Inverse Doppler shift and control field as coherence generators for the stability in superluminal light

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A gain-based four-level atomic medium for the stability in superluminal light propagation using control field and inverse Doppler shift as coherence generators is studied. In regimes of weak and strong control field, a broadband and multiple controllable transparency windows are, respectively, identified with significantly enhanced group indices. The observed Doppler effect for the class of high atomic velocity of the medium is counterintuitive in comparison to the effect of the class of low atomic velocity. The intensity of each of the two pump fields is kept less than the optimum limit reported in [M. D. Stenner and D. J. Gauthier, Phys. Rev. A **67**, 063801 (2003)] for stability in the superluminal light pulse. Consequently, superluminal stable domains with the generated coherence are explored.

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I. INTRODUCTION

The development in theoretical and experimental techniques for the control of the propagation of light pulses in atomic and molecular media has already been made both for slow and fast light [1–11]. This includes the control over the response functions of the multilevel medium through quantum coherence generated by strong laser fields. Such studies led to the generation of the superluminal light pulse that travels faster than the speed of light in vacuum. Using correlation techniques, the existence of such pulse in the optical as well as microwave regimes has been experimentally demonstrated [1,2], which justify the prediction control over light of Garrett and McCumber [3]. The other related works dealing with the control over the group velocity can be found in Refs. [4,5,11]. Nevertheless, in most of these and in the other related studies [6], the retrieved pulse is relatively distorted. Similarly, the scheme of Ref. [7] suggests considerable reduction in probe-field attenuation and distortion, however, ignoring the influence of Doppler broadening. Moreover, the physical mechanisms of the transparency are unexplored. On the other hand, the authors of Refs. [8–10] demonstrated the propagation of the probe pulse in a lossless anomalous dispersion region between two closely spaced gain lines. However, instability due to nonlinear cross modulation and stimulated Raman scattering is developed when the intensity of each of the two pump fields is kept beyond the reported optimum limit for superluminal stability [10].

In this paper, we investigate a gain-based atomic medium for superluminality of a weak probe light. It is found that the response of the medium strongly depends on the strength of the control field. A weak control field results in the development of a broadband transparency window with significantly enhanced negative group index. In the limit of strong control field, multiple controllable transparency bandwidths with a stability for the superluminal light are observed. In comparison to the class of low atomic velocity, the influence of the Doppler effect is counterintuitive on the probe field in the regime of high class atomic velocity. Moreover, the propagating pulse remains undistorted when retrieved. From an application point of view, the coherence due to inverse Doppler shift and the other findings may prove helpful in efficient implementation of various quantum information protocols involving long distances in gaseous media. In addition, it might be of interest for some physicists and chemists working in the areas of laser spectroscopy, quantum optics, and nonlinear optics.

The structure of the paper is as follows. In Sec. II we introduce our model and present susceptibilities for gain process and dispersion of the propagating probe pulse. In Sec. III the susceptibilities for Doppler-broadened medium are given to numerically simulate the results. Subsequently the group indices of the medium and advance ot delay time both for the Doppler-free and Doppler-broadened media are defined later in this section. In Sec. V we discuss our main findings. Main conclusions are summarized in Sec. VI.

II. MODEL AND ELECTRIC SUSCEPTIBILITY

We consider a gain-based four-level atomic system driven by a weak probe field, a control field, and by two coherent pump fields, as shown in Fig. 1. The lower level $|d\rangle$ is coupled with the upper level $|a\rangle$ through two pump fields of Rabi frequencies Ω_1 and Ω_2 in Raman configuration. On the other hand, the lower level $|c\rangle$ is coupled with the upper levels $|a\rangle$ and $|b\rangle$ through a weak probe field of Rabi frequency Ω_p and a control field of Rabi frequency Ω_3, respectively. Under the dipole and rotating wave approximations the Hamiltonian in the interaction picture can be written as

