Spontaneous formation of a doubly quantized vortex in the anomalous component of a trapped Bose gas

Abdelâali Boudjemâa*

Department of Physics, Faculty of Sciences, Hassiba Benbouali University of Chlef, P.O. Box 151, 02000, Ouled Fares, Chlef, Algeria

(Received 2 February 2015; revised manuscript received 7 April 2015; published 29 June 2015)

We study the behavior of an unusual doubly quantized vortex in a harmonically trapped Bose gas at nonzero temperatures by using the time-dependent Hartree-Fock-Bogoliubov equations. This structure, which exhibits nontrivial features, generates spontaneously in the anomalous fraction when phases corresponding to the singly charged vortex are imposed in the condensed and the anomalous components of the gas. Our numerical calculations show that at low temperature, condensed atoms tend to fill the core of the anomalous vortex. We demonstrate that the decay of this vortex is attributed to dissipation induced by the anomalous fluctuations. Excitation frequency and the radius core of the anomalous vortex are also investigated.

DOI: 10.1103/PhysRevA.91.063633 PACS number(s): 03.75.−b, 67.85.−d, 67.25.dk

I. INTRODUCTION

The creation of vortices in Bose-Einstein condensates (BECs) [1–3] sparked many theoretical and experimental investigations of the formation of vortices in Bose gases at finite temperature [4–14]. However, in the majority of these works, the vorticity appears concentrated in a number of singly quantized vortices. This is a consequence of the fact that multiply quantized vortices are dynamically unstable [15–20] and decay into singly quantized vortices. The splitting instability of doubly quantized vortices is, indeed, due to several factors: (i) interatomic interactions, as observed in the experiments of Shin *et al.* [16]; (ii) trap geometry, as was suggested in Ref. [15]; and (iii) thermal fluctuations, which can be another source of instability of the doubly quantized vortex [18,19].

Our aim in this paper is to investigate the formation of a different kind of vortex in a trapped cigar-shaped condensate at finite temperature using our time-dependent Hartree-Fock-Bogoliubov (TDHFB) formalism. Basically, the TDHFB theory is a nonperturbative and nonclassical field approach. It was derived from the so-called Balian-Vénéroni time-dependent variational principle [21]. The main difference between this approach and earlier variational treatments is that, in our variational theory, we do not minimize only the expectation values of a single operator such as the free energy in the variational HF and HFB approximation. Conversely, our variational theory is based on the minimization of an action in addition to a Gaussian variational ansatz (i.e., a Gaussian time-dependent density-like operator). The action to minimize involves two types of variational objects: one related to the observables of interest and another that is akin to a density matrix [21,22]. This leads to a set of coupled nonlinear time-dependent mean-field equations for the condensate, the thermal cloud, and the anomalous average.

The numerical simulation of these equations shows that an unusual doubly quantized vortex can be imprinted willingly in the anomalous fraction of Bose gas without any external perturbations when the condensed and the anomalous phases are inserted into the system. The properties of these vortices are quite different from ordinary quantized vortices. First of all, the anomalous double vortex is surprisingly formed with an angular momentum equal to 1 ($\ell = 1$). Second, it decays into two single vortices in an analogous manner to the usual doubly quantized vortex, due to the dissipation caused by the anomalous fluctuations. In addition, the anomalous vortex disappears at higher temperatures due to the large thermal fluctuations, and it does not exist for a very weakly interacting regime. Another interesting feature of this structure is that it remains robust, i.e., it does not decay with increasing interactions.

After establishing the main features of anomalous vortices in BECs, we may ask the following naive question: How are these vortices physically relevant? Indeed, the study of anomalous vortices provides important insight into the problem of rotations, a subject of great interest in the physics of superfluids. It is worth remarking that superfluidity cannot occur in Bose gases if the anomalous density was neglected, which is in fact natural since both quantities are caused by atomic correlations [23–25]. Moreover, anomalous vortices might give hints about the superradiance phenomenon in ultracold atoms. Superradiance, which is closely related to Hawking radiation (a sonic black hole) [26,27], means that when sound waves are reflected from a vortex, they are reflected with a higher energy than they came in with. One can expect that when a quantized vortex is present in the condensed and anomalous components, this effect will be diminished. In addition, the presence of anomalous vortices in two-dimensional geometry leads to stable quantum Hall states for large interparticle interaction strengths at zero temperature. This can be explained from the fact that the anomalous density is induced by the interacting condensate and grows with increasing interaction strength [28,29], and hence the anomalous vortex becomes robust in such a case (see below). Furthermore, these vortices are analogous to those observed in a Bardeen-Cooper-Schrieffer (BCS) state of strongly interacting Fermi gas [30] since the anomalous density itself characterizes pairing correlations in the noncondensed component of the field. Therefore, this may help to understand the properties of vortices in Fermi gases and superfluidity in the context of neutron stars. Anomalous vortices may also play a key role in the dissipation of transport in superfluids.

