Floquet control of quantum dissipation in spin chains

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Controlling the decoherence induced by the interaction of quantum system with its environment is a fundamental challenge in quantum technology. Utilizing Floquet theory, we explore the constructive role of temporal periodic driving in suppressing decoherence of a spin-1/2 particle coupled to a spin bath. It is revealed that, accompanying the formation of a Floquet bound state in the quasienergy spectrum of the whole system including the system and its environment, the dissipation of the spin system can be inhibited and the system tends to coherently synchronize with the driving. It can be seen as an analog to the decoherence suppression induced by the structured environment in spatially periodic photonic crystal setting. Comparing with other decoherence control schemes, our protocol is robust against the fluctuation of control parameters and easy to realize in practice. It suggests a promising perspective of periodic driving in decoherence control.

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I. INTRODUCTION

As a ubiquitous phenomenon in microscopic world, decoherence is a main obstacle to the realization of any applications of quantum coherence, e.g., quantum information processing [1], quantum metrology [2], and quantum simulation [3]. Many methods, such as feedback control [4,5], decoherence-free subspace encoding [6,7], and dynamical decoupling [8–11], have been proposed to beat this unwanted effect. The dynamical decoupling scheme can be generally described by the so-called spectral filtering theory in the first-order Magnus expansion [12–16], which is valid when the control pulses are sufficiently rapid. Based on the spin echo technique, this scheme is widely exploited to suppress dephasing [8,10,11,13], where the system has no energy exchange with the environment, and classical noises [14–16]. It requires a high controllability to the system due to its sensitivity to the time instants at which the inverse pulses are applied [17]. Furthermore, when dissipation and quantum noises are involved, it generally cannot perform well. Although the dissipation control was partially touched in the original form of the spectral filtering theory [12], whether the first-Markovian approximation used there can capture well the physics for such time-dependent systems is still an open question. This is because that new time scales would be introduced to the systems by the time-dependent control field, which might invalidate the application of the Markovian approximation.

As a main inspiration, we notice that the decoherence of dissipative systems connects tightly with the energy-spectrum characters of the total system consisting of the system and its environment [18–20]. If a bound state residing in the energy band gap of the whole system is formed by changing the environmental spectral density, the decoherence of the system can be suppressed. Thus one can artificially engineer the bound state to suppress decoherence of quantum emitters by introducing spatial periodic confinement in a photonic crystal setting [21–24]. Here the spatial periodic confinement dramatically alters the dispersion relation of the radiation field of the quantum emitter such that a certain band gap structure is present in the spectral density. If the frequency of the quantum emitter resides in the band gap region, then a bound state is formed and thus the decoherence of the emitter can be suppressed. However, in practical solid-state systems, one generally faces that it is hard to manipulate the spectral density via changing the spatial confinement once the material of system is fabricated. Thus a more efficient way in engineering the bound state than changing the spectral density is desired.

Recently, temporal periodic driving has become a highly controllable and versatile tool in quantum control. Many efforts have been devoted to explore nontrivial effects induced by periodic driving on physical systems. It has been proven to play profound role not only in controlling single-quantum-state of microscopic systems [25–32] and implementing geometric phase gates in quantum computation [33,34], but also in generating novel states of matter absent in the original static system [35–42]. Different from static systems, periodically driven systems have no stationary states because the energy is not conserved. Due to Floquet theory, they have well-defined quasistationary-state properties described by the Floquet eigenvalues, which are called quasienergies. The distinguished role of periodic driving in these diverse systems is that the versatility of driving schemes can induce more colorful quasistationary-state behaviors than the static case by controlling the quasienergy spectrum.

In this paper, we explore the possibility of periodic driving on engineering the bound state of a spin-1/2 system interacting with a *XX*-type coupled spin bath. Via manipulating the quasienergy spectrum by periodic driving, we find that a Floquet eigenstate with discrete quasienergy, which we name a Floquet bound state (FBS), can be formed within the band gap of the quasienergy spectrum. We further reveal that the presence of the FBS would dynamically cause the dissipation of the system spin inhibited. The result suggests that we

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can manipulate the periodicity in a temporal domain instead of the one in a spatial domain to suppress decoherence, which relaxes greatly the experimental difficulty in fabricating periodic confinement in a photonic crystal.

Our paper is organized as follows. In Sec. II, we present our model of a periodically driven spin-1/2 particle coupled to a spin chain bath and its exact decoherence dynamics. In Sec. III, the Floquet theory is used to obtain the quasienergy spectrum of the whole system. In Sec. IV, the mechanism of decoherence inhabitation induced by the periodic driving is revealed. The comparisons of this mechanism with the previous methods are also shown in this section. Finally, a summary is given in Sec. V.

II. MODEL AND DYNAMICS

We consider a periodically driven spin-1/2 particle interacting with a one-dimensional spin chain, which is composed of L spin-1/2 particles coupled via XX-type interactions. The Hamiltonian of the total system is $\hat{H}(t) = \hat{H}_S(t) + \hat{H}_I + \hat{H}_E$ with

$$\hat{H}_S(t) = \frac{1}{2}[\lambda + A(t)]\hat{\sigma}_0^z, \quad \hat{H}_I = \frac{g}{2}\left(\hat{\sigma}_0^x\hat{\sigma}_1^x + \hat{\sigma}_0^y\hat{\sigma}_1^y\right), \quad (1)$$

$$\hat{H}_E = \frac{\lambda}{2}\sum_{j=1}^{L}\hat{\sigma}_j^z + \frac{J}{2}\sum_{j=1}^{L-1}\left(\hat{\sigma}_j^x\hat{\sigma}_{j+1}^x + \hat{\sigma}_j^y\hat{\sigma}_{j+1}^y\right), \quad (2)$$

