Resonance of the exchange amplitude of a photon by an electron scattering in a pulsed laser field

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Resonant scattering of a photon by an electron in the presence of the field of the low intensity circularly polarized pulsed laser wave is studied theoretically. The approximation used the case in which a laser-pulse duration is significantly greater than the characteristic oscillation time. The resonance conditions of the exchange diagrams by electron and positron intermediate states were determined. The probability of such a process is calculated. It is demonstrated that the resonant probability may be six to ten orders of magnitude higher than the probability of the Compton effect in the absence of the external field. Obtained results can be verified experimentally in the framework of modern research projects (SLAC, FAIR, XFEL, and ELI).

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I. INTRODUCTION

The characteristic feature of electrodynamics processes of the second order in the fine-structure constant in a laser field is associated with the possibility of their resonant modes. Resonant character relates to the fact that processes of the first order, such as spontaneous emission or one-photon production and annihilation of electron-positron pairs, are allowed in the field of a light wave. Therefore, within a certain range of energy and momentum values, a particle in an intermediate state may fall within the mass shell. Then the considered process of the second order effectively decomposes into two consecutive processes of the first order.

The resonant behavior of the processes of the second order in the fine-structure constant is one of the fundamental problems of quantum electrodynamics in the presence of an external field, whose analysis was started in the mid-1960s (see, for example, reviews [1–5], monographs [6–8], and [9–24]).

Oleinik [9,11] was the first to mention the resonances in the Compton effect in the presence of the field of a plane monochromatic wave but the corresponding analysis was fragmentary. In [19,20] we considered the resonance of the direct and exchange diagrams in the relativistic system for the field of the low-intensity plane monochromatic electromagnetic wave. We considered the resonance of the direct diagram [12,13] and nonresonant Compton scattering [14,15] in the field of the plane pulsed electromagnetic wave.

Generally speaking, strong-field quantum electrodynamics processes can be classified into two categories: either induced or modified by the laser field. Whereas laser-induced processes do not occur in the absence of the field, the laser-modified processes are possible, even though their properties can change significantly under the influence of the field. Thus, the problem of scattering of electrons by a laser pulse (the process of the first order in the fine-structure constant), which in the absence of an external field does not occur, was considered in [25–29].

Experimental verification of QED effects in the laser field was carried out only for the laser-induced processes (processes of the first order in the fine-structure constant) at the facility SLAC National Accelerator Laboratory (Stanford, California, USA) [30,31], and also is included into the scientific program of the FAIR international project based on the laser system PHELIX [32]. Experiments were not carried out for the laser-modified processes because their conduct has a greater complexity. However, for such processes is possible a resonant mode where a particle in an intermediate state may fall within the mass shell. At this rate the resonant probability may exceed the corresponding probability in the external field absence in several orders of the magnitude. That is why studying of the laser-modified processes causes a great scientific interest. High power pulsed lasers whose field cannot be simulated using a model of the plane monochromatic wave are used in modern experiments on verification of quantum electrodynamics effects [30–37]. Thus, theoretical works widely employ the model of the pulsed electromagnetic field that represents the four-potential with an envelope function (see, for example, works studying elementary quantum processes in the presence of the pulsed field [12–15,25–29,38–47], and tunnelling and multiphoton ionization of atoms and ions in the presence of a strong laser field [48–50]).

This paper contains a study of laser-modified Compton scattering. We consider the external field as a circularly polarized pulsed electromagnetic wave, propagating along the z axis with a polarization plane xy. The four-potential of such a field has the form

$$A(\varphi) = g(\phi)A_0(\varphi), \quad A_0(\varphi) = a(e_x \cos\varphi + \delta_{ell} e_y \sin\varphi). \tag{1}$$

Here

$$\varphi = (kr) = \omega t - \mathbf{kr} \tag{2}$$

is the wave phase; $r = (t,\mathbf{r})$ is the four-radius vector; $\phi = \varphi/\varphi_0$, $\varphi_0 = \omega t_{\mathrm{imp}}$, t_{imp} is the pulse duration in the laboratory frame of reference; $a = F/\omega$, F, and ω are the field strength at the center of pulse and wave frequency in the laboratory coordinates; $\delta_{ell} = \pm 1$; $e_{x,y} = (0,\mathbf{e}_{x,y})$ and $k = (\omega,\mathbf{k})$ are the four-polarization vector and four-momentum of the external field photon, such that $k^2 = 0$, $e_{x,y}^2 = -1$, $(e_{x,y}k) = 0$; $g(\varphi)$ is the envelope of potential that satisfies the conditions $g(0) = 1$ and $g \to 0$ at $|\phi| \gg 1$ ($|\varphi| \gg \varphi_0$).

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The relativistic system of units, where $\hbar = c = 1$ and standard metric $(ab) = a_0 b^0 - \mathbf{ab}$, will be used throughout this paper.

The following condition is satisfied in the range of the optical frequency and pulse durations of tens of femtoseconds and above:

$$\varphi_0 = \omega t_{\mathrm{imp}} \gg 1. \quad (3)$$

Thus, the spectral density of the four-potential (1) represents a sharp peak with an amplitude in order with φ_0 and a width in order with φ_0^{-1}. Therefore, it is expedient to consider ω as the frequency of the quasimonochromatic field. For the theoretical analysis, we choose the wave envelope given by

$$g(\phi) = \exp(-4\phi^2). \quad (4)$$

The intensity of the process is governed by the classical relativistic-invariant parameter [51]:

$$\eta = eF/m\omega, \quad (5)$$

where e and m are the electron charge and mass. Note that the parameter is introduced when the elementary quantum processes in the presence of the electromagnetic wave field are studied (see, for example, [1–51]).

In this work, we study the resonance of the exchange amplitude of Compton scattering in the pulsed electromagnetic field.

