Quantum theory for pulse propagation in electromagnetically-induced-transparency media beyond the adiabatic approximation

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Beyond the adiabatic approximation, we develop a quantum theory for optical probe pulses propagating in electromagnetically-induced-transparency (EIT) media by including Langevin noise operators and asking the field operator to satisfy bosonic commutation relation. Influences on the degradation of quantum noise squeezing from optical depth of atomic ensemble, strength of control field, and ground-state decoherence are studied in the slow light, as well as storage and retrieval, for a squeezed probe pulse. Moreover, to give guidelines for realization of quantum interfaces based on EIT media, we demonstrate that the quantum squeezing of output probe pulses could be preserved with a stronger classical control field.

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I. INTRODUCTION

Storage and retrieval of light plays a crucial role both for optical information processing and communication network [1,2]. In particular, through the coherent Raman interference, classical information of probe pulses, its profile and phase, can be controlled with another strong field in the electromagnetically-induced-transparency (EIT) media [3–5]. In addition to classical light sources, nonclassical light sources, such as single-photon states, are investigated first in order to map quantum state of light onto atomic ensembles as a quantum memory device [6–10]. Then, another family of nonclassical light sources, i.e., squeezed states with reduced quantum fluctuation below that of vacuum, are performed in experiments with demonstrations in the slow light [11,12], as well as the storage and retrieval with a squeezed vacuum pulse [13,14].

For quasicontinuous input fields with perturbed quantum fluctuations, it is known that EIT media become opaque for squeezed states [15]. Oscillatory transfer of initial quantum properties between the probe and pump fields is studied to preserve nonseparable entanglement between quantized electromagnetic fields [16]. However, the above theoretical approaches work only for continuous waves, and are not applied for probe pulses in the storage and retrieval process [17]. Within the adiabatic approximation, such a light storage and retrieval process can be clearly illustrated by the picture of dark-state polariton, which is a linear superposition between optical pulse field and atomic polarization state [18–20]. In this framework of dark-state polaritons, photon and atomic states can be mutually transferred by adjusting the control field strength without introducing any extra noises. Even though one can introduce a squeezed operator for these quantized dark-state polaritons, the corresponding squeezed state transfer, quantum correlation, and noise entanglement between probe field and atomic polarization are found as a consequence in the adiabatic limit [21]. Furthermore, in terms of quantizations, photon and atomic states follow different commutation relations. It is thus very desirable to develop a generalized quantum theory to go beyond the adiabatic approximation for pulse propagation in EIT media.

In this work, we develop a quantum theory for optical probe pulses propagating in the EIT media, by introducing Langevin noise operators to go beyond the adiabatic approximation. By requiring the quantized probe field to satisfy bosonic commutation relations, one can find the corresponding quantum noises contributed from all possible dissipative processes. Based on our theory, first we demonstrate quantum fluctuations for the slow-light case when a squeezed vacuum pulse is incident into a EIT medium with a strong classical control field. Then, we study the quadrature noise fluctuations for a squeezed pulse during the storage and retrieval process. Our results not only show the degradation of quantum noise squeezing but also give agreement both to the realizations in the slow light experiment reported in the literature, as well as storage and retrieval of squeezed probe pulses. Based on the quantum theory developed in this work, we reveal that even in atomic systems with a higher optical density, one can preserve the quantum noise squeezing in the output probe field by using a stronger control field. In nonlinear bulk media and optical fibers, the quantum theory for pulse propagation has been applied successfully for the generation of macroscopic nonclassical states exhibiting quadrature squeezing, intrapulse and/or interpulse quantum correlation, and entangled soliton pulses [22–27]. The quantum theory for propagating pulses in EIT media also paves the guideline for quantum memory devices in quantum information and computation.

The remaining part of this paper is organized as follows. In Sec. II, we start from the Maxwell-Bloch equations for a quantized probe pulse and a classical control field in the EIT configuration. Without applying the adiabatic approximation, we derive a propagation equation for the quantized probe pulse by including Langevin noise operators. Then, in Sec. III, quadrature variances in the noise fluctuations for probe pulse are derived both for control fields in time-independent (slow-light) and time-dependent (storage and retrieval) cases. Results based on adiabatic approximations are also shown as a comparison, which clearly reveal a discrepancy on the output noise fluctuations. Moreover, in Sec. IV, we discuss the quadrature noise fluctuations for different optical depths of atomic ensemble and different strengths of control field for slow-light processes. Finally, we give a brief conclusion in Sec. V.

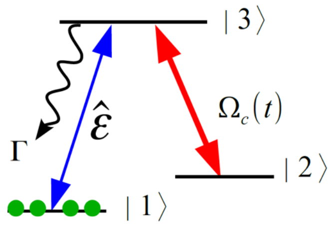

FIG. 1. (Color online) EIT system considered in a single-Λ configuration, where the transitions $|1\rangle \leftrightarrow |3\rangle$ and $|2\rangle \leftrightarrow |3\rangle$ are driven resonantly by a quantized probe field, $\hat{\mathcal{E}}$, and a classical control field denoted by its Rabi frequency, Ω_c, separately. The decay rate in the excited state $|3\rangle$ is denoted by Γ.

II. THEORETICAL MODEL

We begin with the EIT system in a single-Λ configuration, as illustrated in Fig. 1. Here, two copropagating beams pass through a three-level atomic ensemble in the z direction, with the total number of atoms denoted by N. A probe field excites the transition from state $|1\rangle$ to state $|3\rangle$, with the transition frequency ω_{31}, which is treated by a quantum field operator $\hat{\mathcal{E}}(z,t)$ in the slowly varying envelope approximation. The transition between $|2\rangle$ and $|3\rangle$, with the transition frequency ω_{32}, is driven resonantly by a classical control field with the Rabi frequency denoted by $\Omega_c(t)$, which is a time-dependent function during the storage and retrieval process. The two-photon detuning is set as zero to have a transparency window at the frequency ω_{31}. In the Heisenberg picture, the corresponding equations of motion to describe EIT systems can be written as [3,4]

$$\left(\frac{\partial}{\partial t} + c\frac{\partial}{\partial z}\right)\hat{\mathcal{E}} = igN\hat{\sigma}_{13}, \tag{1}$$