where $\hat{\sigma}_j^\alpha$ ($\alpha = x,y,z$) are the Pauli matrices with $j = 0$ and $1, \ldots, L$, respectively, labeling the system spin and the spins in the chain; λ denotes the longitudinal magnetic field exerted homogeneously on all the spins; $A(t)$ is the always-on periodic driving [43,44] only on the system; J and g are, respectively, the coupling strengths between the nearest-neighbor spins of the chain and between the system and the first-site spin of the chain. $\hat{H}_E$ yields a phase transition at the critical point $|\lambda| = 2J$ [45]. This type of system has been widely used to realize quantum state transfer, where the XX-coupling chain is used as a bridge [46,47] and to analyze decoherence caused by a spin bath [48]. Diagonalizing $\hat{H}_E$ in the single-excitation subspace, we can obtain its eigenstate $|\varphi_k\rangle = \sum_{j=1}^{L}\frac{e^{ikjx_0}}{\sqrt{L}}\hat{\sigma}_j^+|\{\downarrow_j\}\rangle$, which is a spin wave with wave vector k, and the eigenenergy $E_k = \lambda + 2J\cos kx_0$ with x_0 being the spatial separation of the two neighbor sites. Here $|\{\downarrow_j\}\rangle$ is the ferromagnetic state of the chain with all its spins pointing to the $-\hat{e}_z$ direction and $\hat{\sigma}_j^+ = (\hat{\sigma}_j^x + i\hat{\sigma}_j^y)/2$. Obviously, the spin chain defines an environment with finite bandwidth $4J$.

We are interested in how the spin chain results in decoherence to the system spin and how it can be suppressed by periodic driving. Since the excitation number $\hat{\mathcal{N}} \equiv \sum_{j=0}^{L}\hat{\sigma}_j^+\hat{\sigma}_j^-$ is conserved, the Hilbert space is divided into independent subspaces with definite $\mathcal{N}$. Consider that the spin chain is initially polarized in a ferromagnetic state and the system is in an up state $|\Psi(0)\rangle = |\phi\rangle \otimes |\{\downarrow_j\}\rangle$ with $|\phi\rangle = |\uparrow_0\rangle$, and its evolution can be expanded as $|\Psi(t)\rangle = e^{i\frac{L\lambda t}{2}}\sum_{j=0}^{L}c_j(t)\hat{\sigma}_j^+|\{\downarrow_j\}\rangle$, where $c_0(t)$ satisfies

$$\dot{c}_0'(t) + i[\lambda + A(t)]c_0'(t) + \int_0^t f(t-\tau)c_0'(\tau)\,d\tau = 0, \quad (3)$$

with $c_0'(t) = c_0(t)e^{-\frac{i}{2}\int_0^t[\lambda+A(\tau)]d\tau}$ and $f(x) \equiv (g^2/L)\sum_k e^{-iE_kx}$ and $c_0(0) = 1$. Denoting the excited-state probability of the system, $|c_0(t)|^2$ characterizes the environmental decoherence effect on the system. Equation (3) provides us with the exact description to the decoherence of the system.

III. FLOQUET QUASI-ENERGY SPECTRUM

For a static system governed by $\hat{H}$, any time-evolved state can be expanded as

$$|\Psi(t)\rangle = \sum_n C_n e^{iE_nt}|\varphi_n\rangle, \quad (4)$$

where $C_n = \langle\varphi_n|\Psi(0)\rangle$, E_n and $|\varphi_n\rangle$ determined by $\hat{H}|\varphi_n\rangle = E_n|\varphi_n\rangle$ are called eigenenergies and stationary states, respectively.

A temporal periodic system governed by $\hat{H}(t) = \hat{H}(t+T)$ can be treated by Floquet theory [49], which, as a powerful approach to map a nonequilibrium system under driving to a static one, can be seen as the application of Bloch theorem in the time domain. According to this theory, the periodic system has a complete set of basis $|u_\alpha(t)\rangle$ determined by

$$[\hat{H}(t) - i\partial_t]|u_\alpha(t)\rangle = \epsilon_\alpha|u_\alpha(t)\rangle \quad (5)$$

such that any state can be expanded as

$$|\Psi(t)\rangle = \sum_\alpha C_\alpha e^{-i\epsilon_\alpha t}|u_\alpha(t)\rangle \quad (6)$$

with $C_\alpha = \langle u_\alpha(0)|\Psi(0)\rangle$. The similar time independence of C_α as C_n in Eq. (4) implies that ϵ_α and $|u_\alpha(t)\rangle$ play the same roles in a periodic system as eigenenergies and stationary states do in static system. Such similarity leads us to call them quasienergies and quasistationary states, respectively. Carrying all the quasistationary-state characters, the quasienergy spectrum formed by all ϵ_α is a key to study periodic system. Note that ϵ_α is periodic with period $2\pi/T$ because $e^{il\omega t}|u_\alpha(t)\rangle$ with $\omega = 2\pi/T$ is also the eigenstate of Eq. (5) with eigenvalue $\epsilon_\alpha + l\omega$.