II. AMPLITUDE OF THE PROCESS

The scattering amplitude of the photon with the four-momentum $k_i = (\omega_i, \mathbf{k}_i)$ by electron with four-momentum $p_i = (E_i, \mathbf{p}_i)$ in the presence of external field (1) is given by (see [12–15], and also Fig. 1)

$$S_{fi} = S_{fi}^{(d)} + S_{fi}^{(e)}, \quad (6)$$

$$S_{fi}^{(e)} = -ie^2 \int d^4r d^4r' \bar{\Psi}_{p_f}(r)\gamma^{\mu} \mathrm{G}(r,r')\gamma^{\nu}\Psi_{p_i}(r') \times A_{\nu}^{*}(k_f r')A_{\mu}(\kappa_i r), \quad (7)$$

where indexes d and e belong to the direct and exchange diagrams, respectively; $p_f = (E_f, \mathbf{p}_f)$ and $k_f = (\omega_f, \mathbf{k}_f)$ are the four-momenta of the final electron and photon; γ^{μ} $(\mu = 0,1,2,3)$ are Dirac matrices. The direct amplitude $S_{fi}^{(d)}$ [Eq. (6)] is obtained from exchange amplitude $S_{fi}^{(e)}$ [Eq. (7)] by replacement $k_i \longleftrightarrow -k_f$. The four-potential of the initial photon is given by

$$A_{\mu}(k_i r) = \sqrt{2\pi/\omega_i}\, e_{\mu} \exp[-i(k_i r)], \quad (8)$$

where e_{μ} is the polarization four-vector of the photon. The four-potential of the final photon $A_{\nu}^{*}(k_f r')$ is obtained from Eq. (8) by complex conjugation, as well as the replacement indices $\mu \to \nu, i \to f$. The wave function of an electron $\Psi_{p_i}(r')$ in the field (1) (see [51,52]) in the zero approximation with respect to parameter ϕ_0^{-1} is given by (see [23,51–54])

$$\Psi_{p_i}(r') = B_{p_i}(r')e^{-iS_{p_i}(r)} u_{p_i}/\sqrt{2V\tilde{E}_i}, \quad (9)$$

$$B_{p_i}(r') = 1 + \frac{e}{2(kp_i)}\hat{k}\hat{A}, \quad (10)$$

$$S_{p_i}(r) = (p_i r) + \eta^2 \frac{m^2}{2(kp_i)} \int_{-\infty}^{\phi} g^2(\phi)d\phi - \frac{m\eta g(\phi)}{(kp_i)}[\delta_{ell}(p_i e_y)\cos\varphi - (p_i e_x)\sin\varphi], \quad (11)$$

and $\bar{\Psi}_{p_f}(r)$ is obtained from $\Psi_{p_i}(r')$ [Eqs. (9) and (11)] by renaming indices and Dirac conjugation. $\mathrm{G}(r,r')$ is the Green's function of the electron in the field (1) (see [51,52]):

$$\mathrm{G}(r,r') = \frac{1}{(2\pi)^4}\int \mathrm{d}^4 q^{(e)} B_{q^{(e)}}(r)\frac{\hat{q}^{(e)} + m}{(q^{(e)})^2 - m^2}\bar{B}_{q^{(e)}}(r') \times \exp\{-i[S_{q^{(e)}}(r) - S_{q^{(e)}}(r')]\}, \quad (12)$$

where hats above notations stand for the scalar product (for example, $\hat{k} = k_{\nu}\gamma^{\nu}$), u_{p_i} and $\bar{u}_{p_f}$ are Dirac bispinors, $V = L_x L_y L_z$ is the normalization volume, L_x and L_y are the normalization lengths in the plane that is perpendicular to the wave vector, L_z is the normalization length along the direction of the wave propagation that must be no less than the pulse duration $L_z \gtrsim t_{\mathrm{imp}}$, $S_{p_i}(r)$ is the classical action whose components correspond to the systematic motion along a smooth curve and the oscillatory motion with frequency ω around the curve [25,53], and $\tilde{E}_i$ is written as

$$\tilde{E}_i = E_i + \eta^2 \frac{m^2}{2(kp_i)}\left(\frac{\varphi_{\mathrm{int}}}{\omega L_z}\right), \quad (13)$$

$$\varphi_{\mathrm{int}} = \int_{-\infty}^{\infty} g^2(\phi)d\varphi = \sqrt{\frac{\pi}{2}}\varphi_0. \quad (14)$$

Let us consider the case of the low-intensity field:

$$\eta^2 \ll 1. \quad (15)$$

In this case, we can neglect the second term in expression (13) and assume that $\tilde{E}_j \approx E_j,\ j = i,f$. The expression for the resonant amplitude of the exchange diagram (6) for condition (15) is written as

$$S_{fi}^{(e)} \approx AI(\beta_{\pm}, l_*)e_{\nu}^{*}e_{\mu}\left(\bar{u}_{p_f}M^{\nu\mu}u_{p_i}\right) \times \delta^{(2)}(\mathbf{p}_{i,\perp} + \mathbf{k}_{i,\perp} - \mathbf{p}_{f,\perp} - \mathbf{k}_{f,\perp}) \times \delta(p_{i,-} + k_{i,-} - p_{f,-} - k_{f,-}), \quad (16)$$

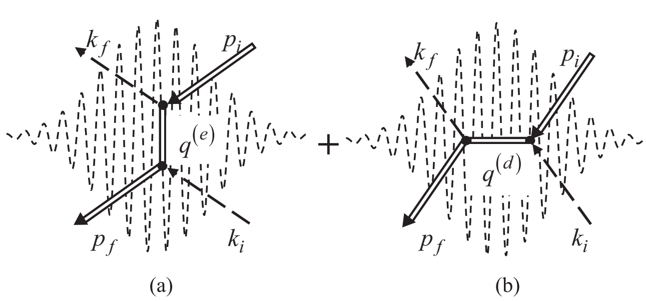


FIG. 1. Feynman diagram for the Compton effect in the field of the pulsed light wave for the direct (a) and exchange (b) parts. Incoming and outgoing double lines correspond to the Volkov function of an electron in initial and final states, and the dashed lines represents the wave function of a photon; inner lines designate the Green's function of an electron in the pulsed field.

$$M^{\nu\mu} = M^{\nu}_{-1}(p_f,q^{(e)})(\hat{q}^{(e)}+m)M^{\mu}_{1}(q^{(e)},p_i), \tag{17}$$

$$A = -\frac{ie^2(2\pi)^3\varphi_0^2}{2\sqrt{\omega_i E_i \omega_f E_f}V^2\omega^2}, \tag{18}$$

where $\mathbf{p}_{i,\perp}$, $\mathbf{k}_{i,\perp}$, $\mathbf{p}_{f,\perp}$, and $\mathbf{k}_{f,\perp}$ are the projections of the corresponding vectors along the wave polarization plane;

$$\begin{aligned} p_{i,-} &= E_i - p_{i,z}, \quad k_{i,-} = \omega_i - k_{i,z},\\ p_{f,-} &= E_f - p_{f,z}, \quad k_{f,-} = \omega_f - k_{f,z} \end{aligned} \tag{19}$$

are differences between the zero components of the corresponding four-momenta and their projections along the wave propagation direction; and $q^{(e)}$ is the momentum of the intermediate particles that correspond to the exchange diagrams in Fig. 1. Using the conservation of four-momenta, we obtain

$$\mathbf{q}^{(e)}_{\perp} = \mathbf{p}_{i,\perp} - \mathbf{k}_{f,\perp}, \quad q^{(e)}_{-} = p_{i,-} - k_{f,-}. \tag{20}$$

The parameter l_* in expression (16) can be determined from the equation

$$p_i + k_i + l_* k = p_f + k_f, \tag{21}$$

$\beta_\pm$ is the resonance parameter:

$$\beta_\pm = \frac{q^{(e)2}-m^2}{4(kq^{(e)})}\varphi_0. \tag{22}$$

Here β_+ corresponds to the positron intermediate state, and β_- corresponds to the electron intermediate state.