$$\frac{\partial}{\partial t}\hat{\sigma}_{13} = -\gamma_{13}\hat{\sigma}_{13} + ig\hat{\mathcal{E}} + i\Omega_c\hat{\sigma}_{12} + \hat{F}_{13}, \tag{2}$$

$$\frac{\partial}{\partial t}\hat{\sigma}_{12} = -\gamma_{12}\hat{\sigma}_{12} + i\Omega_c^*\hat{\sigma}_{13} + \hat{F}_{12}, \tag{3}$$

where $\hat{\sigma}_{\mu\nu} = |\mu\rangle\langle\nu|$ $(\mu,\nu = 1,2,3)$ is used as the collective atomic operator, the constant g denotes the atom-field coupling strength for the transition $|1\rangle \leftrightarrow |3\rangle$, and c denotes the speed of light in the vacuum. Dephasing rate for the dipole transition σ_{13} is denoted by γ_{13}, being equal to $\Gamma/2$, where Γ is the decay rate of the excited state $|3\rangle$. The decoherence rate between two ground states here is γ_{12}. Moreover, in order to satisfy the dissipation-fluctuation theory, Langevin noise operator, $\hat{F}_{13}$ and $\hat{F}_{12}$, is also introduced, which has a zero-mean expectation value $\langle\hat{F}_{\mu\nu}\rangle = 0$ [28,29].

Since the control field is stronger than the probe field, we can safely assume that the atomic population is almost in the ground state. Then, the corresponding dipole transition $\hat{\sigma}_{13}$ changes slowly compared to the excited state decay rate. As a consequence, one can ignore the time derivative term in Eq. (2) and obtain

$$\hat{\sigma}_{13} = \frac{1}{\gamma_{13}}(ig\hat{\mathcal{E}} + i\Omega_c\hat{\sigma}_{12} + \hat{F}_{13}). \tag{4}$$

By substituting the solution shown in Eq. (4) into Eq. (1) and Eq. (3), we have two coupled equations as follows:

$$\left(\frac{\partial}{\partial t} + c\frac{\partial}{\partial z}\right)\hat{\mathcal{E}} = -\frac{g^2N}{\gamma_{13}}\hat{\mathcal{E}} - \frac{gN\Omega_c}{\gamma_{13}}\hat{\sigma}_{12} + i\frac{gN}{\gamma_{13}}\hat{F}_{13}, \tag{5}$$

$$\frac{\partial}{\partial t}\hat{\sigma}_{12} = -\left(\gamma_{12} + \frac{|\Omega_c|^2}{\gamma_{13}}\right)\hat{\sigma}_{12} - \frac{g\Omega_c^*}{\gamma_{13}}\hat{\mathcal{E}} + i\frac{\Omega_c^*}{\gamma_{13}}\hat{F}_{13} + \hat{F}_{12}. \tag{6}$$

From Eqs. (5) and (6), it is clearly illustrated that the decay process in the atomic excited state contributes to the noise fluctuations both for field operator $\hat{\mathcal{E}}$ and atomic dipole moment $\hat{\sigma}_{12}$. Typically, the adiabatic condition is applied for Eq. (6), by assuming $\hat{\sigma}_{12} = -g\,\hat{\mathcal{E}}/\Omega_c$. However, such an adiabatic approximation does not take into account the quantum difference between quantized probe field and atomic polarization, i.e., the former one must obey the bosonic commutation relation, while the latter one follows the fermionic commutation relation.

In Appendix A, through the Laplace transform, the corresponding output probe field can be found in the following form:

$$\begin{aligned}\hat{\mathcal{E}}(z,\tau) &= e^{-\alpha}\hat{\mathcal{E}}(0,\tau) + \sqrt{\alpha}\,e^{-\alpha}\frac{\Omega_c(\tau)}{\gamma_{13}}\\ &\quad\times\int_0^\tau d\tau'\Omega_c^*(\tau')e^{-\gamma_{12}(\tau-\tau')-\kappa(\tau,\tau')}\frac{I_1(\sqrt{4\alpha\kappa(\tau,\tau')})}{\sqrt{\kappa(\tau,\tau')}}\\ &\quad\times\hat{\mathcal{E}}(0,\tau') + \hat{n}(z,\tau).\end{aligned} \tag{7}$$

Here, $I_n(x)$ is the modified Bessel function, resulting from the inverse Laplace transform $\mathcal{L}^{-1}(s^{-(n+1)}e^{as^{-1}}) = (\frac{z}{a})^{n/2}I_n(\sqrt{4az})$. Moreover, $\alpha \equiv \frac{g^2N}{c\gamma}z$ is half of the optical depth $(D_{\rm opt} \equiv 2\alpha)$ for an EIT medium, and $\kappa(\tau,\tau') \equiv \frac{1}{\gamma_{13}}\int_\tau^{\tau'} d\tau''|\Omega_c(\tau'')|^2$ is a dimensionless quantity. The formula for this effective Langevin noise operator $\hat{n}$ can be found in Eq. (A4) for all the details.

In Eq. (7), there are three different terms contributing to the pulse propagation. The first term, following Lambert-Beer's absorption law, corresponds to the attenuation of input probe pulse owing to the decay process in the excited state. The second term is dominant in the EIT system, which shows that the probe field can almost penetrate through atomic ensembles. The last term, $\hat{n}$, can be expressed in terms of the original noise operator $\hat{F}_{13}$ and $\hat{F}_{12}$, as shown in Eq. (A4). Even though the required Langevin noise operator, $\hat{n}$, has a complicated form, nevertheless, we recast this effective Langevin noise operator by asking the field operator $\hat{\mathcal{E}}$ to satisfy the bosonic commutation relation, i.e., $[\hat{\mathcal{E}}(L,\tau),\hat{\mathcal{E}}^\dagger(L,\tau)] = 1$. Due to the fact that the EIT medium is a linear system, we can contribute all possible noises stemmed from the interaction between atomic reservoirs into this effective Langevin noise operator $\hat{n}$. We want to remark that in deriving Eq. (7), the adiabatic approximation that $\hat{\sigma}_{12} \approx -g\,\hat{\mathcal{E}}/\Omega_c$ is not applied here. By including Langevin noise operators, the required commutation relation for atomic polarizations to satisfy is automatically included in this propagation equation for the quantized probe pulse. Base on this equation, we derive the quadrature variances in noise fluctuations for probe pulse in the following section, along with a comparison to the slow-light and light-storage-retrieval processes.

