RAPID COMMUNICATIONS

PHYSICAL REVIEW A **91**, 051603(R) (2015)

Quantum sound-cone fluctuations in cold Fermi gases: Phonon propagation

Jen-Tsung Hsiang,[1,2,*] Da-Shin Lee,[1,†] Chi-Yong Lin,[1,‡] and Ray J. Rivers[3,§]

[1]*Department of Physics, National Dong-Hwa University, Hualien, Taiwan, Republic of China*

[2]*Center for Theoretical Physics, Fudan University, Shanghai, China*

[3]*Blackett Laboratory, Imperial College London, SW7 2BZ, United Kingdom*

(Received 15 April 2014; published 27 May 2015)

We examine the effect of quantum fluctuations in a tunable cold Fermi gas on phonon propagation. We show that these fluctuations can be interpreted as inducing a stochastic space-time. This effect can be displayed in the variation in the travel time of phonons, at its greatest in the crossover region between Bose-Einstein condensate and BCS regimes.

DOI: 10.1103/PhysRevA.91.051603 PACS number(s): 67.85.Lm, 03.65.Yz, 03.70.+k, 05.70.Fh

Massless particles and gapless modes (photons, phonons) propagate causally according to the metrics of their respective light and sound cones. However, the intrinsically quantum-mechanical nature of the particle environments (quantum gravity, condensate fluctuations) makes the cones "fuzzy." In particular, this fuzziness induces fluctuations in particle times of flight. Such an effect is difficult to calculate *ab initio*. Instead, several authors have addressed the simpler problem of deriving this and related effects for photons [1,2] and phonons [3–5] in predetermined phenomenological random media.

In this Rapid Communication we show that, for condensates of cold Fermi gases, the effect of quantum fluctuations on phonon times of flight can be calculated *directly*, without recourse to external sources of fluctuations. Specifically, the diatoms in the gas provide the endogenous stochastic medium in which the phonons move. We recall that the weak-coupling BCS regime of a Fermi gas is dominated by Cooper pairs (in which the atoms are correlated in momentum) with low numbers of diatoms (in which the atoms are correlated in position). The situation is reversed in the strong-coupling Bose-Einstein condensate (BEC) regime, where diatoms dominate. We assume that we can interpolate from one regime to the other by the existence of a tunable *narrow* Feshbach resonance, whose interaction with an external magnetic field enables us to change the strength of atomic interactions or, equivalently, the atomic scattering length. Such is the case for the narrow resonance at $H_0 = 543.25$ G in cold ^{6}Li, discussed in some detail in [6] and used by us elsewhere [7,8], to which we shall turn later.

We show that the fluctuations in the diatom density can be interpreted as inducing a stochastic space-time metric whose effect, as displayed in the variation in the travel time of phonons, is at its greatest in the crossover region between BEC and BCS regimes. Roughly, for the case in hand, the effects are somewhat less than 1% for the propagation time of waves across a typical condensate. As yet this is too small to measure but, nonetheless, is huge in comparison to the relative 10^{-9} fluctuations in photon propagation times in random media [2], which are their nearest equivalent, let alone the infinitesimally small Planck time induced by the fluctuations of quantum gravity, which prompted the analysis [9].

We adopt the notation of our earlier work [7,8] in describing a cold ($T = 0$) Fermi gas, tunable through a narrow Feshbach resonance, by the action ($\hbar = 1$) [10]

$$S = \int dt\, d^3x \left\{ \sum_{\uparrow,\downarrow\downarrow} \psi^*_\sigma(x) \left[i\,\partial_t + \frac{\nabla^2}{2m} + \mu \right] \psi_\sigma(x) + \varphi^*(x) \left[i\,\partial_t + \frac{\nabla^2}{2M} + 2\mu - \nu \right] \varphi(x) - g[\varphi^*(x)\psi_{\downarrow\downarrow}(x)\psi_\uparrow(x) + \varphi(x)\psi^*_\uparrow(x)\psi^*_{\downarrow\downarrow}(x)] \right\} \tag{1}$$

for fermion fields ψ_σ with spin label $\sigma = (\uparrow,\downarrow)$. The diatomic field φ describes the bound-state (Feshbach) resonance with tunable binding energy ν and mass $M = 2m$. Furthermore, the condensate order parameter is the Feshbach resonance field itself $\varphi(x) = -|\varphi(x)|e^{i\theta(x)}$, which leads to a single-fluid model in the hydrodynamic limit with a tunable sound speed [7,8].

