Dissipative Landau-Zener quantum dynamics with transversal and longitudinal noise

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We determine the Landau-Zener transition probability in a dissipative environment including both longitudinal as well as transversal quantum-mechanical noise originating from a single noise source. For this, we use the numerically exact quasiadiabatic path integral, as well as the approximative nonequilibrium Bloch equations. We find that transversal quantum noise in general influences the Landau-Zener probability much more strongly than longitudinal quantum noise does at a given temperature and system-bath coupling strength. In other words, transversal noise contributions become important even when the coupling strength of transversal noise is smaller than that of longitudinal noise. We furthermore reveal that transversal noise renormalizes the tunnel coupling independent of temperature. Finally, we show that the effect of mixed longitudinal and transversal noise originating from a single bath cannot be obtained from an incoherent sum of purely longitudinal and purely transversal noise.

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I. INTRODUCTION

The transition dynamics of quantum systems driven by external fields in the vicinity of avoided crossings of energy levels [1–4] is at the heart of many vastly different physical problems. Examples are the dynamics of chemical reactions [5], the spin reversal dynamics in molecular nanomagnets [6], the nonequilibrium dynamics of glasses at low temperatures [7–10], Bose-Einstein condensates in optical lattices [11–13], and the dynamics of solid-state artificial atoms [14–18]. In any physical realization, a quantum system is influenced by its environment, which exerts fluctuating forces on the system. These result typically in decoherence and relaxation [19,20]. The according relaxation or dephasing times constitute important additional time scales. In fact, it has been shown that they typically compete with the time scales imprinted by the external driving and that they significantly influence the Landau-Zener switching dynamics [21,22].

The Landau-Zener model [1–4] constitutes an archetype problem of driven quantum dynamics around an avoided energy-level crossing. The energy difference of the two diabatic states of a two-level quantum system with the Hamiltonian ($\hbar = 1$)

$$H_S(t) = \frac{\Delta}{2}\sigma_x + \frac{\epsilon(t)}{2}\sigma_z \tag{1}$$

is externally driven by $\epsilon(t) = vt$ with speed v. Here, the $\sigma_{i=x,z}$ are the Pauli matrices. The two diabatic states are coupled due to a nonzero value of Δ, which results in an avoided level crossing at the symmetry point taken here at $t = 0$. The Landau-Zener probability P_0 gives the probability that the quantum system remains in the ground state while the quantum two-level system (TLS) is driven from $t = -\infty$ through the avoided crossing at $t = 0$ to $+\infty$. An analytical solution to the problem was provided independently by Landau [1], Zener [2], Stueckelberg [3], and Majorana [4]. It amounts to the celebrated expression $P_0(v,\Delta) = 1 - \exp[-\pi\Delta^2/(2v)]$ for the Landau-Zener probability.

When the influence of external fluctuating forces is included by coupling the two levels to a bath of harmonic oscillators, only a numerical treatment is possible or approximating assumptions have to be invoked. In general, the dissipative Landau-Zener model is challenging since, first, the driving force generates for most times the dominant contribution in the Hamiltonian. Second, the idealized driving protocol includes infinitely long times such that the dissipative fluctuations can substantially influence the dynamics even for weak system-bath coupling strengths.

Several relevant limiting cases have been studied analytically [23–30]. In the limit of very fast sweeps (or nonadiabatic driving) $v \gg \Delta^2$, it has been found that a thermal heat bath has no influence on the Landau-Zener probability [21–30]. Conversely, a dissipative influence is expected for adiabatic driving when $v \ll \Delta^2$, and in the crossover region when $v \simeq \Delta^2$. In many cases, a dissipative environment causes fluctuations of the energies of the diabatic states and can be denoted as *longitudinal* system-bath coupling. This longitudinal dissipative Landau-Zener model was numerically solved in the full parameter regime of temperatures, driving speeds v, and system-bath couplings [21,22]. At low temperatures, the Landau-Zener probability P is hardly influenced by longitudinal noise [21,22,26]. At zero temperature, it has strictly no influence [27], which not only holds true for bosonic but also for spin-carrying environments [28]. For small sweep velocities and medium to high temperatures, nonmonotonic dependencies on the sweep velocity, temperature, coupling strength, and cutoff frequency are observed [21,22]. This characteristic behavior can be understood as a nontrivial competition between relaxation and driving. Relaxation is limited to a "crossing time window" around the avoided crossing where the energy splitting of the two-level system is smaller than temperature. Outside this window, phonon excitations are suppressed by the Bose factor. For a thermal influence on the Landau-Zener probability, the two-level system must, however, absorb a phonon. Thus, for $v \gg v_{\rm min}$ the time within this crossing window is too short for relaxation to occur and, accordingly, the bath has no influence. In contrast, for $v \ll v_{\rm min}$ relaxation dominates and the system can relax at all times within the crossing time window. Thus, due to the increasing energy splitting beyond the avoided crossing the system relaxes back toward the ground state. Maximal thermal excitation is obtained for $v \simeq v_{\rm min}$, where the crossing time equals the relaxation time. The particular value $v_{\min}$ of the driving speed, for which the Landau-Zener probability exhibits a minimum, depends on temperature, the system-bath coupling strength, and the bath cutoff frequency. To quantitatively underpin this simple picture, the relaxation rates of the two-level system during the Landau-Zener sweep through the avoided crossing must be determined. This can be done by approximate quantum master equations, such as the nonequilibrium Bloch equations [31,32].

