Reversal of relaxation due to a dephasing environment

Marcin Mierzejewski,[1] Janez Bonča,[2,3] and Jerzy Dajka[1,4]
[1] *Institute of Physics, University of Silesia, 40-007 Katowice, Poland*
[2] *Faculty of Mathematics and Physics, University of Ljubljana, 1000 Ljubljana, Slovenia*
[3] *J. Stefan Institute, 1000 Ljubljana, Slovenia*
[4] *Silesian Center for Education and Interdisciplinary Research, University of Silesia, 41-500 Chorzów, Poland*
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We show that any finite quantum system S can be coupled to a dephasing environment in such a way that the internal mechanism responsible for relaxation of observables acting on S can be effectively canceled. By adjusting this coupling, the difference between the initial and the long–time expectation values of any observable on S can be tuned to an arbitrarily small, but nonzero, value. This statement is exemplified and visualized by numerical studies of relaxation in a generic one-dimensional system of interacting fermions.

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I. INTRODUCTION

There is ever-growing research in the field of dynamics of many-body quantum systems [1]. One of the central and still unsolved problems is related to equilibration of closed systems, with a particular emphasis on the relaxation of observables relevant for experimental verification of various theoretical models. Moreover, in large-enough generic quantum systems one expects that equilibration is equivalent to thermalization. It is commonly accepted that the long-time averages of local observables in generic systems coincide with expectation values for the statistical Gibbs ensemble [1–6]. However, our understanding of this process and the condition for its occurrence are far from being complete. For example, it is known that there is a relation between equilibration and integrability of the system [7–10]. Generally, nonintegrable systems are expected to thermalize, while integrable systems do not approach the Gibbs state but rather the generalized Gibbs ensemble (GGE) [11–16]. However, there are quantum systems which do thermalize despite being integrable, provided that they are prepared in certain states [17]. On the other hand, there are nonintegrable quantum systems which do not thermalize [5], and the role of initial entanglement between subsystems seems to be important.

According to common intuition, relaxation of observables is a hallmark of *irreversibility* [18]. If one considers an expectation value of an arbitrary observable O and takes advantage of the spectral decomposition of the Hamiltonian into its eigenstates $H|n\rangle = E_n|n\rangle$, one obtains for an initial state $|\psi\rangle = \sum_m C_m|m\rangle$:

$$\langle O\rangle(t) = \sum_{E_n=E_m} C_n^* C_m \langle n|O|m\rangle + \sum_{E_n\neq E_m} C_n^* C_m e^{i(E_n-E_m)t}\langle n|O|m\rangle. \quad (1)$$

Invoking arguments adopted in the context of the eigenstate thermalization hypothesis [3], one expects that after a sufficiently long time, due to destructive interference of oscillating terms in Eq. (1), the observable can relax to its steady-state value,

$$\lim_{t\to\infty}\langle O\rangle(t) = \lim_{\tau\to\infty}\frac{1}{\tau}\int_0^\tau dt\langle O\rangle(t) = \sum_{E_n=E_m} C_n^* C_m\langle n|O|m\rangle. \quad (2)$$

Although finite closed quantum systems are, strictly speaking, periodic or quasiperiodic, the irreversibility is still manifested as the low (or vanishing) probability of a process which reverses the relaxation. Moreover, if one attaches the quantum system to an infinite environment, transforming a closed system into an *open* system, one expects that the relaxation becomes faster and "more irreversible" due to the *information loss* or dissipation. This seems at least intuitively indisputable. However, in this paper we present a counterexample to this generally accepted mechanism. We demonstrate that there exist environments which cancel internal relaxation mechanisms of an arbitrary quantum system. More precisely, we show that any finite quantum system can be linearly coupled to a continuous set of harmonic oscillators in such a way that evolution of an arbitrary observable O satisfies

$$|\lim_{t\to\infty}\langle O\rangle(t) - \lim_{t\to 0}\langle O\rangle(t)| < \epsilon \quad (3)$$