Matrices $M^{\nu}_{\pm 1}(p,q)$ in expression (17) are the terms which are proportional to the first power of parameter η. They determine the corrections related to the process involving one photon of the wave:

$$\begin{aligned} M^{\nu}_{\pm 1}(p,q) &= \pm\frac{y_0(p,q)}{2}e^{\mp i\chi}\gamma^{\nu} + \frac{m}{2(kq^{(e)})}[\hat{\varepsilon}_{\mp}k^{\nu} - \hat{k}\varepsilon_{\mp\nu}]\\ &\quad + \frac{m}{4}\left[\frac{1}{(kp)} - \frac{1}{(kq)}\right]\hat{\varepsilon}_{\mp}\hat{k}\gamma^{\nu}, \end{aligned} \tag{23}$$

Here $\varepsilon_\pm = e_x \pm i\delta e_y$; $y_0(p,q)$ and $\chi \equiv \chi(p,q)$ are parameters which have the form

$$\begin{aligned} y_0(p,q) &= m\eta\sqrt{-Q^2}, \quad \chi = \delta\varphi',\\ Q \equiv Q(p,q) &= \frac{p}{(kp)} - \frac{q}{(kq)}, \end{aligned} \tag{24}$$

where φ' is the angle between the perpendicular component of the vector $\mathbf{Q}$ and axis x. $M^{\nu}_{-1}(p_f,q^{(e)})$ and $M^{\mu}_{1}(q^{(e)},p_i)$ are obtained from Eqs. (23) and (24) by replacements $p \to p_f, q \to q^{(e)}$ and $p \to q^{(e)}, q \to p_i$, respectively. Function $I(\beta_\pm,l_*)$ in the expression (16) has the form

$$I(\beta_\pm,l_*) = \frac{4}{\pi^2}\int_{-\infty}^{\infty} f_2(\xi,l_*)\frac{1}{(1+\xi)-2\beta_\pm/\varphi_0+i0}f_1(\xi)d\xi. \tag{25}$$

Functions $f_1(\xi)$ and $f_2(\xi,l_*)$ in expression (25) can be written as

$$\begin{aligned} f_1(\xi) &= \int_{-\infty}^{\infty} d\phi g(\phi)\exp\left\{i\varphi_0\left[-(1+\xi)\phi\right.\right.\\ &\quad \left.\left.+\eta^2\frac{v_{fi}}{d_f}\int_{-\infty}^{\varphi} g^2(\phi')d\phi'\right]\right\}, \end{aligned} \tag{26}$$

$$\begin{aligned} f_2(\xi,l_*) &= \int_{-\infty}^{\infty} d\phi g(\phi)\exp\left\{i\varphi_0\left[(\xi - l_* + 1)\phi\right.\right.\\ &\quad \left.\left.-\eta^2\frac{v_{if}}{d_f}\int_{-\infty}^{\varphi} g^2(\phi')d\phi'\right]\right\}, \end{aligned} \tag{27}$$

where

$$\begin{aligned} &v_{if} = \frac{(kk_i)}{(kp_f)}, \quad v_{fi} = \frac{(kk_f)}{(kp_i)}, \quad d_f = \frac{2(k_f p_i)}{m^2},\\ &0 \leqslant v_{if} \leqslant d_f, \quad 0 \leqslant v_{fi} \leqslant d_f, \end{aligned} \tag{28}$$

and ξ is the parameter determined from the equation

$$p_i - k_f = q^{(e)} - \xi k. \tag{29}$$

In the sequel study the wave intensity has to be restricted by a condition that is more rigorous than Eq. (15):

$$\eta^2 \lesssim \varphi_0^{-1}. \tag{30}$$

Then, the simple analytical representation of functions (26) and (27) is written as

$$f_1(\xi) = \tilde{f}(1+\xi), \quad f_2(\xi,l_*) = \tilde{f}(l_* - \xi - 1), \tag{31}$$

where

$$\tilde{f}(x) = \int_{-\infty}^{\infty} g(\phi)\exp(-i\varphi_0 x\phi)d\phi = \frac{\sqrt{\pi}}{2}\exp\left(-\frac{\varphi_0^2 x^2}{16}\right). \tag{32}$$

It follows from expression (32) that the significant range of variables is $\xi \sim \varphi_0^{-1}$. Hence, in the zero approximation with respect to parameter φ_0^{-1} one can assume $l_* = \pm 1$ (for the positron and electron intermediate state, respectively). In this case from Eq. (21) the frequency of the scattered photon is given by

$$\begin{aligned} \omega_f^{(\mp)} &= \frac{(p_i k_i) + l_*[(kp_i)+(kk_i)]}{([p_i + k_i + l_* k]n_{k_f})}\\ &\approx \frac{(p_i k_i) \pm [(kp_i)+(kk_i)]}{([p_i + k_i \pm k]n_{k_f})}, \end{aligned} \tag{33}$$

where

$$n_{k_f} = \frac{k_f}{\omega_f} = (1,\mathbf{n}_{k_f}). \tag{34}$$

The integration with respect to ξ in Eq. (25) yields

$$\begin{aligned} I(\beta,l_*) &= -i\pi\exp\left[-\frac{\varphi_0^2 l^2_* + 8\left(\beta + \frac{\varphi_0 l_*}{4}\right)^2}{16}\right]\\ &\quad \times\left\{\mathrm{erfi}\left[\frac{\sqrt{2}}{2}\left(\beta + \frac{\varphi_0 l_*}{4}\right)\right] + i\right\}, \end{aligned} \tag{35}$$

where erfi(z) is the error function of the imaginary argument.