The general role of environmental fluctuations inducing transitions between the diabatic states for a *transversal* system-bath coupling has been studied very little. Pokrovsky and Sun [30] have employed a perturbative approach and find that transversal noise acts within a much wider time window as opposed to longitudinal noise, which acts mainly closely around the avoided crossing. Thus, neglecting the interplay and possible correlations between longitudinal and transversal noise, Pokrovsky and Sun find an approximative solution for the Landau-Zener probability. At zero temperatures, transversal noise increases the Landau-Zener probability since it renormalizes the tunnel coupling Δ to larger effective values [27,28].

In this work, we treat the full dissipative Landau-Zener model including both longitudinal and transversal environmental fluctuations in the full parameter range of sweep velocities and temperatures and for weak to intermediate damping strengths. We employ the perturbative nonequilibrium Bloch equations (NBEs) [32] and the numerical exact quasiadiabatic propagator path integral (QUAPI) [33,34]. We find that the renormalization of the tunneling coupling due to transversal noise persists for finite temperature. Furthermore, the particular dependence of the upward transition rate for mixed longitudinal and transversal noise originating from a single bath on the energy shows that it cannot be understood as the incoherent sum of the two upward transition rates of either purely longitudinal or purely transversal noise. Hence, longitudinal and transverse noise cannot be treated independently.

Typically, the effect of transversal noise is *less* pronounced, for example, in solid-state artificial atoms [14–18] and in molecular donor-acceptor pairs in solution. In the latter case, however, *weak* transversal noise can dominate the energy transfer dynamics [35]. We observe that for the same value of the system-bath coupling strength the transversal term in the system-bath coupling has a much more pronounced effect on the Landau-Zener probability than the longitudinal coupling term. In other words, to obtain the same modification of the Landau-Zener probability due to external noise, the longitudinal system-bath coupling roughly has to be chosen a factor of 10 stronger than for the corresponding transversal system-bath coupling strength. Thus, the coupling strength alone is not decisive to rule out the relevance of transversal fluctuations in comparison with longitudinal ones.

In the next section, we introduce the model and briefly summarize in the third section the methods employed. Then, we present the results and end with a conclusion.

II. MODEL

A minimal approach to study the dissipative dynamics at avoided crossings is to couple the two-level quantum system (TLS) of Eq. (1) to external harmonic fluctuations. These are conveniently described by quantum harmonic oscillators [19,20] incorporated in the Hamiltonian $H_B = \sum_k \omega_k b_k^\dagger b_k$, with bosonic annihilation b_k and creation $b_k^\dagger$ operators and angular frequencies ω_k. Together with coupling term H_{SB}, the total Hamiltonian $H(t) = H_S(t) + H_{SB} + H_B$ follows. To include both transversal and longitudinal coupling, we use the bilinear form of the coupling:

$$H_{SB} = -\frac{1}{2}(\cos\theta\sigma_z + \sin\theta\sigma_x)\sum_k \lambda_k(b_k + b_k^\dagger). \qquad (2)$$

Therein, the coupling of the oscillators via σ_z to the system describes fluctuations in the driving force and thus generates "longitudinal" noise. In turn, the coupling via σ_x models fluctuations in the coupling between the two diabatic states and thus induces "transversal noise." The mixing angle θ determines the mixing of transversal and longitudinal noise, where $\theta = 0$ corresponds to purely longitudinal and $\theta = \pm\pi/2$ corresponds to purely transversal noise. As usual, the bath influence is captured by the spectral function $J(\omega) = \pi\sum_k \lambda_k^2 \delta(\omega - \omega_k)$, which is typically a smooth function [19] in the frequency range of interest. To be specific, we assume an Ohmic spectrum with $J(\omega) = \gamma\omega\exp(-\omega/\omega_c)$ with cutoff frequency ω_c and the coupling strength γ.