for arbitrary $\epsilon > 0$. In other words, the relaxation cannot be completely eliminated but can be made arbitrarily inefficient. This prediction is in direct contrast to Eq. (2). The main idea behind this result is that a specially tailored environment may cancel the (destructively interfering) oscillations of the of-diagonal matrix elements in Eq. (1). Such environments belong to the class of purely dephasing environments [18]. They preserve the internal conservation laws of the quantum system; hence, they are not generic or even typical. The applicability of the pure dephasing model has been discussed in various areas of quantum and mathematical physics [19–22]. Surprisingly, there are also real systems which can be effectively described by such a simple, highly symmetric model [23]. However, let us stress that pure decoherence remains credible only at time scales significantly smaller than the time scale relevant for exchanging energy between system and its environment [23]. Hence, the applicability of the pure dephasing under experimentally accessible conditions is usually at least disputable.

This paper is organized as follows: In the next section we formulate and prove (in Sec. II A) the central result of our work concerning the reversal of relaxation in dephasing environments. Further, in Sec II B, we provide a simple example to illustrate reversal of relaxation in a generic system of interacting fermions. In Sec. III we present certain generalizations of the main result. Finally, we conclude our work in last section.

II. MODEL AND MAIN RESULT

We consider a quantum system S described by the Hamiltonian

$$H_S = \sum_n E_n |n\rangle\langle n|, \tag{4}$$

with an arbitrary (possibly degenerate) energy spectrum. The system is coupled to an environment B of noninteracting bosons,

$$H_B = \int_0^\infty d\omega\, \omega a^\dagger(\omega) a(\omega), \tag{5}$$

where the fields $a(\omega)$ satisfy $[a(\omega),a^\dagger(\omega')] = \delta(\omega-\omega')$ [24]. The details of the purely dephasing S-B coupling are given by V_S and V_B such that the total Hamiltonian reads [18,25]

$$H = H_S \otimes \mathcal{I}_B + \mathcal{I}_S \otimes H_B + V_S \otimes V_B, \tag{6}$$

where

$$V_B = \int_0^\infty d\omega g(\omega)[a^\dagger(\omega) + a(\omega)], \tag{7}$$

with a real-valued $g(\omega)$ and

$$V_S = \sum_n \gamma_n |n\rangle\langle n|. \tag{8}$$

Note that H_S and V_S commute. This particular property is a hallmark of pure dephasing [18]. We assume also that, initially, the system and its environment are prepared in a separable state

$$\rho(0) = \sum_{n,m} p_{nm} |m\rangle\langle n| \otimes |\Omega\rangle\langle\Omega|, \tag{9}$$

where $|\Omega\rangle$ is the bosonic vacuum. Towards the end of this paper we discuss a more general class of the initial states.

A. Proposition

We consider long-time expectation values of local operators defined for the quantum system $O = O_S \otimes \mathcal{I}_B$. If the system-environment coupling satisfies the proportionality relation

$$\gamma_m^2 - \gamma_n^2 \propto E_m - E_n, \tag{10}$$

then the difference $|\lim_{t\to\infty}\langle O\rangle(t) - \langle O\rangle(0)|$ can be tuned to an arbitrary small value by an appropriate choice of the coupling function $g(\omega)$. Most importantly, a single tuning of $g(\omega)$ holds for all local operators O.