III. KINEMATICS OF RESONANCE OF EXCHANGE AMPLITUDE

Taking into account the condition (30) the resonance parameter (22) is given by (see also [12,13])

$$\begin{aligned}\beta_{+} &= \frac{\varphi_0}{2}\left(l' - \frac{(p_i k_f)}{(kp_i)-(kk_f)}\right),\\ \beta_{-} &= -\frac{\varphi_0}{2}\left(l - l' + \frac{(p_f k_i)}{(kp_i)-(kk_f)}\right),\end{aligned} \tag{36}$$

where l,l' are the integer numbers which determine the total intermediate number of the external wave photons, respectively. It follows from the expression (36) that the resonance of the exchange diagram under $(kp_i) > (kk_f)$ allows the processes, which satisfy the inequality

$$l' > 0, \quad l < l'. \tag{37}$$

In this case the resonance is realized by means of the electron intermediate state. There are the following four-momenta conservation laws:

$$p_i + l'k = q^{(e)} + k_f, \quad q^{(e)} + k_i = p_f - (l-l')k. \tag{38}$$

Resonance occurs for processes satisfying

$$l' < 0, \quad l > l' \tag{39}$$

when $(kp_i) < (kk_f)$ is valid. In this case, the resonance is realized by means of the positron intermediate state and the following four-momenta conservation laws hold:

$$p_i + (-q^{(e)}) = k_f + l'k, \quad k_i + \left(l - l'\right)k = p_f + (-q^{(e)}). \tag{40}$$

For the processes ($l' = \pm 1$, $l = 0$), which allow the resonance of the exchange diagram via electron ($l' = 1$) and positron ($l' = -1$) intermediate states, the resonance parameters $\beta_{\mp}$ are given by

$$\frac{\beta_{\mp}}{\varphi_0} = \frac{1}{2}\frac{v' \pm 1}{[u_f(1-\omega_f^{(\mp)}/\omega_{f,\mathrm{res}}^{(\mp)}) \pm 1]}\left(1 - \frac{\omega_f^{(\mp)}}{\omega_{f,\mathrm{res}}^{(\mp)}}\right), \tag{41}$$

where the upper (lower) sign corresponds to the electron (positron) intermediate state and invariant parameter u_f and frequency of the resulting photon $\omega_{f,\mathrm{res}}^{(\mp)}$ that correspond to the resonance maximum are represented as

$$u_f = \frac{(kk_f)}{(p_i k_f)}, \tag{42}$$

$$\omega_{f,\mathrm{res}}^{(\mp)} = \frac{(kp_i)}{(E_i - \mathbf{p}_i \mathbf{n}_f)(u_f \pm 1)}. \tag{43}$$

It follows from expression (43) that the resonance via the positron intermediate state can be observed under limitations on parameter u_f:

$$u_f > 1. \tag{44}$$

In accordance with conservation law (21), the frequency of the scattered photon at the resonance maximum is

$$\omega_f^{(\mp)} = \frac{(p_i k_i)}{(E_i - \mathbf{p}_i \mathbf{n}_f)(1 \pm d_{if})}. \tag{45}$$

In the case of the resonance through the positronic intermediate state we can rewrite a process as a consequence of two subprocesses: production of an electron-positron pair by the indent photon in the field of the wave with consequent annihilation of the electron-positron pair in the field of the wave. The terms that are proportional to the second power of parameter η^2 determine the corrections related to the process involving two photons of the wave.

Assuming that expressions (43) and (45) are equal, we conclude that the resonance maximum can be observed when the directions of the scattered photon belong to the surface of the cone (see Fig. 2) whose axis coincides with vector $\mathbf{j}^{\mp}$ and the cone angle is $\alpha = \angle(\mathbf{j}^{\mp},\mathbf{n}_f)$:

$$\begin{aligned}&\cos\alpha = j_0^{\mp}/|\mathbf{j}^{\mp}|,\\ &j^{\mp} = (j_0^{\mp},\mathbf{j}^{\mp}) = (kp_i)[p_i + k_i] - (p_i k_i)[k \pm p_i].\end{aligned} \tag{46}$$

The dependence of the resonance polar angle $\tilde{\theta}_f = \angle(\mathbf{k},\mathbf{k}_f)$ of the final photon on the azimuthal angle $\tilde{\psi}_f = \angle(\mathbf{k}_{f\perp},\mathbf{e}_x)$ is given by

$$\tilde{\theta}_f = 2\arctan\left(\frac{\cos\theta_{j^{\mp}}\cos(\psi_{j^{\mp}} - \tilde{\psi}_f) \pm \sqrt{D}}{\cos\alpha + \cos\theta_{j^{\pm}}}\right), \tag{47}$$

$$D = \sin^2\theta_{j^{\mp}}\cos^2(\psi_{j^{\mp}} - \tilde{\psi}_f) + \cos^2\theta_{j^{\mp}} - \cos^2\alpha, \tag{48}$$

where $\theta_{j^{\mp}}$ and $\psi_{j^{\mp}}$ are the polar and azimuthal angles of the vector $\mathbf{j}^{\mp}$.

One can conclude from Fig. 2 that the resonant emission angle of the photon is not determined within the full variation range of the corresponding azimuthal angle. It considerably depends on the initial geometry (whether the axis z occurs within the specified cone or not).

It follows from expression (46) that the four-vector must be spacelike $(j^{\mp})^2 \leqslant 0$ and the following condition must be satisfied:

$$d_i^2(1 - u_i v) \mp 2d_i v + v^2 \leqslant 0, \tag{49}$$

where

$$u_i = \frac{(kk_i)}{(p_i k_i)}, \quad v = \frac{2(kp_i)}{m^2}, \quad d_i = \frac{2(p_i k_i)}{m^2}. \tag{50}$$

In expression (49) sign "−" corresponds to the electron intermediate state, and "+" corresponds to the positron intermediate state. For the electron intermediate state from Eq. (49) we obtain the limitation on the initial parameter d_i under which the resonance of the exchange diagram is observed:

$$\begin{aligned}\frac{v}{1+\sqrt{vu_i}} \leqslant d_i &\leqslant \frac{v}{1-\sqrt{vu_i}}, \quad u_i < v^{-1}\\ d_i &\geqslant \frac{v}{1+\sqrt{vu_i}}, \quad v^{-1} < u_i < v\end{aligned} \tag{51}$$

When parameter d_i does not satisfy conditions (51) the resonance of exchange diagram is absent.