III. METHODS

In order to obtain the Landau-Zener probability $P(v,\Delta,T) = |\langle g|U_{\rm eff}(\infty,-\infty)|g\rangle|^2$ with $|g\rangle$ being the ground state of the quantum two-level system, we need to determine its effective time evolution $U_{\rm eff}(t,t_0)$, thereby tracing out the environmental (or bath) degrees of freedom. Alternatively, we directly determine the reduced density matrix $\rho(t) = U_{\rm eff}(t,t_0)\rho(t_0)$. We employ in the following two approaches: an adiabatic Markovian quantum master equation, i.e., the nonequilibrium Bloch equations (NBEs) [32], and the quasiadiabatic path-integral (QUAPI) approach [33,34]. Throughout this work, we use $k_B = 1$.

A. QUAPI

We briefly summarize the QUAPI approach [33,34] in the following. The algorithm is based on a symmetric Trotter splitting of the short-time propagator $\mathcal{K}(t_{k+1},t_k)$ for the full Hamiltonian $H(t)$ into a part depending on the system Hamiltonian and a part involving the bath and the coupling term. The short-time propagator describes time evolution over a Trotter time slice δt. This splitting is by construction exact in the limit $\delta t \to 0$, but introduces a finite Trotter error for a finite time increment, which has to be eliminated by choosing δt small enough such that convergence is achieved. On the other side, the bath degrees of freedom generate correlations being nonlocal in time. For any finite temperature, these correlations decay exponentially fast at asymptotic times, thereby setting the associated memory time scale. QUAPI now uses an augmented reduced density operator which includes the entire information over the memory time window. This reduced density tensor is then propagated over time via an iteration scheme. Within the memory time window, all correlations are included exactly over the finite memory time $\tau_{\mathrm{mem}} = K\delta t$. In turn, they can safely be neglected for times beyond τ_{mem}. Then, the memory parameter K has to be increased, until convergence is found. The two strategies to achieve convergence are naturally countercurrent (see [36] for details), but nevertheless convergent results can be obtained in a wide range of parameters.

B. Nonequilibrium Bloch equations

Alternatively, a Redfield-type quantum master equation for the dynamics of the driven dissipative system can be derived. One starts by switching into the adiabatic basis $\widetilde{H}_S(t) = R^\dagger H_S(t) R$ with the transformation $R = \exp[i\phi(t)\sigma_y/2]$ with $\phi(t) = \arctan[\epsilon(t)/\Delta]$. The quantum master equation for the reduced density operator involves a memory kernel $\mathcal{M}(t,s)$. The lowest order of Born approximation for $\mathcal{M}(t,s)$ in the system-bath coupling [37,38] can be employed for a weak-system-bath coupling. One obtains

$$\mathcal{M}(t,s) = \mathrm{Tr}\{\mathcal{L}_{SB}(t)\mathcal{U}_0(t,s)\mathcal{L}_{SB}(s)\rho_B^{\mathrm{eq}}\}, \tag{3}$$

with $\rho_B^{\mathrm{eq}} = \exp(-\beta H_B)/\mathrm{Tr}\{\exp(-\beta H_B)\}$ and the free time evolution superoperator $\mathcal{U}_0(t,s) = \exp\{\int_s^t ds'(\mathcal{L}_{LZ}(s') + \mathcal{L}_B(s'))\}$. The Liouvillians $\mathcal{L}_x$ act according to $\mathcal{L}_x O = -i[\widetilde{H}_x, O]$ on operators O and $\widetilde{H}_{LZ}(t) = \widetilde{H}_S(t) + \phi'(t)\sigma_y/2$. Here, $\phi'(t) = \frac{d\phi(t)}{dt}$. Next, we may assume that the memory kernel $\mathcal{M}(t,s)$ is short lived, i.e., $\mathcal{M}(t,s) \ll 1$ for $(t-s) \gg \tau_{\mathrm{mem}}$ with τ_{mem} much shorter than any system time scale Δ^{-1}. Then, a Markovian approximation can be invoked. We note that we also have to assume further that the external driving of the system acts on time scales much larger than τ_{mem} for the Markovian assumption to be valid. This enters formally in the calculation of the memory kernel $\mathcal{M}(t,s)$, when we assume that $\mathcal{L}_{SB}(s) \simeq \mathcal{L}_{SB}(t)$ and $\exp\{\int_s^t ds' \mathcal{L}_{LZ}(s')\} \simeq \exp\{\mathcal{L}_S(t)\cdot(t-s)\}$. Put differently, we assume that the driving is adiabatic with respect to the bath memory time.