III. QUANTUM NOISE REDUCTION IN EIT MEDIA

Now, we apply our theoretical results to the slow-light and light-storage-retrieval cases with a squeezed vacuum pulse propagating in the EIT medium. Here, we define in-phase and out-of-phase quadrature operators for the quantized field $\hat{X} = \hat{\mathcal{E}} + \hat{\mathcal{E}}^\dagger$ and $\hat{Y} = -i(\hat{\mathcal{E}} - \hat{\mathcal{E}}^\dagger)$, which correspond to the amplitude and phase fluctuations, respectively. For a given length of EIT media, $z = L$, this in-phase quadrature operator at the output can be found explicitly:

$$\hat{X}_L(\tau) = e^{-\alpha}\hat{X}_0(\tau) + \int_0^\tau d\tau' f(\tau,\tau')\hat{X}_0(\tau') + \hat{X}_n, \quad (8)$$

where $\hat{X}_n \equiv \hat{n} + \hat{n}^\dagger$ is the corresponding quadrature noise operator, and the shorthand notation $f(\tau,\tau')$ is defined as

$$f(\tau,\tau') = \sqrt{\alpha}\, e^{-\alpha}\frac{\Omega_c(\tau)\Omega_c^*(\tau')}{\gamma_{13}} \times e^{-\gamma_{12}(\tau-\tau')-\kappa(\tau,\tau')}\frac{I_1[\sqrt{4\alpha\kappa(\tau,\tau')}]}{\sqrt{\kappa(\tau,\tau')}}. \quad (9)$$

Based on Eq. (8), we can obtain the quadrature variance for output probe pulse. That is

$$\langle\hat{X}_L^2(\tau)\rangle = e^{-D_{\rm opt}}\langle\hat{X}_0^2(\tau)\rangle + T_p\int_0^\tau d\tau'[f(\tau,\tau')]^2\langle\hat{X}_0^2(\tau')\rangle + T_p e^{-D_{\rm opt}/2} f(\tau,\tau)\langle\hat{X}_0^2(\tau)\rangle + \langle\hat{X}_n^2\rangle, \quad (10)$$

where $D_{\rm opt} = 2\alpha$. We also have applied the equal-space commutation relation for the quantized field, $[\hat{\mathcal{E}}(L,\tau_1),\hat{\mathcal{E}}^\dagger(L,\tau_2)] = T_p\delta(\tau_1 - \tau_2)$, with the time constant T_p used to imply that $T_p^{-1}\int\mathcal{E}^\dagger(z,t)\mathcal{E}(z,t)\,dt$ has physical meaning of photon number inside the whole light pulse. The mathematical formula to obtain a proper time constant, T_p, is shown in Appendix B. In deriving Eq. (10), uncorrelated relation between input field and noise reservoir is also assumed, i.e., $[\hat{X}_0,\hat{X}_n] = 0$. The quadrature variance of noise operator $\hat{n}$ is found by following Eq. (7), along with the bosonic commutation relation for the probe field, i.e.,

$$\langle\hat{X}_n^2\rangle = 1 - e^{-D_{\rm opt}} - T_p\int_0^\tau d\tau'[f(\tau,\tau')]^2 - T_p e^{-D_{\rm opt}/2} f(\tau,\tau) + 2\langle\hat{n}^\dagger\hat{n}\rangle. \quad (11)$$

Here, $\langle\hat{n}^\dagger\hat{n}\rangle$ is proportional to the mean photon number in the bosonic thermal reservoir, which is zero at the absolute zero temperature $T = 0$ K. By substituting Eq. (11) into Eq. (10) with the help of Eq. (9), the output quadrature variance for probe pulse in EIT media has the form

$$\langle\hat{X}_L^2(\tau)\rangle = 1 - e^{-D_{\rm opt}}\left(1 + T_p\frac{|\Omega_c|^2}{\Gamma}(D_{\rm opt})\right)\left(1 - \langle\hat{X}_0^2(\tau)\rangle\right) - T_p\int_0^\tau d\tau'[f(\tau,\tau')]^2\left(1 - \langle\hat{X}_0^2(\tau')\rangle\right) + 2\langle\hat{n}^\dagger\hat{n}\rangle. \quad (12)$$

From Eq. (12), we can see clearly that for a coherent state input, with the $\langle\hat{X}_0^2(\tau)\rangle = 1$, the output variance is above one with additional noise fluctuations from thermal noises in the coupled reservoirs. When the input field is a squeezed light, i.e., $\langle\hat{X}_0^2(\tau)\rangle < 1$, the second and third terms in Eq. (12) will reduce the output variance accordingly, while the last term in Eq. (12) always destroys the quantum squeezing, resulting in a larger quantum variance in the output field.

At $T = 0$ K, even though contributions from thermal noises vanish, quantum fluctuations still remain. In the following calculations, we treat the system at $T = 0$ K, and do not take thermal fluctuations into consideration by excluding any external thermal noises. In the typical experimental conditions to realize EIT phenomena, contribution from the second term in Eq. (12) is quite small for $D_{\rm opt} \gg 1$. In this scenario, the dominant contribution to the degradation in quantum noise squeezing comes from the third term, which can be viewed as the initial squeezed variance weighted by the response function $[f(\tau,\tau')]^2$ through a convolution integral.

Now, we turn to the studies on a squeezed vacuum pulse propagating in EIT media. The incident intensity distribution of probe field is described by a Gaussian profile, $\psi(\tau) = \exp[-\tau^2/\Delta\tau^2]$, with the width of intensity $\Delta\tau$ at e^{-1}, and such a weighting function is imposed on the probe pulse for the initial quantum variance, i.e.,

$$\langle\hat{X}_0^2(\tau)\rangle = 1 - (1 - e^{-2r_0})\psi(\tau), \quad (13)$$

where r_0 is the maximum value for the degree of squeezing. It can be seen that the squeezed variance reaches its maximum value at the center of input pulse, while the background variance reflects that of vacuum states as the standard quantum limit.