As a result of spontaneous symmetry breaking a homogeneous condensate acquires a nonzero $|\varphi(x)| = |\varphi_0|$, about which it fluctuates with fluctuations $\delta|\varphi| = |\varphi| - |\varphi_0|$. The diatom density is $\rho_D = |\varphi|^2$, with fluctuations $\delta\rho_D = 2|\varphi_0|\delta|\varphi|$. We expand in the derivatives of θ and the *small* fluctuations in the condensate density (or, equivalently, $\delta|\varphi|$) always preserving the Galilean invariance of the system. Galilean scalars are the field fluctuation $\delta|\varphi|$ itself (and hence $\delta\rho_D$), $G(\theta) = \dot\theta + (\nabla\theta)^2/4m$, and $D_t(\delta|\varphi|,\theta) = (\dot{\delta|\varphi|}) + \nabla\theta \cdot \nabla(\delta|\varphi|)/2m$, the comoving time derivative in the condensate with fluid velocity $\nabla\theta/2m$.

For a narrow resonance, the mean-field approach adopted in our work can be trusted [10]. The action S is quadratic in the fermion fields. On integrating them out and changing variables to θ and $\epsilon = \kappa^{-1}\delta|\phi|$, a *dimensionless* rescaled condensate fluctuation, the *local* Galilean invariant effective density for the long-wavelength, low-frequency condensate is

*cosmology@gmail.com

†dslee@mail.ndhu.edu.tw

‡Corresponding author: lcyong@mail.ndhu.edu.tw

§r.rivers@imperial.ac.uk

of the form [7,8]

$$S_{\rm eff}[\theta,\epsilon] = S_0[\theta] - \alpha \int d^4x \epsilon G(\theta) + \frac{1}{4}\int d^4x[\eta\dot{\epsilon}^2 - \rho_0(\nabla\epsilon)^2/2m - \bar{M}^2\epsilon^2], \quad (2)$$

where

$$S_0[\theta] = \int d^4x \left[\frac{N_0}{4}\, G^2(\theta) - \frac{1}{2}\rho_0 G(\theta)\right] \quad (3)$$

is the canonical acoustic BCS action [11]. The scale factor κ is chosen so that the coefficients of $(\nabla\epsilon)^2$ and $(\nabla\theta)^2$ in (2) and (3) are identical [11]. The coefficients α, η, etc., are known functions of the scattering length [7,8] and hence of the external magnetic field used to tune the condensate from the BCS to BEC regimes.

The action (2) roughly represents a two-component system of diatoms and Cooper pairs, with a corresponding two-component density in which fermions oscillate from one to the other while maintaining a fixed total number density ρ_0. In the hydrodynamic approximation, where the spatial and temporal variation of ϵ can be ignored in comparison to ϵ itself, density fluctuations ϵ act as sources and sinks to the dynamics of the phase θ and can be eliminated by simply identifying $\epsilon \approx -2\alpha G(\theta)/\bar{M}^2$. The corresponding Euler-Lagrange equation for θ is the continuity equation of a *single* fluid [7,8] from which the fluctuations in the local number density $\delta\rho = \rho - \rho_0 = 2\alpha\epsilon - N_0 G(\theta) = -(N_0 + 4\alpha^2/\bar{M}^2)G(\theta)$ and the number current density $\mathbf{j} = \rho_0\nabla\theta/2m$ lead to the wave equation $\ddot{\theta}(x) - c^2\nabla^2\theta(x) = 0$. The sound speed c will be derived below.

We see immediately that beyond the hydrodynamical approximation ϵ becomes a dynamical field with quantum fluctuations to which the phonons couple. Tracing out the ϵ field will introduce stochasticity in the acoustic metric of the θ field via its Langevin equation.

