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# Scalable quantum computation architecture using always-on Ising interactions via quantum feedforward

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Here, we propose a way to control the interaction between qubits with always-on Ising interaction. Unlike the standard method to change the interaction strength with unitary operations, we fully make use of nonunitary properties of projective measurements so that we can effectively turn the interaction on or off via feedforward. Our scheme is useful for generating two- or three-dimensional cluster states that are universal resources for fault-tolerant quantum computation with this scheme, and it provides an alternative way to realize a scalable quantum processor.

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## I. INTRODUCTION

Quantum computation is a new paradigm of information processing. Known algorithms give superior performance for factoring [1,2], searching an unsorted database [3–5], quantum simulation [6,7], the Deutsch-Jozsa algorithm [8], linear systems of equations [9], an Abelian hidden subgroup [10], group isomorphism [11], welded tree [12], and more. All these algorithms require a large-scale quantum computer. A quantum computer is composed of a sequence of single-qubit gates and two-qubit gates [13–16]. The single-qubit gate denotes a rotation of the qubit around an arbitrary axis and degree. A control-phase gate is one of the typical examples of two-qubit gates. This gate flips the phase of the target qubit if and only if the state of the control qubit is $|1\rangle$. The roles of control and target qubits are reversible for a control-phase gate. Individual qubits should be efficiently addressed, and the interaction between two qubits should be controlled by some external apparatus.

The challenge is how to design and build a quantum computer with realistic technology. This requires quantum architecture. There have been a number of quantum architectures for relevant physical systems, such as the nitrogen-vacancy center [17,18], ion traps [19], and superconducting systems [20]. Many of those have assumed an isolated system and excellent controllability. However, in realistic circumstances, turning the interaction on or off in a reliable way is one of the hardest parts in such architectures. For example, two-qubit gates require turning the interaction between qubits on or off *in situ* via the external control apparatus. Since imperfection of the interaction control tends to induce correlated errors between qubits, sophisticated technology is required to suppress the error rate below the threshold of fault-tolerant quantum computation [21–23]. However, varying the interaction between qubits *in situ* is not possible for all physical systems.

One of the ways to reduce the required level of technology is to use a system with always-on interaction. There are a couple of theoretical proposals for this type of scheme. Zhou *et al.* suggested a system with always-on Heisenberg interaction [24]. They use interaction-free subspace to protect the target encoded qubit from the residual interaction, and they show that only local manipulations in the system actually provide universal quantum computation. Benjamin and Bose also suggested using an always-on Heisenberg interaction system for scalable quantum computation by collectively tuning the qubits [25,26]. These approaches look attractive because of their simplicity, which could reduce potential decoherence from the interaction.

Here, we propose a way to perform universal quantum computation with a system having an always-on Ising interaction. In quantum mechanics, there are two type of operations, unitary operations such as applying microwave pulses and nonunitary operations such as readout of the qubit. While most of the authors of previous papers use unitary operation to control the interaction [24–26], we exploit the nonunitary properties that the projective measurements have. We will assume an always-on Ising interaction between nearest-neighbor qubits and will insert an ancillary qubit between the qubits that process quantum information. We show that it is possible to effectively turn the interaction on or off via quantum measurement and feedforward on the ancillary qubits. Since quantum feedforward technology is becoming a mature technology [27–36], our proposal provides a feasible and reliable way to control the interaction, which is a crucial step for the realization of quantum information processing.

The remainder of this paper is organized as follows. In Sec. II, we review the preliminaries of this paper. Section III presents the details of our scheme to show how always-on interaction is effectively turned on or off via projective measurement to ancillary qubits and quantum feedforward. Section IV concludes our discussion.

## II. CLUSTER STATES AS A RESOURCE FOR QUANTUM COMPUTATION

A two- or three-dimensional cluster state can be a universal resource for measurement-based quantum computation (MBQC) [37–40] and topological quantum computation

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[21–23]. A cluster state is composed of $|+\rangle$ state qubits on the lattice points and the controlled-phase gate operation $\hat{U}_{CZ}$ between each pair of nearest-neighbor qubits. The controlled-phase gate can be realized by Ising-type interaction [37,38]. When we consider qubits A and B and Ising type interaction between A and B, the Hamiltonian to perform the controlled-phase gate is as follows:

$$\hat{H}_{\text{Ising}} = g_{(A,B)} \frac{1+\hat{Z}_A}{2} \frac{1+\hat{Z}_B}{2}, \tag{1}$$

where $g_{(A,B)}$ denotes the interaction strength between qubits A and B. By letting a separable state $|++\rangle_{AB}$ evolve for $g_{(A,B)}t = \pi$ according to this Hamiltonian, the following unitary operator will be applied to the initial state:

$$\exp\left(-i\pi \frac{1+\hat{Z}_A}{2} \frac{1+\hat{Z}_B}{2}\right) = U_{CZ}^{(A,B)}, \tag{2}$$

and hence we can create the controlled-phase gate.