Neglecting further frequency renormalizations, one obtains for the time evolution of the components of the statistical operator $\rho(t) = \frac{1}{2}(\mathbb{1} - \sum_{j=x,y,z} r_j(t)\sigma_j)$ of the system in its adiabatic basis, i.e., $\widetilde{H}_S(t) = E(t)\sigma_x/2$ with $E(t) = \sqrt{\Delta^2 + \epsilon(t)^2}$, the nonequilibrium Bloch equations

$$\begin{aligned}
\partial_t r_x(t) &= \phi'(t) r_z(t) - \gamma_1(t)\big[r_x(t) - r_x^{\mathrm{eq}}(t)\big],\\
\partial_t r_y(t) &= -\gamma_2(t) r_y(t) - E(t) r_z(t),\\
\partial_t r_z(t) &= E(t) r_y(t) - \gamma_2(t) r_z(t) - \phi'(t) r_x(t),
\end{aligned} \tag{4}$$

with time-dependent momentary equilibrium value $r_x^{\mathrm{eq}}(t) = \tanh[\beta\frac{1}{2}E(t)]$ and time-dependent decay rates

$$\gamma_1(t) = \frac{J[E(t)]}{2}\coth\left[\frac{1}{2}\beta E(t)\right]\{u(t)\cos\theta - v(t)\sin\theta\}^2, \tag{5}$$

$$\begin{aligned}
\gamma_2(t) &= \frac{1}{2}\gamma_1(t) + \gamma_{\mathrm{deph}}(t), \quad \text{with}\\
\gamma_{\mathrm{deph}}(t) &= 2\gamma T\{u(t)\sin\theta + v(t)\cos\theta\}^2.
\end{aligned} \tag{6}$$

Herein, $u(t) = \cos\phi(t)$, and $v(t) = \sin\phi(t)$. These equations of motion incorporate the full nonadiabatic behavior in the vicinity of the avoided crossing of the system. They can easily be integrated numerically employing a standard fourth-order Runge-Kutta scheme.

IV. RESULTS

We have applied both the quasiadiabatic path integral and the adiabatic Markovian quantum master equation to obtain results for the Landau-Zener probability for driving speeds from slow (adiabatic) to fast (nonadiabatic) driving, for low ($T \ll \Delta$) to high ($T \gg \Delta$) temperatures, and for system-bath couplings from small, i.e., $\gamma \simeq 10^{-4}$, to intermediate, i.e., $\gamma = 2\times 10^{-2}$. In particular, our study aims at revealing the role of varying the mixing angle θ between longitudinal and transversal noise.

A. Transverse versus longitudinal noise

Figure 1 shows the Landau-Zener probability versus the sweep velocity for a cutoff frequency $\omega_c = 10\Delta$. Figures 1(a) and 1(b) show data for system-bath coupling strength $\gamma = 2\times 10^{-4}$, and Figs. 1(c) and 1(d) show data for $\gamma = 2\times 10^{-3}$. Furthermore, only longitudinal noise, i.e., $\theta = 0$, acts in (a) and (c), and only transversal noise, i.e., $\theta = \pi/2$, is present in (b) and (d). The symbols represent the QUAPI data for various temperatures as indicated (the symbol size corresponds to the typical numerical inaccuracy). The lines represent results determined by the nonequilibrium Bloch equations. The results of both the QUAPI and the NBE method coincide in the regime addressed. For larger sweep velocities $v \gtrsim 3\Delta^2$, the NBE slightly overestimates the exact Landau-Zener probability [32]. Since dissipative effects are suppressed in the nonadiabatic regime, we focus in the following on the adiabatic and crossover regime, i.e., $v \lesssim 3\Delta^2$.

In the cases displayed in Figs. 1(b)–1(d), a minimum in the Landau-Zener probability versus v at a certain v_{min} is observed for temperatures $T \gtrsim \Delta$. With increasing temperature, v_{min} increases and $P(v_{\mathrm{min}})$ decreases. This minimum is the signature of the competition between the Landau-Zener

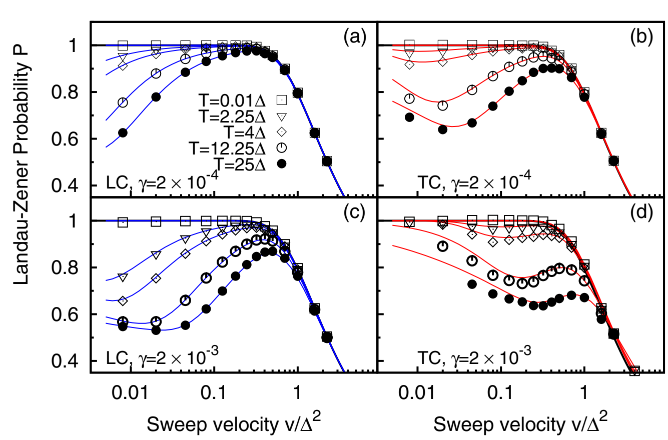

FIG. 1. (Color online) Landau-Zener probability P vs sweep velocity v for various temperatures as indicated for $\omega_c = 10\Delta$. Panels (a) and (c) show P for pure longitudinal system-bath coupling ("LC") with $\theta = 0$, and panels (b) and (c) show P for pure transversal system-bath coupling ("TC") with $\theta = \pi/2$, respectively. The dissipative coupling strength is $\gamma = 2\times 10^{-4}$ for panels (a) and (b) and $\gamma = 2\times 10^{-3}$ for panels (c) and (d).

