Remote one-qubit-state control using the pure initial state of a two-qubit sender: Selective-region and eigenvalue creation

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We study the problem of remote one-qubit mixed state creation using a pure initial state of two-qubit sender and spin-1/2 chain as a connecting line. We express the parameters of creatable states in terms of transition amplitudes. We show that the creation of a complete receiver's state space can be achieved only in the chain engineered for the one-qubit perfect state transfer (PST) (for instance, in the fully engineered Ekert chain); the chain can be arbitrarily long in this case. As for the homogeneous chain, the creatable receiver's state region decreases quickly with the chain length. Both homogeneous chains and chains engineered for PST can be used for the purpose of selective state creation, when only the restricted part of the whole receiver's state space is of interest. Among the parameters of the receiver's state, the eigenvalue is the most hard to create and therefore deserves special study. Regarding the homogeneous spin chain, an arbitrary eigenvalue can be created only if the chain is of no more than 34 nodes. The alternating chain allows us to increase this length to up to 68 nodes.

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I. INTRODUCTION

The problem of remote creation of a particular quantum state is one of the fundamental problems in quantum communication. Its prototype is the pure quantum state transfer problem, which was first formulated in the well-known paper by Bose [1] for the homogeneous ferromagnet spin chain with isotropic Heisenberg interaction. Now the state transfer represents a special direction in quantum information processing. Among the spin systems engineered for the either perfect or high-fidelity (probability) one-qubit pure state transfer, we mention such well-known systems as the spin chains with properly adjusted coupling constants (or the fully engineered spin chains) [2–4] and the homogeneous chains with remote end nodes (the boundary-controlled [5,6] and optimized boundary-controlled [7,8] spin chains). In addition, the experimental realization of the perfect state transfer through the three-qubit chain in trichloroethylene is proposed in [9].

Studying the perfect state transfer (PST) problem in spin chains shows its sensitivity to the chain parameters. Moreover, although it has been achieved for a model system (such as the nearest-neighbor *XY* Hamiltonian in [2–4]), it becomes destroyed by imperfections, such as remote node interactions and quantum noise, which always reduce the state transfer fidelity [8,10–13] so that the original state cannot be perfectly transferred between the ends of a chain. As a consequence, the high-fidelity (probability) state transfer becomes more popular in comparison with the PST, which is justified in numerous papers concerning different aspects of this subject, such as the entanglement [14–18] transfer through a quantum chain [19–23], the entanglement creation between distant qubits [24,25], the so-called ballistic quantum state transfer [26], the high-dimensional state transfer [27,28], and the robustness of state transfer [10–13,29].

Nevertheless, the search for alternative ways of quantum communications free of the destructive effect of imperfections becomes more and more attractive. Thus, the so-called information transfer was proposed in [30]. In this case we take care of transfer of all the state's parameters (instead of the quantum state itself) from the sender to the receiver. These parameters appear linearly in the receiver's state, so that we have to solve a system of linear algebraic equations to obtain these parameters on the receiver's side. In turn, this requires a non-quantum-mechanical tool, which is a price for the robustness of the information transfer. The conclusion about robustness is based on a simple observation that any imperfection of the model changes the coefficients in the above linear system without changing the transferred parameters (as for the noise, also the averaged effect leads to such change of coefficients). Consequently, unlike the state transfer, this process is not sensitive to the parameters of the spin chain as well as to the imperfections of the experimental realization of the proposed model (a nearest-neighbor *XY* Hamiltonian was used in [30]).

In a recent paper [31], the principles of both perfect state transfer [1–6,32–34] and state information transfer [30] were realized in the mixed state-creation algorithm using short homogeneous spin-1/2 chains with nearest-neighbor *XY* interactions. The basic idea of that paper is to handle the parameters of the creatable state of the remote subsystem (receiver) by varying the parameters of another subsystem (sender) through the local unitary transformations of the latter. Notice that most of the earlier experiments realizing the remote state creation use photons as carriers of quantum information [35–41]. State creation based on spin-1/2 chains is suitable for the state and information transfer or creation over the relatively short distances, for instance, inside of a particular quantum device.

In this paper we represent a detailed study of the remote one-qubit mixed state creation of the receiver (the last node of a chain) through the long spin-1/2 chain with a pure initial state of (at most) a two-qubit sender (the first and the second nodes of a chain). We call the parameters varying with the purpose to create the needed receiver's state as the control parameters, while the parameters of the receiver's state are referred to as the creatable parameters. In the case of a one-qubit sender, the time is required as one of the control parameters needed to create a large region of the receiver state space. Therefore this case cannot be considered as one of completely local control (i.e., control through the local parameters of the sender's initial

state) of state creation, because the required time instant must be reported to the receiver's side (perhaps through a classical communication channel). We concentrate on the completely local control achievable using the two-qubit sender (similar to Ref. [31]). In this case the large creatable region can be covered at a properly fixed time instant by just varying the parameters of the sender's initial state. In other words, the time is not included in the list of control parameters. For a pure initial sender's state, we express the parameters of the creatable state in terms of the transition amplitudes and control parameters so that the receiver's density matrix acquires the very simple form. We show that the creation of the complete one-qubit receiver's state space can be achieved in the chain engineered for the one-qubit PST [2–4]. The chain can be arbitrarily long in this case. As for the homogeneous chains, on the contrary, the creatable region decreases very quickly with an increase in the chain length. Apparently, this is an essential disadvantage of homogeneous chains. However, we show that this disadvantage leads to some privileges of such chains in application to the problem of selective-region state creation, when we intend to work with a particular subregion of the receiver's state space. Namely, the homogeneous chains reduce (or even completely remove) the possibility of a "parasitic" state creation outside of the required subregion, if only this subregion is properly selected. Such selective state creation can be considered as the first step in construction of the "branched" communication systems having several senders and one common receiver. Notice that the chains engineered for the PST do not possess this property, although they can be used in the selective-region state creation as well.

Following Ref. [31], by the state of the receiver B we mean the reduced density matrix (the marginal matrix)

$$\rho^B = Tr_{\text{rest}}\rho, \tag{1}$$

where ρ is the density matrix of the whole system and the trace is calculated over the all nodes except for the last one (receiver's node). The one-qubit receiver's state space is parametrized by three parameters. One of them is the eigenvalue and two others are associated with the eigenvectors. In turn, among the two latter parameters, there is the phase which has no restriction for creation [31], i.e., any required value of this phase can be created by a proper choice of the control parameters. Another eigenvector parameter can, in principle, be tuned to the required value by the local unitary transformation of the receiver. On the contrary, the eigenvalue represents the most difficult creatable parameter because it is effected by the evolution and it is not sensitive to the local unitary transformations of the receiver. Thus the eigenvalue creation deserves special attention. It turns out that any eigenvalue can be created in the homogeneous chain of no more than $N_c = 34$ nodes. Any value of this parameter can be created in the fully engineered chains of arbitrary length, but such chains themselves are difficult for practical realization. Alternatively, we consider the alternating chain with even number of nodes for the purpose of remote eigenvalue creation and show that this chain increases the parameter N_c up to 68 nodes.