For the positron intermediate state, the resonance is possible in the range

$$d_i \geqslant d_{i,\mathrm{lim}}, \quad v^{-1} < u_i \leqslant v, \tag{52}$$

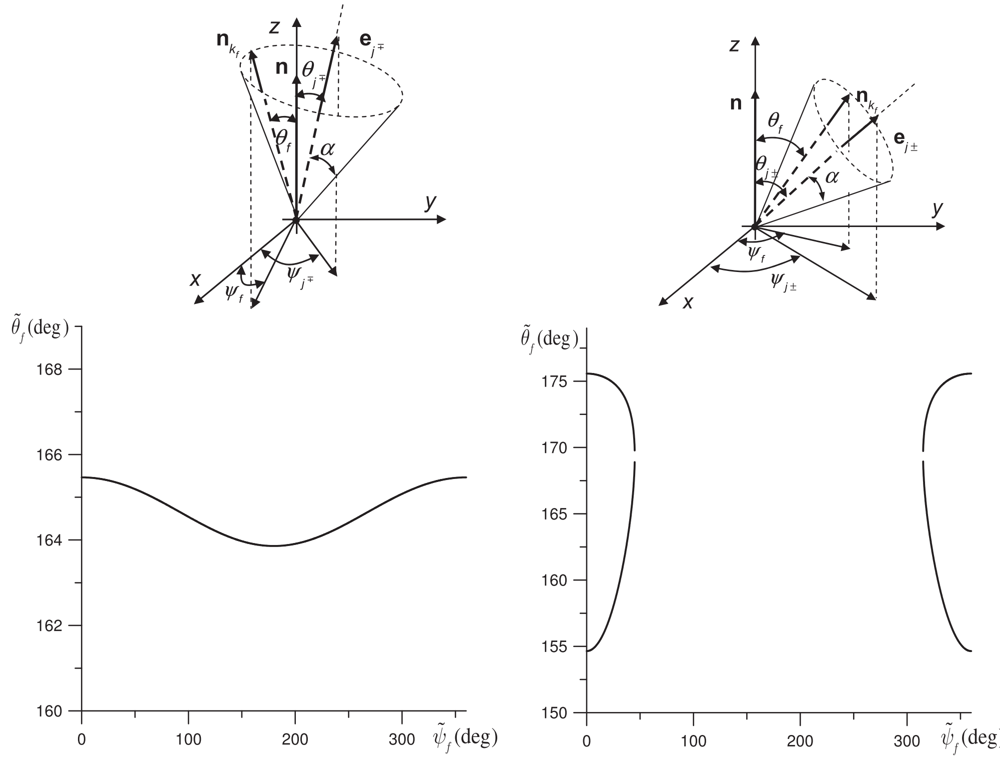


FIG. 2. The geometry of the final photon emission in the case of exchange diagram resonance (top) and corresponding dependencies (bottom) of the polar angle $\tilde{\theta}_f$ on the azimuthal angle of the final photon $\tilde{\psi}_f$ [Eq. (47)] for the polar angle of electron entrance $\theta_i = \angle(\mathbf{k},\mathbf{p}_i) = 163^\circ$ and azimuthal angles of the particle entrance $\psi_i = \tilde{\psi}_i = 0^\circ$. The left illustration corresponds to the photon entrance polar angle $\tilde{\theta}_i = \angle(\mathbf{k},\mathbf{k}_i) = 164^\circ$, and the right one corresponds to $\tilde{\theta}_i = 130^\circ$.

where the threshold value of parameter $d_{i,\lim}$ is

$$d_{i,\lim} = \frac{\upsilon}{\sqrt{\upsilon u_i} - 1}. \tag{53}$$

The resonance region can be represented as the condition for the frequency of the incident photon. In particular, for the electron intermediate state, we have

$$\begin{aligned} \frac{\omega f}{1+\sqrt{\upsilon u_i}} \leqslant \omega_i^{\mathrm{res}} \leqslant \frac{\omega f}{1-\sqrt{\upsilon u_i}}, \quad & u_i < \upsilon^{-1} \\ \omega_i^{\mathrm{res}} \geqslant \frac{\omega f}{1+\sqrt{\upsilon u_i}}, \quad & \upsilon^{-1} < u_i < \upsilon \end{aligned} \tag{54}$$

Here function f is represented as

$$f = \frac{1-\upsilon_i \cos\tilde{\theta}_i}{1-\upsilon_i \cos\theta_i}, \tag{55}$$

where υ_i is electron velocity.

For the positron intermediate state, we have

$$\omega_i^{\mathrm{res}} \geqslant \frac{\omega f}{\sqrt{\upsilon u_i} - 1}, \quad \upsilon^{-1} < u_i < \upsilon. \tag{56}$$

Figure 3 shows the resonant range of the initial photon frequency ω_i^{res} (in the units of the initial electron energy E_i) determined by the system of equations and inequalities (54), (56) at $\omega = 2.36$ eV, $E_i = 48$ GeV, and $\theta_i = 163^\circ$, as a function of the parameter α, which has the form

$$\alpha = (\theta_i - \tilde{\theta}_i)(E_i/m). \tag{57}$$

The chosen geometry of the process is that the momenta of the initial photon and electron and the wave propagation direction belong to the same plane.

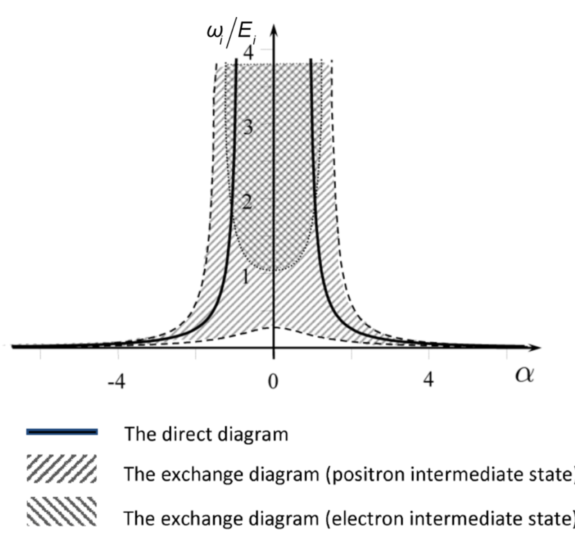


FIG. 3. (Color online) The resonant frequency range of the initial photon ω_i [see Eqs. (54) and (56)] in the units of the initial electron energy E_i as a function of the parameter α [Eq. (57)].

IV. RESONANCE PROBABILITY FOR EXCHANGE AMPLITUDE

Let us determine the differential probability over all the process observation time of the laser-modified Compton scattering using the amplitude (16)–(18):

$$dW^{(\mp)} = \left|S_{fi}^{(e)}\right|^2 \frac{d^3\mathbf{k}_f}{(2\pi)^3}\frac{d^3\mathbf{p}_f}{(2\pi)^3}. \tag{58}$$

We search for the resonance probability of the exchange diagram using the resonance approximation, [i.e. in Eq. (58) assume that $q_{\mp}^{(e)2} = m^2$ except for function $I(\beta_\mp, l_*)$]:

$$\begin{aligned} dW_{\text{res}}^{(\mp)} = {} & \frac{e^4}{16(2\pi)^3\omega_i E_i V (kq_{\mp}^{(e)})^2}\frac{\varphi_0^4}{\omega^4} H^{(\mp)}|I(\beta_\mp, l_*)|^2 \\ & \times \delta^{(2)}(\mathbf{p}_{f\perp} - (\mathbf{p}_{i\perp} + \mathbf{k}_{i\perp} - \mathbf{k}_{f\perp})) \\ & \times \delta(p_{f-} - (p_{i-} + k_{i-} - k_{f-}))\frac{d^3\mathbf{k}_f d\mathbf{p}_{f\perp} dp_{f-}}{\omega_f p_{f-}}. \end{aligned} \tag{59}$$