The remainder of the paper is organized as follows: In Sec. II, we review the main features of the TDHFB equations,

*a.boudjemaa@univ-chlef.dz

which constitute a relevant model to investigate the properties of vortices in Bose gas at finite temperature. In Sec. III, we present the numerical simulation of our TDHFB equations. We show in particular that a doubly quantized vortex is spontaneously formed in the anomalous component of a trapped Bose gas at intermediate temperature. This structure decays at higher temperature due to dissipation induced by the anomalous fluctuations. Our formalism, on the other hand, predicts an important and somehow unexpected result is that the condensed atoms look to occupy the core of the anomalous vortex when including only the anomalous phase in the system without imposing the singly quantized vortex on the condensed phase. Moreover, we discuss the effects of temperature and interactions on the excitation spectrum and on the radius core of the condensed and anomalous vortices. In Sec. IV, we focus on the experimental realization of the anomalous density and its related vortex. Our conclusion and outlook are presented in Sec. V.

II. FORMALISM

At finite temperature, the dynamics of dilute Bose gases is accurately described by the TDHFB equations [31,32], which govern the time evolution of the condensate wave function $\Phi(\mathbf{r}) = \langle\hat\psi(\mathbf{r})\rangle$, the noncondensed density $\tilde n = \langle\hat\psi^+(\mathbf{r})\hat\psi(\mathbf{r})\rangle - \Phi^*(\mathbf{r})\Phi(\mathbf{r})$, and the anomalous density $\tilde m = \langle\hat\psi(\mathbf{r})\hat\psi(\mathbf{r})\rangle - \Phi(\mathbf{r})\Phi(\mathbf{r})$:

$$i\hbar\dot\Phi = \left[-\frac{\hbar^2}{2m}\Delta + V_{\rm ext}(\mathbf{r}) + g(\beta n_c + 2\tilde n)\right]\Phi, \tag{1a}$$

$$i\hbar\dot{\tilde m} = \left[-\frac{\hbar^2}{2m}\Delta + V_{\rm ext}(\mathbf{r}) + 2g(G\tilde m + n)\right]\tilde m, \tag{1b}$$

where m is the atom mass, $g = 4\pi\hbar^2 a/m$ is the coupling constant, with a being the s-wave scattering length, and $n = n_c + \tilde n$ is the total density in a BEC, with $n_c(\mathbf{r}) = |\Phi(\mathbf{r})|^2$ being the condensed density. The dimensionless parameter $\beta = U/g$, where $U = g(1 + \tilde m/\Phi^2)$ is the renormalized coupling constant [31–33] and $G = \beta/4(\beta - 1)$. For $\beta = 1$, i.e., $\tilde m/\Phi^2 = 0$, Eq. (1a) reduces to the HFB-Popov equation, which is safe from all ultraviolet and infrared divergences and thus provides a gapless spectrum. For $0 < \beta < 1$, G is negative and hence $\tilde m$ has a negative sign. For $\beta > 1$, G is positive, and thus $\tilde m$ becomes a positive quantity. For $\beta = 2$, the gas becomes highly correlated and strongly interacting since $\tilde m = \Phi^2$. Therefore, to guarantee the diluteness of the system, β should vary as $\beta = 1 \pm \epsilon$, with ϵ being a small value.

Throughout this paper, the trapping potential is assumed to be of the form

$$V_{\rm ext}(\mathbf{r}) = \tfrac{1}{2}m\omega_\rho^2(\rho^2 + \lambda^2 z^2), \tag{2}$$

where $\rho^2 = x^2 + y^2$, and $\lambda = \omega_z/\omega_\rho$ is the ratio between the trapping frequencies in the axial and radial directions, so that when $\lambda > 1$, the atomic cloud resembles a pancake, and when $\lambda < 1$ it is cigar-shaped.

In our formalism, the normal and the anomalous densities are not independent. By deriving an explicit relationship between them, it is possible to eliminate $\tilde n$ [29,34,35]:

$$I = (2\tilde n + \hat 1)^2 - 4|\tilde m|^2, \tag{3}$$

where $\hat 1$ is the unit operator.

One can easily check by direct substitution that once Eq. (3) holds initially, it remains true during the dynamical evolution. At zero temperature, $I = \hat 1$ [35], and hence Eq. (3) reduces to