For the pure dephasing one easily finds the time-dependent expectation value

$$\langle O\rangle(t) = \sum_{n,m} p_{nm} \langle n|O|m\rangle a_{nm}(t), \tag{11}$$

where

$$a_{nm}(t) = \langle\Omega| \exp[it(E_n + \gamma_n V_B + H_B)] \times \exp[-it(E_m + \gamma_m V_B + H_B)]|\Omega\rangle. \tag{12}$$

Such a simple result occurs due to the block-diagonal structure of the Hamiltonian, where each block describes a set of shifted harmonic oscillators: $E_m + \gamma_m V_B + H_B$. An explicit form of $a_{nm}(t)$ has been derived and used many times in different contexts ranging from mathematical physics [26] to quantum information [19]. An explicit form of the amplitude $a_{nm}(t)$ reads [25]

$$a_{nm}(t) = e^{-i(E_m-E_n)t + i(\gamma_m^2-\gamma_n^2)E(t) - (\gamma_m-\gamma_n)^2\Lambda(t)}, \tag{13}$$

where

$$E(t) = \int_0^\infty d\omega \frac{g^2(\omega)}{\omega^2}[\omega t - \sin(\omega t)], \tag{14}$$

$$\Lambda(t) = \int_0^\infty d\omega \frac{g^2(\omega)}{\omega^2}[1 - \cos(\omega t)]. \tag{15}$$

Utilizing the square integrability of $g(\omega)/\omega$, one finds from the Lebesgue-Riemann lemma [27] in the long-time regime ($t \to \infty$) that

$$E(t) \to \tilde{E}t, \quad \Lambda(t) \to \tilde{\Lambda} = \text{const}, \tag{16}$$

where

$$\tilde{E} = \int_0^\infty d\omega g^2(\omega)\omega^{-1}, \tag{17}$$

$$\tilde{\Lambda} = \int_0^\infty d\omega g^2(\omega)\omega^{-2}. \tag{18}$$

In order to reduce the effects of the internal relaxation one should tune the system-environment coupling in such a way that the oscillatory part in Eq. (13) drops out,

$$E_m - E_n - \tilde{E}(\gamma_m^2 - \gamma_n^2) = 0, \tag{19}$$

while the exponential term $(\gamma_m-\gamma_n)^2\tilde{\Lambda}$ remains small. In order to show that such a particular choice is indeed possible, we introduce a cutoff frequency [18] ω_c for the bosons in the environment and redefine the coupling function

$$g(\omega) = f(\omega/\omega_c). \tag{20}$$

Then, $\tilde{\Lambda} \propto \omega_c^{-1}$ while $\tilde{E} \propto \omega_c^{-0}$. After eliminating the destructive interference of the off-diagonal matrix elements, the exponential damping may become arbitrarily small, of the order of $1/\omega_c$,

$$\log[a_{mn}(t\to\infty)] \propto -\frac{(\gamma_m-\gamma_n)^2}{\omega_c}. \tag{21}$$

While our general scheme does not require any particular form of $g(\omega)$, the results are most transparent for the standard parametrization of the coupling function [18]:

$$g^2(\omega) = \frac{1}{\Gamma(\mu+1)}\left(\frac{\omega}{\omega_c}\right)^{1+\mu} \exp(-\omega/\omega_c), \tag{22}$$

where $\mu > 0$ corresponds to the (mathematically [26]) well-behaving super-Ohmic environment. In the latter case $\tilde{E} = 1$, while $\tilde{\Lambda} = (\mu\omega_c)^{-1}$.

B. Example

In order to exemplify our result with the help of a simple but generic case, we study a one-dimensional system of L sites and $L/2$ spinless fermions with periodic boundary conditions [28–31] (i.e., a ring) with a Hamiltonian given by

$$H_S = -J_h \sum_{j=1}^{L} [\exp(i\phi) c_{j+1}^\dagger c_j + \text{H.c.}] + U_1 \sum_{j=1}^{L} n_{j+1} n_j + U_2 \sum_{j=1}^{L} n_{j+2} n_j, \tag{23}$$

where $n_j = c_j^\dagger c_j$, J_h is the hopping integral, and U_1 and U_2 describe first- and second-nearest–neighbor interactions, respectively. We take J_h as the energy unit, $J_h = 1$, whereas time is expressed in units of $\tau_h = \hbar/J_h$. We take also $U_1 = 1.4$ and $U_2 = 1$. For such parameters H_S describes a generic metal characterized by a featureless response to electromagnetic field and a normal diffusive transport. Moreover, such a system thermalizes even when being decoupled from its surrounding [9]. Numerical studies have been carried out for $L \leqslant 18$. The reason behind introducing U_2 is to stay away from the integrable case, which shows anomalous relaxation [11,13,14,32] and charge transport [33–38].