For the slow-light case, by referring to the experiment conducted by Akamatsu *et al.*, in Refs. [11,12], we apply our quantum theory of pulse propagation to reveal the output variance in a quantized probe field by setting the system parameters as follows. The width of probe field is set as $\Delta\tau \simeq 255/\Gamma$, optical density of the system is $D_{\rm opt} = 10$, and the maximum degree of squeezing in the incident pulse (at the peak) is $r_0 = 0.115$. Here, all the parameters are normalized with respect to the decay rate in the excited state, Γ. The results of output variance calculated by Eq. (12) are shown in Fig. 2(a) for three different strengths of control field, i.e., $\Omega_c = 0.13\Gamma$, 0.07Γ, and 0.04Γ, respectively. The peak position of output pulse follows the same analytical formula for an EIT medium. As the strength of control field decreases, the group velocity of probe pulse goes slowing down accordingly. From Fig. 2(a), one can see that the quantum variances for squeezed probe pulse evolve as that of the classical profile of probe pulse in this slow-light region. In this scenario, the squeezed photons carried by probe pulse propagate as a whole with the same slowing-down group velocity. Moreover, the decay process from the atomic system adds additional fluctuations into the probe pulse, resulting in the degradation of squeezed variance.

One may wonder what would be the results in the adiabatic limit. Here, we also provide the comparison between adiabatic and nonadiabatic approaches. According to the adiabatic approximation, we neglect the time derivative, damping terms, and related Langevin noise operators in Eq. (6). Then, by applying $\hat{\sigma}_{12} = -g\,\hat{\mathcal{E}}/\Omega_c$ and using the Laplace transform

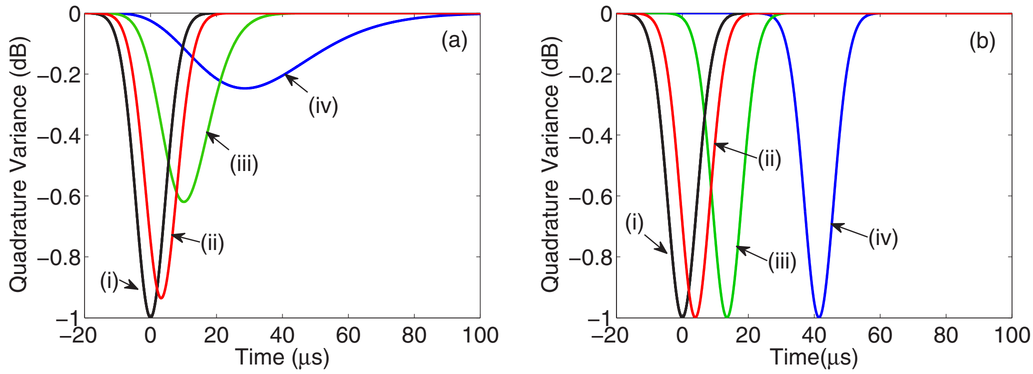

FIG. 2. (Color online) Evolutions of the quadrature noise variance of a squeezed probe pulse propagating in the slow-light region: (a) shows the results beyond the adiabatic approximation, while (b) shows the results under the adiabatic approximation. Here, (i) shows the initial noise fluctuation of an incident squeezed vacuum pulse with squeezing degree $r_0 = 0.115$, while (ii)–(iv) show the noise fluctuations of output pulses. The coupling strengths of control fields are (ii) $\Omega_c = 0.13\Gamma$, (iii) 0.07Γ, and (iv) 0.04Γ, respectively. The optical density ($D_{\rm opt}$) of the system is 10, the width of input squeezed pulse at e^{-1} is $\Delta\tau = 255/\Gamma$, and the ground-state spontaneous decay rate is set as $\gamma_{12} = 0$.

method, we have the output probe field in the form

$$\hat{\mathcal{E}}(z,\tau) = \frac{\Omega_c(\tau)}{\Omega_c(\tau_p)}\hat{\mathcal{E}}(0,\tau_p). \tag{14}$$

According to Eq. (14), the ratio between probe and control fields is fixed, i.e., $\hat{\mathcal{E}}(z,\tau)/\Omega_c(\tau) = \hat{\mathcal{E}}(0,\tau_p)/\Omega_c(\tau_p)$. Thus we can clearly see that the ground-state coherence propagates with the probe field under the adiabatic approximation. The corresponding quadrature noise variance for output probe field can be found as

$$\langle \hat{X}_L^2(\tau)\rangle = 1 - \left[\frac{\Omega_c(\tau)}{\Omega_c(\tau_p)}\right]^2 [1 - \langle \hat{X}_0^2(\tau_p)\rangle] + 2\langle \hat{u}^\dagger \hat{u}\rangle, \tag{15}$$

where τ_p satisfies the relation $\mathcal{G}(\tau_p) = \mathcal{G}(\tau) - \frac{g^2N}{c}z$, and $\mathcal{G}(\tau) \equiv \int_0^\tau d\tau'|\Omega_c(\tau')|^2$. Here, we also introduce an effective noise operator $\hat{u}$, in order to satisfy the required bosonic commutation relation.

Equation (15) gives the output noise variance in the adiabatic limit, based on which we show the results in Fig. 2(b) for the slow-light case. As a comparison, we apply all the same system parameters used in Fig. 2(a). One can see that even though a time delay in the output probe pulse is revealed in the adiabatic limit, the resulting quadrature noise fluctuations remain the same as the input pulse. That is a consequence in the adiabatic limit [21]. With the comparisons to the nonadiabatic results shown in Fig. 2(a), a discrepancy between the nonadiabatic and adiabatic limits can be understood clearly.

