropy, Kogias et al. [60] found in all considered cases that within the general squeezed Bell-like class, the optimal resource state for teleportation of input ensembles of Gaussian states via the gain-optimized VBK scheme actually does always reduce to the TMSVs. In Fig. 8, we draw the teleportation fidelity of coherent states as a function of the EPR correlation and
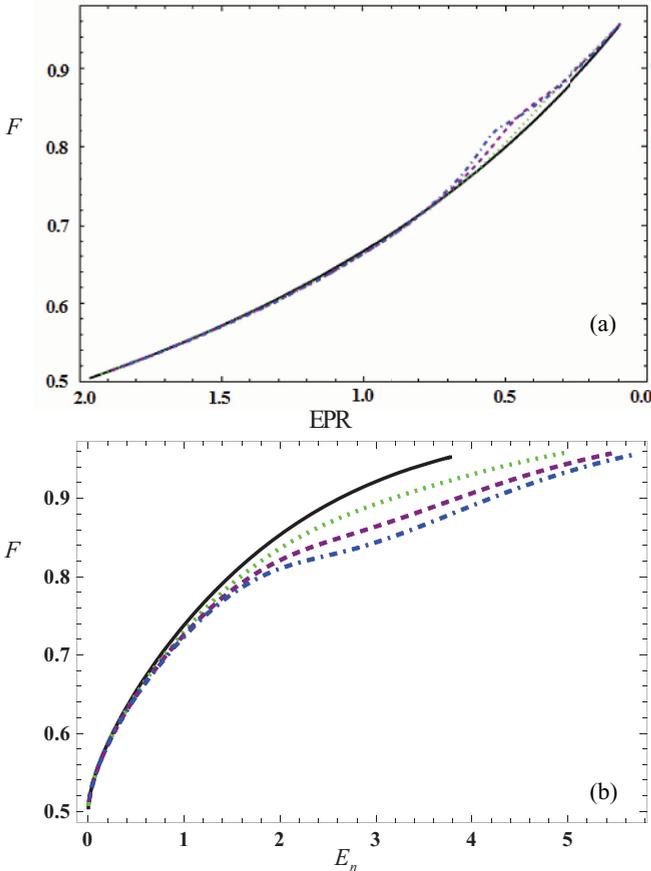
FIG. 8. (Color online) Fidelity of coherent teleportation with the PS-TMSVs entangled resource (a) at the fixed EPR parameter and (b) at the fixed entanglement entropy. These lines correspond to the PS-TMSVs with (0,0) (black line), (1,1) (green dotted line), (2,2) (purple dashed line), and (3,3) (blue dot-dashed line).

the von Neumann entropy, respectively. Figure 8(a) shows that if the EPR correlation parameter $\Upsilon(\rho)$ is smaller than a threshold value (about 0.8), the optimal entangled resource for the teleportation of a coherent state via the ideal VBK scheme reduces to the PS-TMSVs generated by symmetric operation. When $\Upsilon(\rho) > 0.8$, the optimal entangled resource reduces to the TMSVs, for example, $\Upsilon(\rho) = 1.0$ in Table I. On the contrary, at fixed the von Neumann entropy, the optimal entangled resource does always reduce to the TMSVs, as shown in Fig. 8(b), which is consistent with that in Ref. [60]. From those results in Refs. [7,8,33,60] and our results in the present work, it is clearly seen that such conclusion is strongly dependent on the terms of comparison. This is mainly because in the non-Gaussian case the teleportation fidelity depends not only on the entanglement, but also on the degree of non-Gaussianity and the degree of Gaussian affinity [7,8,33]. For an entangled Gaussian resource, the teleportation fidelity depends only on the entanglement, and both quantities are in an exact one-to-one correspondence [24].