Although there are many proposals to realize Ising-type interaction such as ultracold atoms in an optical lattice [41–48], ion traps [49–54], superconducting charge qubits [55], superconducting spin qubits [56], superconducting flux qubits [57], a resonator waveguide [58], the nitrogen-vacancy center [17,59–64], quantum dots [65–68], and electronic spins coupled to the motion of magnetized mechanical resonators [69], the major challenge for experimental realization is switching the interaction on or off with a high fidelity. Only a few experiments have demonstrated a high-fidelity controllable two-qubit gate with a fidelity above the threshold of fault-tolerant quantum computation [70–72]. One of the possible ways to overcome the experimental difficulties for demonstrating high-fidelity two-qubit gates is to use an always-on interaction scheme [24–26,73–75]. Since there is no need for the additional controlling operations to switch the interaction, these scheme may scale well for a large number of qubits. Here, we propose an approach to implement the controlled-phase gate for fault-tolerant quantum computation with always-on interaction by using the nonunitary properties of projective operations and quantum feedforward.

## III. EFFECTIVE INTERACTION CONTROL VIA PROJECTIVE MEASUREMENTS AND QUANTUM FEEDFORWARD

### A. Effectively turning interaction on or off with measurement and quantum feedforward

We introduce a Hamiltonian to realize our scheme to turn the interaction on or off effectively via projective measurements and quantum feedforward. The physical device that we consider is a general solid-state system where every qubit can be individually controlled by a microwave pulse and there are always-on interactions between nearest-neighbor qubits. We assume the following two-qubit Hamiltonian:

$$\hat{H}_{AB} = \hat{H}_{\text{local}} + \hat{H}_{\text{interaction}}, \tag{3}$$

$$\hat{H}_{\text{local}} = \sum_{j=A,B} \left[\frac{\omega_j}{2}\hat{Z}_j + \lambda_j(t)\cos(\omega'_j t + \theta)\hat{X}_j\right], \tag{4}$$

$$\hat{H}_{\text{interaction}} = \frac{g_{(A,B)}}{4}\hat{Z}_A\hat{Z}_B, \tag{5}$$

where $\omega$, $\lambda(t)$, $\omega'$, $\theta$, and $g$ denote the qubit energy, Rabi frequency, microwave frequency, a phase of the microwave, and interaction strength, respectively. In most solid-state systems, it is possible to control the value of $\lambda(t)$ by changing the power of the microwave with much higher accuracy than in the case of two-qubit gates. We move to a rotating frame defined by

$$\hat{U}_{AB} = \exp\left(-i\sum_{j=A,B}\frac{\omega'_j}{2}\hat{Z}_j t\right), \tag{6}$$

where $\omega'_j$ denotes the angular frequency of the rotating frame at site $j$, and we use a rotating-wave approximation so that we can obtain the following Hamiltonian:

$$\hat{H}_{AB} \simeq \sum_{j=A,B}\left(\frac{\omega_j - \omega'_j}{2}\hat{Z}_j + \frac{\lambda_j(t)}{2}\hat{A}^\theta_j\right) + \frac{g_{(A,B)}}{4}\hat{Z}_A\hat{Z}_B, \tag{7}$$

where

$$\hat{A}^\theta = \begin{pmatrix} 0 & e^{-i\theta} \\ e^{i\theta} & 0 \end{pmatrix}. \tag{8}$$

Unless required to perform single-qubit gates, we turn off the microwave and set $\lambda = 0$, and therefore the Hamiltonian introduced here is effectively the same as an Ising model with always-on interaction. On the other hand, for the implementation of accurate single-qubit rotations, we assume a large Rabi frequency, $\lambda \gg g$, so that the coupling strength from the nearest-neighbor qubit can be negligible.