Finally, we mention that the teleportation [36,37,42] can be considered as a prototype of the remote state creation. Teleportation is aimed at the long-distance transfer of an unknown state using the entangled pairs of bits [35,43,44]. An inherent feature of teleportation is that it requires the classical channel of information transfer as a necessary component of the teleportation protocol, unlike the state transfer (creation) problem. A set of modifications of the state teleportation (creation) protocol can be found in [41,45–47].

The structure of this paper is the following. In Sec. II, we represent the basic analytical results concerning the map between the control parameters of the sender's initial state and the creatable parameters of the receiver's state. We show, in particular, that the creatable region covers the whole receiver's state space at the time instant of PST. The state creation in chains governed by the nearest-neighbor XY Hamiltonian is studied in more detail in the same section. In Sec. III we study the state creation inside of the selected subregions of the receiver's state space on the basis of homogeneous and fully engineered Ekert chains. Section IV is devoted to the eigenvalue creation in the long homogeneous and alternating chains, as well as in the chains engineered for the PST. The basic results of the paper are collected in Sec. V.

II. ONE-QUBIT RECEIVER'S STATE CREATION THROUGH PURE TWO-QUBIT INITIAL SENDER'S STATE

A. One-excitation evolution

In this section we introduce two requirements simplifying the spin dynamics calculation.

(1) The Hamiltonian H commutes with the z projection of the total spin momentum:

$$[H, I_z] = 0. \tag{2}$$

(2) The initial state is a superposition of the pure states with up to a single excited spin.

These two requirements allow us to consider the dynamics in the following basis of $N+1$ independent vectors (instead of 2^N independent vectors in the general case of N-qubit dynamics):

$$|0\rangle \equiv |\underbrace{0\ldots0}_{N}\rangle, \quad |n\rangle \equiv |\underbrace{0\ldots0}_{n-1}1\underbrace{0\ldots0}_{N-n}\rangle, \ n = 1,\ldots,N. \tag{3}$$

Next, we can write the Hamiltonian in the following two-block diagonal form:

$$H = \text{diag}(H_0, H_1), \tag{4}$$

where H_0 is written in the basis of a single vector $|0\rangle$ (thus, it is a scalar) and the block H_1 is written in the basis $|n\rangle$, $n = 1,\ldots,N$. Without loss of generality, we take $H_0 = 0$.

Let us consider a pure initial state $|\Psi_0\rangle$ of our quantum system. In accordance with the Schrödinger equation, evolution of this state reads

$$|\Psi(t)\rangle = e^{-iHt}|\Psi_0\rangle. \tag{5}$$

As known, the receiver's state is mixed in general and can be written as (in the case of one excitation)

$$\rho^B \equiv \text{Tr}_{1,2,\ldots,N-1}\rho = \begin{pmatrix} 1-|f_N|^2 & f_N^* f_0 \\ f_N f_0^* & |f_N|^2 \end{pmatrix} = \begin{pmatrix} 1-R_N^2 & R_N R_0 e^{-2\pi i(\Phi_N-\Phi_0)} \\ R_N R_0 e^{2\pi i(\Phi_N-\Phi_0)} & R_N^2 \end{pmatrix}. \tag{6}$$

Here the trace is taken over the nodes $1,\ldots,N-1$, where the asterisk means the complex conjugate value and f_N, f_0 are the probability amplitudes,

$$f_i = \langle i|e^{-iHt}|\Psi_0\rangle = R_i e^{2\pi i\Phi_i},\ \ i=0,\ldots,N, \tag{7}$$

where R_i and Φ_i are real parameters and R_i are positive. Remember a natural constraint

$$|f_N|^2+|f_0|^2\leqslant 1 \ \ \Rightarrow\ \ R_N^2+R_0^2\leqslant 1, \tag{8}$$

where the equality corresponds to the pure state creation because in this case $f_i\equiv 0$ $(i\neq 0,N)$ and the only nonzero eigenvalue equals 1. [The last statement can be directly verified using Eq. (19) derived below.] This phenomenon is equivalent to the PST in the case of a one-qubit sender. Constraint (8) suggests the following parametrization of R_N:

$$R_N=\sqrt{1-R_0^2}R, \tag{9}$$

where

$$0\leqslant R_0\leqslant 1, \tag{10}$$

$$0\leqslant R\leqslant 1, \tag{11}$$

$$0\leqslant\Phi\leqslant 1,\ \ \Phi=\Phi_N-\Phi_0. \tag{12}$$

Thus three parameters R_0, R, and Φ (which are defined by the initial state of our quantum system and by the interaction Hamiltonian) completely characterize the possible creatable receiver's state. However, representation (6) of the density matrix ρ^B is not a preferred one because it does not give us a simple way to estimate whether the whole state space of the receiver is creatable. The following factorized representation allows us to realize this estimation, giving the convenient parametrization of the receiver's state space:

$$\rho^B = U^B\Lambda^B(U^B)^+, \tag{13}$$

where Λ^B is the diagonal matrix of eigenvalues and U^B is the matrix of eigenvectors, which read as follows in our case:

$$\Lambda^B=\mathrm{diag}(\lambda,1-\lambda), \tag{14}$$

$$U^B=\begin{pmatrix}\cos\frac{\beta_1\pi}{2} & -e^{-2i\beta_2\pi}\sin\frac{\beta_1\pi}{2}\\ e^{2i\beta_2\pi}\sin\frac{\beta_1\pi}{2} & \cos\frac{\beta_1\pi}{2}\end{pmatrix}, \tag{15}$$

with λ and β_i $(i=1,2)$ varying inside of the intervals

$$\tfrac{1}{2}\leqslant\lambda\leqslant 1, \tag{16}$$

$$0\leqslant\beta_i\leqslant 1,\ \ i=1,2. \tag{17}$$

Intervals (16) and (17) cover the whole state space of the receiver. Note that the maximally mixed state is characterized by a single parameter $\lambda=\frac{1}{2}$.

Another advantage of representation (13) is that it separates the whole parameter space of the receiver's state into two parts:

The independent eigenvalues of ρ^B: the only parameter λ,

The independent eigenvector parameters of ρ^B: β_1 and β_2 . (18)

Obviously, the parameters λ and β_i, $i=1,2$, are related with R_0, R, and Φ as follows:

$$\lambda=\frac{1}{2}\left(1+\sqrt{\left(1-2R_N^2\right)^2+4R_N^2R_0^2}\right), \tag{19}$$

$$\cos\beta_1\pi=\frac{1-2R_N^2}{\sqrt{\left(1-2R_N^2\right)^2+4R_N^2R_0^2}},\ \ \Rightarrow \tag{20}$$

$$\beta_1\pi=\arccos\frac{1-2R_N^2}{\sqrt{\left(1-2R_N^2\right)^2+4R_N^2R_0^2}}, \tag{21}$$

$$\beta_2=\Phi, \tag{22}$$

with R_N from (9). Clearly, if the triple (R_0,R,Φ) can run all points in cube (10–12), then the triple $(\lambda,\beta_1,\beta_2)$ takes all values inside of the cube (16, 17), and thus the whole receiver's state space is creatable. However, this is possible only in special cases (like the Ekert chain in Sec. II E). Usually, only a part of the receiver's state space can be created (see homogeneous chains in Sec. II E). Formulas (19)–(22) represent the map between the control parameters of the sender (embedded in R_0, R, and Φ) and the creatable parameters λ, β_1, and β_2 of the receiver. Now we specify the dependence on the control parameters introducing a particular sender's initial state.