V. CONCLUSIONS

In summary, we have shown that the symmetric multiple-photon subtraction ($k = l$) can enhance the EPR

correlation, the quadrature squeezing, and the teleportation fidelity of a two-mode squeezed vacuum state (TMSVs) in the whole region of the initial-squeezing parameter r. Those enhancements are more distinct in the low-initial-squeezing regime and increase with the number of subtracted photons. The asymmetric operation generally diminishes the EPR correlation, the two-mode squeezing, the sum squeezing, and the teleportation fidelity, although it can enhance the entanglement. For any values of (k,l), the multiple-photon addition can better increase the degree of entanglement while it diminishes the EPR correlation, the two-mode squeezing, the sum squeezing, and the fidelity for teleporting a coherent state at the same time. Thus, in the multiple-photon-subtraction or multiple-photon-addition schemes, our results clearly show again that the entanglement enhancement does not imply that the teleportation fidelity must be improved. The reason is that the improvement of the fidelity is due to a balancing of three different features: the entanglement content of the resources, their amount of non-Gaussianity, and the degree of Gaussian affinity [7]. When considering the case of teleporting a squeezed vacuum state $|\varepsilon\rangle$, the symmetric photon addition makes somewhat of an improvement on the fidelity for large-squeezing parameters r and ε. For both the PS-TMSVs and PA-TMSVs, the four quantities, including the optimal entanglement, the optimal EPR correlation, the optimal quadrature squeezing, and the optimal teleportation fidelity, always prefer symmetrical arrangements of photon addition or subtraction on the two modes. Our results indicate that the symmetric multiple-photon subtraction may be more useful than the photon addition in continuous-variable quantum information processing.

At present, the best experimentally realized non-Gaussian entangled resource for continuous-variable teleportation is the photon-subtracted squeezed states [6,19]. In a realistic photon-subtraction scenario, the finite transmission coefficient of the beam splitter and the losses in the bosonic channels, as well as the imperfection in photon-detection techniques, have a degrading effect on the output entanglement and the fidelity of the coherent teleportation [3,5,10,11]. Due to these imperfections in these kinds of systems, only when the initial squeezing is below a certain value can the symmetric subtraction be used to improve the entanglement, the EPR correlation, and the fidelity of the coherent teleportation. In the large-initial-squeezing regime, those imperfections in the photon-subtraction scheme can make idealistic models qualitatively wrong: for example, entanglement may decrease instead of increasing and EPR correlations may degrade instead of improving (as shown in Refs. [3,5,10,11]). On the other hand, with the development of the techniques of quantum-state engineering, it can be possible to minimize those imperfections, particularly those imperfections in the lossy transmission channels and the photon-detection techniques. Recently, Dell'Anno *et al.* [61] introduced and discussed a novel set of tunable non-Gaussian entangled resources which contains the theoretical squeezed Bell state, as well as an efficient scheme for their experimental generation. They find that optimized tunable non-Gaussian resources can continue to outperform the corresponding Gaussian resources in the realistic scenario, and even extend to the large-initial-squeezing regime. Therefore, our theoretical results derived in terms of the idealistic multiple-photon-

subtraction and multiple-photon-addition schemes are still meaningful.

In addition, we have analytically proved that the one-mode multiple-photon-subtracted TMSVs is equivalent to that of the one-mode multiple-photon-added one. For the one-mode operation, we have derived analytical expressions of the EPR correlation, two-mode squeezing, sum squeezing, and the teleportation fidelity, respectively. These analytical expressions clearly represent that one-mode multiple-photon operations do not enhance them at all, and even diminish them. Finally, we have proved that the EPR correlation of the PA-TMSVs and PS-TMSVs is four times as much as that of the corresponding quantum fluctuation $(\Delta P)^2$, which indicates that the conditions of the two-mode squeezing and the entanglement become identical, which is an interesting result.