The Hamiltonian described above has the interesting property that an interaction from the other qubit can be turned off by preparing the state of a qubit in a ground state. To explain this, we choose the microwave frequencies as

$$\omega'_A = \omega_A - \tfrac{1}{2}g_{(A,B)}, \quad \omega'_B = \omega_B - \tfrac{1}{2}g_{(A,B)}, \tag{9}$$

and we set

$$\lambda_A = \lambda_B = 0. \tag{10}$$

Then, the Hamiltonian (7) becomes

$$\hat{H}'_{AB} = \sum_{j=A,B}\frac{\omega_j - \omega'_j}{2}\hat{Z}_j + \frac{g_{(A,B)}}{4}\hat{Z}_A\hat{Z}_B \tag{11}$$

$$= \sum_{j=A,B}\frac{g_{(A,B)}}{4}\hat{Z}_j + \frac{g_{(A,B)}}{4}\hat{Z}_A\hat{Z}_B \tag{12}$$

$$= g_{(A,B)}\frac{1+\hat{Z}_A}{2}\frac{1+\hat{Z}_B}{2}. \tag{13}$$

Interestingly, if qubit A is prepared in a ground state, the interaction from qubit A cancels out because of

$$g_{(A,B)}\frac{1+\hat{Z}_A}{2}\frac{1+\hat{Z}_B}{2}|\downarrow\rangle_A = 0. \tag{14}$$

This means that preparing a specific qubit in a ground state effectively turns off the interaction between this qubit and the nearest-neighbor qubit. Therefore, if all nearest-neighbor qubits are in the ground state, the qubit is not affected by any

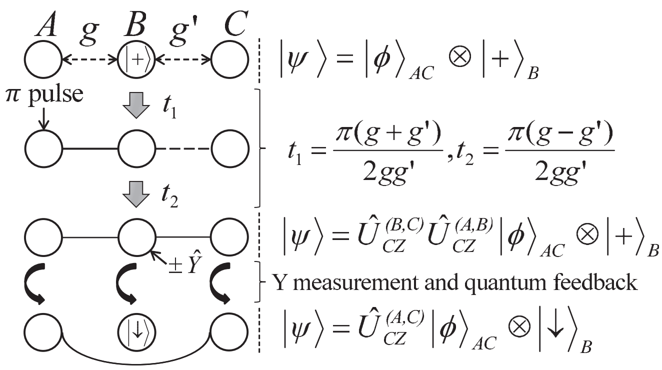


FIG. 1. Schematic of our scheme to implement two-qubit gates via projective measurements and quantum feedforward under the effect of always-on Ising interaction. First, we let the state $|\phi\rangle_{AC} \otimes |+\rangle_B$ evolve for a time $t_1$ according to the Hamiltonian. Second, we perform a $\pi$ pulse on the middle qubit, B. Third, we let the state evolve for a time $t_2$. Finally, we perform a projective measurement and quantum feedforward on qubit B, so that a controlled-phase gate can be implemented between qubits A and C. Due to the engineered Hamiltonian form that we make, the interaction between qubits is turned off as long as qubit B is in a ground state.

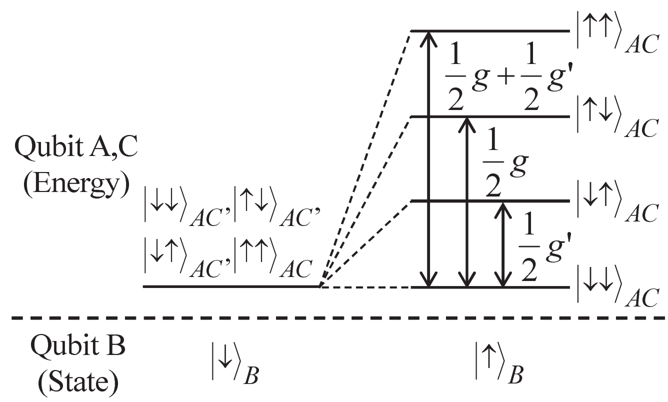


FIG. 2. Energy diagrams of qubits A and C. The energies depend on the state of qubit B. The energies of qubits A and C are degenerate when qubit B is in a ground state. However, once qubit B is excited, degeneracy is removed, so that the energy difference occurs between the states of qubits A and C.

interactions, which is the striking feature of our scheme. Also, if qubit A is prepared in an excited state, the interaction causes an extra phase shift in qubit B.

