Necessary and sufficient conditions for macroscopic realism from quantum mechanics

Lucas Clemente* and Johannes Kofler†
Max Planck Institute of Quantum Optics, Hans-Kopfermann Strasse 1, 85748 Garching, Germany
(Received 6 March 2015; published 4 June 2015)

Macroscopic realism, the classical world view that macroscopic objects exist independently of and are not influenced by measurements, is usually tested using Leggett-Garg inequalities. Recently, another necessary condition called no-signaling in time (NSIT) has been proposed as a witness for nonclassical behavior. In this paper, we show that a combination of NSIT conditions is not only necessary but also sufficient for a macrorealistic description of a physical system. Any violation of macroscopic realism must therefore be witnessed by a suitable NSIT condition. Subsequently, we derive an operational formulation for NSIT in terms of positive operator-valued measures and the system Hamiltonian. We argue that this leads to a suitable definition of "classical" measurements and Hamiltonians, and we apply our formalism to some generic coarse-grained quantum measurements.

DOI: 10.1103/PhysRevA.91.062103 PACS number(s): 03.65.Ta

I. INTRODUCTION

Whether or not the laws of quantum mechanics are universally valid and hold on the level of macroscopic objects is still an open question in the physics community. Some believe that the issue will be settled in favor of quantum theory by the experimental demonstration of Schrödinger-cat-like states [1]. Others hold that some physical mechanism, altering the laws of quantum mechanics [2–4], guarantees a fully classical world on the macroscopic level.

In 1985, Leggett and Garg [5] have put forward macroscopic realism, or macrorealism (MR), a world view encompassing all physical theories which enforce that macroscopic properties of macroscopic objects exist independently of and are not influenced by measurement. While setups such as superconducting devices, heavy molecules, and quantum-optical systems are promising candidates in the race towards an experimental violation of macrorealism, nonclassical effects have so far only been observed for microscopic objects or microscopic properties of larger objects [6–19]. However, a genuine violation of macroscopic realism—with its reference to macroscopically distinct states—requires using solely measurements of macroscopically coarse-grained observables. Note that there are several approaches to quantifying the "macroscopicity" of quantum states and measurements [20–27]. It is also known that usually the restriction to such coarse-grained ("classical") measurements alone already leads to the emergence of classicality [28], unless a certain type of ("nonclassical") Hamiltonian is governing the object's time evolution [29]. Recent investigations have confirmed the intuition that these Hamiltonians are hard to engineer and require a very high control precision in the experimental setup [30–32].

A quantum violation of macrorealism is usually witnessed by the violation of a Leggett-Garg inequality (LGI), which is composed of temporal correlations between sequential measurements of an object undergoing time evolution. Recently, following earlier works [29,33–35], another necessary condition for MR called no-signaling in time (NSIT) was proposed [36]. It can be regarded as a statistical version of the noninvasive measurability postulate.

In Sec. II, we start with the discussion of various instances of NSIT and show that in the correct combination they form a sufficient condition for a macrorealistic description (at a given set of possible measurement times). We also demonstrate that it is impossible to establish such a sufficient condition for a macrorealistic description by combining LGIs involving two-time measurements. Subsequently, in Sec. III, we derive an operational condition for NSIT, based on (projective and nonprojective) measurement operators and the system Hamiltonian. In Sec. IV, we use these results to define the classicality of measurements based on a reference set of *a priori* classical operators and to characterize the classicality of Hamiltonians. Finally, in Sec. V, we apply our formalism to measurements of coherent states, quadratures, and Fock states, and quantify their invasiveness as a function of their coarse graining.

II. NONINVASIVE MEASUREMENTS

Let us start with the definition of macrorealism, consisting of the following postulates [37]: "(1) *Macrorealism per se*. A macroscopic object which has available to it two or more macroscopically distinct states is at any given time in a definite one of those states. (2) *Non-invasive measurability*. It is possible in principle to determine which of these states the system is in without any effect on the state itself or on the subsequent system dynamics. (3) *Induction*. The properties of ensembles are determined exclusively by initial conditions (and in particular not by final conditions)."

In the following, we will first show that a strong reading of noninvasive measurability implies macrorealism per se (Sec. II A). Then we will present various necessary conditions (Sec. II B) and a set of sufficient conditions (Sec. II C) for a macrorealistic description.

A. Macrorealism per se following from strong noninvasive measurability

In this subsection, we assume that the state space of a macroscopic object is split into macroscopically distinct *nonoverlapping* states (macrostates). Consider a macro-observable

*lucas.clemente@mpq.mpg.de
†johannes.kofler@mpq.mpg.de

$Q(t)$ with a one-to-one mapping between its values and the macrostates. Further consider measurements of the macro-observable that enforce a definite postmeasurement macrostate and report the corresponding value as the outcome.

Macrorealism per se (MRps) is fulfilled if $Q(t)$ has a definite value at all times t, prior to and independent of measurement:

$$\forall t : \exists \text{ definite } Q(t). \tag{1}$$

Probabilistic predictions for $Q(t)$ are merely due to ignorance of the observer. Even in cases where $Q(t)$ evolves unpredictably (e.g., in classical chaos) or even indeterministically, it is still assumed to have a definite value at all times.

On top of MRps, the assumption of noninvasive measurability (NIM) in principle allows a measurement at every instant of time, revealing the macrostate without disturbance. NIM guarantees that

$$\forall t : Q(t) = Q_H(t), \tag{2}$$

where H denotes the history of past noninvasive measurements on the system: In order for measurements to be noninvasive, the time evolution of Q must not depend on the history of the experiment [38]. Note that all noninvasive measurements are repeatable; i.e., when performing the same measurement immediately again, the same outcome is obtained with probability 1.

In the literature, NIM is often treated as a necessary condition for macrorealism per se. It is argued that NIM is "so natural a corollary of [MRps] that the latter is virtually meaningless in its absence" [37]. As some others before [36,39,40], we do not adhere to this position. A counterexample to the statement MRps $\Rightarrow$ NIM is given by the de Broglie-Bohm theory, where measurements are invasive, as they affect the guiding field and thus the subsequent (position) state, but MRps is fulfilled, as the (position) state is well defined at all times. In fact, we now argue that there exist two different ways of reading the postulate of NIM in [37].

(1) Weak NIM. Given a macroscopic object is in a definite one of its macrostates, it is possible to determine this state without any effect on the state itself or on the subsequent system dynamics.

(2) Strong NIM (sNIM). It is always possible to measure the macrostate of an object without any effect on the state itself or on the subsequent system dynamics.

Let us now argue that sNIM actually implies MRps. Assuming sNIM, a hypothetical noninvasive measurement can be performed at every instant of time, determining the value of the macro-observable Q. Due to its noninvasive nature, Q must have had a definite value already before the measurement. This ensures that Q has a definite value at all times, giving rise to a "trajectory" $Q(t)$. Therefore,

$$\text{sNIM} \Rightarrow \text{MRps}. \tag{3}$$

Another way of establishing this implication is the following: Assume that MRps fails, i.e., the object is not in a definite macrostate. A measurement leaves the object in a definite macrostate, creating a definite state out of an indefinite one, and therefore does not satisfy sNIM. We thus have $\neg$MRps $\Rightarrow$ $\neg$sNIM, which is equivalent to expression (3).

Note that expression (3) holds even if sNIM is made less stringent, allowing measurements to change the subsequent time evolution, while still determining the macrostate.