$$H(t) = -\frac{\hbar}{2}(\Omega_1 e^{-i\delta_1 t} + \Omega_2 e^{-i\delta_2 t})|a\rangle\,\langle d| - \frac{\hbar}{2}\Omega_3 e^{-i\delta_3 t}\,|b\rangle\,\langle c| - \frac{\hbar}{2}\Omega_p e^{-i\Delta t}|a\rangle\langle c| + \text{H.c.}, \quad (1)$$

where $\omega_1 = \omega_{ad} \pm \delta_1$, $\omega_2 = \omega_{ad} \pm \delta_2$, $\omega_3 = \omega_{bc} \pm \delta_3$ and $\omega_p = \omega_{ac} \pm \Delta$. In Eq. (1), $\delta_1 = \delta_0 - \delta$, and $\delta_2 = \delta_0 + \delta$, are the detuning parameters, with δ being the effective detuning and δ_0 the average detuning of the two pump fields. The Bloch equations are evaluated through the density matrix equation

$$i\hbar\frac{d\rho}{dt} = [H(t),\rho] + \Lambda\rho, \quad (2)$$

where $\Lambda\rho$ is the damping part of the system. With the use of transformation relations between ρ and $\widetilde{\rho}$, for the fast and

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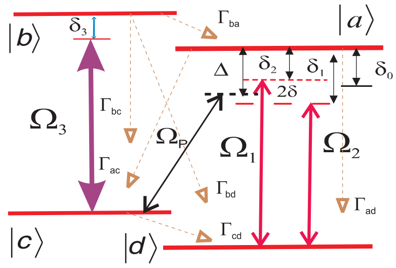

FIG. 1. (Color online) Schematics of the atomic system.

slowly varying amplitudes, the equation for $\widetilde{\rho}_{ac}$ is obtained as follows

$$\widetilde{\rho}_{ac} = -i\Omega_p \text{Ł}(\Delta)\sum_{j=1}^{4}\mathbb{k}_j(\Delta)/8, \tag{3}$$

where the explicit form of the new parameters introduced in Eq. (3) is given in the Appendix. Next, we consider the transmission of a probe pulse through an atomic medium of length L with the model atom being the one displayed in Fig. 1. In the presence of the deriving fields, the effect of the medium on the probe pulse during its transmission can be found from the real and imaginary parts of the susceptibility. Moreover, the presence of the Raman gain processes in the atomic system of the medium requires us to calculate the susceptibility up to the second order in the two pump fields, up to the first order in the probe field, and up to all orders in the control field. Under these approximations the susceptibility becomes

$$\chi = \frac{2N|\wp_{ac}|^2\widetilde{\rho}_{ac}}{\epsilon_0\hbar\Omega_p} = \Omega_p\frac{-i3N\lambda^3\Gamma}{32\pi^3}\sum_{j=1}^{4}\mathbb{k}_j(\Delta), \tag{4}$$

where N is the atomic number density of the medium and the Einstein's coefficient is given as $A = 4|\wp_{ac}|^2\omega_{ac}^3/\epsilon_0\hbar c^3 = 4\Gamma$ with $\omega_{ac} = 2\pi c/\lambda$. Further, it is convenient to measure the gain and dispersion in the units of $N|\wp_{ac}|^2/\epsilon_0\hbar\Omega_p$. The measured value of the prefactor, $N|\wp_{ac}|^2/\epsilon_0\hbar$ is equivalent to the spontaneous decay rate Γ of the sodium D_2 line. In the analysis of our results we use Γ as a unit of the relevant physical quantities and consider the Rabi frequency Ω_p of the probe field as Γ.

Our system is more general than the one given in Refs. [8,9] and hence it presents useful aspects of superluminality in addition as studied later in this paper. In fact, under valid approximations such as $\Omega_3 = 0$, $\Delta_3 = 0$, $\Gamma_{ac} = \Gamma_{ad} = \Gamma$ and setting the other decay rates of the system to zero, with $\delta_0 \approx \Delta$, $\delta/\delta_0, \Gamma/\delta_0 \ll 1$, $\delta_{1,2} = 2\pi(\nu_{1,2} - \nu_{ad})$, $\Delta = 2\pi(\nu_p - \nu_{ac})$, $\nu_{ac} \approx \nu_{ad}$, our result goes directly to $\chi_{\Im} = M_1/[\nu_p - \nu_1 + i\gamma] + M_2/[\nu_p - \nu_2 + i\gamma]$, where $M_{k=1,2} = N|\wp_{ac}|^2|\Omega_{1,2}|^2/8\pi\epsilon_0\hbar\delta_0^2$, $\Gamma = \gamma/2\pi$ and $\wp_{ac}$ is the dipole moment between levels $|a\rangle$ and $|c\rangle$ [8].

