Quantum state transfer along a ring with time-reversal asymmetry

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(Received 9 March 2015; published 18 May 2015)

Time-reversal symmetry breaking can enhance or suppress the probability of success for quantum state transfer (QST), and remarkably it can be used to implement the directional QST. In this paper we study the QST on a ring with time-reversal asymmetry. We show that the system will behave as a quantum state turnplate under some proper parameters, which may serve as time controlled quantum routers in complex quantum networks. We propose realizing the quantum state turnplate in the coupled resonator optical waveguide by controlling the coupling strength and the phase.

DOI: 10.1103/PhysRevA.91.052318 PACS number(s): 03.67.Hk, 03.65.−w, 03.67.Ac, 42.50.Pq

I. INTRODUCTION

Quantum state transfer (QST) is one of the basic tasks in the quantum information process. In the past decade QST has been studied intensively. Several schemes are proposed to achieve it by different channels, i.e., spin chains [1–5], polarized photons in the optical fiber [6,7], coupled-cavity array, and so on. Using the spin chain as a channel many schemes are reported, such as, QST along a one dimensional unmodulated spin chain [1], perfect QST achieved by modulating the coupling strength [3,5,8–18], QST without initialization [19,20], optimizing basis [21,22], generalizing to the high spin QST [23–27], arbitrary perfect state transfer [28–32], and relation with number theory [33,34]. The scheme proposed in Ref. [35] shows that coupled-cavity arrays can be a competitive candidate for the quantum channel, which serve as a high dimensional channel and don't need initialization. Recently time-reversal symmetry breaking is introduced to study the QST [36,37], where the time-reversal symmetry breaking can enhance or suppress the probability of the QST and make the QST directional bias. In this sense time asymmetry is a new resource for exploring the QST.

In Refs. [38,39], it is shown that a synthetic magnetic field can be introduced for photons by differential optical paths in the system of coupled resonator optical waveguides (CROW). In this paper we consider the QST along a ring consisting of coupled cavities or coupled resonator optical waveguides with time-reversal asymmetry. Because of the time-reversal symmetry breaking we hope that the quantum state transfer along the ring one by one periodically like a turnplate of quantum states. The quantum turnplate will be useful in building complex quantum networks where it acts as a quantum router. In the following paper we show that a CROW ring will behave as a quantum state turnplate under some proper parameters.

This article is organized as follows. First we give the physical model and make a general analysis. Then, we analyze the dynamic requirement for the quantum state turnplate in a single excitation model, and give the energy spectrum and symmetry matching condition for the quantum state turnplate. In Sec. IV, we study the spectrum of the system with the c_n symmetry. Then, we discuss the effective Hamiltonian of the system using the perturbation method. We come back to the CROW system in Sec. VI. Finally a summary is given.

II. PHYSICAL MODEL

In the CROW the synthetic magnetic field can be introduced by differential optical paths [38,39]. We consider the CROW system in a ring configuration as shown in Fig. 1(a). The Hamiltonian of the ring is

$$H_R = \sum_{l=1}^{N} (J_l \hat{a}_l \hat{a}_{l+1}^{\dagger} + J_l^{*} \hat{a}_l^{\dagger} \hat{a}_{l+1}), \tag{1}$$

where J_l is the coupling strength of between the sites $l-1$ and l, and $N+1$ is interpreted as 1. $\hat{a}_l$ ($\hat{a}_l^{\dagger}$) is the annihilation (creation) operator. From Ref. [35] we know that the condition for transferring any single mode photon state from node l to node l' is

$$\hat{a}_{l'}(\tau) = \hat{a}_l, \tag{2}$$

where $\hat{a}_{l'}(\tau) = U^{\dagger}(t)\hat{a}_{l'}U(t)$ with $U(t)$ being the time evolution operator. It can be easily verified by noting that the expect value of any operator in the l'th node at time τ is equal to that of the operator in the lth node in the initial state.

Using the Heisenberg equation,

$$\frac{d\hat{a}_l(t)}{dt} = i\,[H_R, \hat{a}_l(t)]\,, \tag{3}$$

and noting that $\hat{a}_l(t)$ can be expressed on the operator bases as

$$\hat{a}_l(t) = \sum_{k=1}^{N} \alpha_k(t)\hat{a}_k,$$

the evolution of the operator $\hat{a}_l(t)$ can be written as

$$i\frac{dA}{dt} = -H(J)A, \tag{4}$$

where $A = [\alpha_1(t), \alpha_2(t), \ldots, \alpha_N]^T$ with T being the transpose operation, and

$$H(J) = \begin{bmatrix} 0 & J_1 & 0 & \cdots & J_N^{*} \\ J_1^{*} & 0 & J_2 & \cdots & 0 \\ 0 & J_2^{*} & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ J_N & 0 & 0 & \cdots & 0 \end{bmatrix}. \tag{5}$$