driving and relaxation [21,22]. In Fig. 1(a), the LZ probability becomes minimal at velocities $v_{\rm min}$ which are smaller than those studied. For $v \gg v_{\rm min}$, the time span during which the system stays in the vicinity of the avoided crossing, is too short for relaxation to occur. The dynamics deviates little from P_0. In contrast, for $v \ll v_{\rm min}$ relaxation dominates and the system then can relax at all times in the crossing region. Thus, due to the increasing energy splitting beyond the avoided crossing, the system relaxes back towards the ground state. In this respect, the minimum occurs when the underlying time scales for passing through the avoided crossing and for relaxation become equal [21,22].

Moreover, the role of purely longitudinal and purely transversal noise is revealed in Fig. 1. It is known for longitudinal noise [21,22] that for increasing γ the minimum $v_{\rm min}$ shifts to larger v. This feature is recovered when comparing Figs. 1(a) and 1(c). This holds true as well for transversal noise, when we compare Figs. 1(b) and 1(d). In that respect, we can conclude that transversal noise resembles longitudinal noise for effective larger system-bath coupling strengths. The Landau-Zener probability versus v at a given temperature differs quantitatively even when comparing cases with stronger longitudinal to cases with weaker transversal noise. However, the Landau-Zener probability, and, specifically, $v_{\rm min}$, is qualitatively the same when we compare a case with longitudinal noise with the coupling strength roughly a factor of 10 larger than for the corresponding case of transversal noise only [see Figs. 1(b) and 1(c)]. Thus, transversal noise generically causes stronger dissipative effects than longitudinal noise at the same nominal strength γ. Since the condition for $v_{\rm min}$ is that the time scales for passing through the avoided crossing and relaxation are equal, transversal noise must either result in effectively larger relaxation rates or it must have an extended crossing time window. For purely classical transversal noise, a similar effect was reported by Pokrovsky and Sun [30]. It was attributed to a larger time window around the avoided crossing where dissipative effects are more efficient for transversal than for longitudinal noise. Since the QUAPI and NBE results coincide for the weak system-bath coupling strengths studied here, we make extended use of the much simpler NBE approach in the following to gain an understanding of the crossing time windows for both noise configurations.

Before we proceed, we note that we have also performed calculations for a smaller value of the cutoff frequency, i.e., $\omega_c = 5\Delta$. The results are essentially the same as those shown in Fig. 1 for $\omega_c = 10\Delta$.

In Fig. 2, we show the momentary energy spectrum at a fixed time t (full black lines) of the quantum two-level system versus t which shows the avoided level crossing around the symmetry point $t = 0$. In order that the Landau-Zener probability is altered, dissipative effects have to excite the two-level system out of the ground state. Within the NBE approach, we may calculate the rate $\gamma_u(t)$ for the upward transition from the ground to the excited adiabatic eigenstate at a given time t. We find

$$\gamma_u(t) = \frac{J[E(t)]}{2}\, n_B\left(\frac{E(t)}{2T}\right)\{u(t)\cos\theta - v(t)\sin\theta\}^2, \quad (7)$$

with the Bose factor $n_B(\omega)$. The dependence of this rate on t is shown in Fig. 2 for a temperature $T = 4\Delta$. The blue dashed line refers to longitudinal noise with $\theta = 0$, and the red dash-dotted line refers to transversal noise with $\theta = \pi/2$. We observe that the action of the longitudinal noise is clearly restricted to the regime close to the avoided level crossing when $|t| \lesssim \Delta/v$. This is due to the prefactor $u^2(t) = \Delta^2/[\Delta^2 + (vt)^2]$. In contrast to this, the prefactor $v^2(t) = (vt)/[\Delta^2 + (vt)^2]$ for the transversal noise vanishes at the symmetry point at the avoided crossing but approaches 1 for large $t \gtrsim \Delta/v$. In turn, the transversal noise has much more time to influence the Landau-Zener probability by dissipative effects.