B. Two-node sender with one excitation initial state

For the purpose of effective remote control of the one-qubit receiver's state, we take the two-node sender with the pure one-excitation initial state of the following general form:

$$\left|\Psi_0^S\right\rangle = a_0|00\rangle+a_1|10\rangle+a_2|01\rangle, \tag{23}$$

$$\sum_{i=0}^{2}|a_i|^2=1, \tag{24}$$

where a_i $(i=0,1,2)$ are the control parameters with constraint (24). Since the common phase of a pure state does not effect the density matrix, we take the real positive a_0 without the loss of generality. Obviously, the above sender's initial state can be obtained from the ground sender's state $|00\rangle$ using the following SU(3) transformation [31]:

$$U^A=\begin{pmatrix}a_0 & -\dfrac{a_1^*}{\sqrt{1-|a_2|^2}} & -\dfrac{a_0a_2^*}{\sqrt{1-|a_2|^2}}\\ a_1 & \dfrac{a_0}{\sqrt{1-|a_2|^2}} & -\dfrac{a_1a_2^*}{\sqrt{1-|a_2|^2}}\\ a_2 & 0 & \sqrt{1-|a_2|^2}\end{pmatrix},\ \ \sum_{i=0}^{2}|a_i|^2=1, \tag{25}$$

i.e., $|\Psi_0^S\rangle = U^A|00\rangle$. This is a five-parameter transformation (one real parameter a_0, two independent amplitudes and two phases of a_1 and a_2) and it represents a particular case of the general eight-parameter SU(3) transformation. We will show that transformation (25) establishes the maximal possible control of the one-qubit receiver state in the framework of our model. Therewith the rest of the quantum system is in the

ground initial state,

$$\left|\Psi_0^{\text{rest}}\right\rangle = \left|\underbrace{0,\ldots,0}_{N-2}\right\rangle. \tag{26}$$

Thus the initial state of the whole system reads

$$|\Psi_0\rangle \equiv \left|\Psi_0^S\right\rangle \otimes \left|\Psi_0^{\text{rest}}\right\rangle = \sum_{i=0}^{2} a_i |i\rangle. \tag{27}$$

We see that the control capability of the sender can be described in two equivalent ways: by the parameters of the initial state [see Eq. (23)] and by the parameters of unitary transformation (25) of the ground sender's state. Since the initial state itself seems to be more physical and more practical in comparison with the unitary transformation, hereafter we focus on formula (23).

Obviously, the control parameters appear linearly in evolution (5) of the state:

$$|\Psi(t)\rangle = e^{-iHt}|\Psi_0\rangle = a_0 e^{-iH_0 t}|0\rangle + \sum_{j=1}^{2} a_j e^{-iH_1 t}|j\rangle$$

$$= a_0|0\rangle + \sum_{j=1}^{2} a_j e^{-iH_1 t}|j\rangle. \tag{28}$$

Consequently, the probability amplitudes appearing in the receiver's state (6) are also linear functions of the control parameters:

$$f_N(t) = \langle N|e^{-iHt}|\Psi_0\rangle = \sum_{j=1}^{2} a_j \langle N|e^{-iH_1 t}|j\rangle = \sum_{j=1}^{2} a_j p_{Nj}(t), \tag{29}$$

$$f_0(t) = \langle 0|e^{-iHt}|\Psi_0\rangle = a_0 \equiv R_0, \tag{30}$$

where

$$p_{kj}(t) = \langle k|e^{-iH_1 t}|j\rangle = r_{kj}(t)e^{2\pi i \chi_{kj}(t)}, \quad k,j > 0, \tag{31}$$

r_{kj} are positive amplitudes and $2\pi\chi_{kj}$ ($0 \leqslant \chi_{kj} \leqslant 1$) are phases of p_{kj}. The meaning of p_{kj} is evident. It is the probability amplitude of the excitation transition from the jth to the kth spin. We emphasize that these probabilities represent the inherent characteristics of the transmission line and do not depend on the control parameters of the sender's initial state.

Thus we see that the parameter R_0 is identical to the parameter a_0 of the initial state [this is a consequence of condition (2)] and does not depend on the particular Hamiltonian. Consequently, the only Hamiltonian-dependent parameter in formulas (19) and (21) is R. Being H dependent, this parameter is not completely controlled by the sender's initial state.

Let us briefly analyze the dependence of λ and β_1 on R. Calculating the derivative of λ with respect to R, we find that λ has the minimum at $R_{\min} = \frac{1}{\sqrt{2}}$,

$$\lambda_{\min} = \tfrac{1}{2}\left(1 + R_0\sqrt{2 - R_0^2}\right), \tag{32}$$

and reaches its maximal value $\lambda_{\max} = 1$ at the boundary points $R = 0,1$. Thus it takes values in the interval

$$\tfrac{1}{2}\left(1 + R_0\sqrt{2 - R_0^2}\right) \leqslant \lambda \leqslant 1, \tag{33}$$

provided that R takes values in its interval (11).

Function $\cos\beta_1$ is a decreasing function of R, taking its maximal value 1 at $R = 0$ and its minimal value $2R_0^2 - 1$ at $R = 1$. Thus

$$2R_0^2 - 1 \leqslant \cos\beta_1 \leqslant 1, \tag{34}$$

provided that R takes values in its interval (11). At the point $R_{\min} = \frac{1}{\sqrt{2}}$ we have

$$\cos\beta_1|_{R=\frac{1}{\sqrt{2}}} = \frac{R_0}{\sqrt{2 - R_0^2}}. \tag{35}$$

C. Analysis of creatable region

To proceed further, we introduce the following parametrization of the sender's initial state (23) [satisfying constraint (24)]:

$$a_0 = \sin\frac{\alpha_1\pi}{2}, \quad a_1 = \cos\frac{\alpha_1\pi}{2}\cos\frac{\alpha_2\pi}{2}e^{2i\pi\varphi_1},$$
$$a_2 = \cos\frac{\alpha_1\pi}{2}\sin\frac{\alpha_2\pi}{2}e^{2i\pi\varphi_2}, \tag{36}$$

therewith

$$0 \leqslant \alpha_i \leqslant 1, \ \ 0 \leqslant \varphi_i \leqslant 1, \ \ i = 1,2. \tag{37}$$