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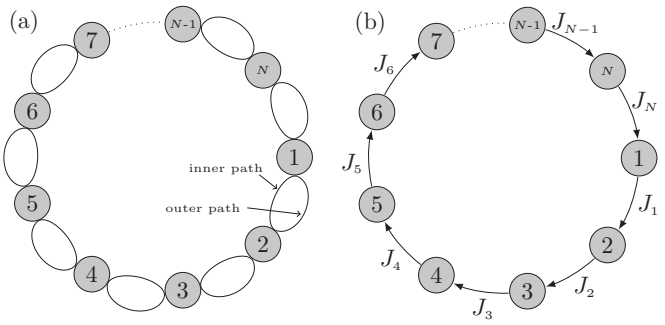


FIG. 1. (a) The CROW ring. The circles mean the site resonators, and the link optical waveguides link the resonators as a ring. Every waveguide between two site resonators has two paths, the outer path and the inner path. The two paths have different lengths which induce the complex coupling strength J. (b) Single excitation graph. The circles correspond to the resonators in the CROW system. The arrow with labels J_l represents the matrix element $J_l|l\rangle\langle l+1|$.

The initial condition is $A(0) = [0, 0, \ldots, 1, \ldots, 0]^T$, where 1 is the lth element. With the above initial condition, Eq. (4) describes the single excitation evolution in the ring with coupling strength $\{-J_i\}$.

So the transfer of any single mode photon state in the CROW ring has the same physical picture as the single excitation model, and we do not need to initialize the state of other sites except the input one.

III. SINGLE EXCITATION RING

First, we consider the single excitation model. To sketch our central idea, we consider the system depicted by a graph in Fig. 1(b) consisting of N sites as a ring, with the Hamiltonian $H(J)$ having the form given in Eq. (5), where J is the set $\{J_l\}$. Let us denote Φ as the set $\{\phi_l\}$, where ϕ_l is the complex phase of J_l. From Ref. [37] we know that the phase Φ can give rise to the time-reversal asymmetry if the graph is nonbipartite graph (N is odd). In this article we concentrate on the cases where N is odd.

Through local unitary operator U^L, the Hamiltonian can be transformed to

$$H(J') = U^L H(J) U^{L\dagger},$$

where $\sum_l \phi'_l = \sum_l \phi_l$ and $|J'_l| = |J_l|$. In other words, only the sum of phases $\sum_l \phi_l$ is relative to the properties of QST. So we can choose a proper operator U^L to make all the phases equal, $\phi'_l = \frac{\sum_l \phi_l}{N}$, with the QST properties in the time evolution unchanged.

Now we consider the question: In what condition would the system behave like a turnplate of quantum state? First, we require that the system have the symmetry of the cyclic group c_n for the turnplate having n ($n \leqslant N$) scales on it, i.e., N/n is an integer. The operators of the c_n group elements can be expressed as

$$\mathcal{T}_n, (\mathcal{T}_n)^2, \ldots, (\mathcal{T}_n)^{n-1}, 1,$$

where

$$\mathcal{T}_n = e^{iL_n \frac{2\pi}{n}},$$

and L_n is a Hermitian operator. From $(\mathcal{T}_n)^n = 1$, we know that L_n has n integer eigenvalues, $l \in \{\lfloor -\frac{n-1}{2} \rfloor, \lfloor -\frac{n-1}{2} \rfloor + 1, \ldots, \lfloor \frac{n-1}{2} \rfloor\}$ where $\lfloor x \rfloor$ is the largest integer not greater than x. Here we only consider the case where n is odd. For the Hamiltonian we have the relation $[H, L_n] = 0$. Let the eigenstate of the system be $|\psi_{l,m}\rangle$, that is,

$$H|\psi_{l,m}\rangle = E_{l,m}|\psi_{l,m}\rangle,$$

and

$$L_n|\psi_{l,m}\rangle = l|\psi_{l,m}\rangle.$$

Now we prove that the system will be a turnplate of quantum states with n scales if the eigenvalues $E_{l,m}$ match the symmetry c_n in the following way:

$$E_{l,m} = \left(\frac{l}{n} + Z_m\right)\epsilon + \epsilon_0, \tag{6}$$

where Z_m is an integer, and ϵ_0 correspond to the phase that is not an observable in physics.

Let the initial state of the system be $|\psi_0\rangle$. It can be easily proved that at time $\tau = \frac{2\pi}{\epsilon}$,

$$|\psi(\tau)\rangle = e^{i\pi\epsilon_0/\epsilon}\mathcal{T}_n|\psi_0\rangle, \tag{7}$$

meaning that the quantum state of the system turns one scale every time interval τ. The energy and symmetry matching condition can be seen as the generalization of the energy and parity matching condition mentioned in Ref. [40], where the parity matching condition corresponds to the case $n = 2$ and $\epsilon = 2E_0$.