In this paper, we implicitly assume induction [the arrow of time (AoT)] [37] and freedom of choice concerning the initial states and measurement times (including whether a measurement takes place at all). Then, sNIM alone is sufficient for macrorealism, and by extension for testable conditions such as the Leggett-Garg inequalities or no-signaling in time [36]:

$$\text{sNIM} \Leftrightarrow \text{MRps} \wedge \text{NIM} \Leftrightarrow \text{MR} \Rightarrow \text{LGI, NSIT}. \tag{4}$$

Let us remark that NIM is in general not as strongly physically motivated as the assumption of locality in Bell's theorem. The so-called "clumsiness loophole" allows violations of NIM to be attributed to imperfections of the measurement apparatus instead of genuine quantum effects. This loophole can be addressed using ideal negative measurements [5] or more involved protocols [41].

B. Necessary conditions for macrorealism

The relationship between LGI and NSIT has previously been discussed in the literature for a number of example systems [29,36,40,42]. Here we consider the archetypal setup depicted in Fig. 1: A system starting in the initial state $\hat{\rho}_0$ evolves under unitary $\hat{U}_{01}$ from t_0 to t_1, and under unitary $\hat{U}_{12}$ from t_1 to t_2. During the evolution, dichotomic measurements may be performed at times t_i for $i \in \{0,1,2\}$. Let us call the outcomes of these measurements $Q_i \in \{-1, +1\}$, and define the correlations $C_{ij} = \langle Q_i Q_j \rangle$. Then, the simplest LGI reads

$$\text{LGI}_{012} : C_{01} + C_{12} - C_{02} \leqslant 1. \tag{5}$$

There exist many other Leggett-Garg inequalities involving more than three possible measurement times or more than two outcomes (for a recent review see [43]). Quantum mechanical experiments are able to violate inequality (5) up to 1.5 for a qubit and, as shown in [44], up to the algebraic maximum 3 for higher-dimensional systems still using dichotomic measurements $Q_i = \pm 1$.

On the other hand, $\text{NSIT}_{(i)j}$ is a statistical version of Eq. (2), requiring that the outcome probabilities $P_j(Q_j)$ of result Q_j

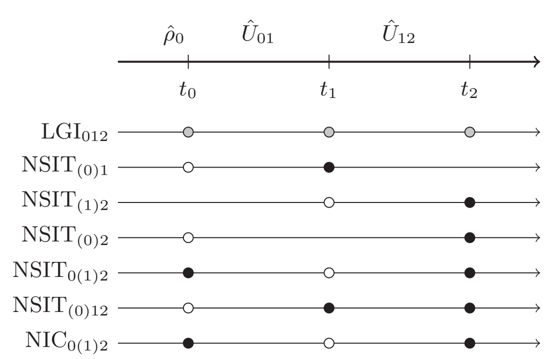

FIG. 1. Different necessary conditions for MR in a system with possible measurements at three points in time. Black filled circles denote measurements that always take place, and unfilled circles denote measurements that may or may not be performed. A pair of measurements is always performed for the LGI, shown with gray filled circles.

measured at time t_j are the same, whether or not a measurement was performed at some earlier time $t_i < t_j$:

$$\mathrm{NSIT}_{(i)j}:\ P_j(Q_j) = P_{ij}(Q_j) \equiv \sum_{Q_i'} P_{ij}(Q_i',Q_j). \tag{6}$$

Note that the probability distributions on both sides of the equation, P_i and P_{ij}, correspond to *different* physical experiments: While P_j is established by measuring only at t_j, P_{ij} is obtained by measuring both at t_i and t_j. Unlike in the LGI in inequality (5), one is not limited to only two outcomes. If it is the later measurement at t_j which may or may not be performed, $\mathrm{NSIT}_{i(j)}$ is an instance of the arrow of time and is therefore fulfilled by both macrorealism and quantum mechanics.

While $\mathrm{NSIT}_{(1)2}$ is a promising condition that is usually able to detect violations of MR more reliably than LGI_{012} [36,42], it fails for particular initial states, where the invasiveness is able to "hide" in the statistics of the experiment (see the discussion below). We can, however, make $\mathrm{NSIT}_{(1)2}$ robust against such cases, by always performing a measurement at t_0. We call the resulting condition

$$\begin{aligned}\mathrm{NSIT}_{0(1)2}:\ P_{02}(Q_0,Q_2) &= P_{012}(Q_0,Q_2)\\ &\equiv \sum_{Q_1'} P_{012}(Q_0,Q_1',Q_2).\end{aligned} \tag{7}$$

$\mathrm{NSIT}_{0(1)2}$ alone is not sufficient for LGI_{012}. Hence, we also introduce the condition

$$\begin{aligned}\mathrm{NSIT}_{(0)12}:\ P_{12}(Q_1,Q_2) &= P_{012}(Q_1,Q_2)\\ &\equiv \sum_{Q_0'} P_{012}(Q_0',Q_1,Q_2).\end{aligned} \tag{8}$$

As was recently shown in [40], a combination of $\mathrm{NSIT}_{(0)12}$, $\mathrm{NSIT}_{0(1)2}$, and the arrow of time (AoT) is sufficient for LGI_{012}:

$$\mathrm{NSIT}_{0(1)2} \wedge \mathrm{NSIT}_{(0)12} \wedge \mathrm{AoT} \Rightarrow \mathrm{LGI}_{012}. \tag{9}$$

The inverse is not true, and moreover the left-hand side is not sufficient for macrorealism (see discussion below).

We further remark that one can also write a condition similar to $\mathrm{NSIT}_{0(1)2}$ in a more intuitive form that we call noninvaded correlations (NIC):

$$\mathrm{NIC}_{0(1)2}:\ C_{02} = C_{02|1}, \tag{10}$$

where $C_{02|1}$ denotes the correlation $\langle Q_0 Q_2\rangle$ given that an additional measurement was performed at t_1. It is shown in Appendix A that $\mathrm{NIC}_{0(1)2}$ follows from $\mathrm{NSIT}_{0(1)2}$.

Figure 1 presents a graphical summary of the conditions that have been discussed in this section.

C. NSITs as sufficient conditions for macrorealism

In the following, we will show that the combination of various NSIT conditions and the arrow of time (AoT) guarantees the existence of a unique global probability distribution $P_{012}(Q_0,Q_1,Q_2)$, which is equivalent to macrorealism evaluated at t_0,t_1,t_2. Let us start by writing all single-measurement probabilities in terms of P_{012}. Once again, note that joint probabilities P with different subscripts correspond to different experimental setups [e.g., $P_2(Q_2)$ is obtained by measuring only at t_2, while $P_{12}(Q_1,Q_2)$ is obtained by measuring at times t_1 and t_2]:

$$P_2(Q_2) = \sum_{Q_1'} P_{12}(Q_1',Q_2) = \sum_{Q_0'}\sum_{Q_1'} P_{012}(Q_0',Q_1',Q_2), \tag{11}$$

where we have used $\mathrm{NSIT}_{(1)2}$ for the first equality and $\mathrm{NSIT}_{(0)12}$ for the second one. Furthermore,

$$P_1(Q_1) = \sum_{Q_2'} P_{12}(Q_1,Q_2') = \sum_{Q_0'}\sum_{Q_2'} P_{012}(Q_0',Q_1,Q_2'), \tag{12}$$

where for the first equality we assumed AoT [i.e., Q_i are (statistically) independent of Q_j for $j > i$], and $\mathrm{NSIT}_{(0)12}$ for the second one. Moreover, we see that

$$P_0(Q_0) = \sum_{Q_1'}\sum_{Q_2'} P_{012}(Q_0,Q_1',Q_2'), \tag{13}$$

where AoT was used twice. Next, the pairwise joint probability functions can be constructed:

