Entanglement control in quantum networks by quantum-coherent time-delayed feedback

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A crucial property of quantum networks is the entanglement between different network nodes. We demonstrate that entanglement in quantum networks can be created and controlled by introducing quantum-coherent time-delayed self-feedback at single nodes.

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I. INTRODUCTION

Future quantum information processing will almost certainly rely on quantum networks [1–3]. In these networks, local operations are performed on the nodes, exploiting local quantities, while communication between the nodes is performed by carriers of quantum information, e.g., photons [4]. In order to utilize the full "quantum toolbox," it is necessary to establish nonclassical statistical correlations between the single nodes, i.e., entanglement [5]. Therefore, it is of utmost importance to understand the generation and transfer of entanglement.

Coupling open quantum systems to a common reservoir is well known to create entangled decoherence-free subspaces [3,6–10], also called "dark states," since they are stable against the reservoir-system interaction. The dynamics of the reservoir-coupled system may start from a large range of initial conditions, but the system reaches a "dark" entangled state in a natural way. The related protocols however depend on a coherent interaction of multiple nodes with the same ("common") reservoir. In view of the need to transfer quantum information between spatially distant nodes, having individual reservoirs, the common reservoir approximation is unrealistic.

Feedback [11] is a typical means for stabilization and control of a plethora of systems. Usually, feedback is not instantaneous, but shows a delay time τ, which in classical nonlinear dynamics is, e.g., used for the stabilization of unstable periodic orbits [12] or fixed points [13]. In quantum mechanics, feedback can either be introduced by measuring the quantum variable and changing the system parameters according to the outcome of the measurement [14–19] or without an external measurement by using a full-quantum-mechanical feedback control mechanism [16,20–22] that includes a quantum-coherent controller. The mechanism to control the entanglement in networks discussed in our paper is of the second kind, and therefore does not introduce any wave function collapse but preserves coherence.

II. QUANTUM-COHERENT TIME-DELAYED FEEDBACK

Time-delayed feedback in its broadest sense consists of a quantum signal leaving the quantum node at some time t and re-entering at $t+\tau$. In order to describe it in a fully quantum-coherent [16,20] manner, we need a reservoir to store the excitation (in our case, a photon) for exactly the period τ. This can be achieved by structuring the reservoir, in our case the photonic density of states [21,23–26]. Structured reservoirs are called "non-Markovian" [27–29] because (quantum) information is not lost instantaneously, but stored for a certain amount of time. Non-Markovian reservoirs are known for their influence on quantum statistics and entanglement [27,28,30–33], e.g., inducing sudden "death" and "birth" of entanglement [30,34,35].

The feedback control we focus on in this paper is of the "Pyragas type," as introduced for classical dynamics in Ref. [12]. It consists of a feedback term in the dynamical equations of the form $-K[c(t)-c(t-\tau)]$, with $c(t)$ a system variable, K the feedback strength, and the delay time τ. The special property of this kind of feedback is that it vanishes for dynamics that are τ periodic, which makes the feedback noninvasive for precisely these dynamics.

The aim of this paper is to discuss how Pyragas-type time-delayed feedback control ("Pyragas control") can be used to create and stabilize entangled states in quantum networks. It is structured as follows: First, we show how Pyragas control can be modeled for single nodes, and also discuss experimental realizations. Second, we turn to two coupled nodes, each individually subject to time-delayed feedback. We show that it is possible to create entanglement between these nodes using Pyragas control. Third, we generalize our results to networks with more nodes and different feedback strengths.

III. SINGLE NODE SUBJECT TO FEEDBACK

To model time-delayed quantum-coherent Pyragas feedback, we need a reservoir that has a precisely defined feedback time τ, after which the emitted radiation interacts with the emitter again. A simple realization of such a feedback reservoir is a one-dimensional photonic reservoir bounded on one side by a mirror. Such a reservoir has been implemented experimentally for atoms as quantum emitters by collecting the emitted photons with a lens and reflecting them back by a mirror at distance L from the atom [36,37] [cf. Fig. 1(a)]. Hemispherical mirrors [25] of radius L were shown to lead to similar dynamics. For quantum nodes consisting of high-quality cavities, e.g., whispering gallery modes [38] or defects in photonic crystals [39–41], the feedback reservoir can be realized by a waveguide [41] or photonic fiber of finite length, coupled weakly to the node [42] [cf. Fig. 1(b)]. If one of the ends of the waveguide is located at distance L, it can provide the necessary time-delayed feedback, while keeping the whole system "open" so that a stable equilibrium can be achieved.