We proceed by constructing the closed time-path (CTP) effective action (e.g., see [12]),

$$S_{\rm CTP}[\theta^+,\epsilon^+;\theta^-,\epsilon^-] = S_{\rm eff}[\theta^+,\epsilon^+] - S_{\rm eff}[\theta^-,\epsilon^-],$$

where $\pm$ denote integration on the upper and lower contours of the path, respectively. It is sufficient to retain only the second power of ϵ. Integrating out the ϵ field then gives an effective nonlocal action for dynamical phonons,

$$S_{\rm eff}[\theta^+;\theta^-] = S_0[\theta^+] - S_0[\theta^-] + \Delta S[\theta^+;\theta^-], \quad (4)$$

where

$$\Delta S[\theta^+;\theta^-] = \frac{\alpha^2}{2}\iint d^4x_1 d^4x_2 \sum_{a,b=+,-} G(\theta^a(x_1)) \times D_\epsilon^{ab}(x_1 - x_2) G(\theta^b(x_2)). \quad (5)$$

In (5) the $D_\epsilon^{\pm\pm}$ denote time-ordered D_ϵ^{++}, and anti-time-ordered D_ϵ^{--} correlators as well as two Wightman correlators $D_\epsilon^{+-} = -\langle\epsilon(x_2)\epsilon(x_1)\rangle$, $D_\epsilon^{-+} = -\langle\epsilon(x_1)\epsilon(x_2)\rangle$, respectively.

We recover the semiclassical phonon field θ and the fluctuating field R about it through the decomposition $\theta^\pm(x) = \theta(x) \pm R/2$. For the purpose of wave propagation we need only to retain terms in $S_{\rm eff}$ linear and quadratic in R. Specifically, $G(\theta^\pm) \approx \dot{\theta} \pm D_t R/2$ at the relevant order. In ΔS, quadratic terms in R are then linearized by the introduction of Gaussian noise ξ with a bilinear coupling $\alpha(D_t R(x))\xi(x)$, and distribution

$$\langle\xi(x)\xi(x')\rangle = D_{\epsilon H}(x - x') = \frac{1}{2}\langle\{\epsilon(x),\epsilon(x')\}\rangle = \int \frac{d^3\mathbf{k}}{(2\pi)^3}\frac{\cos[\omega_k(t - t')]}{\omega_k\eta}\, e^{-i\mathbf{k}\cdot(\mathbf{x}-\mathbf{x}')}. \quad (6)$$

The dispersion relation of the ϵ field is determined by $\omega_k = \sqrt{\rho_0 k^2/2m\eta + \bar{M}^2/\eta}$. The linear term is obtained in terms of the ϵ retarded propagator $D_{\epsilon R}$,

$$D_{\epsilon R}(x - x') = i\theta(t - t')\langle[\epsilon(x),\epsilon(x')]\rangle, \quad (7)$$

and will in turn modify the dynamics of the phonons in the semiclassical approximation.

The semiclassical dynamics of the phonon field in a stochastic background provided by the noise can be explored by the Langevin equation obtained by taking the variation of the resulting action with respect to R,

$$\frac{N_0}{2}\ddot{\theta}(x) - \left(\frac{\rho_0}{4m} - \frac{\alpha\xi}{2m}\right)(\nabla^2\theta) + \alpha^2\int d^4x'\partial_t D_{\epsilon R}(x - x')\dot{\theta}(x') = -\alpha D_t\xi(x). \quad (8)$$

What is crucial for our subsequent discussion is the multiplicative noise term $\xi\nabla^2\theta$, a consequence of the Galilean invariance enforcing covariant derivatives. Behavior of this form is the starting point for the phenomenological stochastic analysis of the papers of [1–4]. However, whereas these authors argue for stochastic behavior on empirical grounds, in our case we see from (6) that the noise ξ is essentially the (known) diatomic fluctuation field ϵ. Equation (8) encodes quantum effects in two distinct ways, through the retarded commutator $D_{\epsilon R}$ and the noise ξ. Although they overlap we shall do our best to treat them separately.