It is worth mentioning that we need precise control of the frequency of the microwave in our scheme. We investigate the effect of a small detuning from the target frequency of the microwave. Supposing that there is a detuning of $\delta\omega_j$ from the target frequency, we have

$$\omega'_j = \omega_j - \frac{g_{(A,B)}}{2} + \delta\omega'_j. \tag{15}$$

In this case, we can rewrite the Hamiltonian (13) as follows:

$$\hat{H}''_{AB} = g_{(A,B)}\frac{1+\hat{Z}_A}{2}\frac{1+\hat{Z}_B}{2} - \sum_{j=A,B}\frac{\delta\omega'_j}{2}\hat{Z}_j. \tag{16}$$

Hence, frequency errors cause a phase shift error on each qubit. Fortunately, due to recent developments in microwave technology, accurate control of the microwave frequency is possible. Therefore, in this paper, we assume that we can choose the exact microwave frequency to avoid this kind of error.

### B. Implementation of controlled-phase gate

We start to illustrate our concept about how to control the effective interaction via projective measurements and quantum feedforward. Suppose that we have three qubits, A, B, and C, in a row and that the coupling strengths between the nearest-neighbor qubits are $g_{(A,B)}$ and $g'_{(B,C)}$, as shown in Fig. 1, where we assume $g > g'$ without loss of generality.

Then, the system Hamiltonian becomes

$$\hat{H} = \sum_{j=A,B,C}\left[\frac{\omega_j}{2}\hat{Z}_j + \lambda_j(t)\cos(\omega'_j t + \theta)\hat{X}_j\right] + \frac{g_{(A,B)}}{4}\hat{Z}_A\hat{Z}_B + \frac{g'_{(B,C)}}{4}\hat{Z}_B\hat{Z}_C \tag{17}$$

$$\simeq g_{(A,B)}\frac{1+\hat{Z}_A}{2}\frac{1+\hat{Z}_B}{2} + g'_{(B,C)}\frac{1+\hat{Z}_B}{2}\frac{1+\hat{Z}_C}{2}, \tag{18}$$

with

$$\omega'_A = \omega_A - \frac{1}{2}g_{(A,B)}, \quad \omega'_B = \omega_B - \frac{g_{(A,B)} + g'_{(B,C)}}{2}, \quad \omega'_C = \omega_C - \frac{1}{2}g'_{(B,C)}, \quad \lambda_A = \lambda_B = \lambda_C = 0. \tag{19}$$

As written in Eq. (17), the state of qubit B changes the energies of qubits A and C. When we set qubit B to the ground state, all eigenstates of qubits A and C degenerate; therefore $\hat{H}$ does not change the system in time. We show the energy diagrams in Fig. 2.

The ancillary qubit induces a conditioned dynamics. The excited state of the ancillary qubit causes the phase rotation on the other qubits, while the ground state of the ancillary qubit does not induce any phase shift on them. Therefore, if we have a superposition of the ancillary qubit, the other two qubits are entangled via such a conditioned dynamics. In order to see this effect more clearly, we describe how such conditioned dynamics occur in the Appendix.

Here, we show the procedure of our scheme for a controlled-phase gate. First, we prepare a separable $|+\rangle$ state for qubit B and prepare an arbitrary state for qubits A and C. An initial state is described by

$$\rho = \rho_{AC} \otimes |+\rangle\langle+|_B. \tag{20}$$

Second, we let the state evolve for a time $t_1$, perform a $\pi$ pulse on qubit $A$, and let the state evolve for a time $t_2$. Here, we adopt a spin-echo technique [76–78] to balance the interaction. In the spin-echo technique, implementation of a $\pi$ pulse can refocus the dynamics of the spin so that the effect of the interaction should be canceled. We therefore introduce

$$t_1 = \frac{\pi(g_{(A,B)} + g'_{(B,C)})}{2g_{(A,B)}g'_{(B,C)}} \tag{21}$$

and

$$t_2 = \frac{\pi(g_{(A,B)} - g'_{(B,C)})}{2g_{(A,B)}g'_{(B,C)}} \tag{22}$$

to satisfy

$$g_{(A,B)}(t_1 - t_2) = g'_{(B,C)}(t_1 + t_2) = \pi. \quad (23)$$

The total unitary evolution $\hat{U}_{CZ}^{(A,B)}\hat{U}_{CZ}^{(B,C)}$ can be described by

$$\hat{U} = \exp\bigg[ -i g_{(A,B)}(t_1 - t_2)\frac{\mathbf{1} + \hat{Z}_A}{2}\frac{\mathbf{1} + \hat{Z}_B}{2} - i g'_{(B,C)}(t_1 + t_2)\frac{\mathbf{1} + \hat{Z}_B}{2}\frac{\mathbf{1} + \hat{Z}_C}{2}\bigg], \quad (24)$$

up to local equivalent, so that we can perform controlled-phase gates even if the coupling strength is asymmetric. The details are explained in the Appendix.