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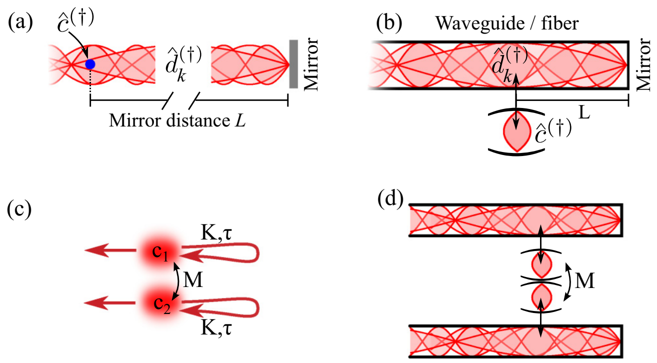

FIG. 1. (Color online) (a) Microscopic modeling of a single node with time-delayed feedback. A quantum node, described by $\hat{c}^{(\dagger)}$ (blue dot), is coupled to a set of one-dimensional bosonic reservoir modes of different wave vector k, described by $\hat{d}_k^{(\dagger)}$. Due to a mirror at distance L, at which the reservoir modes have to vanish, the coupling of the node to the reservoir becomes wave vector dependent. (b) A possible implementation of time-delayed feedback for a quantum node consisting of a high-Q cavity: A photonic waveguide coupled to the node. One end of the waveguide is located at distance L, leading to time-delayed feedback, while the other end is much further away and can be treated as lying at ∞. Due to the weak coupling between the node and the waveguide, the presence of the node does not alter the mode structure of the waveguide. (c) Schematics of a simple network, as discussed first, consisting of two coupled identical nodes. The coupling strength is given by M. Both nodes are subject to time-delayed feedback of equal strength K and equal delay time τ. (d) A possible implementation of the two coupled, feedback-controlled network nodes using cavities and two waveguides.

Waveguides were also proposed to be used in feedback experiments for artificial superconducting atoms [43,44].

We describe the quantum node with the Hamiltonian $\hat{H}_{\text{node}}$, which at the moment is still arbitrary. The transition between two states within the node will be described by the ladder operators $\hat{c}^{(\dagger)}$. Later on, we will choose single mode photonic cavities as nodes, however the mechanism presented can be applied to a large variety of systems, such as atoms [24], artificial atoms [43], or quantum dots in a solid-state environment. The transfer between this excitation within the quantum node (e.g., a photon in a cavity) and the bosonic reservoir mode with wave vector k (creation operator $\hat{d}_k^\dagger$) as well as the free reservoir dynamics [cf. Fig. 1(a)] can be described in the rotating wave approximation by [23,24]

$$\hat{H}_{\text{FB}} = \hbar \int_{-\infty}^{\infty} dk[c_0|k|\hat{d}_k^\dagger \hat{d}_k + (\gamma \sin(kL)\hat{c}^\dagger \hat{d}_k + \text{H.c.})]. \quad (1)$$

Here, γ is the coupling strength. The factor $\sin(kL)$ creates the wave-vector dependent reservoir structure, which will lead to the desired non-Markovian behavior. It takes care of the boundary condition for electric fields at a mirror in distance L to the emitter, leading to a feedback time $\tau = 2L/c_0$, c_0 being the speed of light. It is exactly this factor which is responsible for the appearance of Pyragas-type control terms. Without the sinusoidal prefactor, the coupling would lead to a purely exponential decay of the mode excitation without any feedback, as is well known from Wigner-Weisskopf theory [45].

In the limit that one excitation quantum is in the whole system, the solutions of the Schrödinger equation can be transformed into time-delayed differential equations. We therefore solve the Schrödinger equation with the Hamiltonian $\hat{H} = \hat{H}_{\text{node}} + \hat{H}_{\text{FB}}$, using the state $|\phi\rangle = c|e\rangle|0\rangle + \sum_k d_k|g\rangle|1_k\rangle$. Please note that in our notation operators are marked with a "hat" (e.g., $\hat{c}$) to distinguish them from the probability amplitudes (e.g., c). $|e\rangle$ and $|g\rangle$ denote the excited and ground state of the node, mediated by the ladder operators $\hat{c}^{(\dagger)}$, while $|0\rangle$ describes zero photons in the reservoir and $|1_k\rangle$ describes one photon in the mode k, and no other photons in the reservoir. The resulting differential equations for d_k can be formally integrated and inserted in the ODE of c, which finally gives [24]