Next, we apply our theoretical results to the light storage and retrieval process with a squeezed vacuum pulse, by referring to the experimental parameters reported in the literature [13,14]. Here, the set of system parameters are pulse width of the probe field $\Delta\tau = 255/\Gamma$ and the maximum degree of squeezing in the incident pulse $r_0 = 0.115$. For the probe pulse being fully stored, we choose the optical density $D_{\rm opt} = 50$, the Rabi frequency of control field before and after storage being

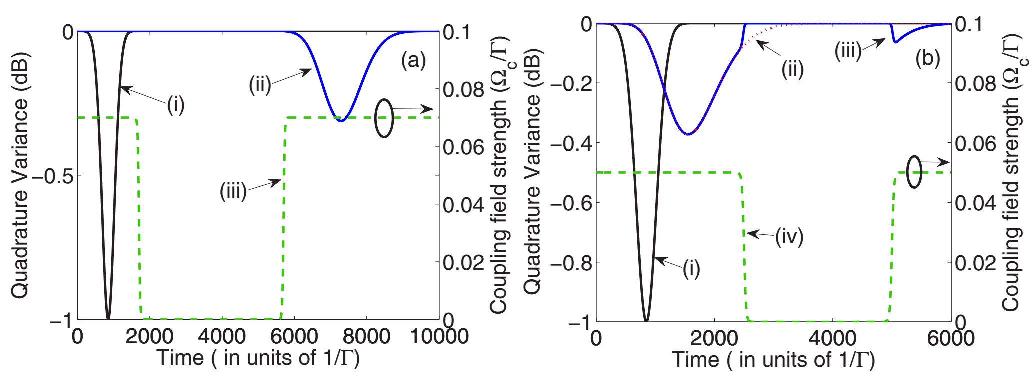

FIG. 3. (Color online) Quadrature noise variance of a squeezed probe pulse during the storage and retrieval process. (a) The case when the probe pulse is fully stored in the EIT medium, with $D_{\rm opt} = 50$. Here, the input noise fluctuation is shown in (i), while the output fluctuation retrieved after a storage period $t_s = 4000(1/\Gamma)$ is shown in (ii). The corresponding control field strength as a function of time is shown in (iii). (b) The case for the storage only a partial of probe pulse, with $D_{\rm opt} = 10$. Here, the initial noise fluctuation of incident squeezed vacuum pulse is shown in (i), the noise fluctuation of output pulse in the slow-light case, with $\Omega_c = 0.05\Gamma$, is shown in (ii), and the corresponding noise fluctuation of output pulse after storage and retrieval process, with the storage time $t_S = 2500(1/\Gamma)$, is shown in (iii). The corresponding control field strength is depicted in (iv). Other parameters used are as follows: the width of input squeezed pulse at the intensity e^{-1} is $\Delta\tau = 255/\Gamma$ and the ground-state decay rate $\gamma_{12} = 0$.

$\Omega_c = 0.07\Gamma$, and the period during the storage $t_s = 4000/\Gamma$. Results on the output quadrature variance in the probe pulse after storage and retrieval process is shown in Fig. 3(a). Degradation on the noise squeezing due to atomic noise fluctuations can be seen clearly for the output pulse. Again, our theoretical formula can be also applied to the case when the probe field is only partially stored, as shown in Fig. 3(b) for $D_{\rm opt} = 10$, the Rabi frequency of control field before and after storage being $\Omega_c = 0.05\Gamma$, and $t_s = 2500/\Gamma$. Based on these, one can expect to have an optimization protocol for the squeezed probe field in the storage and retrieval process.

IV. DISCUSSIONS

For the comparison between nonadiabatic and adiabatic limits, as shown in Eq. (12) and Eq. (15), one can see that the difference comes from the optical density ($D_{\rm opt} = 2\alpha$) of atomic system and the related response function $[f(\tau,\tau')]^2$. It is known that for an atomic system with a higher $D_{\rm opt}$, the decay of classical field goes faster. The decay is proportional to the exponential function $\exp[-D_{\rm opt}]$, which is the same scenario for the quantum noise variance as shown in the second term of Eq. (12). However, the response function $[f(\tau,\tau')]^2$ gives a counterintuitive result. To have a clear picture on this response function, we consider the slow-light case as an example, i.e., $\Omega_c =$ const, and rewrite $|f(\tau,\tau')|^2$ shown in Eq. (9) in terms of $D_{\rm opt}$:

$$[f(\tau,\tau')]^2 \equiv [f(\Delta\tau)]^2 = \frac{(D_{\rm opt})e^{-D_{\rm opt}}}{\Gamma\Delta\tau}|\Omega_c|^2 \times \exp\left(-\frac{4|\Omega_c|^2\Delta\tau}{\Gamma}\right) I_1^2\left[\sqrt{\frac{4(D_{\rm opt})|\Omega_c|^2\Delta\tau}{\Gamma}}\right], \qquad (16)$$

where $\Delta\tau \equiv \tau - \tau'$. From Eq. (16), one can find that the response function, f, in the slow-light case is a function of optical density $D_{\rm opt}$ and control field strength Ω_c. When Ω_c increases, the width of this response function decreases. In this scenario, the interaction time between field and atoms is reduced accordingly, resulting in preserving the output quantum fluctuations easily. On the other hand, the output noise variance suffers less with a smaller optical density.

To illustrate the influence from different optical densities, in Figs. 4(a) and 4(b), we show the quadrature variance of a squeezed probe pulse propagating in the slow-light region for (a) $D_{\rm opt} = 5$ and (b) $D_{\rm opt} = 50$, respectively. Naively, a larger value of the optical density results in a large degradation in the degree of output noise squeezing. However, if the strength of control field Ω_c increases at the same time, such a degradation in the noise squeezing can be avoided. In Fig. 4(c), the minimum variance of quadrature noise fluctuations (the dip in the profile) is shown for different optical depths ($D_{\rm opt}$) of atomic ensembles and different strengths of control field (Ω_c) in the slow-light region. With the same parameter set, the evolutions of quadrature noise variance are depicted for $D_{\rm opt} = 5$, 10, and 50 as a comparison. One can see that as the control field Ω_c is small, a quick change in the quadrature noise variance occurs for all values of $D_{\rm opt}$. However, when the control field strength Ω_c is large enough, the quadrature noise approaches a constant value. It can be seen that different values of $D_{\rm opt}$ result in the same constant value of quadrature noise variance when Ω_c becomes larger. This result reflects that when a wide enough transparent window is supported in EIT media, the output noise variance can be kept as the same as that of the input one for all values of $D_{\rm opt}$'s.