$$\tilde n(\tilde n + \hat 1) = |\tilde m|^2. \tag{4}$$

Equation (4) is in good agreement with that obtained in Refs. [36,37] using the generalized coherent state representation and the Bogoliubov inequality, respectively. Solving (4) for $\tilde n$, one finds $\tilde n = \sqrt{|\tilde m|^2 + \frac14\hat 1} - \frac12\hat 1$. This expression not only renders the set (1) close but also enables us to reduce the number of equations, making the numerical simulation easier. It is also possible to show that Eq. (3) holds if we work in the Bogoliubov quasiparticle space: $\hat a_{\mathbf{k}} = u_k\hat b_{\mathbf{k}} - v_k\hat b^\dagger_{-\mathbf{k}} - \beta_{\mathbf{k}}$, where $\hat b^\dagger_{\mathbf{k}}$ and $\hat b_{\mathbf{k}}$ are operators of elementary excitations and u_k, v_k are the standard Bogoliubov functions. In the quasiparticle vacuum state, $\tilde n$ and $\tilde m$ may be written as $\tilde n = \sum_k[v_k^2 + (u_k^2 + v_k^2)N_k]$ and $\tilde m = -\sum_k[u_k v_k(2N_k + 1)]$, where $N_k = [\exp(\varepsilon_k/T) - 1]^{-1}$ are occupation numbers for the excitations. Employing the orthogonality and symmetry conditions between the functions u,v and using the fact that $2N(x) + 1 = \coth(x/2)$, we obtain

$$I_k = (2\tilde n_k + 1)^2 - 4|\tilde m_k|^2 = \coth^2\left(\frac{\varepsilon_k}{2T}\right). \tag{5}$$

where ε_k is the excitation energy of the BEC.

At $T = 0$, the relation (5) can be shown to be identical to Eq. (4). Physically, I allows us to calculate in a very useful way the superfluid fraction for d-dimensional Bose gas as $n_s = 1 - (2/dTn)\int E_k I_k d^dk/(2\pi)^d$ with $E_k = \hbar^2k^2/2m$ [23].

The wave function Φ, $\tilde n$, and $\tilde m$ are normalized according to $\int d\mathbf{r}|\Phi(\mathbf{r})|^2 = N_c$, $\int d\mathbf{r}\,\tilde n(\mathbf{r}) = \tilde N$, and $\int d\mathbf{r}\,\tilde m(\mathbf{r}) = \tilde M$, where N_c, $\tilde N$, and $\tilde M$ are the condensed number of particles, the number of thermal atoms, and the anomalous integrand, respectively, with $N = N_c + \tilde N$ being the total number of particles. The full details of this self-consistent approach were presented in Refs. [23,29,31,34].

Among the advantages of the TDHFB equations (1), they satisfy the total number of particles and the energy conservation law, and they provide a gapless spectrum [31,32]. Furthermore, the numerical simulation of our equations is relatively easy and is not time-consuming even for large numbers of particles (they do not contain any summation over modes-k) compared to earlier time-dependent HFB equations of Refs. [33,38]. These latter become rapidly unstable for higher modes and for increasing temperatures. For fermionic systems, evidently Eq. (1a) has no analog, while the corresponding equation (1b) is the gap equation.

For a given stationary solution Φ_0 and $\tilde m_0$ of the TDHFB equations (1) with eigenvalue μ, the small-amplitude excitations of the whole system are defined through the random-phase approximation (RPA) as

$$\Phi = \Phi_0 + \left[u_k^c(\mathbf{r})e^{-i\varepsilon_k t/\hbar} + v_k^c(\mathbf{r})e^{i\varepsilon_k t/\hbar}\right]e^{-i\mu t/\hbar},$$

$$\tilde m = \tilde m_0 + \left[u_k^{\tilde m}(\mathbf{r})e^{-i\varepsilon_k t/\hbar} + v_k^{\tilde m}(\mathbf{r})e^{i\varepsilon_k t/\hbar}\right]e^{-i\mu t/\hbar}. \tag{6}$$

We then obtain the extended Bogoliubov–de Gennes equations (BdG):

$$\begin{pmatrix} \mathcal{L}+2g(\beta|\Phi|^2+\tilde{n}) & \mathcal{M} & 0 & 0 \\ -\mathcal{M} & -\mathcal{L}-2g(\beta|\Phi|^2+\tilde{n}) & 0 & 0 \\ 0 & 0 & \mathcal{L}+2g(2G\tilde{m}+n) & 0 \\ 0 & 0 & 0 & -\mathcal{L}-2g(2G\tilde{m}+n) \end{pmatrix} \begin{pmatrix} u_k^c(\mathbf{r}) \\ v_k^c(\mathbf{r}) \\ u_k^{\tilde{m}}(\mathbf{r}) \\ v_k^{\tilde{m}}(\mathbf{r}) \end{pmatrix} = \varepsilon_k \begin{pmatrix} u_k^c(\mathbf{r}) \\ v_k^c(\mathbf{r}) \\ u_k^{\tilde{m}}(\mathbf{r}) \\ v_k^{\tilde{m}}(\mathbf{r}) \end{pmatrix}, \tag{7}$$

where $\mathcal{L}=(-\hbar^2/2m)\Delta+V_{\rm ext}(\mathbf{r})-\mu$ and $\mathcal{M}=g\beta|\Phi|^2$. For $\beta=1$, Eqs. (7) reduce to standard BdG equations.