The Floquet operator acts on an extended Hilbert space named Sambe space, which is made up of the usual Hilbert space and an extra temporal space [50,51]. To calculate the quasienergies, one first expands $|u_\alpha(t)\rangle$ in a complete set of basis of the temporal space, which is generally chosen as $\{e^{ik\omega t}|k \in Z\}$. We have $|u_\alpha(t)\rangle = \sum_k |\tilde{u}_\alpha(k)\rangle e^{ik\omega t}$, with which Eq. (5) is recast into

$$\sum_{k\in Z}[\hat{\tilde{H}}_{l-k} + k\omega\delta_{l,k}]|\tilde{u}_\alpha(k)\rangle = \epsilon_\alpha|\tilde{u}_\alpha(l)\rangle, \quad (7)$$

with $\hat{\tilde{H}}_{l-k} \equiv \int_0^T \hat{H}(t)\frac{e^{-i(l-k)\omega t}}{T}dt$. Then expanding each $\hat{\tilde{H}}_l$ in the complete basis of Hilbert subspace with $\mathcal{N} = 1$, we get an infinite matrix equation. The quasienergies are obtained by truncating the basis of the temporal space to the rank such that the obtained magnitudes converge.

IV. DECOHERENCE INHIBITION BY PERIODIC DRIVING

A. The mechanism of the decoherence inhibition

To reveal the mechanism of decoherence inhibition by the periodic driving, we consider explicitly that the energy
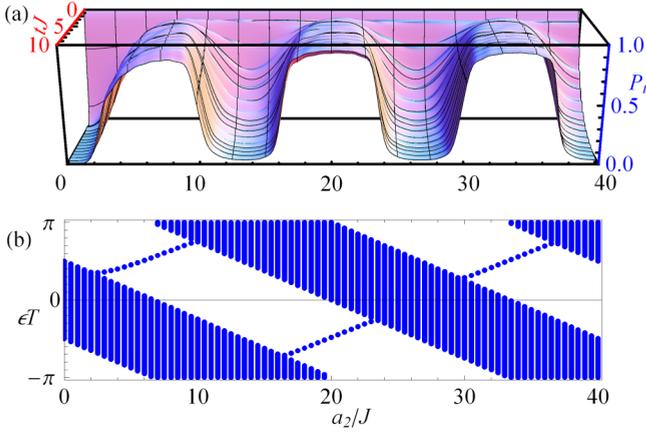
FIG. 1. (Color online) (a) Evolution of the excited-state probability P_t of the system spin in different driving amplitude a_2. (b) Floquet quasienergy spectrum of the whole system with the change of the driving amplitude a_2 in step $\delta a_2 = 0.5J$. The parameters $T = 0.25\pi J^{-1}$, $a_1 = 0$, $\tau = 0.1\pi J^{-1}$, $g = 1.0J$, $\lambda = 20.0J$, and $L = 800$ are used.

splitting of the system is modulated as [48]

$$A(t) = \begin{cases} a_1, & nT < t \leqslant nT + \tau \\ a_2, & nT + \tau < t \leqslant (n+1)T \end{cases}. \quad (8)$$

It is realizable by adding a time-dependent longitudinal magnetic field. Note that although only the driving periodic in this step function is considered, the mechanism revealed in the following is also applicable to other forms. To Eq. (8), we have

$$\hat{\tilde{H}}_l = (\hat{H}_E + \hat{H}_I)\delta_{l,0} + (\omega_l/2)\hat{\sigma}_0^z, \quad (9)$$

$$\omega_l = \frac{a_1(1 - e^{-il\omega\tau}) - a_2(e^{-i2\pi l} - e^{-il\omega\tau})}{2i\pi l}. \quad (10)$$

We first study the asymmetric driving situation by choosing $a_1 = 0$. Figure 1(a) shows the time evolution of the excited-state probability $P_t = |c_0(t)|^2$ with the change of the driving amplitude a_2 via numerically solving Eq. (3). When the driving is switched off, i.e., $a_2 = 0$, P_t decays monotonically to zero, which means a complete decoherence exerted by the spin chain to the system spin. When the driving is switched on, it is interesting to see that, dramatically different from the switch-off case, P_t is stabilized repeatedly with the increase of a_2. To explain this, we plot in Fig. 1(b) the quasienergy spectrum obtained by solving Eq. (7). We can find that an FBS is possible to be formed within the band gap with the increase of a_2. It is remarkable to see that the regimes where the decoherence is inhibited match well with the ones where the FBS is present. To understand the decoherence inhibition induced by the FBS, we, according to Eq. (6), rewrite

$$|\Psi(t)\rangle = e^{i\frac{L\lambda t}{2}}[x e^{-i\epsilon_{FBS}t}|u_{FBS}(t)\rangle$$
$$+ \sum_{\alpha \in \text{Band}} y_\alpha e^{-i\epsilon_\alpha t}|u_\alpha(t)\rangle], \quad (11)$$

where $x = \langle u_{FBS}(0)|\Psi(0)\rangle$ and $y_\alpha = \langle u_\alpha(0)|\Psi(0)\rangle$. Then one can get that P_t evolves asymptotically to
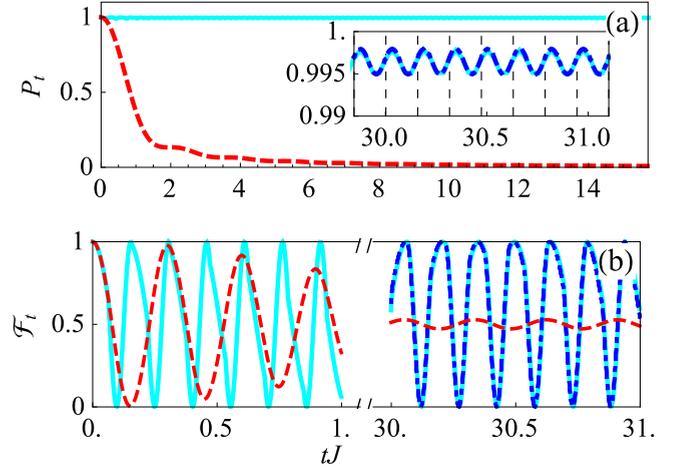
FIG. 2. (Color online) Evolution of P_t for $|\phi\rangle = |\uparrow_0\rangle$ in (a) and $\mathcal{F}_t$ for $|\phi\rangle = (|\uparrow_0\rangle + |\downarrow_0\rangle)/\sqrt{2}$ in (b) when $a_2 = 36.0J$ with the FBS (cyan solid line) and $a_2 = 1.5J$ without the FBS (red dashed line) via numerically solving Eq. (3). The blue dot-dashed lines show the results obtained via analytically evaluating the contribution of the FBS to the asymptotic state, which match the numerical ones. The parameters are the same as Fig. 1 except for $T = 0.05\pi J^{-1}$ and $\tau = 0.02\pi J^{-1}$.