$$P_{01}(Q_0,Q_1) = \sum_{Q_2'} P_{012}(Q_0,Q_1,Q_2') \tag{14}$$

follows from AoT. Using $\mathrm{NSIT}_{0(1)2}$ one obtains

$$P_{02}(Q_0,Q_2) = \sum_{Q_1'} P_{012}(Q_0,Q_1',Q_2). \tag{15}$$

Finally, using $\mathrm{NSIT}_{(0)12}$, we obtain

$$P_{12}(Q_1,Q_2) = \sum_{Q_0'} P_{012}(Q_0',Q_1,Q_2). \tag{16}$$

We have thus shown that there exists a combination of NSIT conditions, whose fulfillment guarantees that all probability distributions in any experiment can be written as the marginals of a unique global probability distribution $P_{012}(Q_0,Q_1,Q_2)$. This is equivalent to the existence of a macrorealistic model for measurements at times t_0,t_1,t_2 (MR_{012}). Note that while MR_{012} cannot prove the world view of MR in general, it implies that no experimental procedure (with measurements at t_0,t_1,t_2) can detect a violation of MR. Let us now write a *necessary and sufficient* condition for MR_{012}:

$$\mathrm{NSIT}_{(1)2} \wedge \mathrm{NSIT}_{0(1)2} \wedge \mathrm{NSIT}_{(0)12} \wedge \mathrm{AoT} \Leftrightarrow \mathrm{MR}_{012}. \tag{17}$$

This set of conditions is not unique: We can, e.g., substitute $\mathrm{NSIT}_{(1)2}$ by $\mathrm{NSIT}_{(0)2}$, as can easily be seen from a graphical representation of all conditions in Fig. 2. We remark that even the combination of all two-time NSIT conditions, $\mathrm{NSIT}_{(0)1} \wedge \mathrm{NSIT}_{(1)2} \wedge \mathrm{NSIT}_{(0)2}$, is sufficient neither for MR_{012} nor for LGI_{012}. Note that LGIs only test for nonclassicalities of the pairwise joint probability distributions. A smaller set of conditions is therefore sufficient for fulfilling all LGIs using two-time correlation functions or probabilities [such as inequality (5) or the so-called Wigner LGIs [42]], see expression (9).

To illustrate these conditions for a qubit, in Table I we show the individual conditions evaluated for a Mach-Zehnder setup (reflectivities R_1,R_2, phase plate φ in one arm) with arbitrary initial state and time evolution. The three possible measurements are which-path measurements before the first beamsplitter (t_0), between the two beamsplitters (t_1), and after

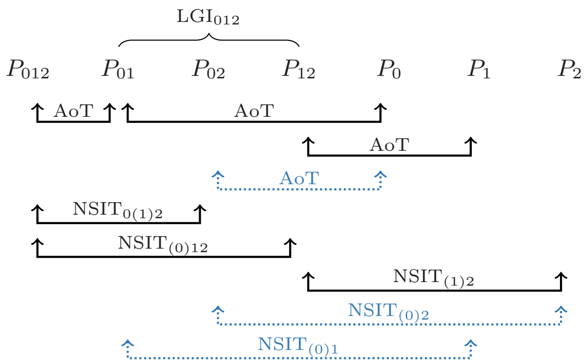

FIG. 2. (Color online) Different combinations of NSIT and AoT conditions are sufficient for guaranteeing that all probability distributions P_i, P_{ij} are the marginals of a unique global probability distribution P_{012}. There are multiple ways of obtaining a sufficient set. The black arrows correspond to one particular choice, and additional conditions are printed for completeness as dotted arrows. Note that the existence of a classical explanation for the pairwise joint probabilities P_{ij} is sufficient for fulfilling LGI$_{012}$, but not for MR$_{012}$.

the second beamsplitter (t_2), respectively. We can easily find cases where LGI$_{012}$ is always fulfilled, but various NSIT conditions still witness a violation of MR, e.g., for $R_1 = R_2 = 1/2, \varphi \neq (n+1/2)\pi$. As discussed above, it is possible for LGI$_{012}$ to be violated with NSIT$_{(1)2}$ fulfilled, e.g., for $R_1 = 1/4, R_2 = 3/4, q = 1/2, \varphi = \pi$. For mixed initial states, NSIT$_{0(1)2}$ reduces to the condition $\varphi = (n+1/2)\pi$ with $n \in \mathbb{N}_0$ and is sufficient for MR$_{012}$, as no interference is possible in this case. For general superposition states, NSIT$_{(0)12}$ can be violated with NSIT$_{0(1)2}$ fulfilled. Moreover, NSIT conditions still allow detecting violations of MR if $R_1 = 0,1$ or $R_2 = 0,1$.

III. NSIT FOR QUANTUM MEASUREMENTS

In the following, we will look at NSIT$_{(0)T}$ in an archetypal quantum experiment. A system has been prepared at $t = 0$ in an initial state $\hat{\rho}_0$. Then, at $t = 0$, a positive operator-valued measure (POVM) $\{\hat{A}_a^\dagger \hat{A}_a\}_a$ with outcomes a is carried out. After the measurement, the system evolves according to a

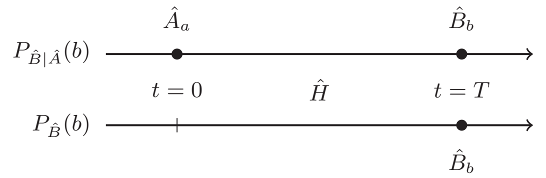

FIG. 3. A system evolves from $t = 0$ to T under Hamiltonian $\hat{H}$. In the first setup, measurements $\hat{A}_a^\dagger \hat{A}_a$ and $\hat{B}_b^\dagger \hat{B}_b$ are performed at $t = 0$ and T, respectively, and in the second setup only a final measurement $\hat{B}_b^\dagger \hat{B}_b$ is performed.

unitary $\hat{U} = e^{-i\hat{H}t}$. At time $t = T$ a second, possibly different POVM $\{\hat{B}_b^\dagger \hat{B}_b\}_b$ with outcomes b is performed.

To determine the effect of the first measurement $\hat{A}_a^\dagger \hat{A}_a$ on the system's state and its subsequent dynamics, we will compare the results of the final measurement with a different experiment, where no measurement was performed at $t = 0$ (or, equivalently, a measurement $\hat{A}_a = \mathbb{1}$ was performed). The two setups are shown in Fig. 3.

The probabilities for obtaining outcome b in the second and first setup are called $P_{\hat{B}}(b)$ and $P_{\hat{B}|\hat{A}}(b)$. They can be calculated as, respectively

$$P_{\hat{B}}(b) = \mathrm{tr}(\hat{B}_b \hat{U}_T \hat{\rho}_0 \hat{U}_T^\dagger \hat{B}_b^\dagger), \tag{18}$$

$$P_{\hat{B}|\hat{A}}(b) = \int da\ \mathrm{tr}(\hat{B}_b \hat{U}_T \hat{A}_a \hat{\rho}_0 \hat{A}_a^\dagger \hat{U}_T^\dagger \hat{B}_b^\dagger), \tag{19}$$

with the integral replaced by a sum if the number of outcomes is countable. NSIT$_{(0)T}$ is fulfilled if the test measurement has no detectable effect on the system, i.e., if $P_{\hat{B}} = P_{\hat{B}|\hat{A}}$:

$$\forall b\colon \mathrm{tr}(\hat{B}_b \hat{U}_T \hat{\rho}_0 \hat{U}_T^\dagger \hat{B}_b^\dagger) = \int da\ \mathrm{tr}(\hat{B}_b \hat{U}_T \hat{A}_a \hat{\rho}_0 \hat{A}_a^\dagger \hat{U}_T^\dagger \hat{B}_b^\dagger). \tag{20}$$

Note that the equality sign in Eq. (20) will often be fulfilled only approximately, even by noninvasive measurements. In practice, one can choose from a variety of error measures and corresponding reasonable error thresholds. However, to simplify notation, we will continue to use the equality sign in the following calculations.