Third, we perform the $\hat{Y}$ basis

$$|\pm 1_{\hat{Y}}\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle \pm i|\downarrow\rangle) \quad (25)$$

measurement on the middle qubit, B. The state after the measurement is written as

$$\rho'_{\pm} = \hat{P}_B^{\pm} e^{-i\hat{H}t}\rho e^{i\hat{H}t}\hat{P}_B^{\pm}, \quad (26)$$

where $\pm$ denotes the measurement result. Here,

$$\hat{P}^{\pm} = \tfrac{1}{2}(1 \pm \hat{Y}) \quad (27)$$

denotes a projection operator on qubit B. Finally, we perform a quantum feedforward operation, which is the implementation of different local operations depending on the measurement results, on qubit B, so that qubit B can be prepared in a ground state. We define the feedforward operator as

$$\hat{F}_{ABC}^{\pm} = \hat{S}_A^{\pm}\hat{U}_B^{\frac{\mp\pi}{2},\hat{X}}\hat{S}_C^{\pm}, \quad (28)$$

where $\hat{S}^{\pm}$ denotes a shift gate defined as

$$\hat{S}^{\pm} = \begin{pmatrix} 1 & 0 \\ 0 & \pm i \end{pmatrix} \quad (29)$$

and $\hat{U}^{\theta,\hat{X}}$ denotes a single-qubit rotating around the $x$-axis rotation with an angle of $\theta$. The state after the quantum feedforward is described as

$$\rho_{\text{final}} = \hat{F}_{ABC}^{+}\rho'_{+}\hat{F}_{ABC}^{+\dagger} + \hat{F}_{ABC}^{-}\rho'_{-}\hat{F}_{ABC}^{-\dagger} \quad (30)$$

$$= \hat{U}_{CZ}^{(A,C)}\rho_{AC}\hat{U}_{CZ}^{(A,C)} \otimes |\downarrow\rangle\langle\downarrow|_B. \quad (31)$$

Therefore, after these operations, controlled-phase operations are performed between qubits A and C, and the state does not evolve anymore because qubit B is prepared in a ground state. As shown in Fig. 2, the states of qubits A and C degenerate, and hence interactions are effectively turned off.

Meanwhile, if we set qubit B in an excited state using a quantum feedforward operation, the final state becomes

$$\rho'_{\text{final}} = e^{-i\hat{H}t'}\big(\hat{U}_{CZ}^{(A,C)}\rho_{AC}\hat{U}_{CZ}^{(A,C)} \otimes |\uparrow\rangle\langle\uparrow|_B\big)e^{i\hat{H}t'} \quad (32)$$

$$= e^{-i\hat{H}'t'}\hat{U}_{CZ}^{(A,C)}\rho_{AC}\hat{U}_{CZ}^{(A,C)}e^{i\hat{H}'t'} \otimes |\uparrow\rangle\langle\uparrow|_B, \quad (33)$$

where $\hat{H}'$ denotes the following Hamiltonian:

$$\hat{H}' = g_{(A,B)}\frac{1 + \hat{Z}_A}{2} + g'_{(B,C)}\frac{1 + \hat{Z}_C}{2}. \quad (34)$$

The energy eigenstates are not degenerate, as shown in Fig. 2, and hence interactions cause the extra phase shift to qubits A and C. In principle, we can correct these extra phases by performing single-qubit rotation. However, unless single-qubit rotation can be perfectly performed, such operations induce another error, which makes it difficult to perform fault-tolerant quantum computation. In addition, it is usually difficult to keep the state in an exited state due to the existence of the energy relaxation. For these reasons, we set qubit B in a ground state after the projective measurement.

It is worth mentioning that, although we introduce a three-qubit case as an example, it is straightforward to generalize this idea to a multiqubit case to create a two- or three-dimensional cluster state.

Since the interaction is of the Ising type, the eigenvectors are represented by the computational basis ($|\uparrow\rangle,|\downarrow\rangle$ basis). This means that the ancillary qubit induces a conditional dynamics such that the target qubits evolve differently depending on the state of the ancillary qubit. If we have a superposition of the ground state and the excited state of the ancillary qubit, it becomes possible to realize the superposition of two such dynamics. This is the key to entangling the ancillary qubit with the target qubits.