$P_\infty \equiv x^2|\langle \Psi(0)|u_{FBS}(t)\rangle|^2$ with all the components in the quasienergy band vanishing due to the out-of-phase interference contributed by the continuous phases (see Appendix A), as confirmed in Fig. 2(a). In the absence of the FBS, although it is dramatically interrupted by the driving, P_t decays to zero finally. Whenever the FBS is formed, P_t would be stabilized to P_∞, which is periodic with period T [see the inset of Fig. 2(a)]. It means that the presence of the FBS would cause P_t to survive in the only component of the FBS and thus synchronize with the driving field [52]. Figure 2(b) plots the performance of the formed FBS in an arbitrary initial state $|\phi\rangle = (|\uparrow_0\rangle + |\downarrow_0\rangle)/\sqrt{2}$. We can see that the decay of the initial-state-fidelity $\mathcal{F}_t \equiv \langle \phi|\text{Tr}_E[|\Psi(t)\rangle\langle\Psi(t)|]|\phi\rangle$, to 50% can be stabilized even as high as the ideal lossless case (i.e., between 0 and 1) with the formation of the FBS. Characterizing the quantum coherence between the two spin states, such a stabilized oscillation means that the quantum coherence is preserved. We can check that $\mathcal{F}_t$ tends to $\mathcal{F}_\infty = \langle \phi|\rho|\phi\rangle$, where

$$\rho = \left(1 - \frac{|x|^2}{2}\right)|\downarrow_0\rangle\langle\downarrow_0| + \frac{|x|^2}{2}\rho_{FBS}(t)$$
$$+ \left\{\frac{x^*}{2}\mu(t)\text{Tr}_E[|\{\downarrow_j\}\rangle\langle u_{FBS}(t)|] + \text{H.c.}\right\} \quad (12)$$

with $\rho_{FBS}(t) = \text{Tr}_E[|u_{FBS}(t)\rangle\langle u_{FBS}(t)|]$ and $\mu(t) = e^{i\int_0^t \frac{\lambda + A(t') + 2\epsilon_{FBS}}{2}dt'}$ (see Appendix B). We plot this $\mathcal{F}_\infty$ with the blue dot-dashed line in Fig. 2(b), which matches with the asymptotical result from numerically solving Eq. (3).

The result reveals that we can manipulate the quasienergy spectrum forming the FBS to suppress decoherence. A prerequisite for forming the FBS is the existence of finite quasienergy gap in the spectrum. We plot in Fig. 3 the Floquet quasienergy spectrum and P_t with the change of τ as well as T. We can see
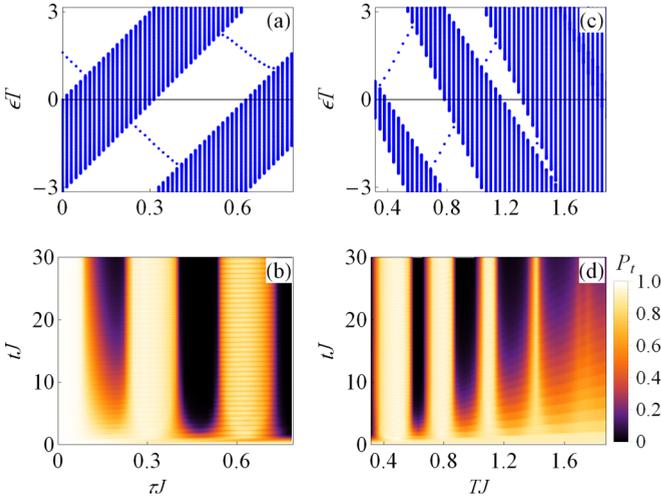
FIG. 3. (Color online) Floquet quasienergy spectrum of the whole system in (a) and evolution of P_t in (b) with the change of τ when $T = 0.25\pi J^{-1}$. The increase step $\delta\tau = 0.03 J^{-1}$ is used in (a). Floquet quasienergy spectrum in (c) and time evolution of P_t in (d) with the change of of T when $\tau = 0.1\pi J^{-1}$. The increase step $\delta T = 0.09 J^{-1}$ is used in (c). Other parameters are the same as Fig. 1.

that, irrespective of which driving parameter is changed, the firm correspondence between the formation of the FBS and the decoherence inhibition can be established. The common character between Fig. 1(b) and Fig. 3(a) is that the width of the formed band gap is kept constant during the change of driving parameters, which is not true for Fig. 3(c). This can be understood in the following way. Periodic in $2\pi/T$, the quasienergy has a full width $2\pi/T$. The energy band of the whole system is $4J$. Therefore, a band gap with finite width $2\pi/T - 4J$ can be present in the quasienergy spectrum only in the high-frequency (i.e., $2\pi/T > 4J$) driving case. This can be tested by Fig. 3(c) where the band gap vanishes whenever $2\pi/T < 4J$. It leads to the continuous energy band of the environment filling up the Floquet spectrum. Thus there is no room for forming the FBS here. Reflecting on P_t in Fig. 3(d), although it is greatly slowed, P_t approaches zero eventually. Therefore, we conclude that the FBS can be present only in the high-frequency driving case $2\pi/T > 4J$, which supplies a necessary condition to stabilize decoherence. It is a very useful criterion on designing a driving scheme for decoherence control.

