Shortcut to adiabatic passage in a waveguide coupler with a complex-hyperbolic-secant scheme

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We propose a directional coupler exploiting the framework of adiabatic passage for two-level atomic systems with a configuration-dependent complex-hyperbolic-secant scheme. A recently developed Shortcut to Adiabatic passage method (STA), which uses a transitionless quantum driving algorithm, is applied to the coupler. The study shows that it is possible to reduce the device length significantly using STA, keeping the efficiency of power transfer at 100%. This shortcut approach shows much superiority in terms of robustness and fidelity in power switching compared to the adiabatic approach.

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I. INTRODUCTION

Among the various popular methods in the field of coherent atomic manipulation, techniques that are based on adiabatic dynamics, such as rapid adiabatic passage (RAP), stimulated Raman adiabatic passage (STIRAP), Stark chirped rapid adiabatic passage (SCRAP), etc., have been studied over a wide range of issues in contemporary physics [1–3]. These methods mainly focus on the transfer of population among the atomic and molecular states efficiently. Many different processes, such as controlling chemical reactions, laser cooling, and nuclear magnetic resonance (NMR), were realized both theoretically and experimentally in the recent past based on adiabatic dynamics [4–6]. Another set of theories in atomic physics, namely, transitionless quantum driving [7] and the counter-diabatic field paradigm [8], has been introduced recently, according to which adiabatic processes can be driven beyond the adiabatic limit without changing the initial and the final states. These theories enable one to drive a quantum system in infinitely short time without losing the robustness of the process. It is worthwhile to mention that Chen *et al.* [9] have proposed a method to speed up the adiabatic passage techniques in the context of two- and three-level atoms. This method is now widely termed as Shortcuts to Adiabatic passage (STA).

Past experiences show that many quantum physics concepts when applied in optics settings can be tested experimentally. To cite some recent examples include parity-time symmetry [10], supersymmetry [11], Anderson localization [12], and so on [13]. Recently, based on the analogies between quantum mechanics and wave optics, many techniques have been proposed to manipulate light in various waveguide structures [13–19]. In this regard, waveguide directional couplers in integrated optics are particularly interesting owing to its tremendous practical applications [20,21]. In general, the function of a waveguide coupler is to split coherently an optical field incident on one of the input ports and direct the two parts to the output ports. As the output is directed in two different directions, couplers are also referred to as directional couplers [20]. Adiabatic following is applied in such devices to study the eigenmode evolution of optical power through the waveguides [22]. For a sufficiently long coupler, where adiabaticity is satisfied, the system follows its initial eigenmode, causing power transfer from one waveguide to the other. Mode-evolution-based studies of directional couplers show robust optical power switching between two, three, or even among an array of waveguides [22–25]. On the flip side, large device length causes higher transmission loss and makes designing practical devices difficult. However, there are significant opportunities to make couplers more efficient and small in dimension using STA [9,26]. Several new studies in this regard have been reported recently [27–33]. In this work we have studied a directional coupler made of two evanescently coupled waveguides. We propose that these waveguides are designed in such a way that the waveguide mismatch parameter Δ and the coupling parameter κ, defined later in the article, follows the so-called Allen-Eberly (AE) scheme. It should be noted that the Allen-Eberly scheme for a pulse-detuning combination is well established and widely used in atomic physics [9]. In fact, the AE scheme is better known as the complex-hyperbolic-secant (CHS) scheme, first proposed by Lamb, Jr. [34,35]. CHS was applied to NMR before it was used in the optical domain [36,37]. Later, the scheme was applied to two-level quantum systems [38,39]. For various adiabatic processes the AE scheme is much faster compared to other model schemes, such as the Landau-Zener scheme [40,41]. This article is organized as follows. Section II discusses adiabaticity in the coupler, while in Sec. III we discuss how to apply the shortcut technique to the proposed coupler. Section IV contains results and discussions followed by conclusions in Sec. V.