In the phonon acoustic limit $\omega = ck$ for which, as $\omega,k \to 0$ in (7), $D_{\epsilon R}(x - x') \to (2/\bar{M}^2)\delta^4(x - x')$, we reproduce the classical mean value speed of sound c, that has been derived elsewhere [7,8] by different means:

$$c^2 = \frac{\rho_0/2m}{N_0 + 4\alpha^2/\bar{M}^2}. \quad (9)$$

If a_S is the s-wave scattering length and k_F,v_F the Fermi momentum and Fermi velocity, respectively, c^2/v_F^2 varies smoothly with $1/k_F a_S$, decreasing monotonically from 1/3 in the BCS regime $(1/k_F a_S < 0)$ to vanishingly small in the BEC regime $(1/k_F a_S > 0)$ [7,8]. More generally, if we take $D_{\epsilon R}(x)$ as follows from (7) we have a Bogoliubov quantum "rainbow" [13] of sound speeds c_k, according to the wavelength k, of the form [7,8]

$$c_k^2 \approx c^2[1 + k^2/K^2 + \cdots], \quad (10)$$

where

$$K^{-2} = \frac{4\alpha^2 c^2}{\bar{M}^4}\left[1 - \frac{c^2\eta}{\rho_0/2m}\right]. \quad (11)$$

In the large momentum limit in the BEC regime we recover [8] the free particle limit for diatoms and molecules $\omega = k^2/4m = k^2/2M$. Provided that the phonons comprise

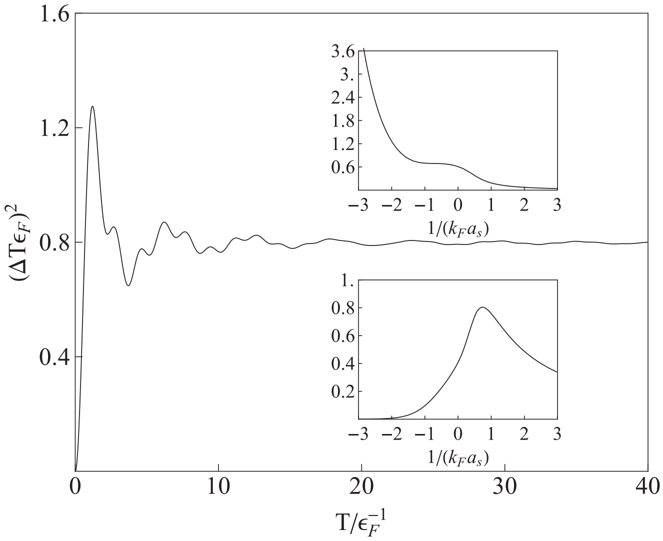

FIG. 1. The figure shows the behavior of $(\epsilon_F \Delta T)^2$ as a function of $\epsilon_F T$ at $1/a_S k_F = 0.7$, given by (15), with the UV cutoff $k_\Lambda = k_F$ for numerically carrying out the momentum integration. The lower inset figure shows the saturation value of fluctuations in time of flight by changing $1/a_S k_F$, also obtained from (15). Its maximum value occurs at $1/a_S k_F = 0.7$ near the crossover regime. The upper inset figure shows the variation of the momentum scale K of Eq. (11) also as a function of $1/a_S k_F$.

a wave packet propagating toward the detector with central momentum k_0 and width Δk_0, with $k_0 + \Delta k_0 < K$ of (11) they all experience approximately the same sound speed c and common fluctuations in times of flight. This is what we now assume [see inset (top) in Fig. 1].

For such long-wavelength phonons Eq. (8) becomes

$$\ddot{\theta}(x) - c^2(1 - 2\alpha\xi/\rho_0)\nabla^2\theta \approx -4m(\alpha/\rho_0)c^2 D_t \xi(x), \quad (12)$$

in terms of the speed of sound c of (9). As a result we can interpret c_ξ where $c_\xi^2 = c^2(1 - 2\alpha\xi/\rho_0)$ as a stochastic speed of sound in the long-wavelength regime.

The noise term in the right-hand side of (12) has no direct effect in fluctuations of the time of flight, and thus will not be considered here.