TABLE I. Different necessary conditions for macrorealism evaluated for a Mach-Zehnder (qubit) experiment [45]. The reflectivity of the first beamsplitter is R_1 and of the second one is R_2. In one path of the interferometer, a phase φ is added. Which-path measurements may be performed before, between and after the beamsplitters. The initial states are $\hat{\rho}_{\text{mix}} = \begin{pmatrix} q & 0 \\ 0 & 1-q \end{pmatrix}$ and $\hat{\rho}_{\text{sup}} = \begin{pmatrix} q & c \\ c^* & 1-q \end{pmatrix}$. The symbol ✓ means that the condition holds for all values of the free parameters. For brevity, $\alpha \equiv \sqrt{R_1 R_2 (1-R_1)(1-R_2)}$. Equation [*] reads $(2i\sqrt{3}c + 6q - 3)\cos\varphi - 2i\sqrt{3}\,\mathrm{Re}(c)(\cos\varphi - 2i\sin\varphi) = 0$, and Eq. [**] reads $\cos\varphi[(2q-1)\alpha + ic(1-2R_1)\sqrt{-(R_2-1)R_2}] + i\sqrt{-(R_2-1)R_2}\,\mathrm{Re}(c)[(2R_1-1)\cos\varphi + i\sin\varphi] = 0$. See main text for discussion.

		LGI$_{012}$	NSIT$_{(1)2}$	NSIT$_{0(1)2}$	NSIT$_{(0)12}$
$\hat{\rho}_{\text{mix}}$:	$R_1 = R_2 = \frac{1}{2}$	✓	$q = \frac{1}{2}$ or $\varphi = (n+\frac{1}{2})\pi$	$\varphi = (n+\frac{1}{2})\pi$	✓
	$R_1 = \frac{1}{4}, R_2 = \frac{3}{4}$	$1 + 3\cos\varphi \geqslant 0$	$q = \frac{1}{2}$ or $\varphi = (n+\frac{1}{2})\pi$	$\varphi = (n+\frac{1}{2})\pi$	✓
	R_1, R_2	$R_1 + \alpha\cos\varphi - R_1 R_2 \geqslant 0$	$q = \frac{1}{2}$ or $\varphi = (n+\frac{1}{2})\pi$ or $\alpha = 0$	$\varphi = (n+\frac{1}{2})\pi$ or $\alpha = 0$	✓
$\hat{\rho}_{\text{sup}}$:	$R_1 = R_2 = \frac{1}{2}$	✓	$2q\cos\varphi = \cos\varphi + 2\,\mathrm{Re}(c)\sin\varphi$	$\varphi = (n+\frac{1}{2})\pi$	$c \in \mathbb{R}$
	$R_1 = \frac{1}{4}, R_2 = \frac{3}{4}$	$1 + 3\cos\varphi \geqslant 0$	[*]	$\varphi = (n+\frac{1}{2})\pi$	$c \in \mathbb{R}$
	R_1, R_2	$R_1 + \alpha\cos\varphi - R_1 R_2 \geqslant 0$	[**]	$\varphi = (n+\frac{1}{2})\pi$ or $\alpha = 0$	$c \in \mathbb{R}$ or $R_1 = 0,1$

A. NSIT without time evolution

Let us start by considering the case $T = 0$ ($\mathrm{NSIT}_{(0)0}$); i.e., the final measurement is performed immediately after the test measurement. In this setup, NSIT can be regarded as a case of joint measurability, a condition previously discussed in the context of compatibility of quantum measurements [46–53]. To rewrite Eq. (20) we use that $\int da\, A_a^\dagger \hat{A}_a = 1$. This yields

$$P_{\hat{B}|\hat{A}}(b) - P_{\hat{B}}(b) = \int da\ \mathrm{tr}[(\hat{A}_a^\dagger \hat{B}_b^\dagger \hat{B}_b \hat{A}_a - \hat{B}_b^\dagger \hat{A}_a^\dagger \hat{A}_a \hat{B}_b)\hat{\rho}_0]. \tag{21}$$

The trace in the above equation can be interpreted as the expectation value of the Hermitian operator $\int da\,(\hat{A}_a^\dagger \hat{B}_b^\dagger \hat{B}_b \hat{A}_a - \hat{B}_b^\dagger \hat{A}_a^\dagger \hat{A}_a \hat{B}_b)$. For $\mathrm{NSIT}_{(0)0}$ to be universally valid, we require that it is zero for *all* initial states $\hat{\rho}_0$. Thus, the operator itself has to be zero:

$$\forall \hat{\rho}_0 : \mathrm{NSIT}_{(0)0} \Leftrightarrow \forall b\colon \int da\,(\hat{A}_a^\dagger \hat{B}_b^\dagger \hat{B}_b \hat{A}_a - \hat{B}_b^\dagger \hat{A}_a^\dagger \hat{A}_a \hat{B}_b) = 0. \tag{22}$$

This equation can be further simplified to $\int da\, \hat{A}_a^\dagger \hat{B}_b^\dagger \hat{B}_b \hat{A}_a = \hat{B}_b^\dagger \hat{B}_b$. Note that for Hermitian operators $\hat{A}_a = \hat{A}_a^\dagger$, $\hat{B}_b = \hat{B}_b^\dagger$ we can rewrite expression (22) using the commutator

$$\forall \hat{\rho}_0 : \mathrm{NSIT}_{(0)0} \Leftrightarrow \forall b\colon \int da\,[\hat{A}_a \hat{B}_b, \hat{B}_b \hat{A}_a] = 0. \tag{23}$$

Furthermore, we have as sufficient conditions the vanishing commutators

$$\forall a,b\colon [\hat{A}_a \hat{B}_b, \hat{B}_b \hat{A}_a] = 0 \Rightarrow \forall \hat{\rho}_0 : \mathrm{NSIT}_{(0)0}, \tag{24}$$

and, consequently,

$$\forall a,b\colon [\hat{A}_a, \hat{B}_b] = 0 \Rightarrow \forall \hat{\rho}_0 : \mathrm{NSIT}_{(0)0}. \tag{25}$$

It is interesting to note that both of these commutator conditions are, generally, only *sufficient* but *not necessary* for $\mathrm{NSIT}_{(0)0}$. In fact, a formulation of $\mathrm{NSIT}_{(0)0}$ must inherently have an asymmetry [52] between the test and final measurements, but both expressions (24) and (25) are symmetric under exchange of $\hat{A}$ and $\hat{B}$ [54].

We can, however, show that vanishing commutators in expressions (24) and (25) are sufficient and necessary when $\hat{A}_a, \hat{B}_b$ are von Neumann projective measurements ($\hat{A}_a^2 = \hat{A}_a, \hat{B}_b^2 = \hat{B}_b$). Let us start by rewriting the equality in expression (22) using $\hat{A}_a = |a\rangle\langle a|$ and $\hat{B}_b = |b\rangle\langle b|$:

$$\int da\, |\langle a|b\rangle|^2 |a\rangle\langle a| = |b\rangle\langle b|. \tag{26}$$

Since $|b\rangle\langle b|$ is a projector, squaring the integral on the left-hand side must leave it unchanged. Using the fact that in order to sum up to identity the $\hat{A}_a$ must be orthogonal projectors, and therefore $\langle a|a'\rangle = \delta(a - a')$, we obtain

$$\left[\int da\, |\langle a|b\rangle|^2 |a\rangle\langle a|\right]^2 = \int da\, |\langle a|b\rangle|^4 |a\rangle\langle a|. \tag{27}$$

Comparing Eqs. (26) and (27), we see that $|\langle a|b\rangle|^2 = |\langle a|b\rangle|^4$ can only be fulfilled if it is nonzero for exactly one a. Thus, $|b\rangle$ is an eigenstate of $\hat{A}_a$, and the commutator is $[\hat{A}_a, \hat{B}_b] = 0$. We have therefore demonstrated that for von Neumann measurements (but not for general POVMs), vanishing commutators in expressions (24) and (25) are both sufficient and necessary for $\mathrm{NSIT}_{(0)0}$.