II. ADIABATICITY IN DIRECTIONAL COUPLER

We consider a directional coupler of length $2L$ consisting of two single-mode waveguides placed in proximity with propagation constants β_1 and β_2. Since we have chosen the guides in close proximity, coupled mode theory can be used to predict the power propagation in the coupler. In fact, it turns out that the prediction of coupled mode theory very much resembles the Schrodinger equation for two-level atomic systems [13]. The coupled equation for the modal amplitudes a_1 and a_2 of the two waveguides can be written as follows:

$$i\frac{da_1(z)}{dz} = \Delta a_1(z) + \kappa a_2(z), \quad (1)$$

$$i\frac{da_2(z)}{dz} = -\Delta a_2(z) + \kappa a_1(z). \quad (2)$$

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Here $\Delta = (\beta_1 - \beta_2)/2$ represents the mismatch between the propagation constants and κ is the coupling between the guides and can be taken to be real without loss of generality. It is easy to see that in the diabatic basis $\{a_j\}$, there exists an operator similar to the Hamiltonian in quantum physics which can be written as

$$H = \begin{pmatrix} \Delta & \kappa \\ \kappa & -\Delta \end{pmatrix}. \quad (3)$$

This Hamiltonian can be diagonalized using unitary transformation to a new basis $\{A_j\}$, which is basically the adiabatic basis, given by

$$\begin{pmatrix} A_1 \\ A_2 \end{pmatrix} = U_0^{-1} \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}, \quad (4)$$

where U_0 is two-dimensional unitary matrix and can be taken as

$$U_0 = \begin{pmatrix} \cos(\theta/2) & -\sin(\theta/2) \\ \sin(\theta/2) & \cos(\theta/2) \end{pmatrix}. \quad (5)$$

Here θ is the angle of mixing and is defined as $\tan\theta = \kappa(z)/\Delta(z)$. The transformed Hamiltonian will be

$$H'(z) = U_0^{-1} H(z) U_0 - i U_0^{-1} \dot{U}_0, \quad (6)$$

where the overdot represents the derivative with respect to z. The second term is regarded as a nonadiabatic correction, owing to fact that the first term is diagonal itself and can drive the system adiabatically alone. When the adiabatic criterion, which can be written as $\dot{\theta}/2 \ll \sqrt{(\Delta^2 + \kappa^2)}$, is satisfied, nonadiabatic corrections generally go to zero. For adiabatic evolution we have followed a coupling-mismatch scheme that is very similar to the famous Allen-Eberly scheme [9] by

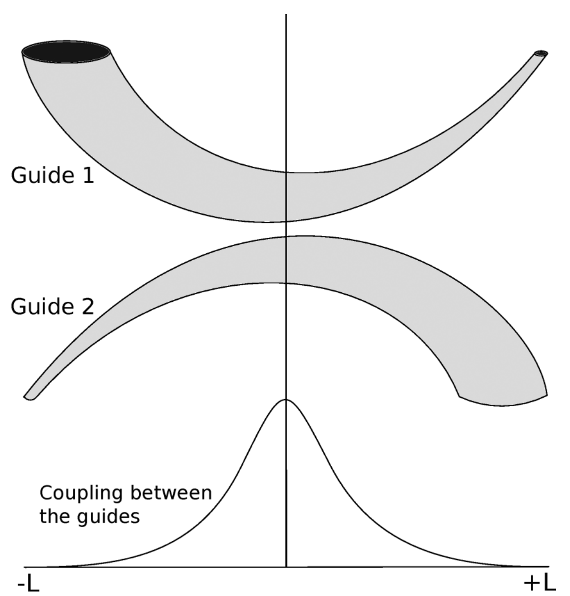

FIG. 1. Schematic for adiabatic directional coupler with Allen-Eberly scheme. β_1 and β_2 are propagation constants for waveguide one and two, respectively. Coupler length is $2L$. Maximum of the coupling occurs at $L = 0$.