The averaging procedure over the initial and summation procedure over final polarizations of photons (k_i, k_f) and electrons (p_i, p_f) gives

$$\begin{aligned} H^{(\mp)} = {} & \frac{1}{4}\text{Sp}\{(\hat{p}_f + m)M_\nu^{\pm 1}(p_f, q_\mp^{(e)})(\hat{q}_\mp^{(e)} + m)M_\mu^{\mp 1}(q_\mp^{(e)}, p_i) \\ & \times (\hat{p}_i + m)M_\nu^{\pm 1}(p_f, q_\mp^{(e)})(\hat{q}_\mp^{(e)} + m)M_\mu^{\mp 1}(q_\mp^{(e)}, p_i)\}. \end{aligned} \tag{60}$$

Calculating Eq. (60) we find

$$H^{(\mp)} = 128\eta^4 m^4 K^{(\mp)}, \tag{61}$$

$$\begin{aligned} K^{(\mp)} = {} & f'(v_{fi}^{(\mp)}, d_f^{(\mp)}) f'(v_{if}^{(\mp)}, d_f^{(\mp)}) + g'(v_{fi}^{(\mp)}, d_f^{(\mp)}) g'(v_{if}^{(\mp)}, d_f^{(\mp)}) \\ & - \frac{2v_{if}^{(\mp)}v_{fi}^{(\mp)}}{(d_f^{(\mp)})^2}(d_i - d_f^{(\mp)}) + \frac{2v_{if}^{(\mp)}v_{fi}^{(\mp)}}{(v_{if}^{(\mp)} \pm 1)(v_{fi}^{(\mp)} \pm 1)} \\ & \times \left(\frac{v_{if}^{(\mp)} + v_{fi}^{(\mp)}}{d_f^{(\mp)}} - \frac{2v_{if}^{(\mp)}v_{fi}^{(\mp)}}{(d_f^{(\mp)})^2}\right), \end{aligned} \tag{62}$$

where parameters $v_{fi}^{(\mp)}$, $v_{if}^{(\mp)}$, and $d_f^{(\mp)}$ [Eq. (28)] have the form

$$\begin{aligned} & v_{fi}^{(\mp)} = 1 \mp \frac{d_i}{v(1 + d_{if})}, \quad v_{if}^{(\mp)} = \frac{u}{1 + u - v_{fi}^{(\mp)}}, \\ & d_f^{(\mp)} = \frac{d_i}{u_f \pm 1}. \end{aligned} \tag{63}$$

Here

$$u = \frac{(kk_i)}{(kp_i)}, \quad d_{if} = \frac{(k_i k_f)}{(p_i k_f)}. \tag{64}$$

Functions f' and g' in Eq. (62) are represented as

$$f'(v_{fi}^{(\mp)}, d_f^{(\mp)}) = 2 + \frac{v_{fi}^{(\mp)2}}{1 + v_{fi}^{(\mp)}} - 4\frac{v_{fi}^{(\mp)}}{d_f^{(\mp)}}\left(1 - \frac{v_{fi}^{(\mp)}}{d_f^{(\mp)}}\right), \tag{65}$$

$$g'(v_{fi}^{(\mp)}, d_f^{(\mp)}) = \frac{(2 + v_{fi}^{(\mp)})(d_f^{(\mp)} - 2v_{fi}^{(\mp)})d_f^{(\mp)}}{2d_f^{(\mp)}(1 + v_{fi}^{(\mp)})}. \tag{66}$$

The integration in expression (59) to $d\mathbf{p}_{f\perp} dp_{f-}$ is relatively simple owing to the presence of three δ functions:

$$\begin{aligned} & \frac{d^3\mathbf{k}_f d\mathbf{p}_{f\perp} dp_{f-}}{p_{f-}\omega_f}\delta^{(2)}(\mathbf{p}_{f\perp} - [\mathbf{p}_{i\perp} + \mathbf{k}_{i\perp} - \mathbf{k}_{f\perp}]) \\ & \quad \times \delta(p_{f-} - [p_{i-} + k_{i-} - k_{f-}]) \to \frac{\omega_f}{p_{f-}} d\omega_f d\Omega_f, \end{aligned} \tag{67}$$

where $d\Omega_f = \sin\tilde{\theta}_f d\tilde{\theta}_f d\tilde{\psi}_f$ is the element of a solid angle in which the final photon is emitted. Taking into account that

$$\omega_f - \omega_i = l_* \frac{(kp_i) + (kk_i) - (kk_f)}{([p_i + k_i]n_{k_f}) - l_*(kn_{k_f})} \approx l_* \frac{(kp_f)}{(p_i k_i)}\omega_i \tag{68}$$

we have

$$d\omega_f \approx \omega_i \frac{(kp_f)}{(p_i k_i)} dl_*. \tag{69}$$

Thus, the differential probability (59) of the process is represented as

$$\begin{aligned} \frac{dW_{\text{res}}^{(\mp)}}{d\Omega_f} = {} & \frac{4e^4 m^2 \eta^4 \tilde{u}_1}{\omega_i E_i V (d_f^{(\mp)})^2}\frac{m^2}{(p_i n_{k_f})^2} \\ & \times \frac{K^{(\mp)}}{(1 + d_{if})^2}\varphi_0^2 P_{\text{res}}(\beta_\mp)\tau_{\text{imp}}, \end{aligned} \tag{70}$$

$$P_{\text{res}}(\beta_\mp) = \frac{1}{2\pi}\int_{-\infty}^{\infty} |I(\beta_\mp, l_*)|^2 d(\varphi_0 l_*). \tag{71}$$

For estimation of the resonance width we expand the function, which determines the profile of the resonant peak, near the resonance maximum $\beta_\mp = 0$. Considering this condition the function (71) can be transformed into the form of the Breit-Wigner formula:

$$P_{\text{res}}(\beta_\mp) = \frac{16a_0(kq^{(e)})^2}{a_2\varphi_0^2}\frac{1}{(q^{(e)} - m^2)^2 + 4m^2\Gamma_\omega^2}, \ \beta_\mp \ll 1, \tag{72}$$

where Γ_ω is transit width, specified by the pulsed character of the external wave:

$$\Gamma_\omega = \frac{2}{\sqrt{a_2}}\frac{(kq^{(e)})}{m}\frac{1}{\varphi_0}, \tag{73}$$

and coefficients are specified by the following expressions:

$$a_0 = \pi P_{\text{res}}(0), \quad a_1 = -\frac{\pi}{2}P_{\text{res}}''(0), \quad a_2 = a_1/a_0. \tag{74}$$