B. NSIT with time evolution

Let us now consider $\mathrm{NSIT}_{(0)T}$ with unitary time evolution $\hat{U} = e^{-i\hat{H}t}$. Analogous to the derivation of expression (22) and defining $\tilde{B}_b^T \equiv \hat{U}_T^\dagger \hat{B}_b \hat{U}_T$, we obtain

$$\forall \hat{\rho}_0 : \mathrm{NSIT}_{(0)T}$$
$$\Leftrightarrow \forall b\colon \int da\,\left(\hat{A}_a^\dagger \big(\tilde{B}_b^T\big)^\dagger \tilde{B}_b^T \hat{A}_a - \big(\tilde{B}_b^T\big)^\dagger \hat{A}_a^\dagger \hat{A}_a \tilde{B}_b^T\right) = 0, \tag{28}$$

and, if $\hat{A}_a, \hat{B}_b$ are Hermitian operators,

$$\forall \hat{\rho}_0 : \mathrm{NSIT}_{(0)T} \Leftrightarrow \forall b\colon \int da\,\left[\hat{A}_a \tilde{B}_b^T, \tilde{B}_b^T \hat{A}_a\right] = 0. \tag{29}$$

Comparing expressions (22) and (28), we can apply the results for $\mathrm{NSIT}_{(0)0}$ derived above, namely,

$$\forall a,b\colon \left[\hat{A}_a \tilde{B}_b^T, \tilde{B}_b^T \hat{A}_a\right] = 0 \Rightarrow \forall \hat{\rho}_0 : \mathrm{NSIT}_{(0)T}, \tag{30}$$

and

$$\forall a,b\colon \left[\hat{A}_a, \tilde{B}_b^T\right] = 0 \Rightarrow \forall \hat{\rho}_0 : \mathrm{NSIT}_{(0)T}. \tag{31}$$

Furthermore, one obtains

$$\forall a,b\colon [\hat{A}_a, \hat{B}_b] = [\hat{A}_a, \hat{U}_T] = 0 \Rightarrow \forall \hat{\rho}_0 : \mathrm{NSIT}_{(0)T}. \tag{32}$$

If $\hat{A}_a, \hat{B}_b$ are von Neumann operators, we have $(\tilde{B}_b^T)^2 = \hat{U}_T^\dagger \hat{B}_b \hat{U}_T \hat{U}_T^\dagger \hat{B}_b \hat{U}_T = \hat{U}_T^\dagger \hat{B}_b \hat{U}_T = \tilde{B}_b^T$. Thus, the results from Sec. III A apply here too: For projectors (but not for general POVMs), vanishing commutators in expressions (30) and (31) are sufficient and necessary for $\mathrm{NSIT}_{(0)T}$.

The above results show that a nonclassical "resource" is required for an experimental violation of NSIT, namely, either highly nonclassical states (equivalent to nonclassical measurements used in their preparation) or nonclassical Hamiltonians (usually requiring an extremely large experimental "control precision" as discussed in [30–32]).

IV. CLASSICALITY

As we have indicated in the Introduction, the coarse graining of "sharp" quantum measurement operators into "fuzzy" classical measurements plays a crucial role in the transition from quantum mechanics to classical physics [28]. However, not every coarse-grained operator can be called classical. As an example, the parity operator (e.g., for large spins or photonic states) only differentiates two macrostates, but is in fact highly nonclassical. Generally speaking, a suitable coarse graining should "lump" together neighboring eigenvalues, independent of a (quantum) experiment's Hamiltonian. However, Hilbert spaces in quantum mechanics possess no inherent measure for the distance between orthogonal states. Such a measure must thus arise solely out of interaction Hamiltonians. Effectively, any definition of classicality must therefore depend on Hamiltonians spontaneously realized by nature, which define a natural order and closeness of states.

In the following, this closeness is established with an *a priori* choice of suitable reference operators. With this reference set, we can write a definition for *classical operators* and *classical Hamiltonians*.

(I) A measurement operator is called *classical* with respect to a reference set iff it fulfills the equality in expression (22) pairwise with every member of the set.

(II) A Hamiltonian is called *classical* with respect to a reference set iff the equality in expression (28) is fulfilled for each combination of measurement operators from the set.

A natural choice for the reference set are coarse-grained versions of quantum operators in phase space. Phase space inherently involves the necessary definition of closeness in a suitable and intuitive way. Several exemplary candidates for different experiments are discussed in the next section.

V. CLASSICALITY OF QUANTUM MEASUREMENTS

In the following, we will apply our results to a number of physical systems. We will focus on the classicality of operators—condition I from the previous section—and always assume either an immediate test measurement or free time evolution in between. To measure the overlap of the undisturbed [Eq. (18)] and the disturbed [Eq. (19)] probability distributions, we make use of the Bhattacharyya coefficient [55], as defined by

$$V = \int db \sqrt{P_{\hat{B}}(b) P_{\hat{B}|\hat{A}}(b)} \in [0,1]. \tag{33}$$

The extreme cases of $V = 0$ and 1 correspond to orthogonal and identical probability distributions, respectively.

A. Quadrature measurements

Let us start with quadrature measurements on pure coherent initial states $\hat{\rho} = |\gamma\rangle\langle\gamma|$. We investigate coarse-grained measurements with unsharpness δ in the X quadrature and unsharpness κ in the P quadrature, as described by the (dimensionless) operators

$$\hat{X}^{\delta}_{x} = \frac{1}{(\delta^2\pi)^{1/4}} \exp\left(-\frac{1}{2\delta^2}(x - \hat{X})^2 \right), \tag{34}$$

$$\hat{P}^{\kappa}_{p} = \frac{1}{(\kappa^2\pi)^{1/4}} \exp\left(-\frac{1}{2\kappa^2}(p - \hat{P})^2 \right). \tag{35}$$

Note that for $\hat{B}_{\beta} = \pi^{-1}|\beta\rangle\langle\beta|$ we recover the well-known Husimi Q distribution [56], since $P_{\hat{B}}(\beta) = \pi^{-2}\,\mathrm{tr}(|\beta\rangle\langle\beta|\hat{\rho}_0|\beta\rangle\langle\beta|) = \pi^{-1}\,\langle\beta|\hat{\rho}_0|\beta\rangle = Q(\beta)$. As an example, choosing $\hat{A} = \hat{X}^{\delta}$ and $\hat{B}_{\beta} = \pi^{-1}|\beta\rangle\langle\beta|$, the Husimi distribution $P_{\hat{B}|\hat{A}}$ is shown in Fig. 4 for several values of δ.

The behaviors for different combinations of $\hat{A},\hat{B} \in \{\hat{X}^{\delta},\hat{P}^{\kappa}\}$ are printed in Table II, and detailed analytic values for the overlaps are listed in Appendix B.