Then

$$f_0(t) = \sin\frac{\alpha_1\pi}{2} \equiv R_0, \tag{38}$$

$$f_N(t) = \cos\frac{\alpha_1\pi}{2}\cos\frac{\alpha_2\pi}{2}r_{N1}(t)e^{2\pi i[\varphi_1 + \chi_{N1}(t)]}$$
$$+ \cos\frac{\alpha_1\pi}{2}\sin\frac{\alpha_2\pi}{2}r_{N2}(t)e^{2\pi i[\varphi_2 + \chi_{N2}(t)]}. \tag{39}$$

The parameter α_1 fixes R_0 inside of interval (10) (thus R_0 does not depend on the time t in our case). The amplitude R_N of f_N reaches its maximal possible value at some time instant t if both terms in Eq. (39) have the same phases at this time instant, i.e., φ_1 and φ_2 satisfy the condition

$$\varphi_1 + \chi_{N1}(t) = \varphi_2 + \chi_{N2}(t). \tag{40}$$

For instance,

$$\varphi_2 = \varphi_1 + \chi_{N1}(t) - \chi_{N2}(t). \tag{41}$$

Therewith we provide any needed phase Φ (12) at any required time instant t ($\Phi_0 \equiv 0$ according to the formula (30) for f_0):

$$\Phi(t) \equiv \Phi_N(t) = \varphi_1 + \chi_{N1}(t). \tag{42}$$

So that any required phase β_2 (22) of the receiver's state can be created.

Owing to phase-relation (40), we have for R

$$R(t) = \cos\frac{\alpha_2\pi}{2}r_{N1}(t) + \sin\frac{\alpha_2\pi}{2}r_{N2}(t). \tag{43}$$

Our purpose is to find such parameters of the Hamiltonian and such a time instant t_0 that R covers the whole interval (11). This is possible only if the chain is engineered for the PST. In general, the following proposition holds.

Proposition 1. Let R take some particular value R_p at a given time instant $t = t_1$. Then R takes values at least in the interval

$$0 \leqslant R \leqslant R_p \tag{44}$$

during the time interval

$$0 \leqslant t \leqslant t_1. \tag{45}$$

Proof. This statement follows from the fact that $R(0) = 0$ and $R(t)$ is a continuous function of t. ■

Obviously, R takes values in the bigger interval

$$0 \leqslant R \leqslant R_{\max} \tag{46}$$

if the value $R_{\max} > R_p$ is achievable over the time interval (45). In other words, the following consequence holds.

Consequence 1. Let R reach the maximal value $R_{\max}$ at some instant t_0 inside of the time interval $[0,T]$, $t_0 \in [0,T]$. Then all receiver's states creatable during the time interval $[0,T]$ can be created during the shorter time interval $[0,t_0]$, therewith R takes values in interval (46).

Thus, if we consider the time t as one of the control parameters of the state-creation process, then we can cover a large region of the receiver's state space even with $\alpha_2 = 0$. However, this type of remote state creation is not completely controlled by the local parameters of the sender's initial state, since the required time instant (as a control parameter) must be transmitted to the receiver's side, which complicates the communication. To avoid this complication, we involve the variable parameter α_2 in the initial state (23, 36). In this case, the large region of the receiver's state space can be created at the properly fixed time instant, thus making the remote state creation completely controlled by the local sender's initial state (the local control of state creation), because the above time instant of state registration can be reported to the receiver's side in advance. For the sake of simplicity, we analyze such control for the case of an XY Hamiltonian with the nearest-neighbor interactions. However, a similar analysis can be elaborated in more complicated cases as well.

D. Evolution governed by nearest-neighbor XY Hamiltonian

The XY Hamiltonian with nearest-neighbor interaction reads

$$\begin{aligned} H &= \sum_{i=1}^{N-1} D_i [I_{ix} I_{(i+1)x} + I_{iy} I_{(i+1)y}] \\ &= \sum_{i=1}^{N-1} \frac{D_i}{2} (I_i^+ I_{i+1}^- + I_i^- I_{i+1}^+), \end{aligned} \tag{47}$$

where D_i are the coupling constants between the nearest neighbors, $I_{j\alpha}$ $(j = 1,\ldots,N,\ \alpha = x,y,z)$ is the jth spin projection on the α axis, and $I_j^{\pm} = I_{jx} \pm i I_{jy}$. Hereafter we assume the symmetry

$$D_i = D_{N-i}, \tag{48}$$

except for the alternating chain with odd N in Sec. IV B. Obviously, condition (2) is satisfied for this Hamiltonian. Now we formulate the following proposition concerning the local control of the remote state creation.

Proposition 2. Let the function $r_{N1}(t)$ take the maximal value $r_{\max}$ at $t = t_0$, $r_{N1}(t_0) = r_{\max}$, and $\chi_{N1}(t_0) \neq \chi_{(N-1)1}(t_0) + n/2$ $(n = 0,\pm 1)$. Then R takes values in the interval

$$0 \leqslant R \leqslant r_{\max} \tag{49}$$

when

$$0 \leqslant \alpha_2 \leqslant 1 \tag{50}$$

at the fixed time instant t_0.

Proof. The evolution of the pure quantum state is governed by the Schrödinger equation

$$i|\Psi\rangle_t = H|\Psi\rangle. \tag{51}$$

Since we deal with the nearest-neighbor Hamiltonian (47), its nonzero elements read

$$H_{i,i+1} = H_{i+1,i} = \frac{D_i}{2}, \quad i = 1, \quad N-1. \tag{52}$$

Let us consider the initial state $|\Psi(0)\rangle = |1\rangle$. Then the last row of Eq. (51) can be written in terms of transition amplitudes (31):

$$i\frac{d}{dt} p_{N1} = \frac{D_{N-1}}{2} p_{(N-1)1}. \tag{53}$$

The complex conjugate of this equation reads

$$i\frac{d}{dt} p_{N1}^* = -\frac{D_{N-1}}{2} p_{(N-1)1}^*. \tag{54}$$

Multiplying Eqs. (53) and (54) by, respectively, p_{N1}^* and p_{N1} and adding them we obtain

$$\frac{d}{dt}(r_{N1})^2 = D_{N-1} r_{N1} r_{(N-1)1} \sin[2\pi(\chi_{(N-1)1} - \chi_{N1})]. \tag{55}$$

At the time instant $t = t_0$ corresponding to the extremum of $r_{N1}(t)$ we have

$$\frac{d}{dt} r_{N1}(t_0) = 0, \quad \text{and consequently} \tag{56}$$

$$r_{N1}(t_0) r_{(N-1)1}(t_0) \sin(2\pi[\chi_{(N-1)1}(t_0) - \chi_{N1}(t_0)]) = 0. \tag{57}$$