Here we look at the particular case $N = n = 3$. The relation $[H, \mathcal{T}_3] = 0$ requires that $J_1 = J_2 = J_3$. In the following of this section we suppose that $|J_1| = |J_2| = |J_3| = 1$. Using the property of the c_3 group we know H and L_n have the same eigenvectors $|\phi_l\rangle = \frac{1}{\sqrt{3}}[1, \omega_3^l, \omega_3^{2l}]^T$ with $\omega_3 = e^{i\frac{2\pi}{3}}$ and $l \in \{0, \pm 1\}$. From Eq. (6) we get the equations,

$$\begin{aligned} E_1 &= \left(\frac{1}{3} + Z_1\right)\epsilon + \epsilon_0, \\ E_0 &= Z_0\epsilon + \epsilon_0, \\ E_{-1} &= \left(\frac{-1}{3} + Z_{-1}\right)\epsilon + \epsilon_0, \end{aligned} \tag{8}$$

where E_1, E_0, and E_{-1} are the eigenvalues for $l = 1, l = 0$, and $l = -1$ respectively. Now we need to find three integers (Z_1, Z_0, and Z_{-1}) and two reals (ϵ, ϵ_0) that make the equations hold. From the characteristic polynomial of Hamiltonian, $\lambda(3 - \lambda^2) + 2\cos(\phi) = 0$, we have

$$E_1 + E_0 + E_{-1} = 0.$$

That is,

$$\epsilon_0 = -\frac{Z_0 + Z_1 + Z_{-1}}{3}\epsilon.$$

So we get two independent equations,

$$\begin{aligned} E_1 &= \left(\frac{1}{3} + Z_1 - \frac{Z_2}{3}\right)\epsilon, \\ E_0 &= \left(Z_0 - \frac{Z_2}{3}\right)\epsilon, \end{aligned} \tag{9}$$

where $Z_2 = Z_0 + Z_1 + Z_{-1}$. Divide both sides, then we get

$$\frac{E_1}{E_0} = \frac{1 + 3Z_1 - Z_2}{3Z_0 - Z_2}. \tag{10}$$

We can easily get the eigenvalues,

$$E_0 = 2\cos(\phi_i), \tag{11}$$

$$E_1 = 2\cos\left(\frac{2\pi}{3} + \phi_i\right). \tag{12}$$

Substituting them into Eq. (10) we get

$$\frac{1 + 3Z_1 - Z_2}{3Z_0 - Z_2} = -\frac{1}{2} - \frac{\sqrt{3}}{2}\tan(\phi_i). \tag{13}$$

Equation (13) gives the condition when the ϕ_i satisfies the energy and symmetry matching condition for the case $N = 3$. Let us see two examples. The first one is that $Z_1 = -1$, $Z_0 = Z_{-1} = 0$ then $\phi_i = \frac{\pi}{6}$. Let $J_1 = J_2 = J_3 = e^{i\frac{\pi}{6}}$, that is, the total phase is $\pi/2$, and the eigenvalues are $E_{l=0} = \sqrt{3}$, $E_{l=1} = -\sqrt{3}$, and $E_{l=-1} = 0$. From Eq. (9) we get $\epsilon = 3\sqrt{3}$ and the time interval $\tau = \frac{2\pi}{3\sqrt{3}}$. We numerically simulate the time evolution of this case and show the probability of the wave function in Fig. 2(a). The second one is that $Z_{-1} = -1$, $Z_0 = 2$, $Z_1 = 1$, then $\phi_i = \arctan(-\frac{2}{\sqrt{3}})$, $\epsilon = \frac{3\cos(\phi_i)}{2}$. We simulate the time evolution of this case in Fig. 2(b).

In the examples, we give the values of Z_{-1}, Z_0, and Z_1 and then find the corresponding ϕ_i. Now the question is whether there is a set of $\{Z_{-1}, Z_0, Z_1\}$ that makes Eq. (13) hold for arbitrary ϕ_i. The answer is no, but we can always find the set of Zs that make Eq. (13) hold approximately with arbitrary high accuracy.