choosing

$$\Delta(z) = \Delta_0 \tanh(2\pi z/L), \quad (7)$$

$$\kappa(z) = \kappa_0 \operatorname{sech}(2\pi z/L). \quad (8)$$

The coupling parameter κ (z) changes with the coupler length $2L$ and also the mismatch coefficient varies from $-\Delta_0$ to $+\Delta_0$. With both Δ and κ being z dependent, the coupler design results in a tapered structure of the waveguides. Moreover, the variation of $\Delta(z)$ should be slow enough to accomplish adiabatic evolution. Also, for the choices in Eqs. (7) and (8), the adiabatic condition is given by $\kappa_0 L \gg \pi$ and hence it demands the coupler length to be large. The schematic of the proposed coupler is shown in Fig. 1.

III. REALIZING SHORTCUT

For shortcuts, we followed Berry's algorithm of transitionless quantum driving [7]. Under the circumstances when the adiabatic criterion cannot be fulfilled, complete power switching does not occur due to the effect of the nonadiabatic term in the Hamiltonian. To overcome this we derive a driving Hamiltonian. The Hamiltonian relevant to our system is simply given by $H_a = i \sum_j |\partial_z A_j\rangle\langle A_j|$, which when transformed back to the basis $\{a_j\}$, eventually turns out to be

$$H_a = \begin{pmatrix} 0 & -i\dot{\theta}/2 \\ i\dot{\theta}/2 & 0 \end{pmatrix}. \quad (9)$$

This additional Hamiltonian should be added back to our original Hamiltonian in order to undo the effects of nonadiabatic terms, which leads to

$$H_{\text{eff}} = \begin{pmatrix} \Delta(z) & \kappa(z) - i\kappa_a(z) \\ \kappa(z) + i\kappa_a(z) & -\Delta(z) \end{pmatrix}. \quad (10)$$

This induces an additional coupling, $\kappa_a = \dot{\theta}/2$, with some phase difference with the original. Also κ_a should be comparable with κ because the dynamics with H does not need to follow the adiabatic condition. However, we can describe it as a combination of an effective coupling and a phase term:

$$H_{\text{eff}} = \begin{pmatrix} \Delta(z) & \kappa_{\text{eff}}(z) e^{-i\phi} \\ \kappa_{\text{eff}}(z) e^{i\phi} & -\Delta(z) \end{pmatrix}, \quad (11)$$

where $\kappa_{\text{eff}} = \sqrt{\kappa^2 + \kappa_a^2}$. Using the following transformation one can eliminate the phase dependence,

$$U_1 = \begin{pmatrix} e^{-i\phi/2} & 0 \\ 0 & e^{i\phi/2} \end{pmatrix}, \quad (12)$$

which again provides a new set of basis $\{A'_j\}$, and now the resulting Hamiltonian becomes

$$H_{\text{eff}} = \begin{pmatrix} \Delta_{\text{eff}}(z) & \kappa_{\text{eff}}(z) \\ \kappa_{\text{eff}}(z) & -\Delta_{\text{eff}}(z) \end{pmatrix}. \quad (13)$$

Here $\Delta_{\text{eff}} = \Delta(z) - \dot{\phi}/2$. It is useful to note that $\{A'_j\}$ is related with the adiabatic basis $\{A_j\}$ by two transformations U_0 and U_1 via the parameters θ and ϕ. However, to keep these bases consistent in terms of the initial and the final states, certain conditions need to be imposed. θ and ϕ have to be adjusted in such a way that $\{A_j\}$ and $\{A'_j\}$ become equivalent at the boundary, which leads to the boundary condition $\dot{\theta}(-L) = \dot{\theta}(L) = 0$.