To see the effect of the fluctuating background on the propagation of phonons, we follow the analysis of [1,2]. For a spatially homogeneous static condensate its operator-valued acoustic metric can be taken as $dt^2 - c_\xi^{-2}\mathbf{dx}^2 = 0$. In conventional formalism the phonon propagates along the sound cone determined by the null-geodesic $c^2dt^2 = d\mathbf{x}^2 + h_{ij}dx^i dx^j$, where $h_{ij} = (2\alpha/\rho_0)\xi\delta_{ij}$. If the spatial separation between the source and the detector is r, then the travel time can be expressed as

$$T = \int_0^T dt \approx \int_0^r dr \frac{1}{c}\left[1 + \frac{1}{2}h_{ij}n^i n^j\right], \quad (13)$$

where $dr = d|\mathbf{x}|$ and $\mathbf{n}^i = dx^i/dr$ is a unit vector along the direction of the sound wave propagation. The local velocity c is evaluated on the unperturbed path of the waves $r(t)$, which we take along the z direction, so that $z(t) = ct$. With $\langle h_{ij}\rangle = 0$, the variance of the travel time is given by

$$\begin{aligned}(\Delta T)^2 &= \langle T^2\rangle - \langle T\rangle^2 \\ &= \frac{1}{4}\int_0^T dt_1 \int_0^T dt_2\, n^i n^j n^l n^m \\ &\quad \times \langle h_{ij}(r(t_1),t_1)\, h_{lm}(r(t_2),t_2)\rangle \\ &= \frac{\alpha^2}{\rho_0^2}\int_0^T dt_1 \int_0^T dt_2 \langle \xi(z(t_1),t_1)\,\xi(z(t_2),t_2)\rangle. \quad (14)\end{aligned}$$

With the noise correlation given by $D_{\epsilon H}$ in (6), Eq. (14) is our key result, but to see whether it can be tested is not straightforward. We have in mind an experiment along the lines of that described in [14], discussed further in [15], in which sound pulses are created by density perturbations. (Note that, for our condensate $c^2 \propto \rho_0$ just as for a condensate of elementary bosons.)

Straightforward substitution of $D_{\epsilon H}$ in (6) gives

$$\begin{aligned}(\Delta T)^2 = \frac{\alpha^2}{\rho_0^2}\int_0^T dt_1 \int_0^T dt_2 \int_0^{k_\Lambda} \frac{k^2 dk}{4\pi^2} \\ \times \frac{\cos[\omega_k(t_1 - t_2)]}{\omega_k \eta}\frac{2\sin[ck|t_1 - t_2|]}{kc|t_1 - t_2|}. \quad (15)\end{aligned}$$

$(\Delta T)^2$ shows a logarithmic UV divergence because of the acoustic approximation and we cut off momentum at $k = k_\Lambda = O(K) = O(k_F)$ in the relevant regime. The initial growth of $(\Delta T)^2$ from zero at time zero is rapid, and when $T_s \sim 1/\omega_{(k=0)} = \sqrt{\eta}/\bar{M}$, the growth halts and $(\Delta T)^2$ saturates to its late time value (Fig. 1).

For large times the k integral in (15) is dominated by large k contributions and can be approximated well by

$$\begin{aligned}(\epsilon_F \Delta T)^2 \approx \frac{\alpha^2}{\rho_0^2}\frac{(mv_F^2)^2}{4\pi^2\eta c^3}\frac{x^2}{(1 - x^2)}\{-\tanh^{-1}[x/\sqrt{1 + y^2}] \\ + x\ln[1 + \sqrt{1 + y^2}/y]\}, \quad (16)\end{aligned}$$

where $y = \sqrt{2m\bar{M}^2/\rho_0 k_\Lambda^2}$ and $x = c/\sqrt{\rho_0/2m\eta} < 1$ across the whole regime from BCS to BEC. We stress that the behavior described above is a consequence of quantum fluctuations and not thermal fluctuations.

Before our numerical study, we need to list the basic attributes of the parameters in the model (See [7,8] for more detail). We find that $0 \leqslant \alpha/\rho_0 \leqslant 1$ increases as we tune the gas from the deep BCS regime ($1/k_F a_S < 0$), when $\alpha/\rho_0 \approx 0$ to the deep BEC regime ($1/k_F a_S > 0$), when $\alpha/\rho_0 \approx 1$. On the contrary, η, N_0, and $\bar{M}^2$ go from finite values to zero as we go from deep BCS to BEC regimes, in each of which $\eta \approx N_0$. As a result of $\bar{M}^2$ vanishing, c^2 falls to *zero* in the deep BEC regime [7,8]. It follows that, for Fermi energy ϵ_F, $(\epsilon_F \Delta T)^2 \to 0$ in the deep BEC regime. Also, $(\epsilon_F \Delta T)^2 \to 0$ in the deep BCS regime since $\alpha \approx 0$ there.