The importance of selecting a complete set of classical reference operators becomes clear when looking at different combinations of coarse-grained $\hat{X}^{\delta},\hat{P}^{\kappa}$. In particular, even a sharp X measurement is revealed by a second (coarse-grained) X measurement only after time evolution. Therefore, $\hat{P}^{\kappa}$ has to be a member of the reference set. On the other hand, a sharp measurement in P can never be detected by another measurement in P under free time evolution $\hat{H} = \hat{P}^2/(2m)$. Therefore, $\hat{X}^{\delta}$ needs to be a member of the set. For $\hat{X}^{\delta}$ and $\hat{P}^{\kappa}$ to fulfill the consistency condition, we further require sufficiently large $\delta \gg 1$ and $\kappa \gg 1$, such that $[\hat{X}^{\delta},\hat{P}^{\kappa}] \approx 0$.

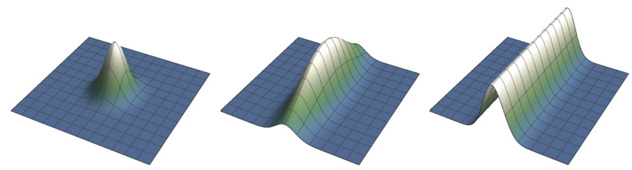

FIG. 4. (Color online) Husimi distribution in the complex plane (mesh with interval 1), immediately after a quadrature measurement with decreasing unsharpness δ. Sharp measurements (small δ) completely destroy the initial state, while unsharp measurements (large δ) keep it intact.

Using the notation $\hat{X}_{\mathrm{CG}}$ ($\hat{P}_{\mathrm{CG}}$) for a sufficiently coarse-grained X (P) measurement and $\hat{X}_{\mathrm{sh}}$ ($\hat{P}_{\mathrm{sh}}$) for a sharp invasive measurement, we can write some candidate reference sets.

(1) $\{\hat{X}_{\mathrm{CG}}\}$ and $\{\hat{X}_{\mathrm{sh}}\}$ do not constitute reference sets, since they cannot detect the invasiveness of a $\hat{X}_{\mathrm{sh}}$ measurement.

(2) $\{\hat{X}_{\mathrm{sh}},\hat{P}_{\mathrm{CG}}\}$ is not a reference set, since the operators do not fulfill expression (22).

(3) $\{\hat{X}_{\mathrm{CG}},\hat{P}_{\mathrm{CG}}\}$ is a possible reference set.

For further discussion about the joint measurability and coexistence of coarse-grained phase-space operators we refer the reader to Refs. [57–59].

B. Coherent-state measurements

As another example, let us now consider coarse-grained operators in coherent-state space:

$$\hat{A}_a = \frac{1}{\pi} \int d\alpha\, f_a(\alpha)\,|\alpha\rangle\langle\alpha|, \tag{36}$$

where $f_a(\alpha)$ are some real and positive envelope functions that define the coarse-grained regions. Again, we consider coherent initial states $\hat{\rho} = |\gamma\rangle\langle\gamma|$ and final measurements $\hat{B}_{\beta} = \pi^{-1}|\beta\rangle\langle\beta|$. An analytical result can be obtained for a measurement $f_a(\alpha) = \delta(a - \alpha)$ for $a \in \mathbb{C}$, yielding

TABLE II. Overlaps (33) between the invaded and the noninvaded probability distributions with different combinations of coarse-grained phase-space quadrature measurements. For final measurements in the momentum quadrature, $\hat{B} = \hat{P}^{\kappa}$, the overlap of the system stays constant, since $\hat{P}^{\kappa}$ commutes with the free Hamiltonian. For analytical values and detailed discussion see Appendix B.

	$\hat{A} = \hat{X}^{\delta}$	$\hat{A} = \hat{P}^{\kappa}$
$\hat{B} = \hat{X}^{\delta}$	$V(0) = 1$ $V(T \to \infty) < 1$	$V(0) < 1$ $V(T \to \infty) = 1$
$\hat{B} = \hat{P}^{\kappa}$	$V(t) = \text{const} < 1$	$V(t) = 1$

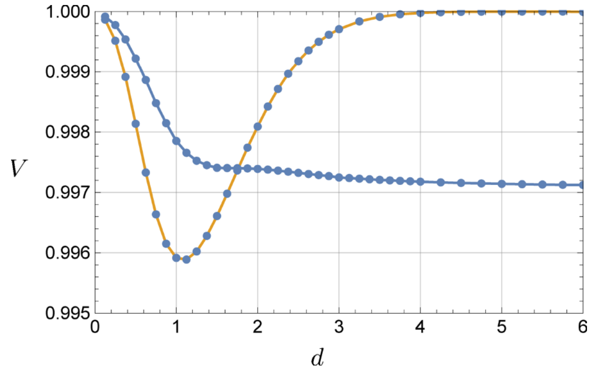

FIG. 5. (Color online) Overlap V vs coarse-graining ring width d. For coherent initial states in the center of the second region $|\gamma = 3d/2\rangle$ the overlap approaches unity as more of the state's probability distribution lies in the region. For initial states located on a border $|\gamma = d\rangle$ the overlap approaches a value close to 0.997. This is due to the artificial sharp boundary between the coarse-grained regions.

$\hat{A}_\alpha = \pi^{-1}|\alpha\rangle\langle\alpha|$. We can now calculate the overlap for $T = 0$:

$$V = \frac{1}{\pi}\int d\beta \left[|\langle\beta|\gamma\rangle|^2 \int d\alpha\, |\langle\beta|\alpha\rangle\langle\alpha|\gamma\rangle|^2\right]^{\frac{1}{2}}$$
$$= \frac{2\sqrt{2}}{3} \approx 0.943. \qquad (37)$$

This overlap provides us with a lower bound, that applies to all coarse-grained measurements based on coherent states. As an example, numerically evaluated overlaps for a ringlike coarse graining [$f_a(r)$ is nonzero for $ad \leqslant r < (a+1)d$, with $a \in \mathbb{N}_0$ and d the ring width] are plotted in Fig. 5.

A choice of reference set, alternative to the previously discussed $\{\hat{X}_{\text{CG}}, \hat{P}_{\text{CG}}\}$, can be made using the coarse-grained coherent-state measurements from Eq. (36), i.e., $\{\hat{A}_a\}$ with suitable envelope functions f_a such that $[\hat{A}_a, \hat{A}_{a'}] \approx 0$.

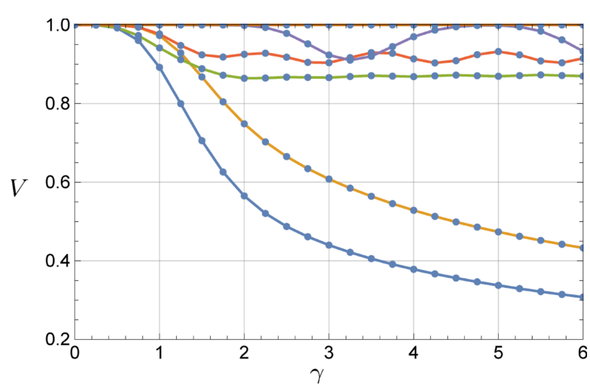

FIG. 6. (Color online) Overlap V [cf. Eq. (33)] vs initial state $|\gamma\rangle$ for coarse-grained Fock measurements with different border functions $g(m)$; from top: $100m^2, 10m^2, 2m^2, m^2, 2m, m$. Quadratic border functions are coarse in the coherent-state space and therefore not as invasive. Linear border functions lead to increasingly sharp measurements. The oscillations are caused by the fact that the presented type of coarse graining works better when the initial state is located in the center of a bin. Dips in the overlap occur when the initial state sits at the border between two bins.