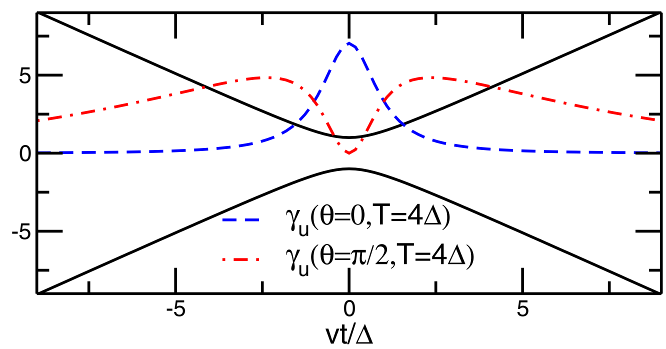

FIG. 2. (Color online) Momentary energy levels (black full lines) of the two-level system (energies and γ in units of Δ) and the momentary upward transition rate $\gamma_u(t)$ of Eq. (7) for longitudinal system-bath coupling (blue dashed line) with $\theta = 0$ and for transversal system-bath coupling (red dash-dotted line) with $\theta = \pi/2$ vs time.

The suppression of the upward transition rates far away from the symmetry point results from a vanishing Bose factor. Phonon excitations in the bath have to be available which could be absorbed by the two-level system in order to get excited from its ground to the excited state. In both cases, the rate is further suppressed once the energy splitting exceeds the bath cutoff frequency ω_c. A large value of the cutoff frequency is also necessary to ensure that the Landau-Zener probability can be determined numerically accurately. This also facilitates numerical convergence. In our calculations, we have $\omega_c \gtrsim T$, so that the influence of ω_c is reduced to a mere quantitative level. To support this observation, we have studied also the case $\omega_c = 5\Delta$ (data not shown) and find qualitatively the same results as in Fig. 1 (up to some minor quantitative differences).

B. Renormalization of tunnel coupling due to transversal noise

At zero temperature, longitudinal noise has no influence on the Landau-Zener probability, whereas transversal noise renormalizes the tunnel coupling [27] according to

$$\Delta^2 \to W^2 = (\Delta - E_R \sin\theta\cos\theta)^2 + S\sin^2\theta, \quad (8)$$

with the reorganization energy $E_R = \int_0^\infty d\omega J(\omega)/\omega = \gamma\omega_c$ and the total spectral weight $S = \int d\omega J(\omega) = \gamma\omega_c^2$. These exact results are strictly valid for $T = 0$. For finite temperature, the influence of longitudinal noise vanishes when $T \ll \Delta$, but for transversal noise the behavior for any finite temperature in the limit $T \to 0$ has not been clarified. Two scenarios are possible. On one hand, the renormalization could be connected to the renormalization of the tunnel coupling in the spin-boson model [19] which emerges at all temperatures. On the other hand, the renormalization might be connected to the quantum

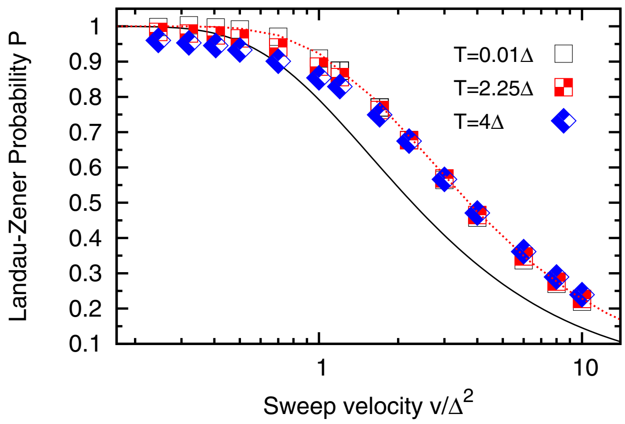

FIG. 3. (Color online) Landau-Zener probability vs sweep velocity for various low-to-intermediate temperatures for transversal system-bath coupling with strength $\gamma = 0.02$ and $\omega_c = 10$. The black full line represents the pure Landau-Zener probability without dissipation whereas the dotted red line marks the Landau-Zener probability with an effective renormalized tunnel coupling.

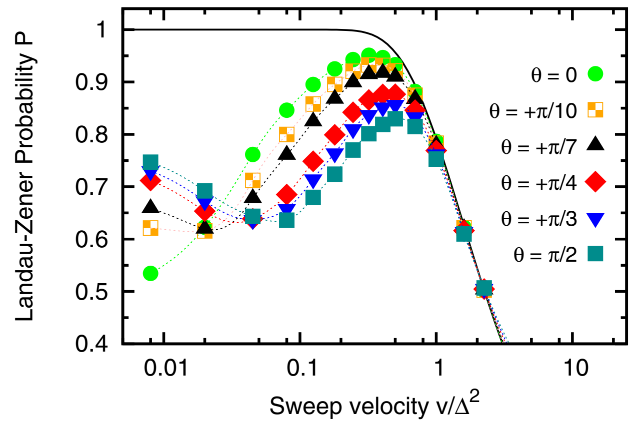

FIG. 4. (Color online) Landau-Zener probability P vs sweep velocity v for various mixing angles θ for $T = 25\Delta$, $\omega_c = 10\Delta$, and $\gamma = 5 \times 10^{-4}$.

phase transition of the spin-boson model [19,20] which is only present strictly at $T = 0$, but vanishes at finite temperatures.