The mechanism of the reversal of relaxation (RR) holds either for all observables or for none. In the following, we show numerical results for a particle current

$$O_S = \sum_{j=1}^{L} i \exp(i\phi) c_{j+1}^\dagger c_j + \text{H.c.} \tag{24}$$

since $\langle O_S \rangle$ vanishes whenever S is in equilibrium; hence, large or even nonzero values of this observable imply that the system is in a nonthermal state. We consider a typical super-Ohmic environment with $\mu = 1$ [see Eq. (22)] when

$$E(t) = t\,\frac{\omega_c^2 t^2}{1+\omega_c^2 t^2}, \quad \Lambda(t) = \frac{1}{\omega_c}\,\frac{\omega_c^2 t^2}{1+\omega_c^2 t^2}. \tag{25}$$

While our qualitative conclusions do not depend on the initial state, we assume that the system is initially in a pure state $p_{nm} = C_n C_m$ [see Eq. (9)], where $C_n = \text{const} > 0$ if $\sum_m \text{Re}\langle n|O_S|m\rangle > 0$ and $C_n = 0$ otherwise. Such a state has a large initial current and has nonzero projection on a large number (approximately one half) of the energy eigenstates. Consequently, it leads to very clear numerical results for the relaxation of the particle current.

In Fig. 1 we present the relaxation of current flowing in an isolated ring, i.e., in a closed system decoupled from any environment. Numerical results show that the fermionic system under consideration is generic and large enough that it relaxes (close) to equilibrium due to internal scattering processes even in the absence of any (dephasing) environment. The corresponding relaxation time is not larger than a few τ_h, and $\langle O_s \rangle(t)$ remains small for $t \gtrsim 10\tau_h$. In Fig. 1 we also show relaxation in the presence of a typical dephasing described by the choice $\gamma_m^2 = \xi_m$ [see Eq. (8)], where ξ_m is a random variable with a uniform distribution in an interval $[0,\xi_0]$. This case is very different from the tuned coupling described by Eq. (10). We notice that such a typical dephasing additionally accelerates the relaxation. Hence, the model under consideration reproduces the expected and intuitive results.

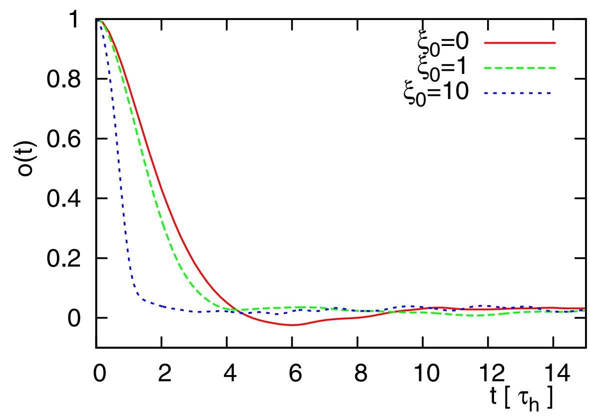

FIG. 1. (Color online) Time dependence of the particle current normalized to its initial value $o(t) = \frac{\langle O_s \rangle(t)}{\langle O_s \rangle(0)}$ for closed generic system ($\xi_0 = 0$) and for generic (random) coupling $\gamma_m^2 = \xi_m$. In the latter case ξ_m is a random variable with flat distribution in $[0,\xi_0]$ and $\omega_c = 1$. For times larger than $14\tau_h$ the current remains approximately as small as in the time window $[10\tau_h, 14\tau_h]$.