Since $\chi_{N1}(t_0) \neq \chi_{(N-1)1}(t_0) + n/2$ $(n = 0,\pm 1)$ by our assumption and $r_{N1}(t_0) > 0$, we obtain

$$r_{(N-1)1}(t_0) = 0. \tag{58}$$

In view of the Hamiltonian symmetry we can write $r_{ij} = r_{ji} = r_{(N-i+1)(N-j+1)}$, and then Eq. (58) yields

$$r_{(N-1)1}(t_0) = r_{N2}(t_0) = 0. \tag{59}$$

This means that condition (40) is automatically satisfied at the extremum point of r_{N1} (because the right-hand side of Eq. (39) has only one term at $t = t_0$). Thus $R(t_0)$, defined by Eq. (43), reads

$$R(t_0) = \cos\frac{\alpha_2 \pi}{2} r_{\max} \tag{60}$$

and takes values in interval (49) if α_2 varies inside of interval (50). ■

In this case we have the completely local state-creation control with three control parameters α_1, α_2, and φ_1. We shall give the following remarks:

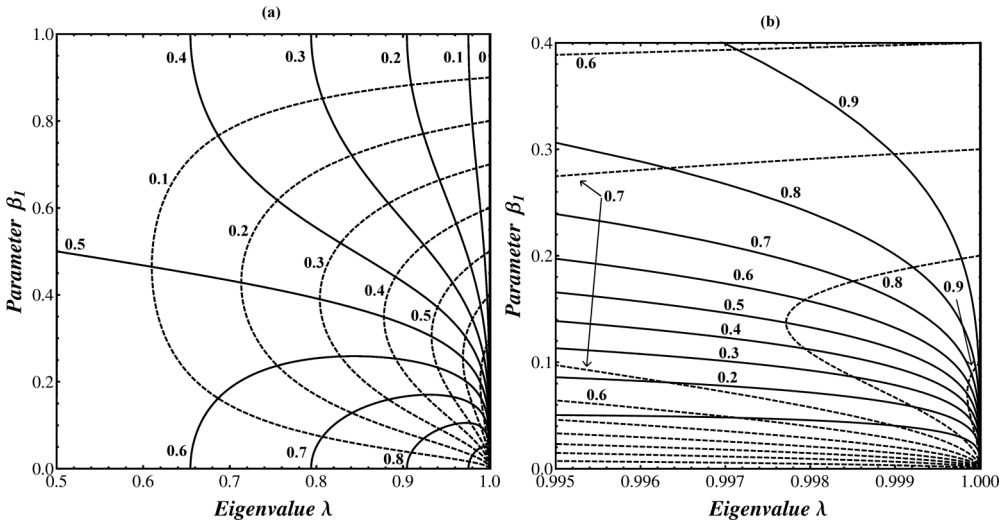

FIG. 1. The map $(\alpha_1,\alpha_2) \to (\lambda,\beta_1)$ in the chains engineered for the pure one-qubit PST (for instance, in the Ekert chain with $t_0 = \pi$). Solid and dashed lines correspond to the constant values of the parameters, respectively, α_2 and α_1; appropriate values of these parameters are indicated in the picture. The variation intervals (37) of α_i are split into ten equal segments, i.e., the parameter increment between the two neighboring lines is 0.1 in both families of curves. The solid line $\alpha_2 = 0$ coincides with the right vertical coordinate axis; the dashed line $\alpha_1 = 0$ is disrupted and coincides with the upper and lower horizontal coordinate axes. The solid and dashed lines with, respectively, $\alpha_2 = 1$ and $\alpha_1 = 1$, shrink to the point $(\lambda,\beta_1) = (1,0)$. The properly scaled neighborhood of this point is depicted in (b). We keep the same gridding in all pictures below.

(1) Hereafter we refer to the above time instant t_0 as the time instant of highest-probability state transfer (therewith the highest-probability state transfer is not always the high-probability state transfer [33] because this probability, i.e., $r_{\max}$, can be far from 1). The creation of the complete receiver's state space is possible if $r_{\max} = 1$, i.e., if the chain is engineered for the pure one-qubit PST.

(2) The conditions of proposition 2 hold for the homogeneous and Ekert chains whose evolution is governed by the XY Hamiltonian (47) with the nearest-neighbor interactions at least over the time intervals $\sim N$ considered here. In this case two control parameters α_1 and α_2 are enough to cover the maximal region in the space (λ,β_1), therewith the parameter φ_1 is responsible for β_2 and has no influence on λ and β_1. In other cases the phases ϕ_i, $i = 1,2$ [see Eqs. (36)], must be included into the set of active control parameters to obtain similar results.

In Secs. II E and III we consider only the two-parameter state-creation control, i.e., the creation of λ and β_1 varying α_1 and α_2, and explore several features of such control. The creation of β_2 is trivial and will not be considered below.

E. State-creation algorithm as a map of control parameter space (α_1,α_2) into creatable parameter space (λ,β_1)

If we need to create a receiver's state inside of a particular subregion of the whole receiver's state space, we need to know the appropriate parameters of the sender's initial state. This prompts us to consider the state-creation algorithm as a map of parameters of arbitrary sender's initial state (the so-called control parameters, α_1 and α_2 in our case) to the set of parameters of the receiver's state space (the so-called creatable parameters, λ and β_1 in our case). This map is depicted in Fig. 1 for the ideal case of the completely creatable receiver's space. As mentioned above, this situation can be realized in the chains engineered for the PST. The Ekert chain can be considered as an example [2], then

$$D_i = \sqrt{i(N-i)} \tag{61}$$

in Eq. (47), therewith $t_0 = \pi$. In the case of a homogeneous spin chain [$D_i \equiv D$, $i = 1,\dots,N-1$, in Eq. (47); we put $D = 1$ for simplicity (dimensionless time t)], the map differs from that shown in Fig. 1. We observe that the creatable region of the receiver's space decreases very quickly with the chain length, as shown in Fig. 2, where chains of 6, 60, and 120 nodes are considered at time instants $t_0 = 7.884$, 63.881, and 124.761, respectively. Figures 1 and 2 help us to select the needed region in the control parameter space (α_1,α_2) to create the state inside of the required region of the creatable parameter space (λ,β_1).