Our finding in the periodically driven system is an analog to the bound-state-induced decoherence suppression revealed in a static system [18–20]. For a static two-level system [19,20] or a harmonic oscillator [53] interacting with an environment, depending on the parameters in the spectral density, the total system may possess a stationary state named as bound state [19] localized out of the continuous energy band of the environment. As a stationary state, the bound state contained as one superposition component in the initial state does not lose its quantum coherence during time evolution. Thus the system evolves exclusively to the time-invariant component of the bound state with other components in the continuous band vanishing due to their out-of-phase interference. This idea was used previously to suppress spontaneous emission of quantum emitters via introducing spatial periodic confinement to the radiation field in a photonic crystal setting [21–24].

The spatial periodicity introduces a band gap structure to the environmental energy spectrum such that an emitter-environment bound state is formed when the frequency of the emitter falls in the band gap. Here we demonstrate that the parallel picture can be set up by introducing temporal periodicity to the system. The benefit of using the temporal periodic driving instead of the spatial periodic confinement is that its high controllability greatly relaxes the experimental difficulty in fabricating the spatial periodic confinement. Thus it is easier to realize in practice.

B. Comparisons with the previous methods

There are several methods in the literature to explore the effects of periodic driving on quantum systems. For example, via neglecting the coupling between different temporal subspaces of the Floquet eigenequation (7) in the high-frequency driving condition, it was shown that the periodic driving can induce the suppressed tunneling of a quantum particle, a phenomenon called coherent destruction of tunneling [25–27], and the decoupling between open system and its environment [30,31]. It was also revealed that, via introducing the first-Markovian approximation to Eq. (3), the dynamics of the open system under periodic control can be characterized by an overlap integration of the noise spectrum and the spectrum of the control, and thus one can craft the filter-transfer function of the control field to suppress decoherence [12]. It is called spectral filtering theory and has been generalized to give a unified description to a dynamical decoupling method [12–16]. In the following we compare our exact treatment with the above approximate methods.

First, our decoherence inhibition mechanism is more robust to the imperfect fluctuation of the driving parameters than the decoupling mechanism revealed in Ref. [30,31], where the decoupling is achieved only in certain single values of the driving parameters. To see this, we resort to the same approximate method as in Refs. [27,30,31]. Expanding $|u_\alpha(t)\rangle$ in a new set of basis of the temporal space as $|u_\alpha(t)\rangle = \sum_k \hat{U}_t e^{ik\omega t} |\tilde{u}_\alpha(k)\rangle$, where $\hat{U}_t = \exp[-(i/2)\int_0^t (A(t') - \bar{A})\hat{\sigma}_z \, dt']$ with $\bar{A} = (1/T)\int_0^T A(t) \, dt$ subtracted to guarantee the periodicity of $|u_\alpha(t)\rangle$, we can obtain a similar form as Eq. (7) but

$$\hat{H}_{l-k} = \left[\frac{\lambda + \bar{A}}{2}\hat{\sigma}_0^z + \hat{H}_E\right]\delta_{l,k} + g(F_{l-k}\hat{\sigma}_1^+\hat{\sigma}_0^- + \text{H.c.}), \quad (13)$$

$$F_{l-k} = \int_0^T \frac{\exp\left\{-i\int_0^t [A(t') - \bar{A}] \, dt'\right\} e^{-i(l-k)\omega t}}{T} \, dt. \quad (14)$$

Using the approximation in Refs. [30,31], we neglect the terms F_{l-k} with $l \neq k$ and keep only F_0. It reduces to a spin system coupled to an environment with the coupling strength renormalized by a factor F_0. In Fig. 4 we plot $|F_0|^2$ and P_t with the change of a_2 in the symmetric driving situation, i.e., $a_1 = -a_2$. It shows that although no FBS is formed, $F_0 = 0$ is achievable in certain values of driving parameters. As expected, it induces the decoherence inhibited [see Fig. 4(b)]. However, the decoupling is sensitive to the driving parameters and any small deviation to the decoupling driving values would cause the asymptotic vanishing of P_t. Different from this, it is a wide parameter regime in our mechanism which makes
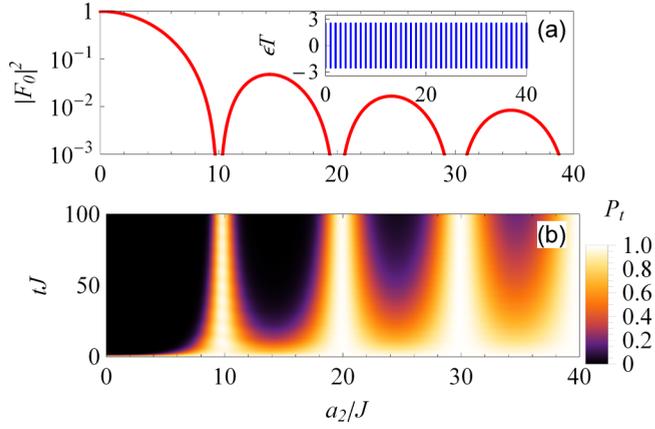
FIG. 4. (Color online) The renomalization factor $|F_0|^2$ in (a) and the evolution of P_t in (b) with the change of a_2 in the symmetric driving case (i.e., $a_1 = -a_2$). The inset of (a) shows the Floquet quasienergy spectrum revealing the absence of the FBS. The parameters are $T = 0.4\pi J^{-1}$ and $\tau = 0.2\pi J^{-1}$, and the others are the same as Fig. 1.

decoherence inhibited (see Figs. 1 and 3), which is more stable to the parameter fluctuation in the practical experiments than the decoupling one.