The situation dramatically changes in the case of the fine-tuned coupling between the ring and its bosonic environment, satisfying Eq. (10). According to the proposition in Sec. II A, we expect a reversal of the relaxation; that is, after sufficiently long evolution time the expectation value of the current should approach its initial value. However, the realistic systems are never perfect, and one can expect that the condition in Eq. (10) is satisfied at most approximately. Let us assume that the optimal achievable tuning is given by

$$\gamma_m^2 = E_m - E_0 + \xi_m, \tag{26}$$

where $\xi_m \in [0,\xi_0]$ is again a uniformly distributed random variable. It describes a degree of quenched or frozen disorder present in our system due its imperfect preparation. The time evolution of current flowing in the ring for different values of ξ_0 is presented in Fig. 2. In particular, for an ideal case the $\xi_m \equiv 0$ condition in Eq. (10) is satisfied, and the RR clearly occurs. For small but nonvanishing values of ξ_0 there is still a wide time window with a significant degree of RR, as indicated in Fig. 2(a). Beyond this time window the current decays towards a much smaller value, as shown in Fig. 2(b). Such nonmonotonic behavior with a wide plateau represents a hallmark of RR that could possibly be observed in simple quantum systems provided their couplings to the environments could be appropriately tuned.

Finally, we discuss the role of the bosonic characteristic (cutoff) frequency ω_c. When this quantity is too small, the coupling to the environment is not beneficial any longer. Although RR is still possible, the off-diagonal matrix elements a_{nm} decay in time due to the exponential terms in Eq. (13). Results shown in Fig. 3 for a perfectly tuned coupling $\gamma_m^2 = E_m - E_0$ indicate that a clear RR effect is observed already when ω_c is one order of magnitude smaller than the typical energy scale of the quantum system, $\sim J_h$. If ω_c is of the same order of magnitude or even larger, then the exponential damping becomes hardly visible.

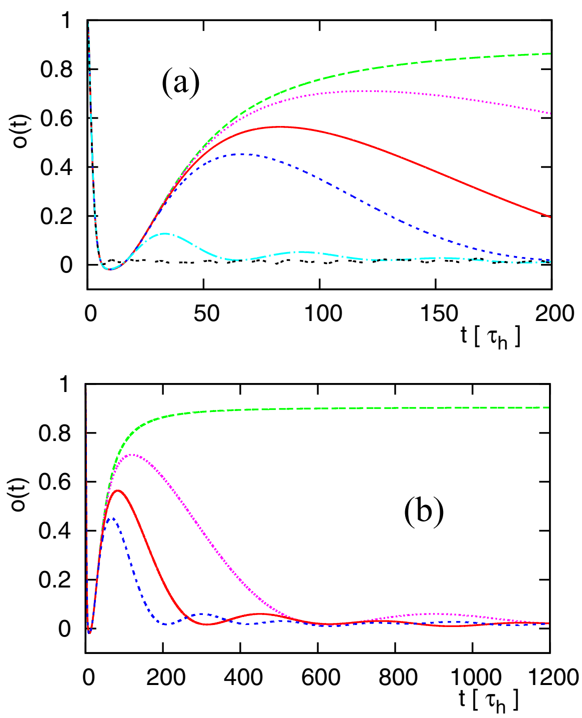

FIG. 2. (Color online) (a) Time dependence of the particle current normalized to its initial value $o(t) = \frac{\langle O_s \rangle(t)}{\langle O_s \rangle(0)}$ for a partially tuned coupling $\gamma_m^2 = E_m - E_0 + \xi_m$. Here ξ_m is random variable with flat distribution in $[0,\xi_0]$ and $\omega_c = 0.1$. The curves from top (green) to bottom (black) are for $\xi_0 = 0, 0.01, 0.02, 0.03, 0.1,$ and 1, respectively. (b) The same results but in a larger time window. In (b) the curves from top (green) to bottom (blue) are for $\xi_0 = 0, 0.01, 0.02,$ and 0.03, respectively.