$$\frac{d}{dt}c = f(c(t)) - K[c(t) - c(t-\tau)]. \quad (2)$$

Here, $f(c(t))$ denotes the internal dynamics of the node due to $\hat{H}_{\text{node}}$, which right now can be chosen arbitrarily. $K = \frac{\pi\gamma^2}{2c_0}$ gives the initial decay rate and also determines the strength of the feedback effects. In the derivation of Eq. (2), we neglected the occurring frequency (Lamb) shift as is frequently done in literature [24,45]. We see that the structure of the continuum leads to the Pyragas control terms. We also see that due to our choice of the structure, no delay terms with $t - 2\tau$, $t - 3\tau$, etc., are present. This means that for Pyragas control there must be no possibility for the photons to spend several round-trip times τ in the reservoir *before* interacting with the node again. Since atoms are very bad mirrors [37], this is automatically the case for atoms as quantum nodes. For high-q cavities, we need to put them outside of the reservoir waveguide structure, as shown in Fig. 1(b), so that the radiation is not reflected back into the reservoir by the cavity mirror without interacting with the cavity mode. However, even in setups that allow for multiple round trips in the reservoir, such as a single cavity mode coupled to a large cavity, a strong frequency dependence of the mode decay dynamics was demonstrated experimentally [46]. Since this will be the important ingredient for our entanglement control scheme, multiple round trips in principle do not deteriorate our approach. However treatments of such systems is beyond the scope of this publication.

For more than one photon, or treatments beyond the Schrödinger equation, e.g., to include further dissipative effects, one has to solve the full dynamics using the Hamiltonian of Eq. (1), including the whole continuum of external modes, instead of being able to use Eq. (2). This is numerically very cumbersome and usually needs further approximations [24,47]. We restrict ourselves to the one-photon case and the structured reservoirs as the only decay channel.

We now discuss the main result of our work, the generation and stabilization of deliberately chosen eigenstates in a quantum network by time-delayed feedback.

IV. TWO COUPLED NODES

We first investigate the simplest of networks, consisting of two nodes, here represented by two coupled single mode cavities of equal angular frequency ω_0 (creation operator $\hat{c}_{1,2}^\dagger$, annihilation operator $\hat{c}_{1,2}$). Both of the nodes are subject to time-delayed feedback of the same strength K and the same feedback delay time τ by coupling to two *separate* structured reservoirs. The two cavities are coupled to each other with a coupling constant M [cf. Fig. 1(c) for a schematic view and (d) for a possible implementation using cavities]. The Hamiltonian of the coupled cavity system, not including the coupling to the reservoirs, reads

$$\hat{H}_{\rm sys} = \hbar\omega_0 \hat{c}_1^\dagger \hat{c}_1 + \hbar\omega_0 \hat{c}_2^\dagger \hat{c}_2 + \hbar M \hat{c}_1^\dagger \hat{c}_2 + \hbar M \hat{c}_2^\dagger \hat{c}_1. \tag{3}$$

This can be realized by cavities either directly coupled through evanescent fields [40,48–50] or through a short, detuned optical fiber [51]. In the latter case, M can be derived from an effective Hamiltonian. In the limit of at most one photon, the coupled cavities can be described by the states $|10\rangle$, $|01\rangle$, and $|00\rangle$, i.e., one photon is in the left cavity and none in the right, or one photon is in the right cavity and none in the left, or there is no photon at all in the cavities, respectively. Solving the Schrödinger equation with a state $|\psi\rangle$ leads us to the following differential equations for the probability amplitudes $c_1 = \langle 10|\psi\rangle$ and $c_2 = \langle 01|\psi\rangle$, including the network node–local reservoir interaction:

$$\frac{d}{dt} c_{1,2}(t) = -i\omega_0 c_{1,2}(t) - iMc_{2,1}(t) - K[c_{1,2}(t) - c_{1,2}(t-\tau)]. \tag{4}$$

The ODE system can be decoupled by transforming into $c_\pm := (c_1 \pm c_2)/\sqrt{2}$. While the states $|10\rangle$ and $|01\rangle$ were separable, the new states $|\pm\rangle := (|10\rangle \pm |01\rangle)/\sqrt{2}$ with the probability amplitudes $c_\pm$ are entangled—a simple manifestation of the fact that a separable state can be regarded as a superposition of entangled states. Our aim is now to choose τ such that one of the entangled states gets stabilized, while the other one decays. Most importantly, we can choose which of the states survives by choosing appropriate feedback. The decoupled system reads as

$$\frac{d}{dt} c_\pm = -i\,(\omega_0 \pm M)\, c_\pm(t) - K[c_\pm(t) - c_\pm(t-\tau)]. \tag{5}$$