Second, we emphasize that our mechanism is substantially different from the spectral filtering theory [12–16]. That theory works only in the first-Markovian approximation, with which the convolution in the exact evolution equation (3) can be removed [12], i.e.,

$$\dot{\alpha}(t) \approx -\alpha(t) \int_0^t d\tau \varepsilon^*(t)\varepsilon(\tau) f(t-\tau) e^{i\omega_a(t-\tau)} \qquad (15)$$

with $\alpha(t) = c_0'(t)e^{i\int_0^t [\lambda + A(\tau)]d\tau}$, $\varepsilon(t) = e^{i\int_0^t [A(\tau) - \bar{A}]d\tau}$, and $\omega_a = \lambda + \bar{A}$. Its solution can be obtained readily as

$$|\alpha(t)| = |c_0(t)| = \exp[-R(t)Q(t)/2], \qquad (16)$$

where

$$R(t) \equiv 2\pi \int_{-\infty}^{+\infty} G(\omega + \omega_a) \frac{|\varepsilon_t(\omega)|^2}{Q(t)} d\omega, \qquad (17)$$

$$Q(t) = \int_0^t d\tau |\varepsilon(\tau)|^2 \qquad (18)$$

with the environmental spectral density $G(\omega)$ relating to its correlation function $f(t-\tau)$ as $f(t-\tau) = \int G(\omega)e^{-i\omega(t-\tau)}d\omega$ and $\varepsilon_t(\omega) = \frac{1}{\sqrt{2\pi}} \int_0^t \varepsilon(\tau) e^{i\omega\tau} d\tau$. Thus it is only under the first Markovian approximation that $|c_0(t)|$ can be denoted by such filtered spectrum form. To check the physics missed by this approximation, we plot in Fig. 5 the comparison of our exact result with the one obtained firmly from the spectral filtering theory. We can see from Fig. 5(a) that the spectral filtering theory shows a complete decoherence to zero because of a dramatic overlap between the environmental spectrum and the control spectrum [see Fig. 5(b)]. However, our exact result in Fig. 5(c) shows a stabilization on decoherence due to the existence of the FBS in the quasienergy spectrum [see Fig. 5(d)]. It means that the spectral filtering theory totally breaks down in describing the
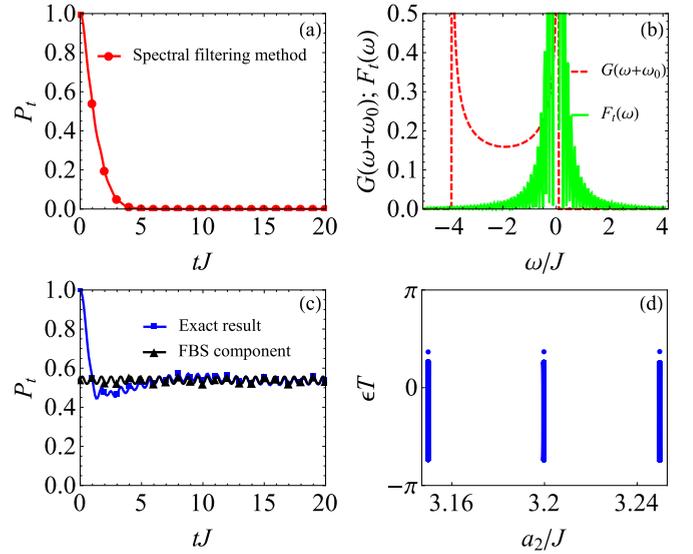
FIG. 5. (Color online) The comparison of P_t calculated by the spectral filtering method in (a) and our exact method in (c). (b) The noise spectrum $G(\omega + \omega_0)$ and the spectrum of the control $F_t(\omega) \equiv |\varepsilon_t(\omega)|^2/Q(t)$ used in the spectral filtering method to determine P_t. The contribution of the formed FBS to P_t is also plotted in (c). The Floquet quasienergy spectrum in (d) shows the existence of the FBS. $a_2 = 3.2J$ is further used in (a–c) and other parameters are same as Fig. 1.

long-time steady state behavior here. To give more evidence on the dominate role of the formed FBS in the steady-state behavior, we plot in Fig. 5(c) the fidelity of the FBS in the time-evolved state, which matches well with P_t in the long-time limit. Therefore, it confirms again that the formed FBS is the physical reason for decoherence inhibition in long-time limit of our model. Thus the decoherence cannot be simply described as an overlap between the noise spectrum and the control field here and the spectral filtering theory is inapplicable to explain our result.

As a final remark, the mechanism revealed in our spin-bath model can also be readily extended to other excitation-number-conserving models, e.g., a two-level system in a coupled cavity array [30,31] and a harmonic oscillator in a bosonic bath model [53].

V. CONCLUSIONS

We have studied the decoherence dynamics of a periodically driven spin-1/2 particle interacting with an *XX* coupled spin chain. It is found that the decoherence of the system can be inhibited by the periodic driving. We have revealed that the mechanism of such decoherence inhibition induced by the periodic driving is the formation of a FBS in the quasienergy spectrum. This can be seen as a close analog of the bound-state induced decoherence suppression in a photonic crystal system, but it relaxes greatly the experimental difficulties of a photonic crystal system in fabricating specific spatial periodicity to engineer a bound state. It opens a door to beat decoherence by tailoring temporal periodicity. Compared with the conventional schemes of decoherence control using periodic driving or pulses, our scheme is robust to the practical

driving parameter fluctuation. Given the fact that periodic driving offers a high controllability to quantum system, our decoherence inhibition mechanism provides us with a promising and realistic way to practical decoherence control.