Clearly, these effects are beyond a mere Markovian description and cannot be observed within the NBE approach. However, we can use QUAPI to determine the Landau-Zener probability for low temperatures and transversal noise with a larger value of the dissipative coupling strength. For this, we set $\gamma = 0.02$ and show in Fig. 3 the results in the relevant regime of sweep velocities. The full line is the pure Landau-Zener probability $P_0(v,\Delta)$ without noise. In addition, the dotted curve depicts the Landau-Zener probability $P_0(v,W)$ without noise, but with a renormalized value of the tunnel coupling according to Eq. (8). The Landau-Zener probability follows the case with an effective renormalized tunnel coupling for all small temperatures $T \leqslant \Delta$ (only the data for $T = 0.01\Delta$ are shown). Once temperature exceeds the tunnel coupling, the dissipative effects due to relaxation, and, in particular, the emergence of the minimum, become sizable. Accordingly, for small v, the Landau-Zener probability decreases and no longer follows $P_0(v,W)$. However, for large v, even at $T \gtrsim \Delta$, the numerical results match $P_0(v,W)$. Hence, we can conclude that the renormalization is a qualitatively similar effect as the renormalization of the tunnel coupling in the spin-boson model. It remains open, however, why renormalization vanishes for longitudinal noise.

C. Asymmetric mixed noise

So far, we have addressed only purely longitudinal or purely transversal quantum noise. In the following, we investigate the case of a finite mixing of both terms. Figure 4 shows the Landau-Zener probability versus v for various mixing angles $0 \leqslant \theta \leqslant \pi/2$ for a fixed temperature $T = 25\Delta$. With increasing mixing angle, the minimum in the probability shifts to larger values of v as could be expected since the character of the total noise becomes more transversal with increasing θ. Transversal noise results in effectively stronger dissipative effects in an enlarged time window in which relaxation is active. Accordingly, $v_{\rm min}$ shifts to larger values.

Pokrovsky and Sun [30] have analyzed mixed noise by assuming that both contributions originate from two different independent baths and can be separated. Such an assumption seems to be valid at first sight, as illustrated by the above discussion of the upward transition rate shown in Fig. 2. Considering only the two cases separately may lead to the conclusion that the transition rates for both cases are nonzero in separated time windows. If both contributions were independent, one would expect that the sign of the respective system-bath coupling terms is not relevant. Thus, we would expect that $P(v,\Delta,\theta) = P(v,\Delta,-\theta)$.

In order to check this, we have calculated the Landau-Zener probability versus v for both signs of the mixing angle θ, but otherwise identical parameters. We should remember that we have a setup with a single noise source. In particular, when we reverse the sign of one term, the two coupling terms no longer commute, i.e., $[O_+,O_-] \neq 0$ for $O_\pm = \frac{1}{2}(\cos\theta\sigma_z \pm \sin\theta\sigma_x)$. Correspondingly, we expect a modified dissipative influence on the Landau-Zener probability in the present configuration. This is shown in Fig. 5 via the Landau-Zener probability versus

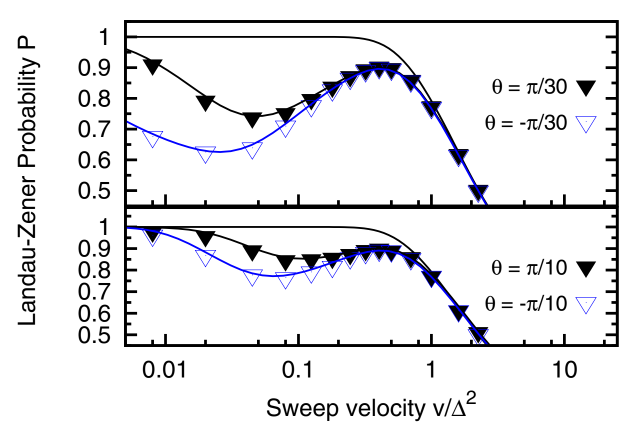

FIG. 5. (Color online) Landau-Zener probability P vs sweep velocity v for mixing angles of opposite sign, $\theta = \pm\pi/30$ (upper figure) and $\theta = \pm\pi/10$ (lower figure) for $T = 4\Delta$, $\omega_c = 10\Delta$, and $\gamma = 0.01$.