III. DOPPLER-BROADENED ELECTRIC SUSCEPTIBILITY

Next, we assume a thermal atomic medium with the atomic velocity relative to the driving fields as v. The control field is counterpropagating to the other fields. Therefore, we replace the detuning parameters $\delta_1 = \delta_1 + k_1 v$, $\delta_2 = \delta_2 + k_2 v$, $\Delta = \Delta + k_p v$, and $\delta_3 = \delta_3 - k_3 v$, in Eq. (3) and Eq. (4). For convenience we consider wave vectors such that $k_1 = k_2 = k_3 = k_p = k$. In the context of thermal medium, the electric susceptibility takes the form

$$\chi(kv) = \Omega_p\frac{-3iN\lambda^3\gamma}{32\pi^3}\sum_{l=1}^{4}\mathbb{R}_l(kv,\Delta), \tag{5}$$

where the relations for the new parameters are given in the Appendix. Integrating $\chi(kv)$ over the atomic velocity distribution leads to the Doppler-broadened susceptibility $\chi^{(d)}$ as follows,

$$\chi^{(d)} = \frac{1}{\sqrt{2\pi V_D^2}}\int_{-\infty}^{\infty}\chi(kv)e^{-\frac{(kv)^2}{2V_D^2}}d(kv), \tag{6}$$

with V_D being the Doppler width and is given by $V_D = \sqrt{K_B T\omega^2/Mc^2}$. The susceptibility in thermal medium becomes

$$\chi^{(d)} = \Omega_p\frac{-3iN\lambda^3\gamma}{32\sqrt{2}\pi^{7/2}V_D}\int_{-\infty}^{\infty}\sum_{l=1}^{4}\mathbb{R}_l(kv,\Delta)e^{-(kv)^2/2V_D^2}d(kv). \tag{7}$$

The group index, $N_g^{(d)}$ (N_g) with (without) Doppler effect is given as

$$N_g^{(d)}(N_g) = 1 + 2\pi\text{Re}[\chi^{(d)}(\chi)] + 2\pi\omega_{ac}\text{Re}\left[\frac{\partial\chi^{(d)}(\chi)}{\partial\Delta}\right]. \tag{8}$$

Consequently, the group advance time τ_{ad} (delay time τ_d), which depends on the negativity (positivity) of the group index N_g^d (N_g) becomes as

$$\tau_{ad}(\tau_d) = L\big[N_g^d(N_g) - 1\big]/c. \tag{9}$$

where L is the length of the vapor cell containing the atomic medium and c is the velocity of light pulse in vacuum.

IV. PULSE TRANSMISSION

Before we proceed, it is important to note that the experimental viability of a superluminal light is conditioned to a less distorted retrieved pulse. To measure the quality $S_{\text{out}}(\omega)$ of the retrieved pulse, we employ the transfer function $H(\omega)$ of the medium through the input pulse as

$$S_{\text{out}}(\omega) = H(\omega)S_{\text{in}}(\omega), \tag{10}$$

where $H(\omega) = e^{-ik(\omega)L}$. The complex wave number $k(\omega)$ can be expanded via Taylor series in terms of the group index as follows

$$K(\omega) = \omega c^{-1}N_g^{(0)} + \frac{c^{-1}}{2!}(\omega-\omega_0)^2\frac{\partial N_g}{\partial\omega}\bigg|_{\omega\to\omega_0} + \frac{c^{-1}}{3!}(\omega-\omega_0)^3\frac{\partial^2 N_g}{\partial\omega^2}\bigg|_{\omega\to\omega_0} + \ldots, \tag{11}$$