Equation (13) is equivalent to

$$-\frac{3}{2} - \frac{\sqrt{3}}{2}\tan(\phi_i) = \frac{1 + 3Z_4}{Z_3}, \tag{14}$$

where Z_3 and Z_4 are two independent integers. $Z_3 = 2Z_0 - Z_1 - Z_{-1}$ and $Z_4 = Z_1 - Z_0$. For a given precision, we can always find two integers P and Q; those make the approximate equation,

$$-\frac{3}{2} - \frac{\sqrt{3}}{2}\tan(\phi_i) \approx \frac{P}{Q},$$

hold. When $Z_3 \gg 1$, we get

$$\frac{P}{Q} \approx \frac{3Z_4}{Z_3}.$$

So $Z_3 = 3Z_5 Q$ and $Z_4 = Z_5 P$ with Z_5 making sure that $Z_3 \gg 1$. From the second equation of Eqs. (9) and Eq. (11) we get

$$\epsilon = \frac{6\cos(\phi_i)}{Z_3}. \tag{15}$$

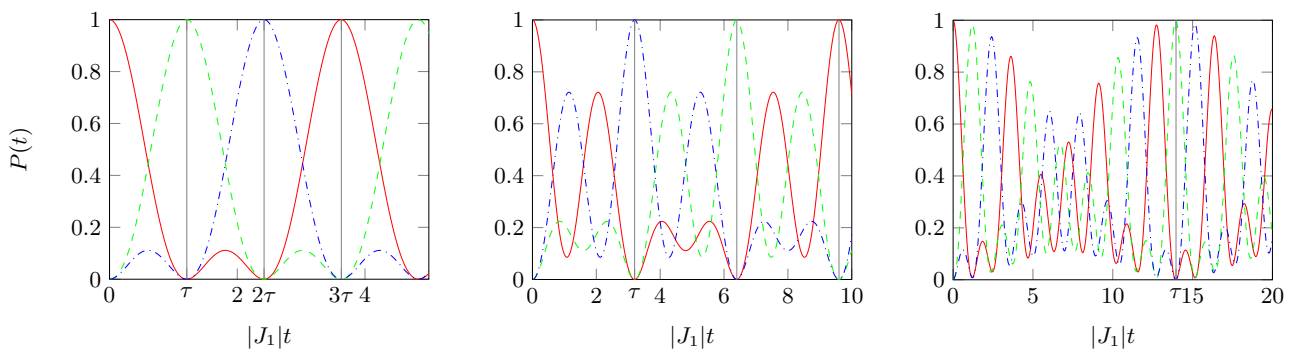
So when $Z_3 \gg 1$, $\tau \gg 1$. It means that for the special case, $N = 3$, the energy and symmetry matching condition can always be satisfied approximately for arbitrary ϕ_i, but it is most likely that we need to wait for a long time τ for a quantum turnplate; see Fig. 2(c). The case that the total phase $\phi = \pi/2 + k\pi$ is one of the best conditions in which we get the turnplate soon compared with the character time of the system.

Given the single excitation condition, it was proved that there is a pretty good state transfer between any two sites of a uniform ring with total phase as $\pi/2 + k\pi$, if the number of the site of the ring is prime [41]. It indicates that the energy spectrum and symmetry matching condition, Eq. (6), is approximately satisfied when the length of the ring is prime.

IV. STRUCTURE OF THE SPECTRUM

In this section we analyze the energy spectrum of the system described by the Hamiltonian in Eq. (5). We start with the definitions of some notations. We denote the characteristic polynomial as A_N, that is, $A_N = \det(H_N - \lambda)$. And denote B_N as $B_N = \det(H_N(J_N = 0) - \lambda)$, where $H_N(J_N = 0)$ represents the Hamiltonian where the coupling between the first site and the last site is zero, i.e., the chain is an open one. From Ref. [42] we know that

$$B_N = \begin{cases} \lambda g(\lambda^2) & \text{if } N \text{ is odd,} \\ g(\lambda^2) & \text{if } N \text{ is even,} \end{cases}$$



(a) $\phi_i = \frac{\pi}{6}$ (b) $\phi_i = \arctan\left(-\frac{2}{\sqrt{3}}\right)$ (c) $\phi_i = \frac{\pi}{7}$

FIG. 2. (Color online) Numerical stimulation of the time evolution of the system consisting of three sites with the parameters $J_1 = J_2 = J_3 = e^{i\phi_i}$ and the initial state $|100\rangle$. (a) $\phi_i = \frac{\pi}{6}$. (b) $\phi_i = \arctan(-\frac{2}{\sqrt{3}})$. (c) $\phi_i = \frac{\pi}{7}$. $P(t)$ is the probability of the wave function. The solid line, dashed line, and dash-dotted line describe the probability at site 1, site 2, and site 3, respectively. At time τ the excitation transfers from site 1 to site 2 and it transfers to site 3 after the next time interval τ. The system acts like a turnplate of the excitation.