Taking into account Eq. (74) we have

$$\Gamma_\omega = 1.7 m d_f \frac{1}{\varphi_0}. \tag{75}$$

We compare the transit width of resonance related to the pulse character of the field [expression (75)] with the radiation width:

$$\Gamma_R = \frac{e^2 m}{4\sqrt{\pi}}\eta^2 F(d_f), \tag{76}$$

where function $F(d_f)$ is given by

$$F(d_f)=\left(1-\frac{4}{d_f}-\frac{8}{d_f^2}\right)\ln(1+d_f)+\frac{1}{2}+\frac{8}{d_f}-\frac{1}{2(1+d_f)^2}. \quad (77)$$

The ratio is

$$\frac{\Gamma_\omega}{\Gamma_R}\approx\frac{8.5}{e^2\varphi_0\eta^2}\frac{d_f}{F(d_f)}\gtrsim 10^3. \quad (78)$$

Hence, the transit width dominates and the radiative broadening can be neglected.

The ratio of the differential resonance probability of the scattering of the photon by electron via exchange diagram (70) and (71) to the differential probability of the Compton effect in the absence of the external field (see [55]) in the same scattering kinematics is

$$R^{(\mp)}=\frac{dW_{\rm res}^{(\mp)}}{d{\rm w}_{\rm Compt}}=\frac{16\eta^4\varphi_0^2}{(d_f^{(\mp)})^2}\frac{K^{(\mp)}}{f(d_{if},d_i)}\frac{\tau_{\rm imp}}{T}P_{\rm res}(\beta_i^{(\mp)}). \quad (79)$$

We underline here that the formula (70) contains the divergence in the infrared spectrum (see [51]), when

$$v/2\lesssim\eta^2\ll 1. \quad (80)$$

Thus, one has to consider the next order of the perturbation theory [51] to escape this divergence when studying processes which are accompanied with emission of long-wavelength photons with energy $\omega\to 0$.

Conditions (80) result in the bottom limit of validity of the formula (70) for the electron energy:

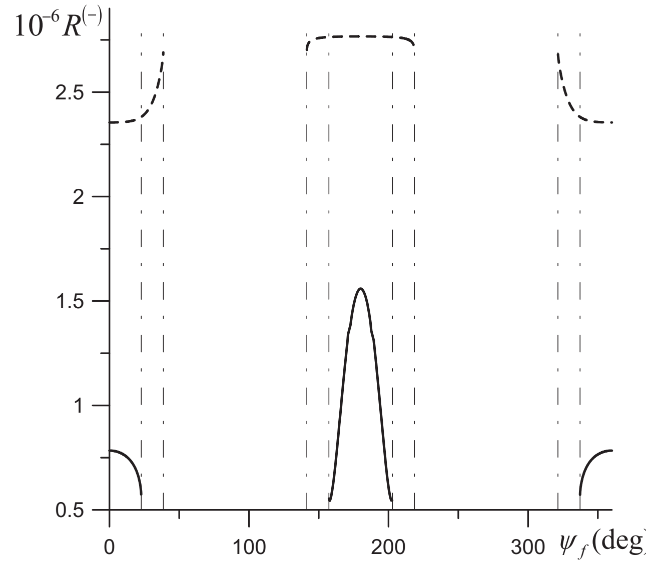
$$\frac{E_i}{m}\gtrsim\eta^2\sim 10\text{–}10^2. \quad (81)$$

In the sequel we consider the case of the electron intermediate state to estimate the possible effects. Figure 4 represents dependencies of the ratio $R^{(-)}$ [Eq. (79)] of the resonant probability of scattering of a photon by an electron in the field of the pulsed wave to the probability of the Compton scattering with absence of the external field influence on the emission azimuthal angles $\tilde{\varphi}_f$ of the photon. The results are given for MeV electron energies and optical petawatt lasers (for example, the PHELIX facility [32]).

Figure 4 demonstrates that the ratio of the probabilities may amount to several orders of the magnitude. Thus, for the electron energy $E_i=5$ MeV the excess of the probability of the studied process in the external field amounts to seven orders; for the electron energy $E_i=10$ MeV it amounts to six orders. If both electron and photon entrance angles are close [see the solid line in Figs. 4(a) and 4(b)], then the effect decreases. The breaks of the graphs are caused by the restriction on the emission polar angle of the photon under the resonance condition [see the formula (47), and graphs in Fig. 2].

It was shown that the variation in the small parameters such as the broadening of the energy and the angular spread of the electron and photon beam did not significantly affect the stability of the resonance peaks.

The obtained results ascertain that the pulsed field character reduces the effect in comparison with the monochromatic wave case (see [20]). Thus, for scattering kinematics and electron energy represented by Fig. 4 the excess of the probability in the pulsed field is one order less than in the monochromatic field.



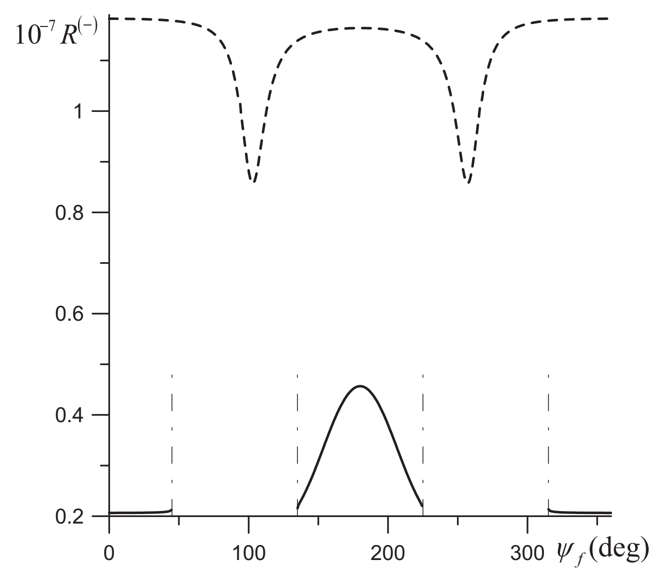


FIG. 4. The ratio (79) of the resonant probability of scattering of the photon by an electron in the field of the pulsed wave ($I=7\times10^{16}$ W cm^{-2}, $\tau/T=1$, $\omega=2.35$ eV) in the resonant peak $\beta_\pm=0$ to the probability of the Compton scattering, when the external field influence is absent, as a function of the photon output azimuthal angle $\tilde{\varphi}_f$. The dashed line corresponds to the photon entrance angle $\tilde{\theta}_i=130^\circ$, and the solid line corresponds to the case $\theta_i=164^\circ$. (a) $E_i=10$ MeV, $\omega_i=12$ eV, $\theta_i=163^\circ$, $\psi_i=\tilde{\psi}_i=0$. (b) $E_i=5$ MeV, $\omega_i=11.7$ eV, $\theta_i=163^\circ$, $\psi_i=\tilde{\psi}_i=0$.