where $N_g^{(0)}$ is the group index of the medium at the central frequency ω_0. It is worth mentioning that the Gaussian pulse is useful for a good retrieved pulse if used as a probe pulse in comparison to other types of laser pulses, for example, the hyperbolic secant pulse. The shortcomings of this pulse if propagated as a probe through the atomic medium in comparison to a Gaussian pulse are discussed in Sec. V. Here, we consider a Gaussian light pulse of the form

$$S_{\rm in}(t) = \exp\left[-t^2/\tau_0^2\right]\exp[i(\omega_0+\xi)t], \tag{12}$$

as the probe pulse for our system with τ_0 being the pulse width and ξ is the shift in frequency at the boundary of the medium. The Fourier transform of this function is

$$\begin{aligned} S_{\rm in}(\omega) &= \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} S_{\rm in}(t)e^{i\omega t}dt \\ &= \frac{\tau_0}{\sqrt{2}}\exp[-((\omega-\omega_0-\xi)\tau_0)^2/4]. \end{aligned} \tag{13}$$

With the help of convolution theorem, the output pulse can be written as

$$S_{\rm out}(t) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} S_{\rm int}(\omega)H(\omega)e^{i\omega t}d\omega, \tag{14}$$

whose integration results in

$$\begin{aligned} S_{\rm out}(t) &= \frac{\tau_0\sqrt{c}}{D^{1/2}}\left[1-\frac{i(6FG+G^2)Ln_2}{48cF^3}\right]\exp\left[-\frac{\xi^2\tau_0^2}{4}\right] \\ &\quad\times\exp\left[i\left(t-\frac{Ln_0}{c}\right)\omega_0+G+\frac{cG^2}{D}\right], \end{aligned} \tag{15}$$

where the new parameters are defined as

$$F = \left(2iLn_1 + c\tau_0^2\right)/4c,$$

$$G = \left(c\xi\tau_0^2 - 2iLn_0 + 2ict\right)/2c,$$

$$D = \sqrt{2iLn_1 + c\tau_0^2},$$

with $n_1 = \partial N_g/\partial\omega$ and $n_2 = \partial^2 N_g/\partial\omega^2$.

V. DISCUSSION

In this section, we focus on the analysis of our results for stability of superluminal light pulse propagation through a gain-based atomic medium at room temperature in a vapor cell. The main role for the stability is simultaneously played by the coherence of the control field and the inverse Doppler broadening of the system. The prominent contribution to the Doppler shift, $(\delta_{av}-\Delta-\delta_3)+[(\omega_{av}-\omega_p)-\omega_3]v/\lambda$ comes from the terms in the square brackets. The atomic velocity of the thermal medium is the main parameter, which significantly influence the dynamics of the traditional Doppler broadening. We consider different classes of averaged atomic velocities through coherence-preserving collisions of the atoms of the gain medium with the atoms of a buffer gas when added to the vapor cell under suitable pressure. Such a phenomenon of coherence-preserving collisions at room temperature and a pressure of 6.8 kPa between the atoms of ^{87}Rb/^{85}Rb as the medium and the neon atoms as the buffer gas is demonstrated [12]. In addition, the atoms of the buffer gas are considered to be lighter than the atoms of the gain medium. For example, the atoms of He or H may be used as the