where $g(x)$ is an arbitrary function of x. So the determinate of A_N with odd N is

$$\begin{aligned}\det A_N &= -\lambda \det B_{N-1} - J_1^2 \det B_{N-2} \\ &\quad + \prod_{l=1}^{N} J_l e^{i\phi} + \prod_{l=1}^{N} J_l e^{-i\phi} - J_N^2 \det B_{N-2} \\ &= \lambda f(\lambda^2) + \lambda g(\lambda^2) + 2\prod_l J_l \cos\phi \\ &= \lambda F(\lambda^2) + \prod_l J_l 2\cos\phi,\end{aligned}$$

where ϕ is the total phase. When $\phi = \frac{\pi}{2} + k\pi, k = 0, \pm 1, \pm 2, \ldots$, the spectrum has the structure $\{0, \pm E_l\}$, that is, the spectrum is symmetric around 0. Let us consider the system that has the C_n symmetry and contains $N = n \times p$ sites. The eigenvalues of the operator L_n is $0, \pm 1, \ldots, \pm\frac{n-1}{2}$, which we label as l, and every eigenvalue has p-fold degeneracy. We can easily write out the eigenvector of L_n,

$$\begin{aligned}|l,i\rangle = &\frac{1}{\sqrt{n}}\left(|i\rangle + \omega_n^l |i+p\rangle + \omega_n^{2l}|i+2p\rangle\right. \\ &\left. + \cdots + \omega_n^{(n-1)l}|i+(n-1)p\rangle\right).\end{aligned}$$

So L_n can be expressed explicitly as

$$L_n = \sum_{l=-\frac{n-1}{2}}^{\frac{n-1}{2}} \sum_{i=1}^{p} l|l,i\rangle\langle l,i|. \tag{16}$$

Let $P_l = \sum_i |l,i\rangle\langle l,i|$. From the relation $[H, L] = 0$, we know that

$$P_l H P_{l'} = 0 \quad \text{for } l \neq l'.$$

This means that the Hamiltonian is block diagonalized under the bases $\{|l,i\rangle\}$. Because the system has the c_n symmetry, we have the relation $J_i = J_{i+p}$. So there are only p parameters, $J_1, J_2, \ldots, J_p$. We can always let $J_1 = 1$ and other parameters be the ratio to J_1. Then the property of the system doesn't change up to the time scales. So there are $p-1$ parameters that we need to consider.

Using the bra ket form of the Hamiltonian,

$$\begin{aligned}H = &J_N|N\rangle\langle 1| + J_N^*|1\rangle\langle N| \\ &+ \sum_{k=1}^{N-1}(J_k|k\rangle\langle k+1| + J_k^*|k+1\rangle\langle k|),\end{aligned} \tag{17}$$

and acting the projector P_l on both sides of Eq. (17) we can directly give

$$\begin{aligned}P_l H P_l = &\omega_n^l J_p|p\rangle\langle 1| + \omega_n^{l*} J_p^*|1\rangle\langle p| \\ &+ \sum_{k=1}^{p-1}\left(J_k|k\rangle\langle k+1| + J_k^*|k+1\rangle\langle k|\right).\end{aligned} \tag{18}$$

Comparing Eqs. (17) and (18) we get that in every block the Hamiltonian is equivalent to the Hamiltonian of the ring with length p and the moduli of the coupling strength are not changed just with the total phase changing from ϕ/n to $\phi_l = \frac{\phi}{n} + \frac{2l\pi}{n}$ (see Fig. 3). So the characteristic function can

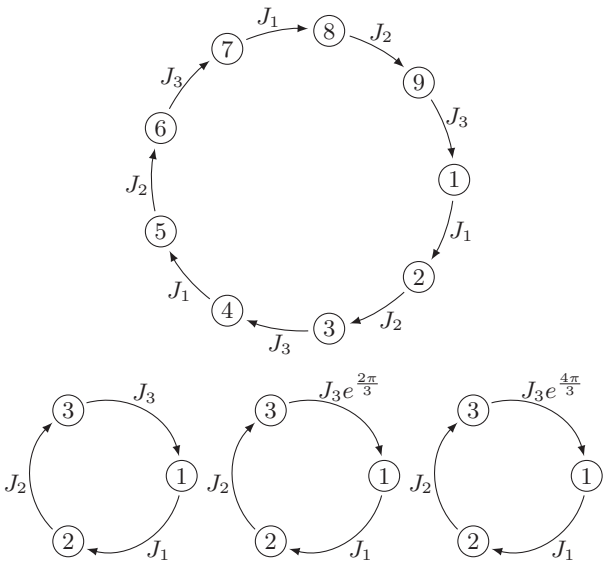


FIG. 3. The ring with nine nodes and c_3 symmetry. Under the bases $|l,i\rangle$ the Hamiltonian is block diagonalized consisting of three blocks which is implied by c_3. In every block the Hamiltonian represents a ring with length 3 and the coupling strength is the same as the original ring in the site bases with the total phase being $\phi/3$, $\phi/3 + 2\pi/3$, and $\phi/3 + 4\pi/3$ where ϕ is the total phase of the original ring.

be written as

$$\lambda F(\lambda^2) + 2\prod_{k=1}^{p} J_k \cos\phi_l = 0,$$

where $l = -\frac{n-1}{2}, \ldots, \frac{n-1}{2}$. Every function means a curve that crosses with the axis of the variable p times corresponding to the p roots; see Fig. 4(a). All the curves have the same shape. When the total phase $\phi = \frac{\pi}{2} + k\pi$, the curve corresponding to $l = -\frac{n+1}{4}$ (or $l = \frac{n-1}{4}$) crosses the original point and we call it curve 0. Other curves can be obtained from the curve 0 by translating $2\prod_k J_k \cos\phi_l$ along the vertical axis. So if $\prod_{k=1}^{p} J_k$ are little enough the spectrum of the Hamiltonian has the shape indicated in Fig. 4(b), that is, the spectrum consists of separated groups.