Since the problem is cylindrically symmetric and the system has an angular momentum κ per particle, the excitations can be labeled by an angular momentum quantum number ℓ relative to that of the condensate and the anomalous density, such that

$$\begin{aligned} u_k(\rho,\theta,z) &= e^{i(\kappa+\ell)\theta}u_k(\rho,z), \\ v_k(\rho,\theta,z) &= e^{i(\kappa-\ell)\theta}v_k(\rho,z). \end{aligned} \tag{8}$$

The Bogoliubov eigenvalue problem thus splits up into a block-diagonal matrix where the blocks corresponding to different ℓ are decoupled. This allows us to treat each ℓ value separately. For the problem at hand, $\ell=1$.

III. RESULTS AND DISCUSSIONS

Before analyzing the doubly anomalous vortex, it is convenient to shed some light on the behavior of the singly anomalous vortex and looking how the condensed atoms behave? Including the centrifugal potential related to the anomalous density in Eq. (1b) without imposing the singly quantized vortex on the condensed phase. The centrifugal potential forces the solution of $\tilde{m}$ to be zero along the z axis for nonzero angular momentum. We have then performed the numerical simulation by scaling the length with $a_0=\sqrt{\hbar/m\omega_\rho}$ and energy with $\hbar\omega_\rho$. One obtains dimensionless TDHFB equations, which we discretize using the finite-difference method. The trap parameters are the same as in the experiment of Ref. [16], i.e., the radial trap frequency is $\omega_\rho=220$ Hz and the axial frequency is $\omega_z=3$ Hz. The number of particles is 1.5×10^6 ^{23}Na atoms, and the interaction strength is $an_z=5.6$.

Figure 1 shows that the anomalous density forms a local minimum in the center of the trap and appears as a tornado (vortex) surrounds the condensate. It is easy to see that the anomalous vortex preserves the same shape as the usual vortex. Importantly, we observe that the condensed atoms are located in the core of the anomalous vortex. In fact, the formation of the anomalous vortex occurs first due to the centrifugal forces on the gas and to the correlations between pairs of condensed and noncondensed atoms. Note that these interactions between pairs lead also to the formation of a dip in the anomalous density and in the thermal cloud near the edge of the condensate, even in the absence of centrifugal forces [29,39].

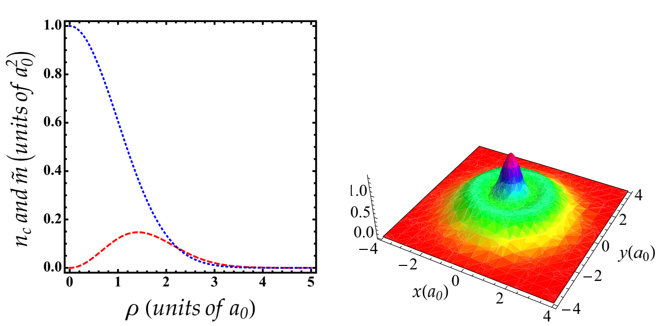

FIG. 1. (Color online) Condensed density (blue dotted lines) and anomalous vortex (red dashed lines) vs the radial distance for $N_c/N=60\%$ and for $\beta=1.05$ (left panel). Two-dimensional (2D) densities, integrated along the z direction for the condensed atoms (red/dark blue) and anomalous vortex (green/yellow) (right panel).

We turn now to investigate the behavior of the anomalous double vortex. To this end, we solve our TDHFB equations with the same experimental values keeping fixed the phases in both the condensate fraction and in the anomalous density. The resulting equations contain centrifugal potentials that force the solution of Φ and $\tilde{m}$ to be zero along the z axis.

As is shown in Fig. 2, a doubly quantized vortex spontaneously forms in the anomalous fraction of the Bose gas even with $\ell=1$, as we have already expected. The process starts precisely at intermediate temperature ($N_c/N=55\%$), where the anomalous density reaches its maximal value. At this range of temperature, the quantized vortex starts to decay, which explains the lower energy cost of gathering particles from the vortex to its surrounding bath. Consequently, the centrifugal force associated with the anomalous fraction leads to the nucleation of a supplementary vortex on the anomalous