Moreover, in practical experiments, spontaneous decay in the ground states, γ_{12}, can be another decoherence mechanism for the degradation in the output noise squeezing. Here,

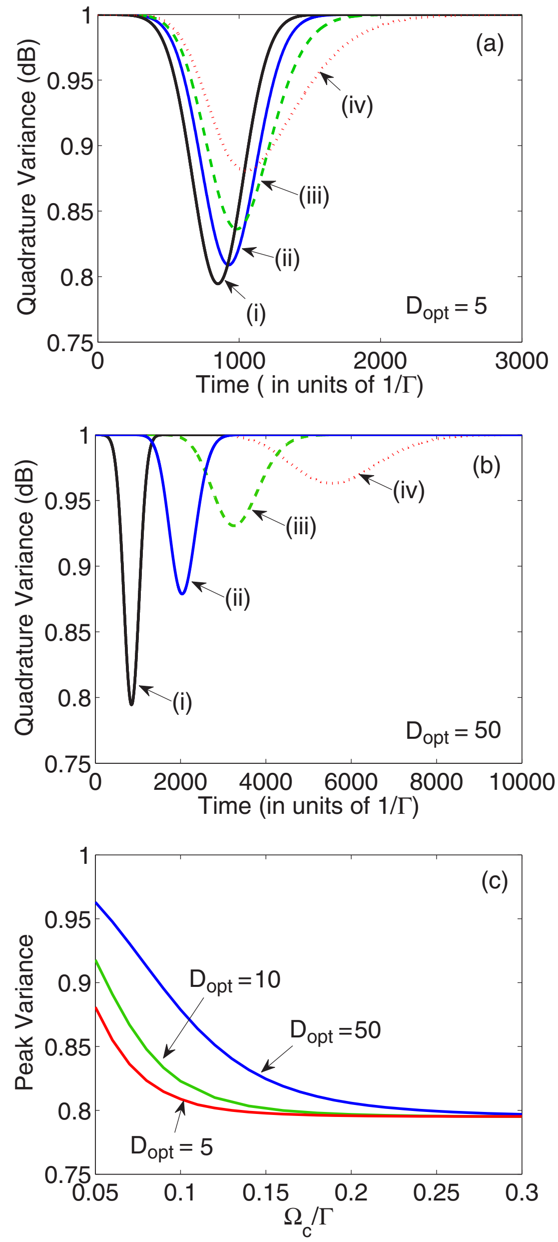

FIG. 4. (Color online) Quadrature noise variance of a squeezed probe pulse propagating in the slow-light region for different optical densities: (a) $D_{\rm opt} = 5$ and (b) $D_{\rm opt} = 50$. (i) The quadrature variance of the incident squeezed pulse; while (ii)–(iv) show the quadrature variance of output pulses under different control field strengths. (ii) $\Omega_c = 0.1\Gamma$, (iii) $\Omega_c = 0.07\Gamma$, and (iv) $\Omega_c = 0.05\Gamma$, respectively. The peak variance of quadrature noise (the dip in the profile) is shown in (c) as a function of Ω_c, for $D_{\rm opt} = 5$, 10, and 50, respectively.

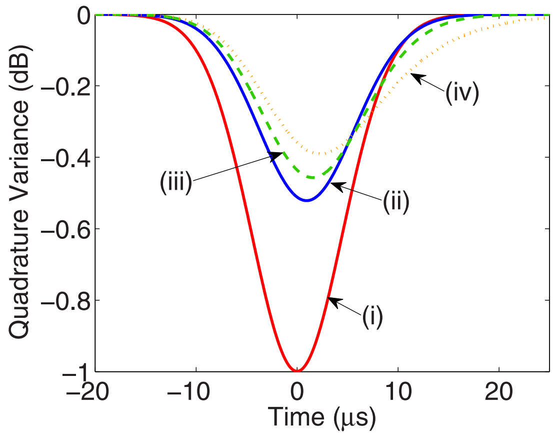

FIG. 5. (Color online) Quantum noise variance of a squeezed probe pulse with the introduction of ground-state decoherence γ_{12}. The input variance is shown in (i), while the output variances are depicted for (ii) $\Omega_c = 0.1005\Gamma, \gamma_{12} = 0.0033\Gamma$, (iii) $\Omega_c = 0.0715\Gamma, \gamma_{12} = 0.0019\Gamma$, and (iv) $\Omega_c = 0.495\Gamma, \gamma_{12} = 0.00087\Gamma$, respectively.

we take the ground-state decoherence into consideration by referring to the experiments conducted by Akamatsu *et al.*, who have performed measurements on the noise squeezing in the slow-light case, both for continuous waves [11] and for pulses [12]. By adopting experimental parameters from the measured photon flux shown in Fig. 4(c) of Ref. [12], we calculate the output quadrature noise variance of a squeezed probe pulse in Fig. 5. Good agreement between our theoretical results and experimental observations can be found easily. For all the three cases shown in Fig. 5, theoretical results on the output noise squeezing is slightly better than those from experimental measurement. Such a small discrepancy on the values comes from the reason that only quantum noises are considered, while all other possible noise sources from the environment are neglected. It is noted that one can expand Eq. (12) with nonresonance processes by replacing γ_{13} by $\gamma_{13} - i\Delta_p$ and γ_{12} by $\gamma_{12} - i\delta$, respectively, where Δ_p and δ are the corresponding one- and two-photon detunings in EIT media.

V. CONCLUSION

In summary, we have developed a quantum theory for the quantized probe pulse propagating in EIT media beyond the adiabatic limit. By requiring the bosonic commutation relation for quantized probe field to be satisfied, we derive the required Langevin noise operator, which contributes to the quadrature variance in the output field. Our results show the degradation of quantum noise squeezing, which is missing in the adiabatic limit, both for the slow-light and light-storage-retrieval processes with a squeezed probe pulse. Such a quantum mechanical approach gives a clear physical insight to the quantum noise variance in the output probe field, related to the control field strength and optical density of atomic ensemble. This work provides a deeper understanding in the quantum memory with squeezed light sources, and can also be applied to the relevant quantum information processing.