IV. RESULTS AND DISCUSSION

In order to study the power evolution in the coupler we have numerically solved the master equation [1],

$$\dot{\rho} = -i\,[H,\rho], \tag{14}$$

for both Hamiltonians in Eqs. (3) and (13). ρ is the density matrix with matrix elements $\rho_{ij} = a_i a_j^*$. Diagonal elements $\rho_{ii} = |a_i(z)|^2$ represent the power in the ith waveguide, while the off-diagonal elements refer to the coherence between the waveguides. Here the dephasing rate Γ has not been considered, as we have considered the waveguides to be lossless.

For the adiabatic coupler forms of κ and Δ are taken as in Eqs. (7) and (8). The shortcut approach has been achieved by choosing κ_a as follows:

$$\kappa_a = \kappa_0 \exp\left(-z^2/z_0^2\right). \tag{15}$$

Here z_0 is the width of the Gaussian and is well adjusted with the varying coupler length so that the boundary conditions for θ are satisfied at the boundary. Figures 2(a) and 2(b) depict the spatial variation of the mismatch and coupling parameters

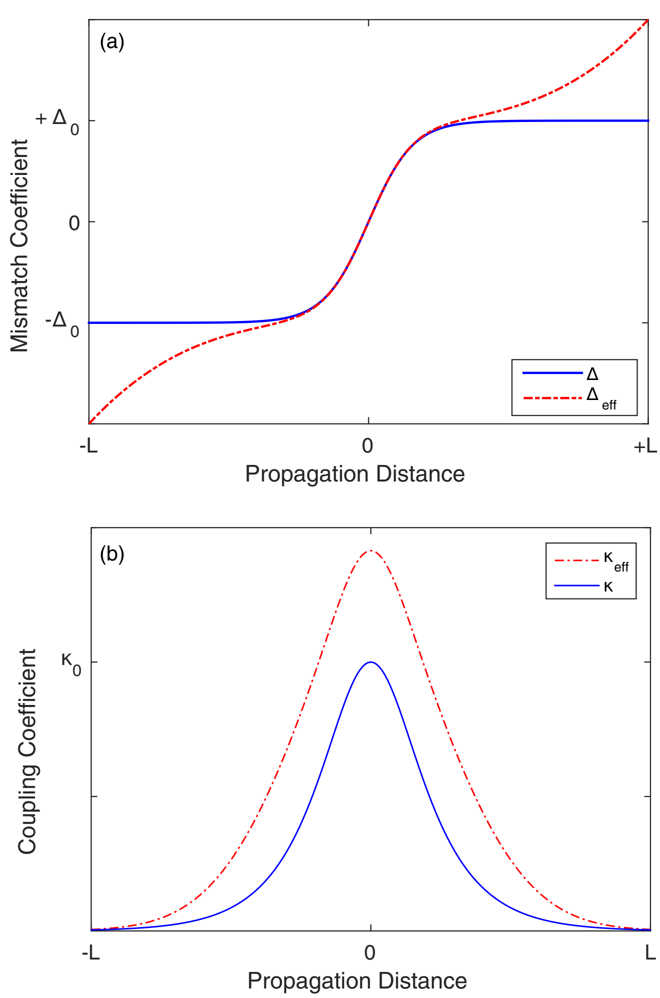

FIG. 2. (Color online) Spatial variation of (a) mismatch coefficient Δ and Δ_{eff}. Δ varies from $-\Delta_0$ to $+\Delta_0$, but Δ_{eff} is greater towards the ends. (b) Coupling coefficients κ and κ_{eff}. Amplitude of κ_{eff} is greater than κ.