To be concrete, consider a cold ^{6}Li condensate of 3×10^5 atoms tuned by the narrow resonance at $H_0 = 543.25$ G [6], mentioned earlier. The narrowness of the resonance width is best determined by the dimensionless width $\gamma_0 \approx \sqrt{\Gamma_0/\epsilon_F}$, where the resonance width Γ_0 [10] is mainly given by H_ω, the so-called "resonance width" of the central field H_0 required to achieve infinite scattering length (the unitary limit).

We take the number density $\rho_0 = k_F^3/3\pi^2 \approx 1 \times 10^{11}$ cm^{-3} [6], for which $\epsilon_F \approx 7 \times 10^{-12}$ eV ($\epsilon_F/\hbar \approx 10$ ms^{-1}) and $\gamma_0 \approx 0.6$. In terms of the dimensionless coupling $\bar{g}$, where $g^2 = (64\epsilon_F^2/3k_F^3)\bar{g}^2$ [10], ^{6}Li at the density above corresponds to $\bar{g}^2 = 0.8$. In the inset (bottom) to Fig. 1 we plot the saturation value of $(\epsilon_F \Delta T)^2$ obtained from (15) on varying $1/k_F a_S$, where we take the UV cutoff $k_\Lambda = k_F$ for carrying out the momentum integration in (15) numerically. The maximum travel time fluctuation occurs near the crossover regime at $1/a_S k_F \approx 0.7$. The main figure in Fig. 1 shows the evolution of ΔT for this value of $1/a_S k_F$, achieving its saturation value of $\Delta T \approx 0.9\epsilon_F^{-1} \approx 0.1$ ms, in agreement with Eq. (16), after $T_s \sim \sqrt{\eta}/\bar{M} \approx 1.0\epsilon_F^{-1} \approx 0.1$ ms. In particular (see upper inset), $K \approx 0.3k_F$ at $1/a_S k_F \approx 0.7$ with the central momentum $k_0 \approx 0.1k_F$ determined by the sound speed $c \approx 0.1v_F$. With $k_F \approx 1/\mu$m the width of the density fluctuations moving on a condensate of size $L \approx 100k_F^{-1} \approx 100$ μm can be of the order of several micrometers. With $c \approx 1.4$ μm/ms the time of flight from the center of the condensate is approximately 30 ms, whence the 1% or less fluctuation effect cited initially. Unfortunately, the effect is not yet testable since experimentalists most easily measure the (saturated) fluctuations $\Delta r = c\Delta T \approx 0.14$ μm in the position of the propagating wave front. Currently, such uncertainty is well within the noise by between one and two orders of magnitude [16].

Unlike the case for light-cone fluctuations, where $\Delta T \propto T$ [2], the saturation of ΔT here, and hence the vanishing of $\Delta T/T$ for large T, makes comparison difficult. Nonetheless, the result $\Delta T = O(\epsilon_F^{-1})$ is as we would expect by analogy with quantum gravity [1], as discussed in [2,4]. In quantum gravity, at best $(\Delta T/T)^2 \sim \ell_P^2 \lambda_c^2 U$ where ℓ_P is the Planck length, and U is the energy density of a bath of gravitons with a characteristic wavelength λ_c. If, for example, we take the energy density and typical wavelength of gravitons to be of the order of those of microwave background radiation in the present Universe, we find $\Delta T/T \approx 10^{-33}$, immeasurably small. By analogy with gravity, on dimensional grounds $(\Delta T)^2$ can be parametrized as $(\Delta T)^2 \sim U\omega_c/k_c^3$ in which the effective energy density U is due to the condensate fluctuations with a typical frequency ω_c and momentum k_c. We estimate U as $U \approx k_F^3 \epsilon_F$ with the frequency $\omega_c \approx \bar{M}/\sqrt{\eta} \approx \epsilon_F$, and the momentum $k_c \approx \sqrt{2m\bar{M}^2/\rho_0} \approx k_F$ near crossover regime, leading to the relatively large value of $\Delta T = O(\epsilon_F^{-1})$ results.