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APPENDIX A

In this Appendix, we provide the formula to derive Eq. (7) by solving the coupled equation of Eqs. (5) and (6). First of all, we define the interaction window both in space and time. Here, at $t = 0$, the probe field enters the EIT medium with the length of atomic cloud L, and light-atom interactions take place only inside the atomic ensemble, i.e., $0 < z < L$ and $t > 0$. Furthermore, we also transform our system into the moving frame, by changing the variables to $\tau = t - z/c$ and $z' = z$, respectively. It should be noted that the control field, Ω_c, propagates with the speed close to light speed, or equivalently $\Omega_c(z,t) = \Omega_c(t - z/c) = \Omega_c(\tau)$. Since the initial condition of input probe field is given, we apply the *Laplace transform* by changing the coordinate space $z \to s$, i.e., by defining $\hat{E}(s,\tau) = \mathcal{L}[\hat{\mathcal{E}}(z,\tau)]$, $\hat{P}(s,\tau) = \mathcal{L}[\hat{\sigma}_{12}(z,\tau)]$, $\tilde{F}_{13}(s,\tau) = \mathcal{L}[\hat{F}_{13}(z,\tau)]$, and $\tilde{F}_{12}(s,\tau) = \mathcal{L}[\hat{F}_{12}(z,\tau)]$. Then in the s domain, Eqs. (5) and (6) become

$$\hat{E}(s,\tau) = (s')^{-1}\left[-\frac{gN\Omega_c(\tau)}{c\gamma_{13}}\hat{P}(s,\tau) + \hat{\mathcal{E}}(0,\tau) + i\frac{gN}{c\gamma_{13}}\tilde{F}_{13}(s,\tau)\right], \tag{A1}$$

$$\frac{\partial}{\partial\tau}\hat{P}(s,\tau) = -\left(\gamma_{12} + \frac{|\Omega_c|^2}{\gamma_{13}}\right)\hat{P}(s,\tau) - \frac{g\Omega_c^*(\tau)}{\gamma_{13}}\hat{E}(s,\tau) + \tilde{F}(s,\tau), \tag{A2}$$

where $s' = s + \frac{g^2N}{c\gamma_{13}}$, and $\tilde{F}(s,\tau) = i\frac{\Omega_c^*}{\gamma_{13}}\tilde{F}_{13}(s,\tau) + \tilde{F}_{12}(s,\tau)$. By substituting Eq. (A1) into Eq. (A2) and solving the differential equation, we can obtain the solution as shown below:

$$\begin{aligned}\hat{P}(s,\tau) = {} & \hat{P}(s,0)\exp\left[-\gamma_{12}\tau - \kappa(\tau,0) + \frac{g^2N}{c\gamma_{13}}\kappa(\tau,0)(s')^{-1}\right] \\ & - \frac{g}{\gamma_{13}}\int_0^\tau d\tau'\Omega_c^*(\tau')e^{-\gamma_{12}(\tau-\tau')-\kappa(\tau,\tau')}\hat{\mathcal{E}}(0,\tau')(s')^{-1} \\ & \times \exp\left\{\frac{g^2N}{c\gamma_{13}}\kappa(\tau,\tau')(s')^{-1}\right\} \\ & + \int_0^\tau d\tau' e^{-\gamma_{12}(\tau-\tau')-\kappa(\tau,\tau')}\exp\left\{\frac{g^2N}{c\gamma_{13}}\kappa(\tau,\tau')(s')^{-1}\right\} \\ & \times \left(\tilde{F}(s,\tau') - i\frac{g^2N}{c\gamma_{13}}\frac{\Omega_c^*}{\gamma_{13}}(s')^{-1}\tilde{F}_{13}(s,\tau')\right).\end{aligned} \tag{A3}$$

The first term in Eq. (A3) can be neglected because we assume that there is no polarization initially. We substitute this solution into Eq. (A1) and perform the inverse Laplace transform. Then, we can have the output probe field propagating through an EIT medium as shown in Eq. (7). Here, the corresponding noise operator $\hat{n}(z,\tau)$ in Eq. (7) is

$$\hat{n}(z,\tau) = i\frac{gN}{c\gamma_{13}}\int_0^z dz' \exp\left[-\alpha\left(1-\frac{z'}{z}\right)\right]\hat{F}_{13}(z',\tau)$$
$$- i\frac{gN}{c\gamma_{13}}\frac{\Omega_c(\tau)}{\gamma_{13}}\int_0^\tau d\tau'\int_0^z dz'\Omega_c^*(\tau')e^{-\gamma_{12}(\tau-\tau')-\kappa(\tau,\tau')}\exp\left[-\alpha\left(1-\frac{z'}{z}\right)\right]I_0\left(\sqrt{4\alpha\kappa(\tau,\tau')\left(1-\frac{z'}{z}\right)}\right)\hat{F}_{13}(z',\tau')$$
$$+ i\frac{gN}{c\gamma_{13}}\frac{\Omega_c(\tau)}{\gamma_{13}}\int_0^\tau d\tau'\int_0^z dz'\Omega_c^*(\tau')e^{-\gamma_{12}(\tau-\tau')-\kappa(\tau,\tau')}\exp\left[-\alpha\left(1-\frac{z'}{z}\right)\right]\sqrt{\frac{\alpha(1-\frac{z'}{z})}{\kappa(\tau,\tau')}}I_1\left(\sqrt{4\alpha\kappa(\tau,\tau')\left(1-\frac{z'}{z}\right)}\right)\hat{F}_{13}(z',\tau')$$
$$- \frac{gN}{c\gamma_{13}}\Omega_c(\tau)\int_0^\tau d\tau'\int_0^z dz' e^{-\gamma_{12}(\tau-\tau')-\kappa(\tau,\tau')}\exp\left[-\alpha\left(1-\frac{z'}{z}\right)\right]I_0\left(\sqrt{4\alpha\kappa(\tau,\tau')\left(1-\frac{z'}{z}\right)}\right)\hat{F}_{12}(z',\tau'). \quad \text{(A4)}$$

APPENDIX B

In Eq. (12), one can calculate the output variance of a quantized probe field with any given noise distribution initially, $\langle\hat{X}_0(\tau)\rangle$. The dominant term in the output variance is the convolution integral from the response function f and related input variance. To find the proper time constant, T_p, defined in Eq. (12), we look at the steady state in the slow-light case by taking the input probe field as a continuous wave, i.e., $\langle\hat{X}_0(\tau)\rangle = \hat{X}_0$ and $\Omega_c = \text{const}$. In this scenario, one can directly reduce Eq. (12) into the following form for the output variance:

$$\langle\hat{X}_L^2\rangle = 1 - \left[e^{-D_{\text{opt}}}\left(1 + T_p\frac{|\Omega_c|^2}{\Gamma}(D_{\text{opt}})\right) + T_p\int_0^\infty d\Delta\tau\,[f(\Delta\tau)]^2\right]\left(1-\langle\hat{X}_0^2\rangle\right). \quad \text{(B1)}$$