Due to the coupling, there exist two modes (entangled in the old basis) with different frequencies $\omega_\pm = \omega_0 \pm M$. Let us focus on the terms describing the coupling to the reservoirs. While the first term, $-Kc_\pm(t)$, describes a decay, the influence of the feedback-induced time-delayed term $+Kc_\pm(t-\tau)$ depends on the state of the system at $t-\tau$. In the case that $c_\pm(t-\tau) = c_\pm(t)$, it cancels the influence of the decay term and leads to a stabilization of the quantum state. An equivalent way to formulate this is that the necessary condition for a stabilization of an oscillation with angular frequency ω is that $\omega\tau$ is an integer multiple of 2π. In contrast, if $c_\pm(t-\tau) \approx -c_\pm(t)$, the decay is enhanced after τ. Usually, the situation (increase or decrease of the decay) differs for the two different angular eigenfrequencies $\omega_\pm$. In particular, one can set τ such that we get a stabilization only for one of the two modes, while the other mode is destabilized. This gives us the ability to control whether a mode is "dark" or "bright" by simply tuning the delay time τ. Since the eigenmodes $|+\rangle$ and $|-\rangle$ are entangled, by stabilizing only one of them we can single out this entangled component from any initial state, even from a fully separable one such as $|10\rangle$. This mechanism is fundamentally different from coupling to a common reservoir [6], where usually only antisymmetric states are dark and therefore do not decay. In our case, we are able to control which state becomes dark and which bright. For specific experimental realizations, especially in the case of atoms as quantum nodes, there might of course be *common* decay channels as well, which also contribute to different decay dynamics for $|+\rangle$ and $|-\rangle$. This is independent of the controllable decay proposed here and can either enhance or decrease the effectiveness of our mechanism. Additionally, the single nodes will experience losses through further decay channels. These losses should be minimized, since they set an upper bound to the lifetime of the entangled states.

In our numerical simulations (see Fig. 2), we choose $\omega_0 = 1\ \text{fs}^{-1}$, $M = 10\ \text{ns}^{-1}$, and $K = 0.52\ \text{ns}^{-1}$, which leads to strong coupling between the cavities ($M > K$) but is

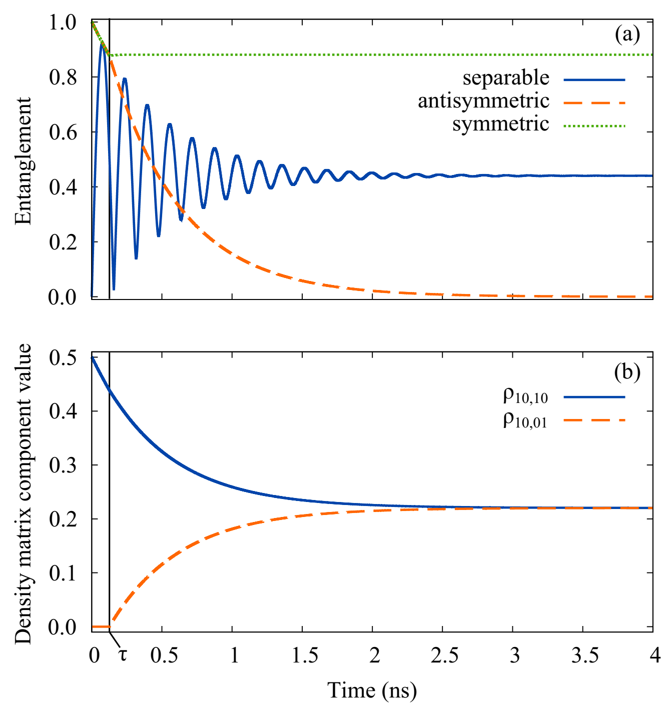

FIG. 2. (Color online) Numerical results of the entanglement between the two cavities discussed in the first section, calculated by the concurrence. Assumed values are given in the text. (a) Entanglement dynamics, starting from different states. Blue, solid line: Starting from the separable state $|10\rangle$, the entanglement reaches its final value slightly above 0.4 after a series of oscillations. Orange, dashed line: Starting from the antisymmetric state $|-\rangle$, which is maximally entangled, the concurrence decreases exponentially with an even higher rate after the feedback time. Green, dotted line: The maximally entangled symmetric state $|+\rangle$ gets stabilized after the feedback delay time. (b) Density matrix elements, starting with a 1:1 incoherent mixture of $|10\rangle$ and $|01\rangle$. Blue, solid line: The diagonal matrix element $\rho_{10,10} = \langle 10|\rho|10\rangle$ first decreases and finally reaches a stable value due to feedback stabilization. Orange, dashed line: The off-diagonal matrix element $\rho_{10,01} = \langle 10|\rho|01\rangle$ stays zero up to the delay time τ, since we are dealing with an incoherent ensemble. After τ, coherence is built up due to the feedback, and $\rho_{10,01}$ increases steadily to its final value slightly above 0.2. This demonstrates the importance of the feedback for the creation of entanglement between the two systems.

in the range of experiments [39,40,48]. We choose $\tau = 4\pi \times 10^4/(\omega_0 + M) \approx 126$ ps, which makes $\omega_+\tau$ an integer multiple of 2π. Therefore, $|+\rangle$ is stabilized while $|-\rangle$ is destabilized. In order to get the most accurate results, we directly solve the Schrödinger equation, using the Hamiltonian of Eq. (1). The reservoir is discretized using 1000 modes equally spaced around ω_0/c_0.