ACKNOWLEDGMENTS

APPENDIX: DERIVATION OF EQS. (2) AND (8)

In the Fock space, the TMSVs can be written as

$$S_2(\xi)|00\rangle = \frac{1}{\cosh r} \exp[a^\dagger b^\dagger \tanh r]|00\rangle. \quad \text{(A1)}$$

The expectation value of a general product of operators $a^p a^{\dagger q} b^h b^{\dagger j}$ in the TMSVs reads

$$C_{p,q,h,j} = Tr(a^{\dagger q} b^{\dagger j} S_2(\xi)|00\rangle\langle00|S_2^\dagger(\xi)a^p b^h). \quad \text{(A2)}$$

Substituting Eq. (A1) into (A2) and inserting the completeness relation of the two-mode coherent state, as well as using the integral formulas

$$\int \frac{d^2 z}{\pi} \exp[\zeta|z|^2 + \xi z + \eta z^*] = -\frac{1}{\zeta}\exp\left[-\frac{\xi\eta}{\zeta}\right], \quad \text{(A3)}$$

whose convergent condition is $\text{Re}(\zeta) < 0$, after doing straightforward calculation, we obtain

$$
\begin{aligned}
C_{p,q,h,j} &= \frac{1}{\cosh^2 r}\int \frac{d^2 z_1 d^2 z_2}{\pi^2} z_1^p z_2^h z_1^{*q} z_2^{*j}\exp[\tanh r(z_1^* z_2^* + z_1 z_2) - (|z_1|^2 + |z_2|^2)] \\
&= \frac{1}{\cosh^2 r}\int \frac{d^2 z_1 d^2 z_2}{\pi^2}\frac{\partial^{p+q+h+j}}{\partial f^p \partial s^q \partial t^h \partial \tau^j}\exp[-(|z_1|^2 + |z_2|^2) + f z_1 + s z_1^* + t z_2 + \tau z_2^* + \tanh r(z_1^* z_2^* + z_1 z_2)]|_{f,s,t,\tau=0} \\
&= \frac{\partial^{p+q+h+j}}{\partial f^p \partial s^q \partial t^h \partial \tau^j}\exp\left[(fs + t\tau)\cosh^2 r + (ft + s\tau)\frac{\sinh 2r}{2}\right]\Bigg|_{f,s,t,\tau=0}. \quad \text{(A4)}
\end{aligned}
$$

By the binomial theorem, we further obtain Eq. (2),

$$
\begin{aligned}
C_{p,q,h,j} &= \frac{\partial^{q+h}}{\partial s^q \partial t^h}\left(s\cosh^2 r + t\frac{\sinh 2r}{2}\right)^p\left(s\frac{\sinh 2r}{2} + t\cosh^2 r\right)^j\Bigg|_{s,t=0} \\
&= \sum_{m}^{\min[p,h]}\frac{p!q!h!j!(\cosh^2 r)^{p+h-2m}}{m!(p-m)!(h-m)!}\frac{\left(\frac{\sinh 2r}{2}\right)^{j-h+2m}\delta_{p+j,q+h}}{(j-h+m)!}. \quad \text{(A5)}
\end{aligned}
$$

Next, the expectation value of a general product of operators $a^{\dagger q} a^p b^{\dagger j} b^h$ in the TMSVs reads

$$N_{p,q,h,j} = \text{Tr}[a^p b^h S_2(\xi)|00\rangle\langle00|S_2^\dagger(\xi)a^{\dagger q} b^{\dagger j}]. \quad \text{(A6)}$$

By using the same approach as that to derive Eq. (A5), substituting Eq. (A1) into (A6), and inserting the completeness relation of the two-mode coherent state for two times, we have

$$N_{p,q,h,j} = \frac{\partial^{p+q+h+j}}{\partial f^p \partial s^q \partial t^h \partial \tau^j}\exp\left[(fs + t\tau)\sinh^2 r + (ft + s\tau)\frac{\sinh 2r}{2}\right]\Bigg|_{f,s,t,\tau=0}. \quad \text{(A7)}$$

Similarly, we finally obtain Eq. (6),

$$N_{p,q,h,j} = \sum_{m}^{\min[p,h]}\frac{p!q!h!j!(\sinh^2 r)^{p+h-2m}}{m!(p-m)!(h-m)!}\frac{\left(\frac{\sinh 2r}{2}\right)^{j-h+2m}\delta_{p+j,q+h}}{(j-h+m)!}. \quad \text{(A8)}$$

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