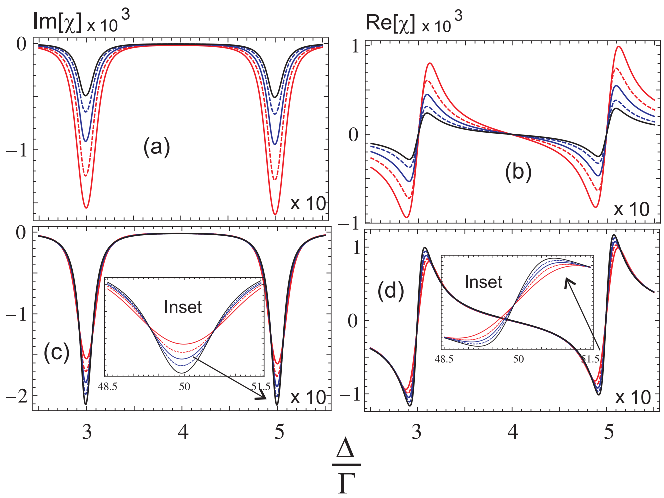

FIG. 2. (Color online) Im[χ], Re[χ] vs $\frac{\Delta}{\Gamma}$ with $\lambda = 586.9nm$, $\omega_{ac} = 10^3\Gamma$, $\delta_3 = 0$, $\delta_2 = 50\Gamma$, $\delta_1 = 30\Gamma$. The radiative decay rates are selected as $\Gamma = 1\,\mathrm{MHz}$, $\Gamma_{ad} = \Gamma_{ac} = \Gamma_{bd} = \Gamma_{bc} = 2.05\Gamma$ whereas $\Gamma_{cd} = \Gamma_{ab} = 0.5\Gamma$ represents the decay rates due to atomic collisions. Also, $\Omega_1 = 2.5\Gamma$, $\Omega_2 = 2.55\Gamma$, $\Omega_3 = 2\Gamma$. Further, in (a) and (b) [(c) and (d)] $V_D = 0$ (0) (red), 4Γ (2Γ) (red dashed), 6Γ (3Γ) (blue), 9Γ (4Γ) (blue dashed), 12Γ (5Γ) (black). Note that in this figure and in what follows $\frac{\Delta}{\Gamma}$ is shown as multiple of 10. The insets in (c) and (d) are, respectively, the gain and dispersion around $\frac{\Delta}{\Gamma} = 50$.

buffer gas when the gain atomic medium for our system is considered atomic sodium, ^{23}Na. Under this condition, the dependence of the collisional rate on the atomic velocity of the thermal gain-based medium can safely be neglected [12]. We also consider the two pump fields to be weak. Otherwise, Raman scattering is initiated where the polarization of the scattering modes appear orthogonal to the two pump fields, which in turn adds to the probe field as gain [10]. For this reason, the intensity $2\pi\times 6.250\times 10^{10}\mathrm{Hz}^3$ (natural unit) that corresponds to $\Omega_1 = 2.5\Gamma$ and $\wp_{ac} = 2.4923\times a_0$ for ^{23}Na with a_0 being the Bohar radius is less than $2.793\times 10^{11}\mathrm{Hz}^3$ $2\pi\times 2.793\times 10^{10}\mathrm{Hz}^3$ used in Ref. [10]. Furthermore, unlike Ref. [13], the probe field we consider is in the classical limit. In what follows, the effect of the strength of the control field on the stability of the superluminal light pulse is studied in the regimes of both weak and strong control field.

In the weak control-field regime and in the absence of Doppler broadening, a lossless and broadband transparency window with anomalous dispersion is developed as shown in Figs. 2(a), 2(b). The behavior of the dispersion, gain, and the group index of our system coincides with those of Refs. [8–10] when the control field is switched off (not shown graphically). Although the physical process appears similar to the way described in Refs. [8–10], however, a significant enhancement in the negative group index occurs in the system through the increase of the control field in its weak-field regime [see Fig. 2(b)]. In the study of Refs. [8–10], the two strong pump fields manipulate the transparency window and the group index but at the cost of generating the decoherence process [10]. In our case, this mechanism is avoided while the coherence, which is developed through the strength of the control field, is used for controlling the transparency window