We define the entanglement via the concurrence C, which can be easily calculated as $C = 2|c_1^* c_2|$ [5,52], leading to a value between 0 (no entanglement) and 1 (maximal entanglement). Starting with the separable state $|10\rangle$, we see in Fig. 2(a) (blue, solid curve), that the entanglement C gets stabilized at a value slightly below 0.5 after a series of Rabi oscillations. Analogous to other dissipative entanglement creation mechanisms [3,6–10], the maximally achievable value is $C = 0.5$. Without feedback, C would also show a series of oscillations, however it approaches 0 in the long-time limit. The mechanism of entanglement generation and stabilization becomes clear by looking at the other two curves of Fig. 2(a), where we plot the entanglement dynamics depending on different initial states. Starting as a maximally entangled antisymmetric state $|-\rangle$, we see that the probability of being in the state, and therefore the entanglement, decays rapidly after the feedback delay time τ. The symmetric state $|+\rangle$, however, gets stabilized. Since the separable state $|10\rangle$ is a superposition of these two states, we single out the symmetric component after a set of Rabi oscillations and thereby create stable entanglement between the two cavities.

Stability can be reached, since the system is still "open" despite the introduction of a feedback mirror: The radiation can still escape to infinity traveling away from the mirror [e.g., to the left-hand side in Figs. 1(a), 1(b), and 1(d)]. If the Pyragas condition is met, the radiation in this decay channel however interferes destructively, which leads to a nonzero stable photon density in the nodes.

Figure 2(a) suggests that there is an "average" entanglement in the system also for small times, even below τ, if the concurrence is averaged over one Rabi oscillation period. The real power of our mechanism becomes visible by looking at the dynamics beyond pure state dynamics, i.e., incoherent mixtures. We start with an initially incoherent 1:1 mixture of $|10\rangle$ and $|01\rangle$, described by the density matrix $\rho(t=0) = \frac{1}{2}(|10\rangle\langle 10| + |01\rangle\langle 01|)$. The results are depicted in Fig. 2(b). We see a decay of the diagonal density matrix element $\rho_{10,10} = \langle 10|\rho|10\rangle$ (blue, solid line) due to the coupling to a reservoir. The entanglement C is, however, dependent on the off-diagonal component $\rho_{10,01} = \langle 10|\rho|01\rangle$ (orange, dashed line) and can in our case be calculated as $C = 2|\rho_{10,01}|$. The component $\rho_{10,01}$ starts and stays at zero up to the delay time τ. Afterwards, it strongly increases until it reaches its final value slightly above 0.2 for large times. This clearly demonstrates that *time-delayed feedback induced by local reservoirs is the important quantity to establish nonclassical coherence between the two cavities*. Without feedback, $\rho_{10,01}$, and therefore C, would have stayed 0 all the time. However, even with feedback, both photons will end up in the reservoir with more than 50% probability, leading to $\rho_{00,00} \geqslant 0.5$ (in order to preserve the trace of ρ). Therefore, we will not reach entanglement larger than 0.5 when starting from a fully disentangled state.

Important quantities are the final entanglement $C(t \to \infty)$ and the time the separation of $|+\rangle$ and $|-\rangle$ needs, since a fast separation leads to faster entanglement stabilization. Both quantities can be calculated analytically by a Laplace transform of $c_\pm$ and using it to solve Eq. (5). The separation time is given by the decay rate λ of the unstable component (in the above case, the antisymmetric state), with the constraint that the other component fulfills the Pyragas condition and therefore gets stabilized. This decay rate can be interpreted as a Lyapunov exponent for the entanglement stabilization. It can be calculated as [13,53]

$$\lambda = \frac{1}{\tau}(\mathrm{Re}\{\mathcal{W}[K\tau \exp(K\tau + 2iM\tau)]\} - K\tau), \tag{6}$$

using the Lambert-$\mathcal{W}$ function. For the important long-time behavior, the solution of Eq. (6) with smallest absolute value is dominant. In Fig. 3, we plotted the scaled Lyapunov exponent λ/K over the scaled delay time $K\tau$. We use $M/K = 20$, which is approximately fulfilled in our numerical simulations. We see a series of dips of λ/K. At the largest negative values, the entanglement stabilization mechanism works best. Here the two components can be discriminated very well by time-delayed feedback. We see an optimum at about $K\tau = 0.25$. However, when $\lambda/K = 0$, both components are stabilized. Therefore we do not create stable entanglement, but stabilize Rabi oscillations between the components instead [21].