III. GENERALIZATIONS

The assumptions behind the proposition in Sec. II A are not very restrictive; nevertheless, they limit the applicability of our main result to a certain class of problems. Here we discuss which assumptions can be relaxed without a significant modification of Eq. (3). First, one may consider a more general class of initial separable states, Eq. (9), where the bosonic subsystem is in the coherent state $|\Xi_\zeta\rangle = D(\zeta)|\Omega\rangle$, where D is the displacement operator [18,39] where $\zeta(\omega)$ is square integrable and the product $\zeta(\omega)g(\omega)$ satisfies the Lebesgue-Riemann lemma. This condition is satisfied, e.g., for a nonnegative and square-integrable $\zeta(\omega)$. Of course, for $\zeta(\omega) \equiv 0$ one arrives back at the proposition.

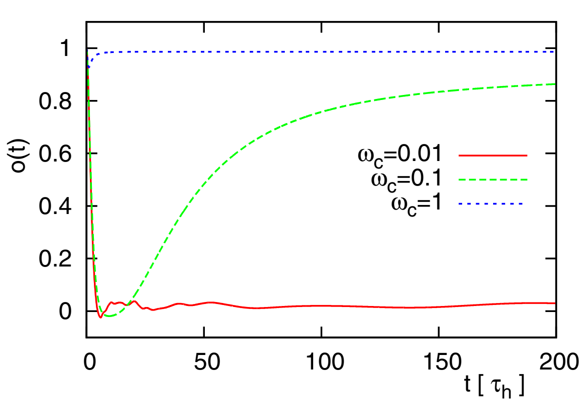

FIG. 3. (Color online) Time dependence of the particle current normalized to its initial value $o(t) = \frac{\langle O_s \rangle(t)}{\langle O_s \rangle(0)}$ for a perfectly tuned coupling $\gamma_m^2 = E_m - E_0$ and various ω_c.

It is known that the initial system-environment entanglement can affect thermalization processes [5]. A simple example of an entangled pure initial (not normalized) state is given by $|n\rangle \otimes |\Xi_\zeta\rangle + |m\rangle \otimes |\Xi_\chi\rangle$, where $|\Xi_\chi\rangle$ and $|\Xi_\zeta\rangle$ are noncolinear. Such a simple initial preparation of the pure dephasing models still allows us to find the exact time dependence of the total system plus environment composite [40]. Again, the reversal of relaxation can occur provided that both $\zeta(\omega)g(\omega)$ and $\chi(\omega)g(\omega)$ satisfy the Lebesgue-Riemann lemma.

IV. CONCLUSIONS

The effect of an environment attached to a small system may occasionally be counterintuitive or even unpredictable. Let us mention only two notable examples: stochastic resonance [41] (both classical and quantum) and environment-induced entanglement [42]. In our work we present another example of a counterintuitive effect, which is the reversal of relaxation due to a coupling to the environment. We show that relaxation of an arbitrary observable acting on a finite quantum system can effectively be reversed by attaching the system to an infinite super-Ohmic dephasing reservoir. Since this mechanism requires fine-tuning the coupling between the energy levels and the bosonic bath, we expect that it could possibly be realized in simple quantum systems rather than in complex generic cases. For the latter systems our finding will most likely remain only a matter of principle, particularly since such a coupling represents a nonlocal interaction. However, even for an imperfect tuning of the interaction between the system and the environment, one still finds a partial reversal of relaxation. This mechanism may be realized in open quantum systems when the time scale of energy exchange with the environment is large when compared to other time scales of the system. Such an approximation has been effectively applied to describe quantum optical systems coupled to a single electromagnetic mode [23]. The effect studied in our work shows up as a broad time window in which the expectation values of observables are close to the values in the initial state. Even though the proposed mechanism for the reversal of relaxation is at this stage rather theoretical, it may have an important impact on the area of quantum computing, where one of the greatest challenges is controlling or removing quantum decoherence among interacting quantum systems.

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