III. SELECTION OF CREATABLE SUBREGIONS

We have considered a problem of maximal possible covering of the receiver's state space. It is shown for Hamiltonian (47) that the maximal creatable region corresponds to the time instant t_0 of the highest-probability state transfer. Any deviation from t_0 reduces the creatable region. However, this unpleasant phenomenon turns out to be useful if we would like to work only with a restricted subregion of the receiver's state space without interacting with its remaining part. For instance, this problem appears in a "branched" communication line when we need to share the creatable region among several senders, so that each sender works only with its own subregion. In fact, Fig. 2 shows that the creatable region of a homogeneous chain of 120 nodes is restricted, roughly speaking, by the

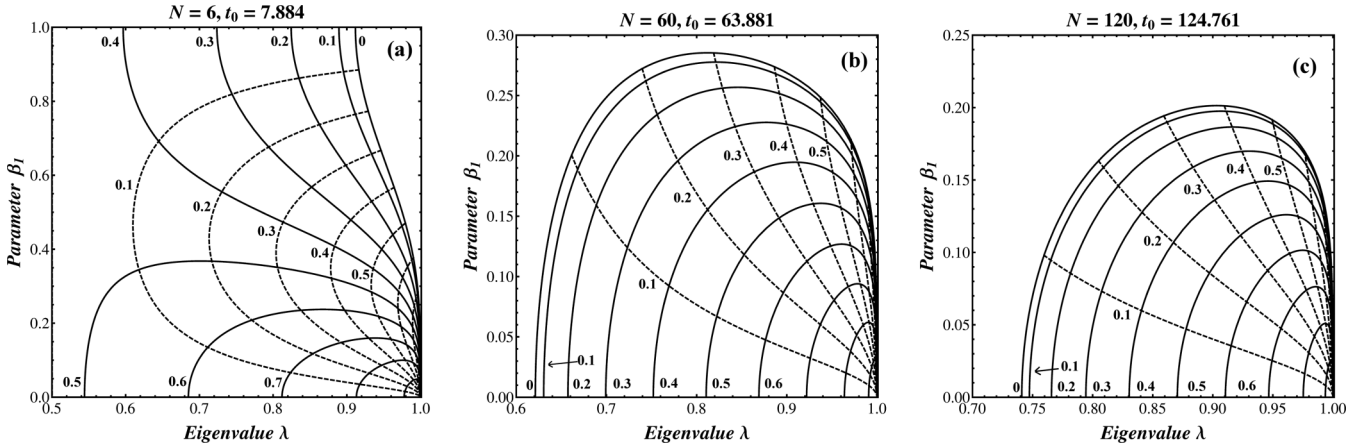

FIG. 2. The map $(\alpha_1,\alpha_2) \to (\lambda,\beta_1)$ in the homogeneous chains with $N = 6$, 60, and 120 at time instants t_0 corresponding to the maximum p_{N1} (the highest-probability state transfer).

rectangle $0.74 < \lambda \leqslant 1, 0 \leqslant \beta_1 < 0.21$, so the region outside of this rectangle can be safely used for other purposes.

Now we describe the separation of several nonoverlapping creatable subregions. Our results are based on the following observation. If we take $t_1 < t_0$, then the conditions of Proposition 2 are broken so that we do not cover the maximal creatable region varying the control parameters α_1 and α_2. Moreover, the parameter R in formulas (19) and (21) can not take all values in interval (44) (remember that t is fixed here, unlike Proposition 1). In general, the lower and upper boundaries appear:

$$q_{\min}(t_1) \leqslant R \leqslant q_{\max}(t_1). \tag{62}$$

Thus, considering K chains of different lengths N_i, $N_1 < \cdots < N_K$, and appropriate time instants t_i such that

$$q^i_{\min}(t_i) > q^{i+1}_{\max}(t_{i+1}), \quad i = 1,\ldots,K-1, \tag{63}$$

we may select K nonoverlapping creatable subregions in the receiver's state space. All these regions have the only common point $(\lambda,\beta_1) = (1,0)$.

First, we consider the selective state creation using the homogeneous chains. In this case we use the time instants t_i and the chain lengths N_i as parameters selecting the proper creatable subregions. Combining both these parameters, we can vary the creatable subregion in a needed way. Examples of two particularly selected creatable subregions corresponding to $(N,t) = (6,9.375)$ and $(60,62.7)$ are shown in Fig. 3(a).

Next, we perform the above selection using the Ekert chain [2]. In this case we can create different subregions using the chains of the same prescribed length N and varying the time instants t_i of the state registration. Examples of three creatable subregions corresponding to the chain of $N = 120$ spins and the registration time instants $t = 2.994$, 2.895, and 2.816 are shown in Fig. 3(b).

The privilege of homogeneous chains is that their creatable regions are restricted as shown in Fig. 2, which reduces the possibility of "parasitic" state creation from an "alien" sender. For instance, the sender responsible for the lower region in Fig. 3(a) ($N = 60$) cannot create the states in the upper selected region (corresponding to $N = 6$), regardless of the

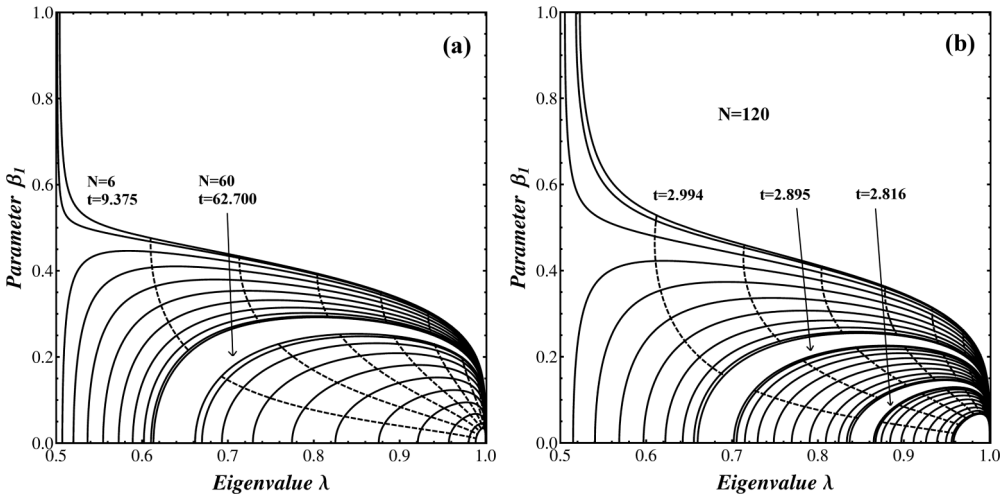

FIG. 3. Selectivity in the remote state creation. By choosing the proper chain lengths and/or the time instants of the state registration we obtain different braid-shaped creatable regions. Gridding lines inside of each braid are the same as in Fig. 1, i.e., the parameters α_i, $i = 1,2$ take values in interval (37). (a) The homogeneous spin chains of $N = 6$ and 60 with the state-detecting time instants $t = 9.375$ and 62.7, respectively. (b) The Ekert chain with $N = 120$ and the state-detecting time instants $t = 2.994$, 2.895, and 2.816.

values of the control parameters, this conclusion follows from comparison of Figs. 3(a) and 2(b). However, the opposite is not true and a six-node chain can create the "parasitic" states in the lower subregion of Fig. 3.

IV. LONG-DISTANCE EIGENVALUE CREATION IN HOMOGENEOUS AND NONHOMOGENEOUS CHAINS

A. Three types of creatable parameters

In Secs. II C and II E we show that the whole receiver's state space [31] cannot be remotely created using an arbitrary spin chain. But are there equivalent obstacles for creation of each of the three parameters λ, β_1, and β_2 of the receiver's state space? It seemed that all these parameters behave differently in the state-creation process.