The poles of the Laplace transform of $c_\pm$ give the long-term dynamics of the system. We find, that when starting from a pure separable state the concurrence equilibrates at

$$C(t \to \infty) = \frac{1}{2(1 + K\tau)^2}. \tag{7}$$

This shows that τ should be kept small to reach high values of entanglement. However, this has to be balanced against the vanishing Lyapunov exponent λ for $K\tau \to 0$, leading to

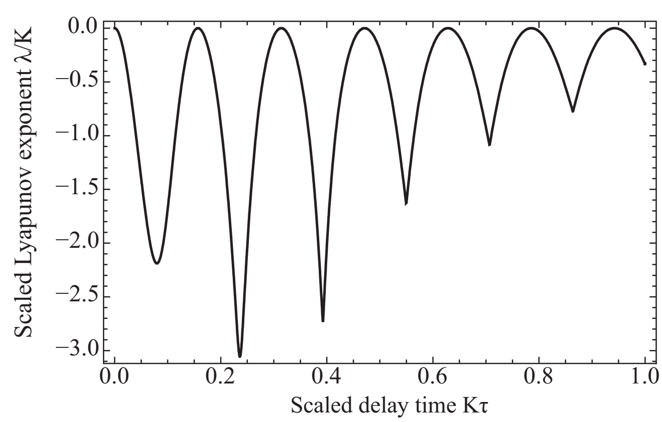

FIG. 3. Scaled Lyapunov exponent λ/K of the stabilization mechanism, calculated using Eq. (6), depending on the scaled feedback delay time $K\tau$. At the largest negative values, the separation between different states is the fastest, which means the entanglement stabilization mechanism works best. When $\lambda = 0$, both components get stabilized, and in our ideal case we never reach an equilibrium, but get stabilized Rabi oscillations instead [21].

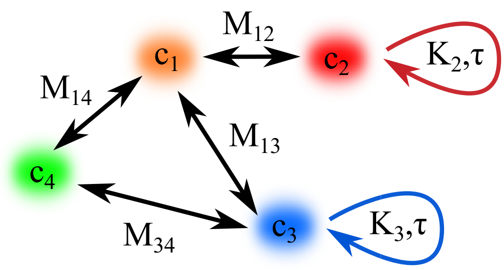

FIG. 4. (Color online) A complex network with many nodes. The several nodes, here depicted as single mode cavities, may have different resonance frequencies. The network is described by the coupling between the nodes via the matrix **M**. Some nodes are subject to time-delayed feedback with individual outcoupling strength K_i. The delay time is assumed to be equal for all feedback loops.

very long separation times. In the limit of $K\tau = 0$, the value $C = 0.5$ is never stabilized. Therefore, time delay is crucial for our mechanism. We also see that very strong coupling $M \gg K$ is not necessary for finite entanglement, however strong coupling leads to an easier and faster separation of the two components and therefore larger negative Lyapunov exponents λ [cf. Eq. (6)].

V. MANY NODE NETWORKS

We now extend our analysis to larger networks containing multiple nodes with different frequencies, which may or may not be coupled to an external feedback reservoir (see Fig. 4). We define the vector $\vec{c}$ such that the nth component c_n is the probability amplitude of node n being in the excited state while all the other nodes are in the ground state. The matrix Ω is a diagonal matrix containing the eigenfrequencies of the nodes, while the matrix **M** gives the couplings between the nodes. The coupling strength to a feedback bath at the different nodes is given by **K**. Diagonal elements of **K** model individual feedback reservoirs that couple only to a single node, while we can include the coherent decay of multiple nodes into *the same* feedback reservoir by off-diagonal elements [3]. We choose the same delay time for all the feedback. The dynamics of $\vec{c}$ is described by

$$\frac{d}{dt}\vec{c} = -i\Omega\vec{c}(t) - i\mathbf{M}\vec{c}(t) - \mathbf{K}[\vec{c}(t) - \vec{c}(t-\tau)]. \quad (8)$$

The kernel ker(**K**) describes decoherence-free subspaces [3,6] also used for entanglement creation. Our aim is to stabilize states not contained in ker(**K**).