APPENDIX A: THE CONTRIBUTION OF THE FORMED FBS TO THE LONG-TIME STEADY STATE

For the initial state $|\Psi(0)\rangle = |\uparrow\rangle \otimes |\{\downarrow_1 \cdots \downarrow_L\}\rangle$, $|\Psi(t)\rangle$ can also be expanded in the Floquet basis as

$$|\Psi(t)\rangle = e^{i\frac{L\lambda t}{2}}[xe^{-i\epsilon_{\text{FBS}}t}|u_{\text{FBS}}(t)\rangle + \sum_{\alpha \in \text{B}} y_\alpha e^{-i\epsilon_\alpha t}|u_\alpha(t)\rangle],$$

(A1)

where $|u_{\text{FBS}}(t)\rangle$ is the formed FBS with quasienergy ϵ_{FBS}, $|u_\alpha(t)\rangle$ are the Floquet eigenstates in the continuous band with quasienergies ϵ_α, $x = \langle u_{\text{FBS}}(0)|\Psi(0)\rangle$, and $y_\alpha = \langle u_\alpha(0)|\Psi(0)\rangle$. Then we can calculate the probability of the system spin keeping in an up state as

$$\begin{aligned} P_t = &|x|^2|\langle\Psi(0)|u_{\text{FBS}}(t)\rangle|^2 \\ &+ \sum_{\alpha,\beta \in \text{B}} y_\alpha^* y_\beta e^{-i(\epsilon_\beta - \epsilon_\alpha)t}\langle\Psi(0)|u_\beta(t)\rangle\langle u_\alpha(t)|\Psi(0)\rangle \\ &+ \sum_{\alpha \in \text{B}}[xy_\alpha^* e^{-i(\epsilon_{\text{FBS}} - \epsilon_\alpha)t}\langle\Psi(0)|u_{\text{FBS}}(t)\rangle\langle u_\alpha(t)|\Psi(0)\rangle \\ &+ \text{c.c.}] \end{aligned}$$

(A2)

Due to the out-of-phase interference contributed from $e^{-i(\epsilon_\beta - \epsilon_\alpha)t}$ with $\alpha \neq \beta$ and $e^{-i(\epsilon_{\text{FBS}} - \epsilon_\alpha)t}$, P_t tends to

$$\begin{aligned} P_\infty &= |x|^2|\langle\Psi(0)|u_{\text{FBS}}(\infty)\rangle|^2 + \sum_{\alpha \in \text{B}}|y_\alpha|^2|\langle\Psi(0)|u_\alpha(\infty)\rangle|^2 \\ &= |x|^2|\langle\Psi(0)|u_{\text{FBS}}(\infty)\rangle|^2 + \sum_{\alpha \in \text{B}}|y_\alpha|^4|\langle u_\alpha(0)|u_\alpha(\infty)\rangle|^2, \end{aligned}$$

(A3)

where the orthogonality of Floquet eigenstates has been used. Noticing the fact that $\sum_{\alpha \in \text{B}}|y_\alpha|^2 = \sum_{\alpha=1}^{L}|y_\alpha|^2 \sim 1$ (because we have L Floquet eigenstates forming the continuous quasienergy band), we can estimate that $|y_\alpha|^2 \sim 1/L$. In the thermodynamics limit $L \Rightarrow \infty$, the last term tends to zero. Thus we have

$$P_\infty = |x|^2|\langle\Psi(0)|u_{\text{FBS}}(\infty)\rangle|^2.$$

(A4)

From the above analysis, we can see that the preserved excited-state probability is determined by the weight of $|u_{\text{FBS}}(0)\rangle$ in the initial state $|\Psi(0)\rangle$ and the excited-state
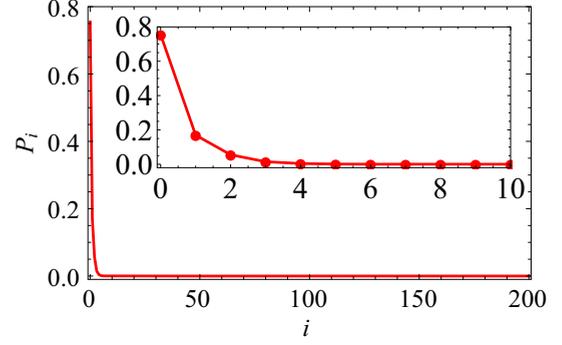
FIG. 6. (Color online) The distribution of excited-state population of the formed Floquet bound state at time $t = T/4$ over the spin sites. The parameters used are $T = 0.25\pi J^{-1}$, $\tau = 0.1\pi J^{-1}$, $a_2 = 3.2J$, and $a_1 = 0$.

probability of the system spin in $|u_{\text{FBS}}(\infty)\rangle$ itself. In Fig. 6 we plot the distribution of excited-state population of the formed FBS at time $t = T/4$ over the spin sites. We can see that its excited-state population is mainly confined in the site of the system spin, which acts as an impurity in the whole system.