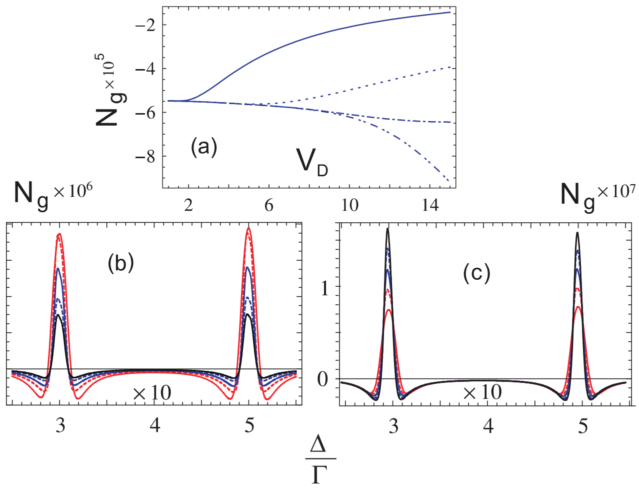

FIG. 3. (Color online) N_g vs $\frac{\Delta}{\Gamma}$ for average v (a) 1.5×10^3 m/s (b) 5×10^3 m/s. The V_D for (a) and (b) are chosen from Fig. 2, correspondingly. However, vs V_D (c) v for the classes of 5×10^3 m/s (solid), 10×10^3 m/s (dotted) and for the class of 15×10^3 m/s (dot-dashed).

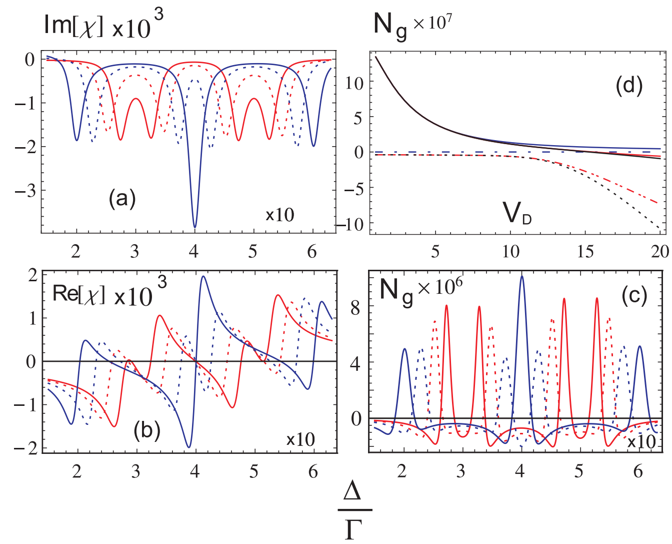

FIG. 4. (Color online) In (a) Im[χ], in (b) Re[χ] and in (c) N_g vs $\frac{\Delta}{\Gamma}$ with $\Omega_1 = 2.5\Gamma$, $\Omega_2 = 4.3\Gamma$ and with $\Omega_3 = 6\Gamma$ (red), 10Γ (red dashed), 15Γ (blue dashed), 20.5Γ (blue). The other parameters are chosen from Fig. 2. In (d) N_g vs V_D with $\Omega_3 = 6\Gamma$ (red) for average v of 5×10^3 m/s and $\Delta = 40\Gamma, 30\Gamma$ (blue, blue dot-dashed), 15×10^3 m/s and $\Delta = 40\Gamma, 30\Gamma$ (red, red dashed) and for average v of 500×10^3 m/s and $\Delta = 40\Gamma, 30\Gamma$ (black, black dotted).

and for advancement of the superluminal light pulse in the medium.

It is worth mentioning that in our gain-based system, the Doppler effect on the probe pulse for the class of high atomic velocity is counterintuitive (negative shift) in comparison to the effect of the class of low atomic velocity. The comparison of Figs. 2(a)–2(b) and Fig. 3(b) with Figs. 2(c)–2(d) and Fig. 3(c), respectively, reflects the validity of our claim. In the former (latter) case the gain lines and the lines associated with the group index become sharp (broad) and tall (short) with the increase in the Doppler width. The inverse shift becomes more prominent for selection of the class of relatively more high average atomic velocity as shown in Fig. 3(a). Traditionally, the Doppler broadening plays the role of decoherence and its effect becomes dominant with increase in temperature of the medium. On the contrary, our findings show narrowing of the gain lines, which results from the proper selection of the class of high atomic velocity through coherence-preserving collision between the atoms of the medium and the buffer gas [14]. The effect of the inverse Doppler broadening on the gain mechanisms of our system, in general, agrees with the studies of the negative-index metamaterials [15], photonic crystals [16], and cavity photon absorption [12].