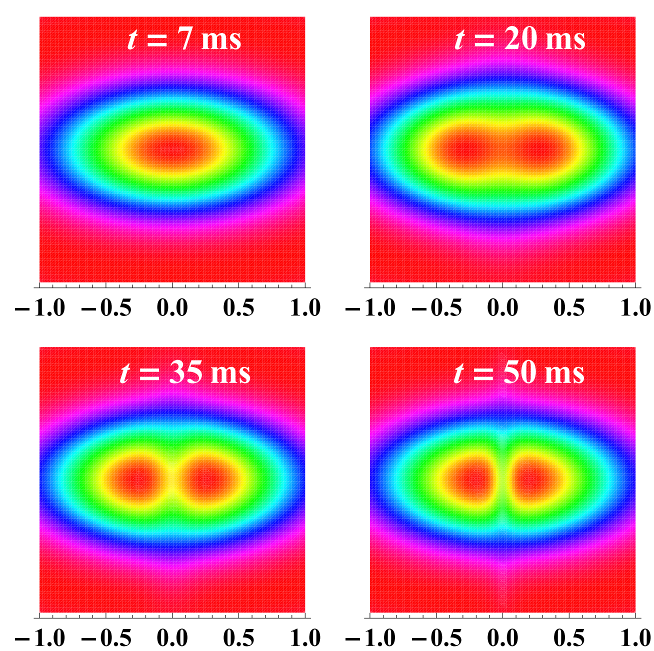

FIG. 2. (Color online) Time evolution of the nucleation process of a doubly quantized vortex in the anomalous component at $N_c/N=55\%$. Axial images for interaction strength $an_z=5.6$.

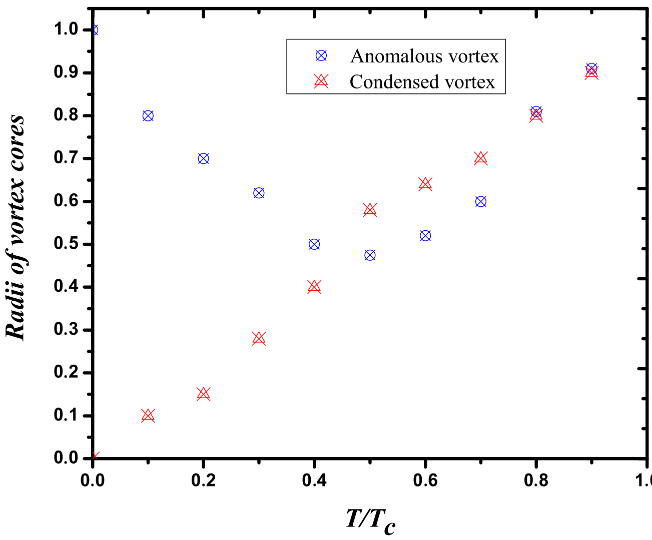

FIG. 3. (Color online) Radii of vortex cores as a function of the reduced temperature. Parameters are the same as in Fig. 1.

component. In addition, it is clearly seen from the same figure that, at $t = 20$ ms, the generation of the anomalous double vortex has already begun, and at $t = 35$ ms the vortex cores begin to disentangle. Not that at very low temperature, the condensed vortex dominates the anomalous one, and thus it will be difficult to detect such a double vortex. At higher temperatures when the system ends with only a thermal cloud, this vortex decays and disappears. The decay is attributed to dissipation induced by the anomalous fluctuations, in contrast to the case of a quantized vortex, where the decay is mainly a consequence of dynamical instability [16,17].

In Fig. 3, we compare the radius core of the anomalous vortex with that of the quantized vortex; we find that the former is larger than the latter at temperatures $T \leqslant 0.5T_c$. One can see also that the radius of the anomalous vortex is decreasing with temperature and has a minimum at $T \sim 0.5T_c$, which is indeed natural since at such a temperature the anomalous fraction reaches its maximum. At higher temperature ($T > 0.5T_c$), both radii increase and confront each other near the transition. This can be attributed to the fact that the anomalous density is proportional to the condensed density. Both densities tend to zero, and hence their contribution becomes automatically negligible at $T \sim T_c$ [23,29]. Since the anomalous density subsists only in a narrow regime and its vortex core radius is too small ($\sim 10^{-7}$ m), direct *in situ* observation of these vortices is difficult. Note that even the radius core of a quantized vortex is small for direct visual direction. On the other hand, one can expect also that the condensed vortex becomes widespread as interactions rise due to the dissipation, unlike the anomalous vortex core. This latter becomes narrower and deeper for strong interactions. We can conclude that anomalous vortices, regardless of their charge (singly, doubly, or multiply), grow with interactions. In noninteracting gases, these vortices cannot survive anymore.

For completeness, we restrict ourselves now to analyzing the frequency of vortices by explicitly solving the extended BdG equations (7). Figure 4 displays the vortex frequency as a function of the reduced temperature. The observed increase

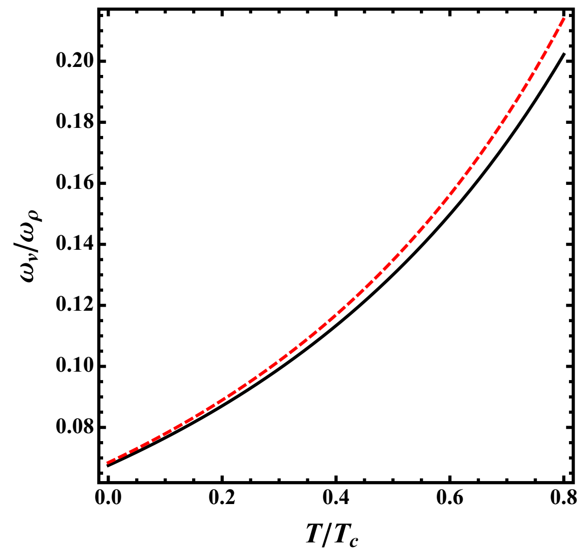

FIG. 4. (Color online) Frequency of the anomalous vortex as a function of the reduced temperature. Parameters are the same as in Fig. 1. Solid line, $\beta = 1.05$; red dashed lines, $\beta = 1$ (HFB-Popov approximation). Here we have followed the method outlined in [29,33,39] to calculate the reduced temperature.