C. Fock state measurements

Instructive examples for observing the effect of coarse graining are different combinations of Fock measurements on coherent initial states. We look at coarse-grained von Neumann measurement operators defined by different border functions $g(m)$:

$$\hat{A}_m = \sum_k \begin{cases} |k\rangle\langle k| & \text{if} \quad g(m) \leqslant k < g(m+1) \\ 0 & \text{else} \end{cases}. \qquad (38)$$

For $g(m) = cm^2$ with $c > 0$, the region corresponding to each operator is constant sized in the coherent-state space, since the average photon number is $\bar{n} = |\alpha|^2$. For sufficiently large c the measurement is therefore sufficiently coarse grained. Measurements with constant-sized regions in Fock space, $g(m) = cm$, correspond to increasingly sharp measurements in coherent-state space. The resulting overlap for different choices of $g(m)$ can be calculated numerically and is discussed in Fig. 6.

VI. CONCLUSION AND OUTLOOK

In contrast to a still widespread belief, we showed that the assumption of macrorealism per se is implied by a strong interpretation of noninvasive measurability. Moreover, no-signaling in time (NSIT), i.e., noninvasiveness on the statistical level, is in general a more reliable witness for the violation of macrorealism than the well-known Leggett-Garg inequalities, which are based on two-time correlation functions. In fact, we demonstrated that the right combination of various NSIT conditions serves not only as a necessary but also a sufficient condition for a macrorealistic model for measurements at the predefined time instants accessible in the experiment. We then derived operational criteria for the measurement operators and the system Hamiltonian, whose fulfillment guarantees that no violation of macrorealism can in principle be observed. We argued that these conditions can be used to define the "classicality" of measurements and by extension of the system's time evolution. Finally, we showed that the classicality of measurements is arbitrarily well fulfilled by suitably coarse-grained versions of quantum measurements.

While our results suggest that an experimental demonstration of nonclassicalities requires either very precise measurements or a complex time evolution, a general proof of this tradeoff (in terms of experimental control parameters) is still missing. Moreover, coarse graining, which leads to the classicality of measurements, already requires the notion of "closeness" or "neighborhood" of eigenvalues, and thereby an understanding of classical phase space. This notion itself stems from Hamiltonians that are spontaneously realized in nature and govern our physical world. The present definition of classicality mitigates this circularity with the choice of an *a priori* set of classical measurements. However, it is an open question whether the presupposition of classical phase space can be avoided, or whether it is a fundamental requirement for understanding the quantum-to-classical transition.

ACKNOWLEDGMENTS

We thank Guido Bacciagaluppi, Gemma de las Cuevas, Owen J. E. Maroney, and Chris G. Timpson for fruitful discussions and helpful comments on the manuscript. We acknowledge support from the European Union Integrated Project Simulators and Interfaces with Quantum Systems.

APPENDIX A: PROOF THAT $\mathrm{NSIT}_{0(1)2}$ IS SUFFICIENT FOR $\mathrm{NIC}_{0(1)2}$

Let us use the short notation $P_i(\pm_i) \equiv P_i(Q_i = \pm)$. Then, the correlations in $\mathrm{NIC}_{0(1)2}$ can be written as

$$\begin{aligned} C_{02} = &+P_{02}(+_0,+_2) + P_{02}(-_0,-_2) \\ &- P_{02}(+_0,-_2) - P_{02}(-_0,+_2), \end{aligned} \tag{A1}$$

and, for the variant with a measurement at t_1,

$$\begin{aligned} C_{02|1} = &+P_{012}(+_0,+_2) + P_{012}(-_0,-_2) \\ &- P_{012}(+_0,-_2) - P_{012}(-_0,+_2). \end{aligned} \tag{A2}$$

Using $\mathrm{NSIT}_{0(1)2}$, i.e., $P_{02}(Q_0,Q_2) = P_{012}(Q_0,Q_2)$, we immediately see that $\mathrm{NSIT}_{0(1)2}$ is sufficient for $C_{02} = C_{02|1}$, and therefore for $\mathrm{NIC}_{0(1)2}$.

APPENDIX B: OVERLAPS FOR QUADRATURE MEASUREMENTS

In the following we will give analytical values for the overlap for different combinations of coarse-grained $\hat{X}^\delta$ and $\hat{P}^\kappa$ measures, as defined by Eqs. (34) and (35), acting on a particle with initial state $\langle x|\psi\rangle = \pi^{-1/4}\sigma^{-1/2}\exp[-x^2/(2\sigma^2)]$. In between the measurements we apply a unitary generated by a free Hamiltonian $\hat{U}_T = \exp(-it\hat{p}^2/2m)$. There are four combinations.

(1) $\hat{A} = \hat{X}^\delta, \hat{B} = \hat{X}^\delta$. Here the overlap starts at $V(0) = 1$, but approaches the value

$$\lim_{t\to\infty} V(t) = \frac{4\delta^2(\delta^2+\sigma^2)}{(2\delta^2+\sigma^2)^2}. \tag{B1}$$

The effect of the measurement only becomes apparent with time evolution.

(2) $\hat{A} = \hat{P}^\kappa, \hat{B} = \hat{X}^\delta$. The overlap starts at

$$V(0) = \frac{4\kappa^2(\delta^2+\sigma^2)[\kappa^2(\delta^2+\sigma^2)+1]}{[2\kappa^2(\delta^2+\sigma^2)+1]^2} \tag{B2}$$

and approaches 1 for $t \to \infty$. The momentum measurement changes the spatial distribution once, but with wave-packet expansion the impact becomes less apparent.

(3) $\hat{A} = \hat{X}^\delta, \hat{B} = \hat{P}^\kappa$. The overlap is constant in time at the value

$$V = \frac{4\delta^2(\kappa^2\sigma^2+1)[\delta^2(\kappa^2\sigma^2+1)+\sigma^2]}{[2\delta^2(\kappa^2\sigma^2+1)+\sigma^2]^2}, \tag{B3}$$

since $[\hat{P}^\kappa,\hat{H}] = 0$.

(4) $\hat{A} = \hat{P}^\kappa, \hat{B} = \hat{P}^\kappa$. The overlap is constant at 1, and a measurement in $\hat{P}$ cannot be detected by a second $\hat{P}$ measurement, as again $[\hat{P}^\kappa,\hat{H}] = 0$.

These examples reaffirm the importance of the selection of multiple final measurements.