In the strong control-field regime, the gain of the system exhibits a pair of a doublet with almost lossless anomalous transparency windows [see Fig. 4]. The bandwidths of these windows, which are controlled by the control field, may easily be made compatible with the probe-pulse width for a best quality retrieved pulse [see Figs. 4(a), 4(b)]. Here, the control field splits the ground state $|c\rangle$ into a doublet. The photons of the two pump fields then adds to the probe field through the two created paths. The strong control field controls the spacing of each doublet whereby the gain lines close to the center are overlapped at the center with a normal behavior for the dispersion. However, the side anomalous dispersions become optimally broadened with negligible gain. The corresponding group indices, which are physically consistent with the absorptions and dispersions of Figs. 4(a), 4(b), are shown in Fig. 4(c). The influence of the inverse Doppler broadening on the group index of the medium in strong control-field regime is shown in Fig. 4(d). It can be seen that the inverse Doppler shift depends on the large Doppler width and on the different classes of high average atomic velocities. At $\Delta = 40\Gamma$, the probe pulse is subluminal decreasing (agreeing with Ref. [17]) as compared with the superluminal enhancing behavior at $\Delta = 30\Gamma$ ($\Delta = 50\Gamma$). In the latter case, the transparency windows become optimally broadened. It is worth mentioning that the enhancement of negative group index due to the inverse Doppler shift is in addition to one developed by the weak as well as by the strong control field of the system.

Note that the analytical formula, Eq. (15) is derived for analysis of the behavior of the retrieved pulse when a Gaussian pulse is transmitted through the medium. The behavior of this propagating pulse when retrieved, is displayed in Figs. 5(a)–5(c), where the transition frequency $\omega_{ac} = 1000\Gamma$, the probe-field frequency $\omega_0 = 970\Gamma$, the detuning $\Delta = \omega_{ac} - \omega$ ($\omega = 2\pi\nu_p$), and the location $\Delta = 30\Gamma$ are used. A smooth transmission of the pulse is ensured when its pulse width is kept smaller than the width of the transparency window [see Fig. 5(a)]. This lossless characteristics of the medium agree with Ref. [9]. In this case, the resonance between the probe pulse and the gain lines around the window is easily avoided. The symmetric expansion or compression of the probe pulse as shown in Figs. 5(b), 5(c), is due to relatively large changes in its pulse width and/or upshifting-to-medium frequency. This expansion characteristic of the retrieved pulse with the corresponding parameters agrees with Ref. [9]. However, we note that the waist of the hyperbolic secant pulse is broader than the one of the Gaussian pulse. In comparison to the Gaussian pulse, it will be hard to avoid resonance of the pulse and

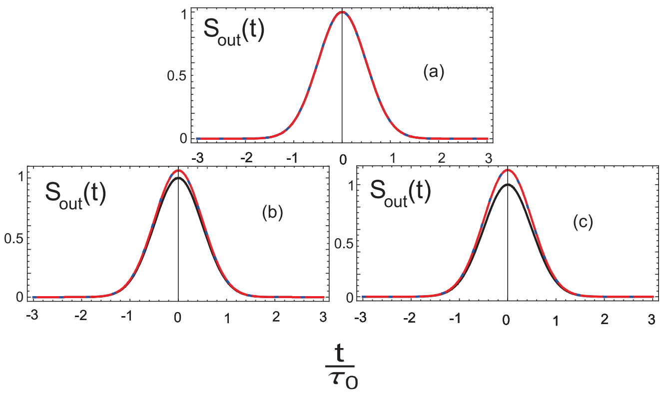

FIG. 5. (Color online) The intensity of a Gaussian pulse at input and output versus the normalized time for (a) $\xi = 0$, (b) $\xi = 0.005$ MHz, (c) $\xi = 0.01$ MHz, $\tau_0 = 3.50\mu s$, $L = 0.03m$, $\omega_0 = 970\Gamma$, $V_D = 6\Gamma$, $\Omega_1 = 2.5\Gamma$, $\Omega_2 = 4.3\Gamma$, $\Omega_3 = 8\Gamma$. The values of the other parameters are the same as in Fig. 2

the gain lines, if used as a probe. Therefore, the hyperbolic secant pulse may get relatively distorted when retrieved under identical parameters. Although this analysis is qualitative, a quantitative estimation similar to the Gaussian pulse when used as a probe pulse, is required. Nonetheless, the underlying physics is expected to remain unaffected.