We introduce the matrix **T** which diagonalizes $\Omega + \mathbf{M}$ such that

$$\mathbf{T}(\Omega + \mathbf{M})\mathbf{T}^{-1} := \Xi \quad (9)$$

is diagonal. With this, we can define a differential equation for the vector $\vec{\xi} = \mathbf{T}\vec{c}$:

$$\frac{d}{dt}\vec{\xi} = -i\,\Xi\vec{\xi} - \mathbf{TKT}^{-1}[\vec{\xi}(t) - \vec{\xi}(t-\tau)]. \quad (10)$$

With the elements of Ξ, we have the eigenfrequencies Ξ_{ii} of the system at hand, which we can use to choose τ. If we set τ such that it is an integer multiple of the period given by an eigenfrequency, we stabilize the respective eigenstate $\vec{\xi}_{\Xi_{ii}}$ since the term $(\vec{\xi}_{\Xi_{ii}}(t) - \vec{\xi}_{\Xi_{ii}}(t-\tau))$ vanishes when the system equilibrates. These eigenstates are usually highly entangled and can be used, as shown for two coupled cavities, to create entanglement from a separable starting state. We also recognize that we would not have needed to couple both nodes of our simple two node model in the paper equally to the feedback reservoirs. This symmetry made the coupling matrix $\mathbf{TKT}^{-1}$ diagonal, leading to a full decoupling of Eq. (5). However the whole term describing the reservoir interaction vanishes if the Pyragas condition is met, independent of the properties of $\mathbf{TKT}^{-1}$. The only important property is that there is at least one feedback reservoir, providing a delay time τ.

VI. CONCLUSION AND OUTLOOK

We have shown that we can use time-delayed feedback on single nodes in quantum networks to select which of the eigenstates shall be dark and which shall be bright. Since the eigenstates are usually entangled, this can be used to create entangled states from separable states and tune the entanglement within the network.

Due to the conceptual simplicity, our proposal should be applicable to a large variety of realizations of quantum nodes [36,37,43]. Our analysis only covered the case of one excitation in the network. It is certainly interesting to expand the scheme to a higher number of excitations and also include nonlinearities that can lead to a large number of additional network modes.

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[1] H. J. Kimble, Nature (London) **453**, 1023 (2008).
[2] G. Rempe, EPJ Web Conf. **57**, 03001 (2013).
[3] G. Manzano, F. Galve, G. L. Giorgi, E. Hernández-García, and R. Zambrini, Sci. Rep. **3**, 1 (2013).
[4] J. I. Cirac, P. Zoller, H. J. Kimble, and H. Mabuchi, Phys. Rev. Lett. **78**, 3221 (1997).
[5] R. Horodecki, P. Horodecki, M. Horodecki, and K. Horodecki, Rev. Mod. Phys. **81**, 865 (2009).
[6] P. Zanardi and M. Rasetti, Phys. Rev. Lett. **79**, 3306 (1997).
[7] E. del Valle, F. P. Laussy, and C. Tejedor, Europhys. Lett. **80**, 57001 (2007).
[8] J. Li and G. S. Paraoanu, New J. Phys. **11**, 113020 (2009).