ω_v with rising T is consistent with results obtained using the related HFB-Popov [33] at very low temperature, i.e., $T < 0.3T_c$. At $T \geqslant 0.3T_c$, our result deviates from those of the preceding theory due to the inclusion of the anomalous density.

Figure 5 clearly depicts that the frequency of the anomalous vortex is decreasing with the anomalous fraction, which means that pair correlations may enhance the vortex frequency. One should mention at this level that the vortex frequency has never been explored before in terms of the anomalous fraction.

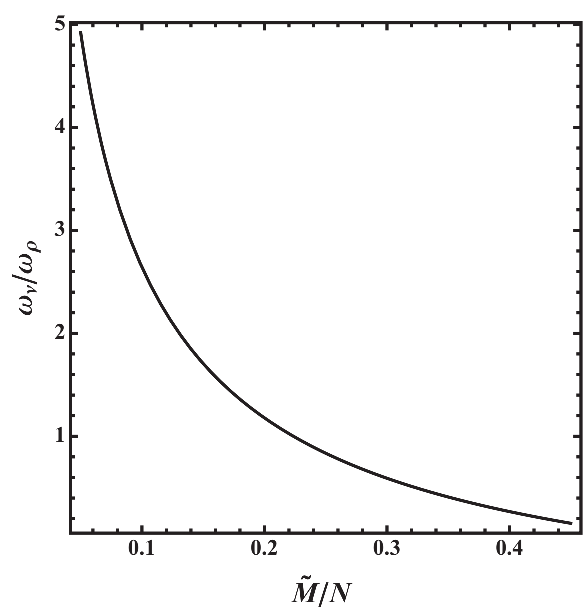

FIG. 5. Vortex frequency as a function of the anomalous fraction. Parameters are the same as in Fig. 1.

IV. EXPERIMENTAL REALIZATION

Let us now discuss the feasibility of observing the obtained results experimentally. It was shown in Refs. [40,41] that the anomalous correlations of either bosonic or fermionic (pairing states) systems can be detected using two different schemes of interference experiments. The first one is based on the phase-sensitive detection employed earlier in the condensed-matter systems, and the other technique deals with two superfluids weakly coupled by interlayer tunneling. In the case of bosonic superfluids, the anomalous correlations play a crucial role in their occurrence [see Eq. (3)] [25,31], and they have an unusual property is that they increase with the separation between quasiparticles [41]. The anomalous average also influences a number of observables, including almost all thermodynamic characteristics, the phase-transition order, and dynamical properties. Moreover, the anomalous density manifests itself into a second-order correlation function as $G^{(2)}(r) = \langle \hat{\psi}^\dagger(r)\hat{\psi}^\dagger(r)\hat{\psi}(r)\hat{\psi}(r)\rangle = n_c^2 + \tilde{m}^2 + 2\tilde{n}^2 + 4\tilde{n}n_c + 2\tilde{m}n_c$ [24,42]. In our opinion, this latter quantity constitutes the best candidate to measure the anomalous density experimentally. To illustrate this, Perrin *et al.* [43,44] have pointed out that the experiment using four-wave mixing of the collision of two BECs of metastable helium atoms, which produces a cloud of scattered atoms, may yield detailed information about the atomic pair correlations. In such an experiment, the Raman transition transfers the atoms into an untrapped magnetic substate. The transferred atoms thus expand freely, falling onto a microchannel plate (MCP) detector that allows the three-dimensional reconstruction of the position of single atoms. Knowing the positions of individual atoms, the initial momenta and the second-order momentum correlation function of the cloud of scattered particles can be computed. In this sense, anomalous correlations can be extracted or even observed, although indirectly.