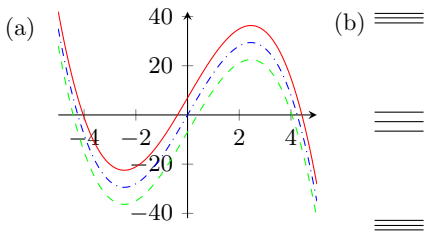


FIG. 4. (Color online) Spectrum structure of the Hamiltonian H_9 with length $N = 9$ and C_3 symmetry. The coupling strength is $J_1 = J_3 = e^{\frac{\pi}{18}}$, $J_2 = 4J_1$. So the total phase is $\phi = \pi/2$. (a) Pictures of the characteristic polynomials of three equivalent Hamiltonians obtained from the Hamiltonian H_9. The dash-dotted line is the curve 0 corresponding to $l = -1$, and the other two lines can be obtained from curve 0 by translating along the vertical axis. (b) Spectrum of the Hamiltonian H_9. Every energy lever corresponds to one cross point of the curve and the horizontal axis in (a).

V. EFFECTIVE HAMILTONIAN

Now we introduce the approximate method based on the spectrum structure. To discuss concretely, we consider the system with nine sites ($N = 9$) and the c_3 symmetry. Its configuration is shown in Fig. 3. The eigensystem of the Hamiltonian H is equivalent to the Hamiltonian H' with

$$J'_k = |J_k|, \quad \text{for} \quad k \neq 1,$$
$$J'_1 = |J_1|e^{i\phi},$$

where $\phi = \sum_k \phi_k$. So we consider the Hamiltonian H'. Let $J'_2 \gg J'_{1,3}$ and write the Hamiltonian H' into two terms, $H' = H'_0 + V$. Given that the H' is represented in the site basis $|i\rangle$, H'_0 consists of the terms containing J'_2, and V consists of the terms containing $J'_{1,3}$. V is the perturbation compared with H'_0.

The eigenvalues of the Hamiltonian H'_0 are $\alpha \in \{0, \pm J'_2\}$ and every energy level has threefold degeneracy. So the energy levels are separated into three groups (manifold) corresponding to three αs. Using i to label the different bases we denote the three manifolds as $|i, \alpha\rangle$. In the manifold with $\alpha = 0$ the three bases are

$$|1\rangle, |4\rangle, |7\rangle.$$

And the manifolds with $\alpha = \pm|J_2|$ are spanned by the bases,

$$|X^+_{23}\rangle, |X^+_{56}\rangle, |X^+_{89}\rangle,$$

and

$$|X^-_{23}\rangle, |X^-_{56}\rangle, |X^-_{89}\rangle,$$

respectively, where

$$|X^+_{ij}\rangle = \frac{|0\rangle_i + |1\rangle_j}{\sqrt{2}}, \quad \text{and} \quad |X^-_{ij}\rangle = \frac{|0\rangle_i - |1\rangle_j}{\sqrt{2}}.$$

We take V as perturbation then compute the effective Hamiltonian in the manifold $\alpha = 0$. And the effective Hamiltonian is

$$H^{\alpha=0}_{\text{eff}} = -\begin{bmatrix} 0 & ge^{i\phi} & g \\ ge^{-i\phi} & 0 & g \\ g & g & 0 \end{bmatrix},$$

where $g = \frac{J_1 J_3}{J_2}$. $H^{\alpha=0}_{\text{eff}}$ is identical to the representation of the Hamiltonian of the ring consisting of three nodes with uniform coupling strength g and total phase ϕ. From the analysis in Sec. III we know that when $\phi = \frac{\pi}{2} + k\pi$ the system is a turnplate of a quantum state with three scales and the time interval of the transfer state from one node to the next is $\frac{2\pi}{3g\sqrt{3}}$. We numerically simulate the time evolution of the system with parameters $J_1 = J_3 = 1$, $J_2 = 100$ and $\phi = -\frac{\pi}{2}$ from the initial state $|\psi(0)\rangle = |1\rangle$ in Fig. 5. At time $\tau = 120.92$ the excitation transfers from site 1 to site 4 and after the same time interval it transfers to site 7 then back to site 1 circularly. The sites, except the sites 1, 4, and 7 can't be excited.