[9] H. Krauter, C. A. Muschik, K. Jensen, W. Wasilewski, J. M. Petersen, J. I. Cirac, and E. S. Polzik, Phys. Rev. Lett. **107**, 080503 (2011).
[10] K. G. H. Vollbrecht, C. A. Muschik, and J. I. Cirac, Phys. Rev. Lett. **107**, 120502 (2011).
[11] J. Bechhoefer, Rev. Mod. Phys. **77**, 783 (2005).
[12] K. Pyragas, Phys. Lett. A **170**, 421 (1992).
[13] P. Hövel and E. Schöll, Phys. Rev. E **72**, 046203 (2005).
[14] H. M. Wiseman and G. J. Milburn, *Quantum Measurement and Control* (Cambridge University Press, Cambridge, England, 2009).
[15] C. Sayrin, I. Dotsenko, X. Zhou, B. Peaudecerf, T. Rybarczyk, S. Gleyzes, P. Rouchon, M. Mirrahimi, H. Amini, M. Brune, J.-M. Raimond, and S. Haroche, Nature (London) **477**, 73 (2011).
[16] A. Serafini, ISRN Opt. **2012**, 15 (2012).
[17] N. Yamamoto, H. I. Nurdin, M. R. James, and I. R. Petersen, Phys. Rev. A **78**, 042339 (2008).
[18] G. Kiesslich, C. Emary, G. Schaller, and T. Brandes, New J. Phys. **14**, 123036 (2012).
[19] C. Emary, Philos. Trans. R. Soc. London A **371**, 20120468 (2013).
[20] S. Lloyd, Phys. Rev. A **62**, 022108 (2000).
[21] A. Carmele, J. Kabuss, F. Schulze, S. Reitzenstein, and A. Knorr, Phys. Rev. Lett. **110**, 013601 (2013).
[22] A. L. Grimsmo, arXiv:1502.06959.
[23] R. J. Cook and P. W. Milonni, Phys. Rev. A **35**, 5081 (1987).
[24] U. Dorner and P. Zoller, Phys. Rev. A **66**, 023816 (2002).
[25] G. Hétet, L. Slodička, A. Glätzle, M. Hennrich, and R. Blatt, Phys. Rev. A **82**, 063812 (2010).
[26] S. M. Hein, F. Schulze, A. Carmele, and A. Knorr, Phys. Rev. Lett. **113**, 027401 (2014).
[27] A. Rivas, S. F. Huelga, and M. B. Plenio, Phys. Rev. Lett. **105**, 050403 (2010).
[28] S. F. Huelga, A. Rivas, and M. B. Plenio, Phys. Rev. Lett. **108**, 160402 (2012).
[29] T. Tufarelli, M. Kim, and F. Ciccarello, Phys. Scr. **2014**, 014043 (2014).
[30] B. Bellomo, R. Lo Franco, and G. Compagno, Phys. Rev. Lett. **99**, 160502 (2007).
[31] S. Maniscalco, F. Francica, R. L. Zaffino, N. Lo Gullo, and F. Plastina, Phys. Rev. Lett. **100**, 090503 (2008).
[32] H. Xiang, S. Jin-Qiao, S. Jian, and Z. Shi-Qun, Commun. Theor. Phys. **57**, 29 (2012).
[33] R. Lo Franco, B. Bellomo, S. Maniscalco, and G. Compagno, Int. J. Mod. Phys. B **27**, 1345053 (2013).
[34] L. Mazzola, S. Maniscalco, J. Piilo, K.-A. Suominen, and B. M. Garraway, Phys. Rev. A **79**, 042302 (2009).
[35] T. Yu and J. H. Eberly, Science **323**, 598 (2009).
[36] J. Eschner, C. Raab, F. Schmidt-Kaler, and R. Blatt, Nature (London) **413**, 495 (2001).
[37] G. Hétet, L. Slodička, M. Hennrich, and R. Blatt, Phys. Rev. Lett. **107**, 133002 (2011).
[38] M. Witzany, T.-L. Liu, J.-B. Shim, F. Hargart, E. Koroknay, W.-M. Schulz, M. Jetter, E. Hu, J. Wiersig, and P. Michler, New J. Phys. **15**, 013060 (2013).
[39] A. Yoshihiro, A. Takashi, S. Bong-Shik, and N. Susumu, Nature (London) **425**, 944 (2003).
[40] A. R. A. Chalcraft, S. Lam, B. D. Jones, D. Szymanski, R. Oulton, A. C. T. Thijssen, M. S. Skolnick, D. M. Whittaker, T. F. Krauss, and A. M. Fox, Opt. Express **19**, 5670 (2011).
[41] P. Yao and S. Hughes, Phys. Rev. B **80**, 165128 (2009).
[42] J.-T. Shen and S. Fan, Phys. Rev. A **79**, 023837 (2009).
[43] K. Koshino and Y. Nakamura, New J. Phys. **14**, 043005 (2012).
[44] I. C. Hoi, A. F. Kockum, L. Tornberg, A. Pourkabirian, G. Johansson, P. Delsing, and C. M. Wilson, arXiv:1410.8840.
[45] M. O. Scully and M. S. Zubairy, *Quantum Optics* (Cambridge University Press, Cambridge, England, 1997).
[46] C.-Y. Jin, R. Johne, M. Y. Swinkels, T. B. Hoang, L. Midolo, P. J. van Veldhoven, and A. Fiore, Nat. Nano **9**, 886 (2014).
[47] F. Schulze, B. Lingnau, S. M. Hein, A. Carmele, E. Schöll, K. Lüdge, and A. Knorr, Phys. Rev. A **89**, 041801 (2014).
[48] D. O'Brien, M. D. Settle, T. Karle, A. Michaeli, M. Salib, and T. F. Krauss, Opt. Express **15**, 1228 (2007).
[49] M. J. Hartmann, F. G. S. L. Brandao, and M. B. Plenio, Nat. Phys. **2**, 849 (2006).
[50] M. Hartmann, F. Brandao, and M. Plenio, Laser Photon. Rev. **2**, 527 (2008).
[51] J. Cho, D. G. Angelakis, and S. Bose, Phys. Rev. A **78**, 022323 (2008).
[52] W. K. Wootters, Phys. Rev. Lett. **80**, 2245 (1998).
[53] S. Yanchuk, M. Wolfrum, P. Hövel, and E. Schöll, Phys. Rev. E **74**, 026201 (2006).