[1] E. Schrödinger, Naturwissenschaften **23**, 807 (1935).
[2] G. C. Ghirardi, A. Rimini, and T. Weber, Phys. Rev. D **34**, 470 (1986).
[3] L. Diósi, Phys. Rev. A **40**, 1165 (1989).
[4] R. Penrose, Phil. Trans. R. Soc. A **356**, 1927 (1998).
[5] A. J. Leggett and A. Garg, Phys. Rev. Lett. **54**, 857 (1985).
[6] J. R. Friedman, V. Patel, W. Chen, S. K. Tolpygo, and J. E. Lukens, Nature (London) **406**, 43 (2000).
[7] B. Julsgaard, A. Kozhekin, and E. S. Polzik, Nature (London) **413**, 400 (2001).
[8] A. Palacios-Laloy, F. Mallet, F. Nguyen, P. Bertet, D. Vion, D. Esteve, and A. N. Korotkov, Nature Phys. **6**, 442 (2010).
[9] S. Gerlich, S. Eibenberger, M. Tomandl, S. Nimmrichter, K. Hornberger, P. J. Fagan, J. Tüxen, M. Mayor, and M. Arndt, Nat. Commun. **2**, 263 (2011).
[10] J. Dressel, C. J. Broadbent, J. C. Howell, and A. N. Jordan, Phys. Rev. Lett. **106**, 040402 (2011).
[11] M. E. Goggin, M. P. Almeida, M. Barbieri, B. P. Lanyon, J. L. O'Brien, A. G. White, and G. J. Pryde, Proc. Natl. Acad. Sci. USA **108**, 1256 (2011).
[12] A. Fedrizzi, M. P. Almeida, M. A. Broome, A. G. White, and M. Barbieri, Phys. Rev. Lett. **106**, 200402 (2011).
[13] G. Waldherr, P. Neumann, S. F. Huelga, F. Jelezko, and J. Wrachtrup, Phys. Rev. Lett. **107**, 090401 (2011).
[14] G. C. Knee, S. Simmons, E. M. Gauger, J. J. L. Morton, H. Riemann, N. V. Abrosimov, P. Becker, H.-J. Pohl, K. M. Itoh, M. L. W. Thewalt, G. A. D. Briggs, and S. C. Benjamin, Nat. Commun. **3**, 606 (2012).
[15] A. I. Lvovsky, R. Ghobadi, A. Chandra, A. S. Prasad, and C. Simon, Nature Phys. **9**, 541 (2013).
[16] P. Sekatski, M. Aspelmeyer, and N. Sangouard, Phys. Rev. Lett. **112**, 080502 (2014).
[17] R. Ghobadi, S. Kumar, B. Pepper, D. Bouwmeester, A. I. Lvovsky, and C. Simon, Phys. Rev. Lett. **112**, 080503 (2014).
[18] A. Asadian, Č. Brukner, and P. Rabl, Phys. Rev. Lett. **112**, 190402 (2014).
[19] C. Robens, W. Alt, D. Meschede, C. Emary, and A. Alberti, Phys. Rev. X **5**, 011003 (2015).
[20] W. Dür, C. Simon, and J. I. Cirac, Phys. Rev. Lett. **89**, 210402 (2002).
[21] J. I. Korsbakken, K. B. Whaley, J. Dubois, and J. I. Cirac, Phys. Rev. A **75**, 042106 (2007).
[22] C.-W. Lee and H. Jeong, Phys. Rev. Lett. **106**, 220401 (2011).
[23] S. Nimmrichter and K. Hornberger, Phys. Rev. Lett. **110**, 160403 (2013).
[24] M. Mraz, J. Sperling, W. Vogel, and B. Hage, Phys. Rev. A **90**, 033812 (2014).
[25] H. Jeong, M. Kang, and H. Kwon, Opt. Commun. **337**, 12 (2015).

[26] A. Laghaout, J. S. Neergaard-Nielsen, and U. L. Andersen, Opt. Commun. **337**, 96 (2015).
[27] F. Fröwis, N. Sangouard, and N. Gisin, Opt. Commun. **337**, 2 (2015).
[28] J. Kofler and Č. Brukner, Phys. Rev. Lett. **99**, 180403 (2007).
[29] J. Kofler and Č. Brukner, Phys. Rev. Lett. **101**, 090403 (2008).
[30] T. Wang, R. Ghobadi, S. Raeisi, and C. Simon, Phys. Rev. A **88**, 062114 (2013).
[31] H. Jeong, Y. Lim, and M. S. Kim, Phys. Rev. Lett. **112**, 010402 (2014).
[32] P. Sekatski, N. Gisin, and N. Sangouard, Phys. Rev. Lett. **113**, 090403 (2014).
[33] R. K. Clifton, in *Symposium on the Foundations of Modern Physics 1990*, edited by P. Lahti and P. Mittelstaedt (World Scientific, Singapore, 1991).
[34] S. Foster and A. Elby, Found. Phys. **21**, 773 (1991).
[35] F. Benatti, G. Ghirardi, and R. Grassi, Found. Phys. Lett. **7**, 105 (1994).
[36] J. Kofler and Č. Brukner, Phys. Rev. A **87**, 052115 (2013).
[37] A. J. Leggett, J. Phys.: Condens. Matter **14**, R415 (2002).
[38] Let us now assume the existence of hidden parameters $\lambda(t)$ that define all physical properties. MRps is fulfilled if the macro-observable is a deterministic function $Q = Q(\lambda(t))$. There are two conceivable scenarios: (i) *deterministic time evolution* of λ, causing deterministic time evolution of the macro-observable $Q(\lambda(t))$; and (ii) *stochastic time evolution* of λ, where some intrinsic randomness generates random jumps in λ. We still have a deterministic dependency $Q(\lambda)$, but $Q(\lambda(t))$ *appears* stochastic. In both cases MRps is fulfilled, since the system is in a single macrostate, as described by $Q = Q(\lambda(t))$, at all times. The condition for NIM then reads $Q(\lambda(t)) = Q(\lambda_H(t))$, where $\lambda_H(t)$ are the hidden parameters after a history H of noninvasive measurements.
[39] G. Bacciagaluppi, Int. J. Quantum Found. **1**, 1 (2015).
[40] O. J. E. Maroney and C. G. Timpson, arXiv:1412.6139.
[41] M. M. Wilde and A. Mizel, Found. Phys. **42**, 256 (2012).
[42] D. Saha, S. Mal, P. K. Panigrahi, and D. Home, Phys. Rev. A **91**, 032117 (2015).
[43] C. Emary, N. Lambert, and F. Nori, Rep. Prog. Phys. **77**, 016001 (2014).
[44] C. Budroni and C. Emary, Phys. Rev. Lett. **113**, 050401 (2014).
[45] M. Reck, A. Zeilinger, H. J. Bernstein, and P. Bertani, Phys. Rev. Lett. **73**, 58 (1994).
[46] P. Busch, Phys. Rev. D **33**, 2253 (1986).
[47] P. Lahti, Int. J. Theo. Phys. **42**, 893 (2003).
[48] W. Son, E. Andersson, S. M. Barnett, and M. S. Kim, Phys. Rev. A **72**, 052116 (2005).
[49] S. T. Ali, C. Carmeli, T. Heinosaari, and A. Toigo, Found. Phys. **39**, 593 (2009).
[50] M. M. Wolf, D. Perez-Garcia, and C. Fernandez, Phys. Rev. Lett. **103**, 230402 (2009).
[51] S. Yu, N.-l. Liu, L. Li, and C. H. Oh, Phys. Rev. A **81**, 062116 (2010).
[52] T. Heinosaari and M. M. Wolf, J. Math. Phys. **51**, 092201 (2010).
[53] M. Banik, Md. R. Gazi, S. Ghosh, and G. Kar, Phys. Rev. A **87**, 052125 (2013).
[54] A simple example for this are the Pauli matrices with $\hat{A} = \hat{A}\sigma_x, \hat{B} = \hat{\sigma}_y$. Then, $[\hat{A},\hat{B}] = 2i\hat{\sigma}_z$ and $[\hat{A}\hat{B},\hat{B}\hat{A}] = 0$. Although the first commutator is nonzero, $\mathrm{NSIT}_{(0)0}$ is trivially fulfilled. The physical interpretation of a $\hat{\sigma}_x$ measurement (or rather, its corresponding POVM element $\mathbb{1}$) is a single-qubit operation without a meaningful measurement outcome.
[55] A. Bhattacharyya, Bull. Calcutta Math. Soc. **35**, 99 (1943).
[56] K. Husimi, Proc. Phys. Math. Soc. Jpn. **22**, 264 (1940).
[57] P. Busch and P. J. Lahti, Found. Phys. **26**, 875 (1996).
[58] P. Busch, Found. Phys. **39**, 712 (2009).
[59] P. Busch and G. Jaeger, Found. Phys. **40**, 1341 (2010).