The correspondence between the propriety of the system with site number $N = n \times p$ and $N = n$, where n is decided by the symmetry of the large system, can be generalized to the general case. Let $J_{1(p)} \ll J_{l\neq 1(p)}$, then the Hamiltonian can be written as $H = H_0 + V$, where V is the perturbation term consisting of the terms containing $J_{1(p)}$, and H_0 is the other term. The spectrum of the Hamiltonian H_0 has the form depicted in Fig. 6. The zero energy level is n-fold degeneracy with degenerate ket $|m^{(0)}\rangle$, where $m = 1 + l \times p$ and $l = 1, 2, \ldots, n-1$. All the $|m\rangle$s span the manifold $\mathcal{M}_{\alpha=0}$. The energies greater and less than zero distribute two sides of the zero-energy level symmetrically with an energy gap and every energy level is n-fold degeneracy. From the perturbation theory in the degenerate case we know that up to the first order the eigenkets of Hamiltonian H corresponding to the manifold $\mathcal{M}_{\alpha=0}$ are

$$|\phi\rangle = \sum_m c_m |m\rangle - \sum_{k\notin\mathcal{M}} \frac{|k^{(0)}\rangle V_{k,m}}{E^{(0)}_k},$$

where $V_{k,m} = \langle k|V|m\rangle$ and $|k^{(0)}\rangle$ is the eigenket of H_0 which is not in the manifold $\mathcal{M}_{\alpha=0}$. So when $V_{k,m}$ is much less than $E^{(0)}_k$ (the gap on the zero energy level), the manifold $\mathcal{M}_{\alpha=0}$ is close, i.e., if the initial state is $|m\rangle$ then the system is governed by the effective Hamiltonian $H^{\alpha=0}_{\text{eff}}$ which represents a Hamiltonian of the n-site cycle. So the system with $N = n \times p$ and c_n symmetry can be reduced to the system with $N = n$ and with c_n symmetry.

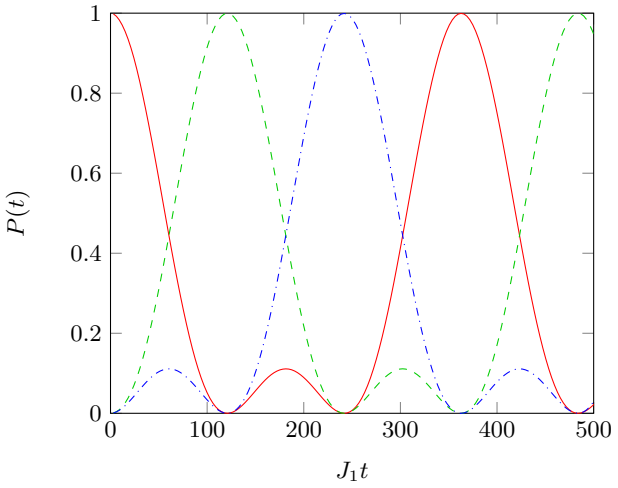


FIG. 5. (Color online) Numerically simulate the time evolution of the system with nine sites. $J_1 = J_3 = 1$, $J_2 = 100$, and $\phi = -\frac{\pi}{2}$. The initial state is $|\psi(0)\rangle = |100000000\rangle$ with $|1\rangle$ as the input state. The solid line presents the fidelity of the state transfer for the state of the first site at a different time. The dashed line and the dash-dotted line corresponding to the fidelity for the seventh site and fourth site, respectively.

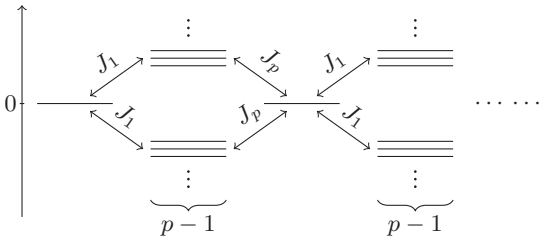


FIG. 6. Spectrum of the H_0 which is the main part of the Hamiltonian. Every zero energy level denotes the Hilbert space of the sites labeled by number $(1 + l \times p)$, and every grouped $p - 1$ level denotes the Hilbert space of the sites from $(2 + l \times p)$ to $p + (l \times p)$, respectively, where $l = 0, 1, \ldots, n-1$.

VI. QUANTUM TURNPLATE ON THE CROW RING

Now we come back to the physical system, the CROW ring. For the ring containing three resonators, they can be connected by three identical connecting waveguides, which contribute the same coupling strength $|J_l|$ and phase ϕ_l. In order to make a quantum turnplate we just need to modify the optical path to make total phase $\phi = \pi/2 + k\pi$. Initially we input the photonic state to node 1. Then we will see that the photonic state will transfer from node 1 to node 2, node 3, and back to node 1 cyclicly with perfect fidelity every time interval τ. For the ring containing $N = 3 \times p$ resonators, which has the c_3 symmetry, we need to modify the coupling strength between resonators and connecting waveguides to satisfy the condition $J_{1(p)} \ll J_{l\neq 1(p)}$ and change the optical path to make the total phase be $\phi = \pi/2 + k\pi$. Then photonic states can be transferred among the site 1, $p+1$, and $2p+1$ cyclicly with high fidelity.