Here, we have extended the integration from zero to infinity for such a time-independent input variance. Moreover, the integration inside the bracket in Eq. (B1) can be solved analytically by using the integral identity: $\int_0^\infty dx\frac{1}{x}e^{-ax}I_1^2(\sqrt{bx}) = e^y[I_0(y)-I_1(y)]-1$, where $y=b/2a$. The resulting quadrature variance can be found as

$$\langle\hat{X}_L^2\rangle = 1 - \exp\left[-\frac{(D_{\text{opt}})\gamma_{12}\Gamma}{\gamma_{12}\Gamma+2|\Omega_c|^2}\right]\left(1-\langle\hat{X}_0^2\rangle\right). \quad \text{(B2)}$$

In Eq. (B2), one can see that the exponential term corresponds to the intensity damping due to the ground-state decoherence, γ_{12}. In the derivations shown above, the commutation relation is required to satisfy

$$[\hat{\mathcal{E}}(0,t),\hat{\mathcal{E}}^\dagger(0,t')] = T_p\,\delta(t-t'), \quad \text{(B3)}$$

based on which we have the explicit form for the time constant, T_p,

$$T_p = \frac{\Gamma\sqrt{D_{\text{opt}}}}{|\Omega_c|^2}\frac{(e^\xi - e^{-\xi})(D_{\text{opt}})^{-3/2}}{I_0(\xi)-I_1(\xi)}, \quad \text{(B4)}$$

with the shorthanded notation $\xi = \frac{(D_{\text{opt}})|\Omega_c|^2}{\gamma_{12}\Gamma+2|\Omega_c|^2}$.

[1] L.-M. Duan, M. D. Lukin, J. I. Cirac, and P. Zoller, Nature (London) **414**, 413 (2001).
[2] A. Kuzmich, W. P. Bowen, A. D. Boozer, A. Boca, C. W. Chou, L.-M. Duan, and H. J. Kimble, Nature (London) **423**, 731 (2003).
[3] S. E. Harris, Phys. Today **50**(7), 36 (1997).
[4] M. Fleischhauer, A. Imamoglu, and J. P. Marangos, Rev. Mod. Phys. **77**, 633 (2005).
[5] Y. F. Chen, C. Y. Wang, S. H. Wang, and I. A. Yu, Phys. Rev. Lett. **96**, 043603 (2006).
[6] M. D. Lukin, Rev. Mod. Phys. **75**, 457 (2003).
[7] C. Liu, Z. Dutton, C. H. Behroozi, and L. V. Hau, Nature (London) **409**, 490 (2001).
[8] D. F. Phillips, A. Fleischhauer, A. Mair, R. L. Walsworth, and M. D. Lukin, Phys. Rev. Lett. **86**, 783 (2001).
[9] T. Chaneliere, D. N. Matsukevich, S. D. Jenkins, S.-Y. Lan, T. A. B. Kennedy, and A. Kuzmich, Nature (London) **438**, 833 (2005).
[10] M. D. Eisaman, A. Andre, F. Massou, M. Fleischhauer, A. S. Zibrov, and M. D. Lukin, Nature (London) **438**, 837 (2005).
[11] D. Akamatsu, K. Akiba, and M. Kozuma, Phys. Rev. Lett. **92**, 203602 (2004).
[12] D. Akamatsu, Y. Yokoi, M. Arikawa, S. Nagatsuka, T. Tanimura, A. Furusawa, and M. Kozuma, Phys. Rev. Lett. **99**, 153602 (2007).
[13] K. Honda, D. Akamatsu, M. Arikawa, Y. Yokoi, K. Akiba, S. Nagatsuka, T. Tanimura, A. Furusawa, and M. Kozuma, Phys. Rev. Lett. **100**, 093601 (2008).
[14] J. Appel, E. Figueroa, D. Korystov, M. Lobino, and A. I. Lvovsky, Phys. Rev. Lett. **100**, 093602 (2008).
[15] P. Barberis-Blostein and M. Bienert, Phys. Rev. Lett. **98**, 033602 (2007).
[16] Y.-L. Chuang and R.-K. Lee, Opt. Lett. **34**, 1537 (2009).
[17] A. B. Matsko, Y. V. Rostovtsev, O. Kocharovskaya, A. S. Zibrov, and M. O. Scully, Phys. Rev. A **64**, 043809 (2001).
[18] M. Fleischhauer and M. D. Lukin, Phys. Rev. Lett. **84**, 5094 (2000).
[19] M. Fleischhauer and M. D. Lukin, Phys. Rev. A **65**, 022314 (2002).

[20] A. Dantan and M. Pinard, Phys. Rev. A **69**, 043810 (2004).
[21] Y.-L. Chuang, I. A. Yu, and R.-K. Lee, J. Opt. Soc. Am. B **32**, 1384 (2015).
[22] M. Rosenbluh and R. M. Shelby, Phys. Rev. Lett. **66**, 153 (1991).
[23] Y. Lai and S. S. Yu, Phys. Rev. A **51**, 817 (1995).
[24] S. R. Friberg, S. Machida, M. J. Werner, A. Levanon, and T. Mukai, Phys. Rev. Lett. **77**, 3775 (1996).
[25] E. Schmidt, L. Knoll, D.-G. Welsch, M. Zielonka, F. Konig, and A. Sizmann, Phys. Rev. Lett. **85**, 3801 (2000).
[26] R.-K. Lee and Y. Lai, Phys. Rev. A **69**, 021801(R) (2004).
[27] Y. Lai and R.-K. Lee, Phys. Rev. Lett. **103**, 013902 (2009).
[28] M. O. Scully and M. S. Zubairy, *Quantum Optics* (Cambridge University Press, New York, 1997).
[29] R.-K. Lee, Y. Lai, and B. A. Malomed, Phys. Rev. A **70**, 063817 (2004).