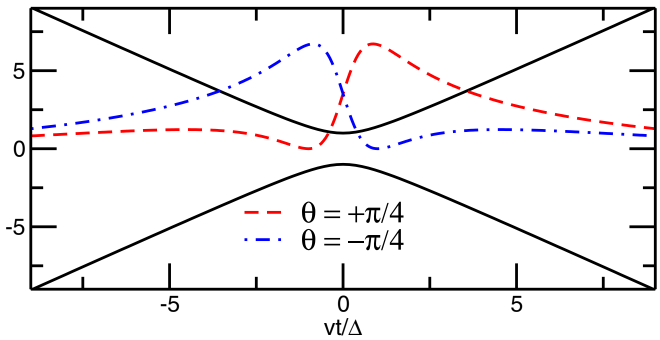

FIG. 6. (Color online) Momentary energy levels (black full lines) of the two-level system (energies and γ in units of Δ) and the momentary upward transition rate $\gamma_u(t)$ of Eq. (7) for $\theta = -\pi/4$ (blue dashed line) and $\theta = \pi/4$ (red dash-dotted line) vs time.

v for both signs of θ but otherwise identical parameters. Indeed, we find significant differences which emerge specifically in the adiabatic regime with $v \lesssim v_{\rm min}$. In contrast, for $v > v_{\rm min}$ and in the nonadiabatic regime, the sign of θ is irrelevant.

This asymmetric behavior with respect to the sign of θ is again a result of the time-dependent upward transition rates and the associated time window for the action of dissipation. The system Hamiltonian and the system-bath coupling term only commute at a single time t_c. This time is determined according to $[H_S(t_c), \cos\theta\sigma_z + \sin\theta\sigma_x] = 0$, which is fulfilled when $vt_c = \Delta\tan(\pi/2 - \theta)$. For $\theta > 0$, we have that $t_c > 0$ which thus occurs beyond the avoided crossing. At this time, the upward transition rate vanishes. The upward transition rate is asymmetric in time with respect to the symmetry point $t = 0$ for all $0 < \theta < \pi/2$. We plot in Fig. 6 the upward transition rate for the two mixing angles $\theta = \pm\pi/4$. We observe a single global maximum for $t < 0$ ($t > 0$) for $\theta < 0$ ($\theta > 0$). Thus, the rate is also well suppressed for $t > 0$ ($t < 0$) when $t_c > 0$ ($t_c < 0$). Now, dissipative effects before the avoided crossing, i.e., when $t < 0$, mainly support the excitation out of the ground state since driving can only excite the system close to the avoided crossing when $|vt| \lesssim \Delta$. Relaxation beyond the avoided crossing rather induces a repopulation of the ground state. Thus, the minimum in the Landau-Zener probability is expected to be more pronounced for $t_c < 0$ or when $\theta < 0$. This is indeed what we find.

V. CONCLUSIONS

We have investigated the dissipative Landau-Zener model including both longitudinal and transversal environmental fluctuations in the full parameter range of sweep velocities and temperatures and for weak to intermediate damping strengths. We have employed the perturbative nonequilibrium Bloch equations (NBEs) [32] and the numerically exact quasiadiabatic propagator path integral (QUAPI) [33,34]. We have found that the renormalization of the tunnel coupling at zero temperature due to transversal noise persists up to finite temperatures. Furthermore, we have shown that the upward transition rate for mixed longitudinal and transversal noise originating from a single bath is not the incoherent sum of the two upward transition rates of purely longitudinal and purely transversal noise.

Most importantly, we have found that for equal system-bath coupling strength the influence of transversal quantum fluctuations on the Landau-Zener probability is much stronger. In detail, for equal modifications of the Landau-Zener probability, the transversal system-bath coupling has roughly to be a factor of 10 stronger than the longitudinal system-bath coupling. This can be understood in terms of the time-dependent momentary relaxation rates emerging from an adiabatic-Markovian treatment. From this, we see that the relaxation rates due to longitudinal noise act only within a time window determined by $|vt| \lesssim \Delta$. In turn, transversal noise effects are sizable over a much larger time window around the avoided crossing. We note that these results hold for an Ohmic spectral function for the environmental fluctuations. For super-Ohmic fluctuations as, for example, resulting from lattice vibrations, both longitudinal as well as transversal noise would act in the broader time window defined by temperature. Thus, the difference between both kinds of fluctuations is expected to be smaller. However, both will act more strongly on the Landau-Zener probability than longitudinal Ohmic noise for otherwise identical parameters. Thus, we can conclude furthermore that the system-bath coupling strength alone is not the decisive parameter to rule out the relevance of one type of noise against another one for driven quantum dynamics around an avoided level crossing. This observation can be rather important for the interpretation and the analysis of experiments, specifically for artificial solid-state atoms and devices designed to process quantum information.

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