We simulate the QST along the CROW ring consisting of nine resonators with c_3 symmetry. The parameters are $|J_1| = |J_3| = 1$, $|J_2| = 100$ and the total phase $\phi = \pi/2$. Initially the state $|\psi\rangle = (|0\rangle + |1\rangle + |2\rangle)/\sqrt{3}$ is input into the first resonator. Then we observe transfer of $|\psi\rangle$ along the ring. In Fig. 7 we plot the time evolution of the fidelity,

$$F(t) = \langle\psi|\rho_i(t)|\psi\rangle,$$

of sites 1, 4, and 7 and it behaves as a quantum state turnplate. ρ_i means the reduced density matrix of site i.

VII. DISCUSSION AND SUMMARY

Using the similar method we used to get Eq. (7) we have the equation,

$$\hat{a}_l(\tau) = e^{i\pi\epsilon_0/\epsilon}\hat{a}_{l-p}, \tag{19}$$

for the annihilation operator of the photon in the CROW ring. So the basis for the quantum state in different sites should be identified. For example, in the ring with nine sites the bases for site 1 and site 4 should be $\{|0\rangle, |1\rangle, |2\rangle, \ldots\}$ and $\{|0\rangle, e^{i\pi/3}|1\rangle, e^{i2\pi/3}|2\rangle, \ldots\}$, respectively. Light scattered from the resonators can be imaged using an infrared camera. Directly the quantum turnplate will be observed from the image of the camera.

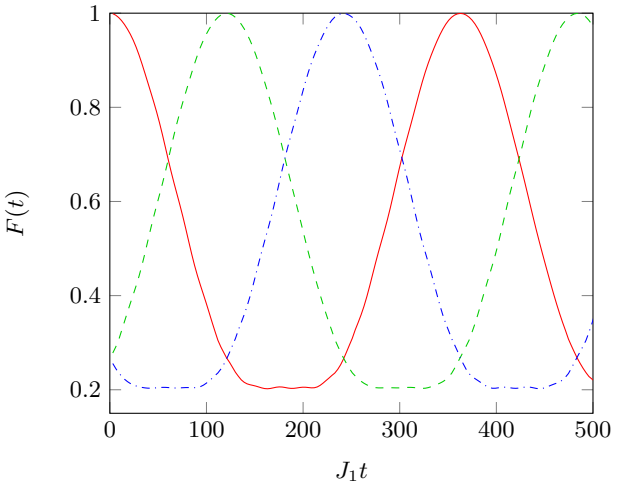


FIG. 7. (Color online) Numerically simulate the time evolution of the fidelity of the CROW system with nine resonators. $J_1 = J_3 = 1$, $J_2 = 100$, and $\phi = \frac{\pi}{2}$. The initial state is $\frac{1}{\sqrt{28}}(|0\rangle + 2|1\rangle + 3|2\rangle)(|1000000\rangle + |1100000\rangle)$ with $\frac{1}{\sqrt{14}}(|0\rangle + 2|1\rangle + 3|2\rangle)$ as the input state. The solid line represents the fidelity of the state transfer for the state of the first site at a different time. The dashed line and the dash-dotted line correspond to the fidelity for the fourth site and seventh site, respectively.

In summary, we study the QST on the ring of coupled cavities with time-reversal asymmetry. The transfer of any single mode photon state in the CROW ring has the same physical picture as the single excitation model, and we do not need to initialize the state of other sites except the input one. To act as a quantum state turnplate the eigenvalues of the equivalent single excitation model should satisfy the matching condition Eq. (6). We study the case $N = 3$ in detail. It is shown that the matching condition can be satisfied approximately for arbitrary phase ϕ and if the total phase $\phi = \pi/2 + k\pi$, the matching condition is satisfied exactly with good scale time τ. Furthermore, we study the structure of the spectrum of the single excitation ring in general condition and prove the QST equivalent between the ring consisting of n sites and the one consisting of $n \times p$ sites with c_n symmetry. Utilizing the time-reversal asymmetry the CROW consisting of $3 \times p$ resonators can sever as a quantum turnplate without initialization, which can also be observed in experiments. Quantum state turnplates would be useful to build a complex quantum network.

ACKNOWLEDGMENTS

This work is supported by NSF of China (Grants No. 11475254 and No. 11175247) and NKBRSF of China (Grants No. 2012CB922104 and No. 2014CB921202).

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