With regard to vortices associated with the anomalous fraction, these structures can be nucleated during a controlled merging of three independent BECs with uncorrelated phases, which is analogous to the Kibble-Zurek mechanism [45–47]. This mechanism, which has been successfully utilized to create the thermal vortex [48], is appealing because of its potential for characterizing a wide variety of phase transitions, irrespective of the microscopic processes involved. The main difference between the thermal vortex observed in [48] and the anomalous vortex predicted in the present paper is that the former is generated during the condensation process when the temperature is somehow higher, whereas the anomalous vortex is generated at intermediate temperatures, as we have already shown in Sec. III. Anomalous vortices can be formed also by considering a self-localization of a neutral impurity atom embedded in a dilute Bose gas. This can be realized using a species-selective dipole potential [49], giving rise to spatially localizing the impurities in the center of the BEC, exhibiting a vortex-like behavior in both condensate and anomalous components. Our recent analytical and numerical results based on the TDHFB equations (1) [32,50] showed that impurities distort the anomalous density and form a vortex in the case of repulsive impurity-host interaction. Despite the fragility of these vortices, and the difficulties inherent in observing them, they can be stabilized by a suitable localized pinning potential or the addition of quartic confinement. Numerical simulations based on the Gross-Pitaevskii equation revealed that these techniques are also efficient to stabilize ordinary multiply charged vortices [51,52].

V. CONCLUSION

In this paper, we have studied the properties of the anomalous double vortex in a trapped Bose gas at nonzero temperatures by solving numerically the TDHFB equations. The outcomes of our simulation are numerous. First of all, we have shown that the condensed atoms fill the core of the anomalous vortex when only the anomalous phase is inserted into the system without imposing the singly quantized vortex on the condensed phase. In addition, the anomalous double vortex is generated spontaneously in Bose gases if phases corresponding to the singly charged vortex are imposed in the condensed and anomalous components. We have demonstrated that the vortex decay is mainly driven by dissipation caused by anomalous fluctuations, thus enabling a better understanding of the splitting process. At higher temperatures, this vortex disappears. As a result, the decay time of such a vortex depends on the temperature. Furthermore, we have found that the core of the quantized vortex increases with both interactions and temperature. In contrast, the size of the anomalous vortex core decreases at low temperature and for strong interactions. Moreover, we have pointed out that pair correlations may enhance the spectrum frequency of the anomalous vortex.

It is worth stressing that our formalism permits us to investigate, in a useful manner, the formation of vortices in the thermal cloud component, since this component is related to the anomalous fraction via Eq. (3). However, this vortex remains largely unexplored both theoretically and experimentally. Our continuing work will investigate in greater detail the static properties and dynamics of the thermal vortex.

Finally, the experimental realization of the anomalous vortex is of immense interest for a very broad scientific community striving to gain more insight into what is indeed happening regarding this type of vortex. Theoretically, the results obtained in this paper are important since they clarify the generation and the decay process of these vortices in terms of temperature and interactions.

ACKNOWLEDGMENTS

We thank Brian Anderson and Anatoli Polkovnikov for fruitful discussions. We are grateful to Makoto Tsubota and Alexander Fetter for comments on the manuscript.

[1] M. R. Matthews, B. P. Anderson, P. C. Haljan, D. S. Hall, C. E. Wieman, and E. A. Cornell, Phys. Rev. Lett. **83**, 2498 (1999).

[2] K. W. Madison, F. Chevy, V. Bretin, and J. Dalibard, Phys. Rev. Lett. **86**, 4443 (2001).