APPENDIX B: THE EFFECT OF PERIODIC DRIVING ON THE INITIAL SUPERPOSITION STATE

For the general initial state $|\Psi(0)\rangle = |\phi\rangle \otimes |\{\downarrow_{j\neq0}\}\rangle$ with $|\phi\rangle = \alpha|\uparrow_0\rangle + \beta|\downarrow_0\rangle$ under $|\alpha|^2 + |\beta|^2 = 1$, its evolved state $|\Psi(t)\rangle$ can be expanded as

$$|\Psi(t)\rangle = e^{i\frac{L\lambda t}{2}}\left[\alpha \sum_{j=0}^{L} c_j(t)\hat{\sigma}_j^+|\{\downarrow_j\}\rangle + \beta e^{i\int_0^t \frac{\lambda + A(t')}{2}dt'}|\{\downarrow_j\}\rangle\right],$$

(B1)

where $c_0(t)$ satisfies Eq. (3) in the main text. The fidelity of the system in its initial state $|\phi\rangle$ can be calculated as

$$\begin{aligned} \mathcal{F}_t &= \langle\phi|\text{Tr}_{\text{E}}[|\Psi(t)\rangle\langle\Psi(t)|]|\phi\rangle \\ &= ||\alpha|^2 c_0(t)e^{-i\int_0^t \frac{\lambda + A(t')}{2}dt'} + |\beta|^2|^2 \\ &+ |\alpha\beta|^2|[1 - |c_0(t)|^2]. \end{aligned}$$

(B2)

Since $|\phi\rangle$ is not an eigenstate of the system even in the absence of the environmental influence, $\mathcal{F}_t$ is a temporally oscillating function even in the long-time limit. To qualitatively reflect the performance of the periodic driving on suppressing decoherence, we use the maximal value $\mathcal{F}_t$ to characterize it. This happens at a set of times τ_n such that $c_0(\tau_n)e^{-i\int_0^{\tau_n} \frac{\lambda + A(t')}{2}dt'} = |c_0(\tau_n)|$. Under this condition, Eq. (B2) has the form

$$\begin{aligned} \mathcal{F}_{\tau_n} &= 1 - |\alpha|^4[1 - |c_0(\tau_n)|^2] - |\alpha\beta|^2[1 - |c_0(\tau_n)|]^2 \\ &\geqslant |\beta|^2 + |\alpha|^2|c_0(\tau_n)|^2. \end{aligned}$$

(B3)

When the FBS is absent, $|c_0(\infty)| = 0$ and thus $\mathcal{F}_{\tau_n} = |\beta|^2$. This corresponds to the complete decoherence (i.e., the system spin decays totally to its low-energy spin down state). Whenever the FBS is formed, a nonzero $|c_0(\infty)|$ would be achieved. Then we could have $\mathcal{F}_{\tau_n} > |\beta|^2$ in the steady state. From this analysis, we can see that the preserved probability

for arbitrary initial state is determined by the same long-time behavior of $|c_0(\infty)|$ as the one for the spin up initial state. This proves well that our mechanism of dissipation suppression can also be applied to the initial superposition state.

More precisely, we can evaluate the contribution of the formed FBS to the steady state. $|\Psi(t)\rangle$ can also be expanded in the Floquet basis as

$$|\Psi(t)\rangle = e^{i\frac{L\lambda t}{2}}\left\{\beta e^{i\int_0^t \frac{\lambda+A(t')}{2}dt'}|\{\downarrow_j\}\rangle + \alpha[xe^{-i\epsilon_{\text{FBS}}t}|u_{\text{FBS}}(t)\rangle + \sum_{\gamma\in\text{B}}y_\gamma e^{-i\epsilon_\gamma t}|u_\gamma(t)\rangle]\right\}.$$ (B4)

Due to the out-of-phase interference, the reduced density matrix tends to

$$\rho(\infty) = \text{Tr}_{\text{E}}[|\Psi(\infty)\rangle\langle\Psi(\infty)|] = |\beta|^2|\downarrow_0\rangle\langle\downarrow_0| + |\alpha|^2\{|x|^2\rho_{\text{FBS}}(t) + \sum_\gamma |y_\gamma|^2\text{Tr}_{\text{E}}[|u_\gamma(t)\rangle\langle u_\gamma(t)|]\}$$

$$+ \{\beta\alpha^*x^*\mu(t)\text{Tr}_{\text{E}}[|\{\downarrow_j\}\rangle\langle u_{\text{FBS}}(t)|] + \text{H.c.}\},$$ (B5)

where $\rho_{\text{FBS}}(t) = \text{Tr}_{\text{E}}[|u_{\text{FBS}}(t)\rangle\langle u_{\text{FBS}}(t)|]$ and $\mu(t) = e^{i\int_0^t \frac{\lambda+A(t')+2\epsilon_{\text{FBS}}}{2}dt'}$. Noticing the fact that $\text{Tr}_{\text{E}}[|u_\gamma(t)\rangle\langle u_\gamma(t)|]$ is dominated by $|\downarrow_0\rangle\langle\downarrow_0|$ and $\sum_\gamma |y_\gamma|^2 + |x|^2 = 1$, we have $\sum_\gamma |y_\gamma|^2\text{Tr}_{\text{E}}[|u_\gamma(t)\rangle\langle u_\gamma(t)|] \approx (1-|x|^2)|\downarrow_0\rangle\langle\downarrow_0|$. Thus the asymptotic state of the system spin is

$$\rho(\infty) = (1-|\alpha|^2|x|^2)|\downarrow_0\rangle\langle\downarrow_0| + |\alpha|^2|x|^2\rho_{\text{FBS}}(t) + \{\beta\alpha^*x^*\mu(t)\text{Tr}_{\text{E}}[|\{\downarrow_j\}\rangle\langle u_{\text{FBS}}(t)|] + \text{H.c.}\}.$$ (B6)

Then the analytical form of the fidelity in the long-time limit can be calculated by $\mathcal{F}_\infty = \langle\phi|\rho(\infty)|\phi\rangle$. It gives the contribution of the formed FBS to the asymptotical state and can be used to check the validity of our FBS theory in explaining the dynamics of the system spin.

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