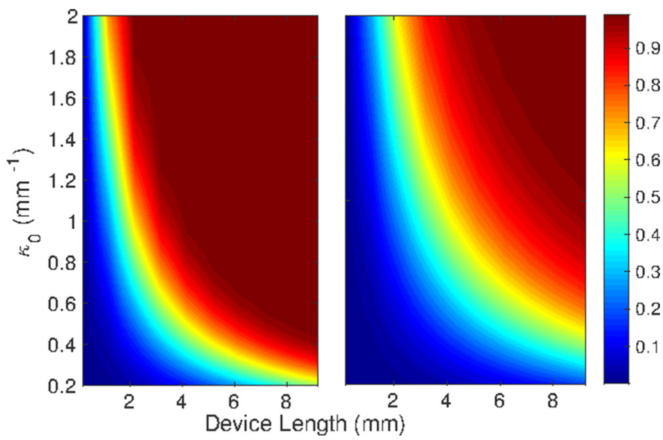

FIG. 3. (Color online) Contour plots for output power with varying κ_0 and device length L. Shortcut (left) shows high fidelity over adiabatic coupler (right).

for the adiabatic and the STA coupler. The geometry of the coupler depends on the coupling between the waveguides and mismatch coefficient. Stronger coupling refers to larger separation distances between the waveguides towards the ends of the coupler, which indicates a significant decrement in device length. On the other hand, the extent of tapering of the coupler is controlled by the mismatch coefficient.

As Δ goes higher, β_1 and β_2 tend to change more rapidly throughout the length $2L$. As far as adiabaticity is concerned, larger $\kappa\ (z)$ is preferable for power transfer as it is required to satisfy the condition $\kappa_0 L \gg \pi$. However, that does not contribute to shortening of the coupler length, whereas in STA couplers, a significant amount of coupling length can be reduced with little enhanced coupling. These facts can readily be seen in Fig. 3, where we have plotted final power output as a function of device length and the coupling amplitude. With a particular choice of $\Delta_0 = 1\ \text{mm}^{-1}$, contours reveals that for large variation of κ_0 a shortcut approach shows much superiority in terms of robustness and fidelity in power switching. For any given value of κ_0, the minimum distance required to transfer power between the waveguides with an adiabatic coupler is much greater, at least two times, than that of the STA coupler. Figure 4 depicts the spatial evolution of fractional power, defined as $P_2(z)/P_1(-L)$, in the coupler. In our simulation, the input power in the first waveguide is taken to be unity, i.e., $P_1(-L) = 1$, while the input power in the second waveguide is kept empty. Other parameters are chosen as $\Delta_0 = \kappa_0 = 1\ \text{mm}^{-1}$.

For smaller propagation distance, say $z < 4$ mm or so, the fractional power at the second waveguide, using adiabatic dynamics, never reaches unity. It only shows high transfer probability at large propagation distances, say $z > 10$ mm or so. However, one can achieve nearly 100% power transfer to the second waveguide using the shortcut approach. Coupling efficiency calculation also supports our previous results. Figure 5 illustrates the efficiency of both the adiabatic and the STA coupler with respect to device length. It is quite clear from the plot that the STA coupler achieves 100% efficiency with much shorter distance compared to the adiabatic coupler. With regard to the practical implementation of the scheme, one may design or fabricate a silica (SiO_2)-based