In our model the speed of sound (9) vanishes in the BEC regime because of the absence of direct diatomic self-interactions in the Lagrangian density in (1), but the qualitative behavior shown in Fig. 1 does not rely on this fact. Suppose, as in [17], we include such a term

$$L(\varphi) = -u_B|\varphi(x)|^4/4 \tag{17}$$

in the integrand of (1). The effect in $S_{\text{eff}}(\theta,\epsilon)$ of (4) is just to replace $\bar{M}^2$ by $\mathcal{M}^2 = \bar{M}^2 + 6u_B\kappa^2|\varphi_0|^2$ in all results following (4). [The term linear in ϵ, which corresponds to making the replacement $\alpha G_0 \to \alpha G_0 + u_B\kappa|\varphi|^3$ in (4) has no effect, since it always contributes to total derivatives in the calculations which follow.] This leaves c^2 unchanged in the BCS regime because $\alpha \approx 0$ there, but since $\kappa^2|\varphi_0|^2 \neq 0$ it permits c^2 to tend to a nonzero limit in the deep BEC regime. However, the vanishing of α in the deep BCS regime and the vanishing of η in the deep BEC regime are sufficient for fluctuations to have no effect there. In the intermediate regime there will be a reduction in ΔT due to the increase in $\bar{M}^2$ in Eq. (16) for the crossover regime. The effect is not dramatic but the details will depend on parameter choice. Qualitatively the behavior shown in Fig. 1 will persist.

This work was supported in part by the Ministry of Science and Technology, Taiwan.

[1] B. L. Hu and K. Shiokawa, Phys. Rev. D **57**, 3474 (1998).
[2] L. H. Ford, V. A. de Lorenci, G. Menezes, and N. F. Svaiter, Ann. Phys. (N.Y.) **329**, 80 (2013).
[3] V. Gurarie and A. Altland, Phys. Rev. Lett. **94**, 245502 (2005).
[4] G. Krein, G. Menezes, and N. F. Svaiter, Phys. Rev. Lett. **105**, 131301 (2010).
[5] C. Gaul, N. Renner, and C. A. Müller, Phys. Rev. A **80**, 053620 (2009).
[6] K. E. Strecker, G. B. Partridge, and R. G. Hulet, Phys. Rev. Lett. **91**, 080406 (2003).
[7] C.-Y. Lin, D.-S. Lee, and R. J. Rivers, Phys. Rev. A **84**, 013623 (2011).
[8] J.-T. Hsiang, C.-Y. Lin, D.-S. Lee, and R. J. Rivers, J. Phys.: Condens. Matter **25**, 404211 (2013).
[9] L. H. Ford, Int. J. Theor. Phys. **44**, 1753 (2005).
[10] V. Gurarie and L. Radzihovsky, Ann. Phys. (N.Y.) **322**, 2 (2007).
[11] I. J. R. Aitchison, P. Ao, D. J. Thouless, and X.-M. Zhu, Phys. Rev. B **51**, 6531 (1995).
[12] D.-S. Lee and D. Boyanovsky, Nucl. Phys. B **406**, 631 (1993).
[13] S. Weinfurtner, P. Jain, M. Visser, and C. W. Gardiner, Class. Quantum Grav. **26**, 065012 (2009).
[14] M. R. Andrews, D. M. Kurn, H.-J. Miesner, D. S. Durfee, C. G. Townsend, S. Inouye, and W. Ketterle, Phys. Rev. Lett. **79**, 553 (1997).
[15] G. M. Kavoulakis and C. J. Pethick, Phys. Rev. A **58**, 1563 (1998).
[16] J. Joseph, B. Clancy, L. Luo, J. Kinast, A. Turlapov, and J. E. Thomas, Phys. Rev. Lett. **98**, 170401 (2007).
[17] E. Timmermans, K. Furuya, P. W. Milonni, and A. K. Kerman, Phys. Lett. A **285**, 228 (2001).