[3] J. R. Abo-Shaeer, C. Raman, J. M. Vogels, and W. Ketterle, Science **292**, 476 (2001).
[4] D. S. Rokhsar, Phys. Rev. Lett. **79**, 2164 (1997).
[5] S. M. M. Virtanen, T. P. Simula, and M. M. Salomaa, Phys. Rev. Lett. **86**, 2704 (2001).
[6] B. Jackson, J. F. McCann, and C. S. Adams, Phys. Rev. Lett. **80**, 3903 (1998).
[7] P. O. Fedichev and G. V. Shlyapnikov, Phys. Rev. A **60**, R1779 (1999).
[8] A. L. Fetter, Rev. Mod. Phys. **81**, 647 (2009).
[9] B. Jackson, N. P. Proukakis, C. F. Barenghi, and E. Zaremba, Phys. Rev. A **79**, 053615 (2009).
[10] B. G. Wild and D. A. W. Hutchinson, Phys. Rev. A **80**, 035603 (2009).
[11] E. A. L. Henn, J. A. Seman, G. Roati, K. M. F. Magalhaes, and V. S. Bagnato, Phys. Rev. Lett. **103**, 045301 (2009).
[12] Z. Hadzibabic, P. Kruger, M. Cheneau, B. Battelier, and J. Dalibard, Nature (London) **441**, 1118 (2006).
[13] D. V. Freilich, D. M. Bianchi, A. M. Kaufman, T. K. Langin, and D. S. Hall, Science **329**, 1182 (2010).
[14] A. J. Allen, E. Zaremba, C. F. Barenghi, and N. P. Proukakis, Phys. Rev. A **87**, 013630 (2013).
[15] M. Mottonen, T. Mizushima, T. Isoshima, M. M. Salomaa, and K. Machida, Phys. Rev. A **68**, 023611 (2003).
[16] Y. Shin, M. Saba, M. Vengalattore, T. A. Pasquini, C. Sanner, A. E. Leanhardt, M. Prentiss, D. E. Pritchard, and W. Ketterle, Phys. Rev. Lett. **93**, 160406 (2004).
[17] A. M. Mateo and V. Delgado, Phys. Rev. Lett. **97**, 180409 (2006).
[18] T. Karpiuk, M. Brewczyk, M. Gajda, and K. Rzażewski, J. Phys. B: At. Mol. Opt. Phys. **42**, 095301 (2009).
[19] K. Gawryluk, T. Karpiuk, M. Brewczyk, and K. Rzażewski, Phys. Rev. A **78**, 025603 (2008).
[20] J. A. M. Huhtamaki, M. Mottonen, T. Isoshima, V. Pietila, and S. M. M. Virtanen, Phys. Rev. Lett. **97**, 110406 (2006).
[21] R. Balian and M. Vénéroni, Ann. Phys. (N.Y.) **164**, 334 (1985); **187**, 29 (1988).
[22] M. Benarous and H. Flocard, Ann. Phys. (N.Y.) **273**, 242 (1999).
[23] A. Boudjemâa, Phys. Rev. A **86**, 043608 (2012).
[24] A. Boudjemâa, J. Phys. B **48**, 035302 (2015).
[25] V. I. Yukalov and E. P. Yukalova, Phys. Rev. A **90**, 013627 (2014).
[26] S. Basak and P. Majumdar, Class. Quant. Grav. **20**, 2929 (2000).
[27] F. Federici, C. Cherubini, S. Succi, and M. P. Tosi, Phys. Rev. A **73**, 033604 (2006).
[28] A. Griffin and H. Shi, Phys. Rep. **304**, 1 (1998).
[29] A. Boudjemâa and M. Benarous, Phys. Rev. A **84**, 043633 (2011).
[30] M. W. Zwierlein, J. R. Abo-Shaeer, A. Schirotzek, C. H. Schunck, and W. Ketterle, Nature (London) **435**, 1047 (2005).
[31] A. Boudjemâa, Phys. Rev. A **88**, 023619 (2013).
[32] A. Boudjemâa, J. Phys. A **48**, 045002 (2015).
[33] D. A. W. Hutchinson, R. J. Dodd, K. Burnett, S. A. Morgan, M. Rush, E. Zaremba, N. P. Proukakis, M. Edwards, and C. W. Clark, J. Phys. B **33**, 3825 (2000).
[34] A. Boudjemâa and M. Benarous, Eur. Phys. J. D **59**, 427 (2010).
[35] C. Martin, Ann. Phys. (N.Y.) **271**, 294 (1999).
[36] V. Chernyak, S. Choi, and S. Mukamel, Phys. Rev. A **67**, 053604 (2003).
[37] V. I. Yukalov and E. P. Yukalova, Laser Phys. Lett. **2**, 506 (2005).
[38] S. Giorgini, Phys. Rev. A **57**, 2949 (1998).
[39] T. M. Wright, N. P. Proukakis, and M. J. Davis, Phys. Rev. A **84**, 023608 (2011).
[40] A. Polkovnikov, E. Althman, E. Demler, and V. Gritsev, Proc. Natl. Acad. Sci. (USA) **103**, 6125 (2006).
[41] V. Gritsev, E. Demler, and A. Polkovnikov, Phys. Rev. A **78**, 063624 (2008).
[42] M. Naraschewski and R. J. Glauber, Phys. Rev. A **59**, 4595 (1999).
[43] A. Perrin, H. Chang, V. Krachmalnicoff, M. Schellekens, D. Boiron, A. Aspect, and C. I. Westbrook, Phys. Rev. Lett. **99**, 150405 (2007).
[44] J. Chwedenczuk, P. Zin, M. Trippenbach, A. Perrin, V. Leung, D. Boiron, and C. I. Westbrook, Phys. Rev. A **78**, 053605 (2008).
[45] T. W. B. Kibble, J. Phys. A **9**, 1387 (1976).
[46] W. H. Zurek, Nature (London) **317**, 505 (1985).
[47] J. R. Anglin and W. H. Zurek, Phys. Rev. Lett. **83**, 1707 (1999).
[48] C. N. Weiler, T. W. Neely, D. R. Scherer, A. S. Bradley, M. J. Davis, and B. P. Anderson, Nature (London) **455**, 948 (2008).
[49] J. Catani, G. Lamporesi, D. Naik, M. Gring, M. Inguscio, F. Minardi, A. Kantian, and T. Giamarchi, Phys. Rev. A **85**, 023623 (2012).
[50] A. Boudjemâa, Phys. Rev. A **90**, 013628 (2014).
[51] E. Lundh, Phys. Rev. A **65**, 043604 (2002).
[52] T. P. Simula, S. M. M. Virtanen, and M. M. Salomaa, Phys. Rev. A **65**, 033614 (2002).