First of all, we shall emphasis the principal difference between the eigenvalue λ and the eigenvector parameters β_i, $i = 1,2$. The latter have an advantage in that they, in principle, can be tuned to the required values by the local unitary transformation of the receiver (assuming that unitary transformations are applicable on the receiver side), which is a quantum-mechanics operation. Indeed, if we have created the receiver's state in the form of Eq. (13), i.e., $\rho^B = U^B \Lambda^B (U^B)^+$, while the required state is $\rho^{\rm req} = \tilde{U}^B \Lambda^B (\tilde{U}^B)^+$ [for the one-qubit receiver, the unitary transformation $\tilde{U}^B$ has the form (15) with different parameters], then $\rho^{\rm req} = \tilde{U}^B (U^B)^+ \rho^B U^B (\tilde{U}^B)^+$, i.e., the above-mentioned local transformation of the receiver reads

$$U^{\rm loc} = \tilde{U}^B (U^B)^+. \tag{64}$$

Notice that the transformation $U^{\rm loc}$ depends on the sender's control parameters, which are included in U^B. Consequently, the receiver needs information about (some of) the control parameters to apply the proper $U^{\rm loc}$. This information must be transferred from the sender to the receiver using some additional (classical) communication channel, similar to the teleportation algorithm. This means that by inserting $U^{\rm loc}$ into the state-creation algorithm, we lose the completely remote control of all parameters of the receiver state, except for the eigenvalues (matrix Λ^B), which cannot be changed by $U^{\rm loc}$. In this paper we do not consider the local transformations of the receiver as a part of the state-creation algorithm.

It is also shown in Sec. II C that the most reliable parameter is the phase β_2 [31], because any of its values can be created using the phases φ_i, $i = 1,2$, of the sender's initial state. Moreover, this property of the parameter β_2 does not depend on the Hamiltonian governing the spin dynamics (this can be simply demonstrated). All this suggests that we consider this parameter as a preferable candidate for the carrier of quantum information.

Thus the eigenvalue λ turned out to be the most defenseless parameter, because (i) we are not able to create its arbitrary value (in general) and (ii) it cannot be changed by the local unitary transformations of the receiver. Therefore the eigenvalue is completely defined by the sender's initial state and by the interaction Hamiltonian, and consequently, the eigenvalue creation deserves special study.

Let us consider the λ creation in more detail using three types of chains: the homogeneous chain, the alternating chain, and the chain engineered for the one-qubit PST (Ekert chain).

B. Eigenvalue creation using an Ekert chain, homogeneous, and alternating chains

Considering the state creation based on a spin chain of general position, the maximal variation interval (16) for λ becomes compressed. The reason is pointed out in Sec. II C, where the expression for λ as a function of R is represented [see Eq. (19)]. It was shown that the left boundary of eigenvalue $\lambda_l = \frac{1}{2}$ can be created if $R > R_{\rm min} = \frac{1}{\sqrt{2}}$ [see Eq. (32)]. Consequently, this boundary is achievable if only $R_{\rm min} \leqslant r_{\rm max}$ in Eq. (49). This prompts us to introduce the parameter $\lambda^{\rm cr}_{\rm min}(N)$, indicating the minimal eigenvalue creatable on the receiver site is a characteristic of the chain.

Considering the homogeneous spin chain [$D_i = D_1 = 1$, $i = 1,\ldots,N-1$ in Eq. (47)], we see from Fig. 2 that $\lambda^{\rm cr}_{\rm min}(N) = \frac{1}{2}$ only if the chain is short enough [Fig. 2(a)], unlike the long chains [Figs. 2(b) and 2(c)]. The general dependence of $\lambda^{\rm cr}_{\rm min}$ on N is shown in Fig. 4, indicating that there is such a critical length $N_c^{\rm hom} = 34$ that

$$\lambda^{\rm cr}_{\rm min} > \tfrac{1}{2} \quad \text{for} \quad N > N_c^{\rm hom}. \tag{65}$$

Of course, $\lambda^{\rm cr}_{\rm min}(N) = \frac{1}{2}$ and it does not depend on the chain length in the case of an Ekert chain ($N_c^{\rm PST} = \infty$). However, such chains are hard to realize, so we are forced to look for alternative ways of increasing the parameter N_c.

A simple way to do that is using an alternating chain. In this case $D_i = D_1 = 1$, $i = 1,3,5,\ldots$, and $D_i = D_2 = d$, $i = 2,4,6,\ldots$ in Eq. (47). Therewith d is called the alternation parameter. The results of our calculations for the chain with an even number of nodes are collected in Fig. 5. To simplify calculations we put $\alpha_2 = 0$ in this subsection. Using the variable parameter α_2, we would only slightly modify the figures without changing the parameter N_c.

The chain with an even number of nodes is considered in Fig. 5. The parameter R responsible for the λ creation takes its critical value $\frac{1}{\sqrt{2}}$ inside of different time intervals depicted in Figs. 5(a) and 5(b). The lines (or spots) mean that any λ from the interval $\frac{1}{2} \leqslant \lambda \leqslant 1$ can be created for the proper N and d. The envelopes of these figures give the parameter $N_c^{\rm even}$ as a function of d. The most reasonable time interval, $0 \leqslant t \leqslant 1.3 \min(d, \frac{1}{d})$, is depicted in Fig. 5(a): the alternation allows us to increase the length N_c until $N_c^{\rm even} = 36$. A more significant increase in N_c is observed over the second time interval

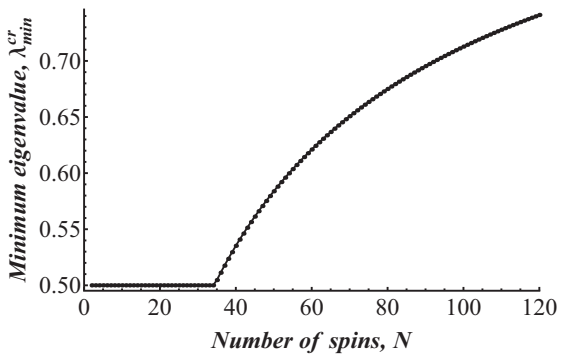

FIG. 4. The homogeneous spin chains: the minimal creatable eigenvalue $\lambda^{\rm cr}_{\rm min}$ as a function of chain length N.