## IV. CONCLUSION

Here, we show a way to perform a controlled-phase gate operation with always-on Ising interaction. Our method uses projective measurements and quantum feedforward to effectively turn the interaction in this system on or off. Importantly, direct control of the interaction is not required in our scheme. Therefore our protocol would provide a practical way to implement two-qubit gates for a system where the interaction is always on, which is an important step for scalable quantum computation.

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## APPENDIX: THE DETAILS OF IMPLEMENTATION OF CONTROLLED-PHASE GATE

In this Appendix, we show the details of our scheme to perform a controlled-phase gate. We set the initial state of the system as follows:

$$|\Psi_1\rangle = |+\downarrow+\rangle_{ABC}. \quad (A1)$$

From the Hamiltonian $\hat{H}'$ in Eq. (34), we define the following local Hamiltonians:

$$\hat{H}'_A = g_{(A,B)}\frac{1 + \hat{Z}_A}{2}, \quad \hat{H}'_C = g'_{(B,C)}\frac{1 + \hat{Z}_C}{2}. \quad (A2)$$

First, we perform a $\frac{\pi}{2}$ pulse on qubit B, so that we can obtain the following state:

$$|\Psi_2\rangle = |+++\rangle_{ABC}. \quad (A3)$$

Second, we let the state evolve for a time $t_1$ in Eq. (21). The state becomes

$$|\Psi_3\rangle = \frac{1}{\sqrt{2}}(|\downarrow\rangle_B + e^{i(\hat{H}'_A+\hat{H}'_C)t_1}|\uparrow\rangle_B) \otimes |++\rangle_{AC}. \quad (A4)$$

Third, we perform a $\pi$ pulse on qubit A to balance the effects of interaction strengths.

$$|\Psi_4\rangle = \frac{1}{\sqrt{2}}(|\downarrow\rangle_B|++\rangle_{AC} + \hat{X}_A e^{i(\hat{H}'_A+\hat{H}'_C)t_1}|\uparrow\rangle_B|++\rangle_{AC}). \tag{A5}$$

Fourth, we let the state evolve for a time $t_2$ in Eq. (22). Controlled-phase gates are performed between two pairs of qubits as follows:

$$|\Psi_5\rangle = \frac{1}{\sqrt{2}}(|\downarrow\rangle_B|++\rangle_{AC} + e^{i(\hat{H}'_A+\hat{H}'_C)t_2}\hat{X}_A e^{i(\hat{H}'_A+\hat{H}'_C)t_1}|\uparrow\rangle_B|++\rangle_{AC}) \tag{A6}$$

$$= \frac{1}{\sqrt{2}}(|\downarrow\rangle_B|++\rangle_{AC} + \hat{X}_A e^{i\hat{H}'_A(t_1-t_2)}e^{i\hat{H}'_C(t_1+t_2)}|\uparrow\rangle_B|++\rangle_{AC}) \tag{A7}$$

$$= \frac{1}{\sqrt{2}}(|\downarrow\rangle_B|++\rangle_{AC} - |\uparrow\rangle_B|--\rangle_{AC}). \tag{A8}$$

Finally, we measure qubit B in the $Y$ basis. According to the measurement result, the states becomes

$$|\Psi_6^{\pm}\rangle = \frac{1}{2}[(|\uparrow\rangle_B \pm i|\downarrow\rangle_B)|++\rangle_{AC} \mp i(|\uparrow\rangle_B \pm i|\downarrow\rangle_B)|--\rangle_{AC}] \tag{A9}$$

$$= \frac{1}{\sqrt{2}}|\pm 1_{\hat{Y}}\rangle_B \otimes (|++\rangle_{AC} \mp i|--\rangle_{AC}). \tag{A10}$$

The operation of quantum feedforward is determined according to the measurement result. These operations are equivalent to performing a controlled-phase gate between qubits A and C as follows:

$$|\Psi_7^{\pm}\rangle = \hat{F}^{\pm}_{ABC}|\Psi_6^{\pm}\rangle \tag{A11}$$

$$= \frac{1}{\sqrt{2}}\hat{U}_B^{\frac{\mp\pi}{2},\hat{X}}|\pm 1_{\hat{Y}}\rangle_B \otimes \hat{S}_A^{\pm}\hat{S}_C^{\pm}(|++\rangle_{AC} \mp i|--\rangle_{AC}) \tag{A12}$$

$$= \frac{1}{\sqrt{2}}(|\uparrow -\rangle_{AC} - |\downarrow +\rangle_{AC}) \otimes |\downarrow\rangle_B \tag{A13}$$

$$= \hat{U}_{CZ}^{(A,C)}|\Psi_1\rangle. \tag{A14}$$


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