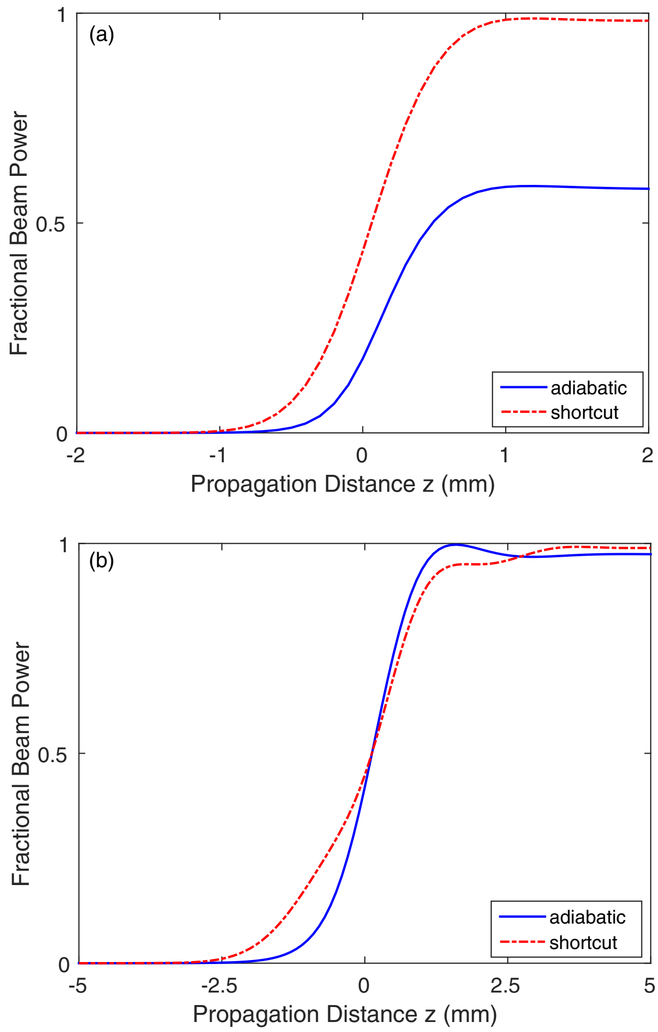

FIG. 4. (Color online) Fractional power output vs propagation distance for $\Delta_0 = \kappa_0 = 1$ (a) for $2L = 4$ mm and (b) for $2L = 10$ mm.

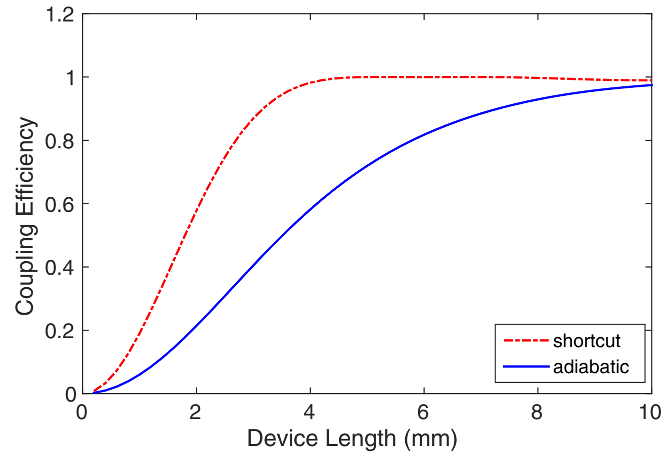

FIG. 5. (Color online) Coupling efficiency for adiabatic and STA coupler with varying device lengths. Parameters are the same as in Fig. 4.

fiber coupler [20] using the proposed scheme. The effective coupling coefficient (κ_{eff}), which is the most critical parameter in realizing the proposed waveguide, could be manipulated with judicious choice of core radius, center-to-center separation between the waveguides, and the refractive index difference between the two waveguides. One may choose the parameter z_0 to obtain the effective κ_{eff} theoretically. And then, applying the appropriate mathematical relation between κ_{eff} and the coupler parameters, derivable using the coupled mode theory, one can decide upon the other coupler parameters [42].

V. CONCLUSION

In conclusion, drawing inspiration from quantum optics, we have proposed a directional coupler based on the complex-hyperbolic-secant scheme. The variation in propagation constants $\beta_1(z)$ and $\beta_2(z)$ (and thereby Δ) can be achieved by varying the cross-sectional area of the waveguides along the direction of propagation. On the other hand, the coupling parameter κ can be adjusted by controlling the adjacent distance between the waveguides. The coupler is studied in the adiabatic regime followed by application of recently developed shortcuts to the adiabatic passage technique to the coupler. It turns out that by using shortcuts one can reduce the length of the coupler significantly, keeping the power transfer efficiency nearly 100%. This study may open new possibilities of exploiting the STA and AE schemes for various applications in integrated optics, specifically within the context of photonic circuits.

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