For both ultrarelativistic energy $E_i\gg m$ and the optical frequency range $\omega\ll m$ when the electron moves within the narrow cone with the direction of the external wave propagating, the following characteristic parameter occurs:

$$\delta_i=\frac{E_i\theta_i}{m}. \quad (82)$$

Consequently, quantities v, u_i [Eq. (50)] can be represented in the form

$$v = \frac{\omega}{E_i}\left(1+\delta_i^2\right), \quad (83)$$

$$u_i = \frac{\omega}{E_i}\frac{(1-\cos\tilde{\theta})}{(1-\cos\theta)}, \quad (84)$$

where $\theta = \angle(\mathbf{p}_i,\mathbf{k}_i)$.

Considering Eqs. (54) and (55) we obtain that the resonant frequency of the initial photon belongs to the narrow interval and is determined as

$$f\omega(1-\tilde{\delta}) \leqslant \omega_{i,\mathrm{res}} \leqslant f\omega(1+\tilde{\delta}), \quad (85)$$

where

$$\tilde{\delta} = \frac{\omega}{E_i}\sqrt{\frac{(1+\delta_i^2)(1-\cos\tilde{\theta})}{1-\cos\theta}}. \quad (86)$$

The condition (80) results in the restriction on the parameter δ_i and, consequently, the restriction on the small entrance angle of the electron:

$$\delta_i \gtrsim \eta\sqrt{\frac{2E_i}{\omega}}\ . \quad (87)$$

Thus, for $E_i = 0.5$ GeV and the optical frequency range,

$$\theta_i \sim 10^{-1}. \quad (88)$$

For the case of both electron ultrarelativistic energy and small entrance angles (88), the expression (79) is simplified considerably:

$$R^{(\mp)} = \frac{dW_{\mathrm{res}}^{(\mp)}}{d\mathrm{w}_{\mathrm{Compt}}} = \frac{16\eta^4\varphi_0^2}{v^2} \times \frac{2f'(v_{if},v)+vg'(v_{if},v)}{f'(u,v)}\frac{\tau_{\mathrm{imp}}}{T}P_{\mathrm{res}}(\beta_i^{(\mp)}), \quad (89)$$

where

$$u = \frac{2E_i\omega_i(1-\cos\tilde{\theta})}{m^2(1+\delta_i^2)}, \quad v_{if} = \frac{u}{1+u}, \quad (90)$$

and the parameter v is determined by the expression (83). Figure 5 represents the dependence of the ratio $R^{(-)}$ [Eq. (89)] on the parameter δ_i [Eq. (82)].

One can conclude from Fig. 5 that, for the range of ultrarelativistic energy of the electron moving within the narrow cone with the direction of the external wave propagating, the excess of the resonant probability in the external field over the Compton scattering probability may amount to ten orders of magnitude.

The study of the resonant Compton scattering was started long enough ago [9,11], but there is still no experimental verification. The reason is either that it is very difficult to observe or nobody is interested in carrying out the measurements. In our view there are currently the necessary conditions for the experimental verification of resonant laser-modified Compton scattering. We do not give in this paper the experiment details; these could be made into a separate paper. We note the following. In order to apply the model of a plane nonmonochromatic wave [Eq. (1)] the size R of the focal spot must be much greater than the characteristic wavelength λ of the laser radiation: $R/\lambda \gg 1$. Moreover, the pulse must be sufficiently short so as to be able to neglect the spreading of the packet [25].

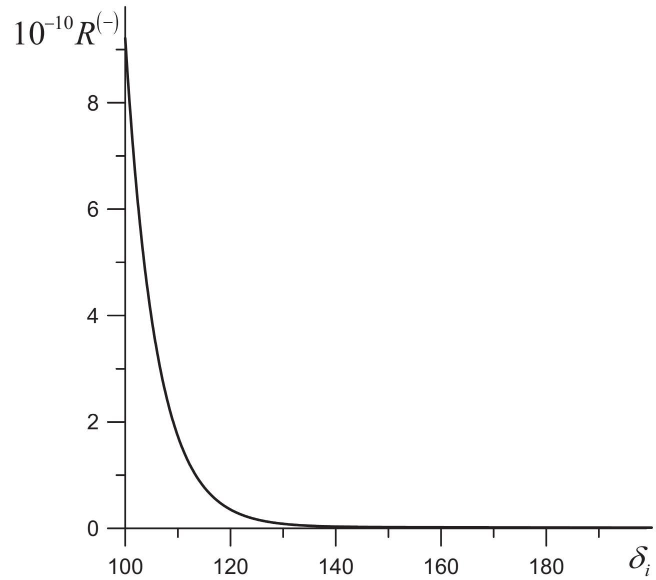


FIG. 5. The ratio of probabilities $R^{(-)}$ [Eq. (89)], as a function of the parameter δ_i [Eq. (82)] in the resonance peak $\beta_{\pm} = 0$ under $\tau/T = 1$, $I = 7 \times 10^{16}$ W cm^{-2}.

It should be noted that, in the experiment SLAC E144 [31], in principle, in secondary processes (when an emitted photon is scattered by an electron in the laser pulse) resonant laser-modified Compton scattering could occur. Therefore, verification of the studied process could be carried out by modifying the SLAC experiment. We also emphasize that good prospects for testing the resonance Compton effect appear in the framework of megaprojects XFEL, FAIR, and ELI.

V. CONCLUSIONS

Analysis of laser-modified Compton scattering through the exchange diagram has demonstrated the following.

(1) The resonant probability of the Compton scattering in the field of the weak intensity wave may exceed the corresponding probability in the external field absence in several orders of the magnitude. Thus, for the electron energy $E_i = 5$ MeV, the photon frequency $\omega_i = 12$ eV, the intensity in the pulse peak $I = 7 \times 10^{16}$ W cm^{-2}, and arbitrary angles of the entrance of both electron and photon, the ratio $R^{(-)}$ [Eq. (79)] amounts to seven orders of the magnitude.

(2) The excess of the resonant probability of the Compton effect for the case of ultrarelativistic energy of the electron moving in the narrow cone with the direction of the external wave propagating, but considering the condition (88), may amount to ten orders of the magnitude.

Obtained results can be verified experimentally in the framework of modern research projects (SLAC, FAIR, XFEL, and ELI).

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