VI. CONCLUSION

In conclusion, superluminal stable domains are identified in the proposed gain-based atomic system with the coherence of the counterintuitive inverse Doppler shift and control field. In regimes of weak and strong control field, a broadband window and multiple controllable transparency windows are, respectively, observed with significantly enhanced group indices. The intensity of each of the two pump fields is kept less than the optimum limit reported in Ref. [10] for stability in superluminal light pulse. The influence of the Doppler effect on the probe pulse in the medium for the class of high atomic velocity is counterintuitive. Therefore, it generates coherence effect on the probe pulse in addition to the effect of the coherence of the control field. The findings of our work may prove helpful in processing information carried by the light pulse through its high superluminal group velocity [10] and other processes such as imaging and cloaking [18,19]. Generally, it might be of interest for researchers working in the areas of laser spectroscopy, quantum optics, and nonlinear optics.

APPENDIX

The parameters in Eq. (3) are given as

$$Ł(\Delta) = (\Gamma_{ac} - i\Delta)[\Gamma_{ab} - i(\Delta - \delta_3)] + \frac{\Omega_3^2}{4},$$

$$\mathbb{k}_{1(2)}(\Delta) = \frac{|\Omega_{1(2)}|^2\{2\Gamma_{ad}[\Gamma_{ab} - i(\Delta - \delta_3)]\}}{(\Gamma_{ad} + \Gamma_{ac})\left(\Gamma_{ad}^2 + \delta_{1(2)}^2\right)},$$

$$\mathbb{k}_{3(4)}(\Delta) = \left| \frac{\Omega_{1(2)}|^2\left\{\Xi[\Gamma_{ab} - i(\Delta - \delta_{1(2)} - \delta_3)] - \frac{\Omega_3^2}{4}\right\}}{(\Gamma_{ad} + i\delta_{1(2)})\left(Æ_{1(2)} + \frac{\Omega_3^2}{4}\right)} \right.$$

$$Æ_{1(2)} = [\Gamma_{cd} - i(\Delta - \delta_{1(2)})][\Gamma_{bd} - i(\Delta - \delta_{1(2)} - \delta_3)]$$

$$\Xi = [\Gamma_{ab} - i(\Delta - \delta_3)]$$

where Γ_{mn} $(n.m = a,b,c,d)$ with the subscript nm represents the transition rate from an energy level $|n\rangle$ to the energy level $|m\rangle$.

The parameters in Eq. (5) are given as

$$\mathbb{R}_{1(2)}(kv,\Delta) = \frac{2|\Omega_{1(2)}|^2\Gamma_{ad}[\Gamma_{ab} - i(2kv + \Delta - \delta_3)]}{\left(\beta + \frac{\Omega_3^2}{4}\right)(\Gamma_{ad} + \Gamma_{ac})\left[\Gamma_{ad}^2 + (kv + \delta_{1(2)})^2\right]},$$

$$\mathbb{R}_{3(4)}(kv,\Delta) = \frac{\Omega_{1(2)}^2\left[\alpha\Gamma_{bd} - i\alpha(kv + \Delta - \delta_{1(2)} - \delta_3) - \frac{\Omega_3^2}{4}\right]}{[\Gamma_{ad} + i(kv + \delta_{1(2)})]\left(\beta + \frac{\Omega_3^2}{4}\right)\eta_{1(2)}},$$

$$\beta = [\Gamma_{ac} - i(kv + \Delta)][\Gamma_{ab} - i(2kv + \Delta - \delta_3)],$$

$$\alpha = [\Gamma_{ab} - i(2kv + \Delta - \delta_3)].$$

with

$$\eta_{1(2)}(kv,\Delta) = [\Gamma_{cd} - i(\Delta - \delta_{1(2)})][\Gamma_{bd} - i(kv + \Delta - \delta_{1(2)} - \delta_3)] + \frac{\Omega_3^2}{4},$$

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