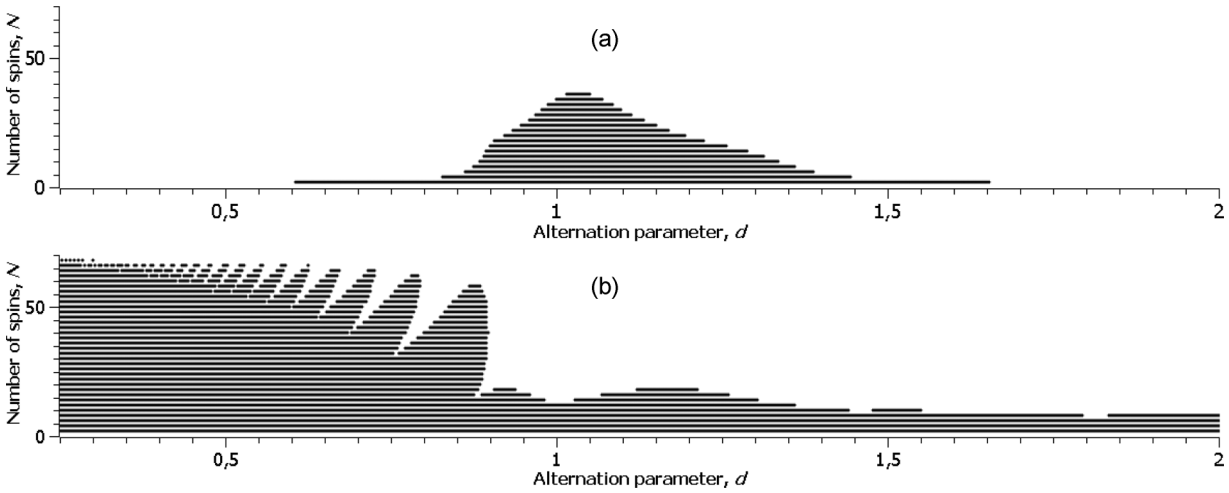

FIG. 5. The eigenvalue creation in the alternating spin chains with even N over the different time intervals. Lines (or spots) correspond to the creation of any λ from the interval $\frac{1}{2} \leqslant \lambda \leqslant 1$ for the appropriate N and d. The envelope of each figure represents the critical length N_c^{even} as a function of the dimerization parameter d: (a) $0 \leqslant t \leqslant 1.3N \min(d, \frac{1}{d})$, $(N_c^{\text{even}})_{\max} = 36 > N_c^{\text{hom}} = 34$; (b) $1.3 \min(d, \frac{1}{d}) < t \leqslant 1.5N \max(d, \frac{1}{d})$, $(N_c^{\text{even}})_{\max} = 68 > N_c^{\text{hom}}$.

$1.3 \min(d, \frac{1}{d}) < t \leqslant 1.5 \max(d, \frac{1}{d})$ shown in Fig. 5(b). (The numerical coefficients 1.3 and 1.5 are empiric.) In this case $N_c^{\text{even}} = 68$, which is twice bigger than N_c^{hom}. This results from the chain "dimerization" with a decrease in the alternation parameter d. We see that the function $N_c^{\text{even}}(d)$ is not unique for $d < 1$ if R achieves its critical value $\frac{1}{\sqrt{2}}$ over the time interval corresponding to Fig. 5(b).

The case of odd N is not interesting because it does not yield any increase of the critical length in comparison with the homogeneous chain ($N_c^{\text{odd}} = 33 < N_c^{\text{hom}}$), as shown in Fig. 6.

V. CONCLUSIONS

In this paper we study several aspects of the remote state creation using the homogeneous, Ekert, and alternating spin-1/2 chains. To simplify calculation, we require the commutation condition (2) for the Hamiltonian and one-spin excitation initial state. Based on these requirements are the following results:

(1) The receiver's density matrix can be simply expressed in terms of the probability amplitudes. These amplitudes are the characteristics of the transmission line known in advance (Secs. II A and II B).

(2) Three parameters of the creatable one-qubit-state space can be referred to as the phase and amplitude of the eigenvector and the eigenvalue. We show that an arbitrary eigenvector's phase can be created using the proper values of the control parameters (Sec. II C), the eigenvector's amplitude can be tuned by the unitary transformation of the receiver, while the eigenvalue is most difficult to create and thus deserves special consideration (Sec. IV).

(3) Being the most reliably creatable, the eigenvector's phase (parameter β_2) is a preferable candidate for the quantum information carrier in quantum communication lines.

In addition, the following results were obtained for the nearest-neighbor XY Hamiltonian (47):

(1) The arbitrary parameters of the two-qubit pure sender's initial state are the control parameters establishing the complete local control of the receiver's creatable region at the properly fixed time instant of the state registration (Secs. II and III).

(2) The maximal creatable region corresponds to the time instant associated with the highest-probability state transfer (although this probability may be far from unit, i.e., "highest" does not mean "high"). The creatable region decreases very quickly with the chain length of a homogeneous chain. As anticipated in [31], the complete state space of the one-qubit receiver can be created only in the spin chain engineered for the pure one-qubit PST (Secs. II C and II E).

(3) The map (control parameter space) $\rightarrow$ (creatable parameter space) is numerically described and depicted in Figs. 1 and 2, thus helping one to choose the control parameters needed to create a particular receiver's state (Sec. II E). Everywhere in Figs. 1–3 we use the same gridding of the parameter space (α_1, α_2).

(4) By choosing different lengths of the homogeneous chain and different time instants of the state registration we can

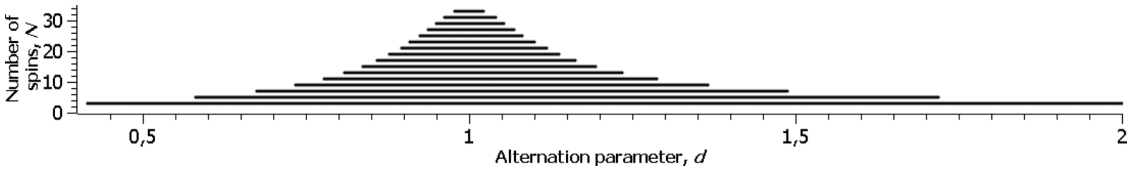

FIG. 6. The same as Fig. 5 for the chain with odd N and the time interval $0 \leqslant t \leqslant 3N \min(d, \frac{1}{d})$; $(N_c^{\text{odd}})_{\max} = 33 < N_c^{\text{hom}}$.

select the needed creatable subregion. A similar result can be achieved using the Ekert chain of a fixed length and different time instants of the state registration (Sec. III).

(5) Considering the process of the remote eigenvalue creation, we show that the arbitrary eigenvalue can be created through the homogeneous spin chain of up to 34 nodes, through the alternating chain of up to 68 nodes, and through the Ekert chain of arbitrary length (Sec. IV B).

Among the aspects deserving deeper study, we mention (i) the transformation of created states using the tool of local (nonunitary) operations; (ii) the robustness of state creation with respect to chaotic permutations and model imperfections, in particular, the effect of remote spin interactions has to be clarified; and (iii) the model with a two-excitation initial state (instead of the single-excitation one) has to be explored.

We also notice that the creation and evolution of quantum correlations is another direction of quantum information processing stimulating intensive investigations (for instance, see Refs. [48–53]). Currently, the quantum entanglement [14,15,17,18] and the quantum discord [54–57] are widely accepted measures of quantum correlations. The remote creation of entangled quantum states and states with quantum discord is one more problem postponed for further study.

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