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Generalized Gibbs ensembles for quantum field theories

F. H. L. Essler,[1] G. Mussardo,[2,3] and M. Panfil[2]

[1]*The Rudolf Peierls Centre for Theoretical Physics, University of Oxford, Oxford OX1 3NP, United Kingdom*

[2]*SISSA and INFN, Sezione di Trieste, via Beirut 2/4, I-34151 Trieste, Italy*

[3]*International Centre for Theoretical Physics (ICTP), I-34151 Trieste, Italy*

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We consider the nonequilibrium dynamics in quantum field theories (QFTs). After being prepared in a density matrix that is not an eigenstate of the Hamiltonian, such systems are expected to relax *locally* to a stationary state. In the presence of local conservation laws, these stationary states are believed to be described by appropriate generalized Gibbs ensembles. Here we demonstrate that in order to obtain a correct description of the stationary state, it is necessary to take into account conservation laws that are not (ultra)local in the usual sense of QFTs, but fulfill a significantly weaker form of locality. We discuss the implications of our results for integrable QFTs in one spatial dimension.

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I. INTRODUCTION

The past decade has witnessed dramatic progress in realizing and analyzing isolated many-particle quantum systems out of equilibrium [1–6]. Key questions that emerged from these experiments is why and how observables relax toward time-independent values, and what principles underlie a possible statistical description of the latter [7–29]. It was demonstrated early on that nonequilibrium dynamics is strongly affected by dimensionality, and that conservation laws play an important role. In particular, the experiments of [2] on trapped ^{87}Rb atoms established that three-dimensional condensates rapidly relax to a stationary state characterized by an effective temperature, whereas constraining the motion of atoms to one dimension greatly reduces the relaxation rate and dramatically changes the nature of the stationary state. The suggestion that this unusual steady state is a consequence of (approximate) conservation laws motivated a host of theoretical studies investigating the role played by conservation laws. We may summarize the results of these works as follows: given an initial state $|\Psi\rangle$ and a translationally invariant system with Hamiltonian $H\equiv I_0$ and conservation laws I_n such that $[I_n,I_m]=0$, the stationary behavior of n-point functions of local operators $O_a(x)$ *in the thermodynamic limit* is described by a *generalized Gibbs ensemble* (GGE), as proposed by Rigol *et al.* in a seminal paper [9],

$$\lim_{t\to\infty}\langle\Psi(t)|\prod_{j=1}^{n}O_j(x_j)|\Psi(t)\rangle=\mathrm{Tr}\left(\rho_{\mathrm{GGE}}\prod_{j=1}^{n}O_j(x_j)\right). \quad (1)$$

Here $|\Psi(t)\rangle=\exp(-iHt)|\Psi\rangle$ and

$$\rho_{\mathrm{GGE}}=\frac{1}{Z}\exp\left(-\sum_n\lambda_n I_n\right), \quad (2)$$

where the values of the Lagrange multipliers λ_n are fixed by the requirement that the expectation values of the conserved charges must be the same at time zero and in the stationary state, i.e., $\lim_{V\to\infty}\langle\Psi|I_n|\Psi\rangle/V=\lim_{V\to\infty}\mathrm{Tr}[\rho_{\mathrm{GGE}}I_n]/V$. Very recently it has become clear that the question of which conservation laws I_n need to be included in the definition of (2) is quite subtle [30–34]. Here we address this issue for continuum quantum field theories (QFTs), in both relativistic and nonrelativistic cases. This is of fundamental importance as a problem in QFT per se. It is also a pressing concern due to the crucial role QFT has played in establishing the current theoretical understanding of the nonequilibrium dynamics of isolated quantum systems, providing key insights [35–37] of experimental relevance [6,38]. We show that it is generally necessary to include "quasilocal" charges in the definition of the GGE. This can already be seen for the simplest possible example, namely noninteracting QTFs, to which we turn next.

II. FREE MAJORANA FERMION

Let us consider a general quantum quench in the free Majorana fermion theory with Hamiltonian density,

$$\mathcal{H}=\frac{iv}{2}[R(x)\partial_x R(x)-L(x)\partial_x L(x)]+imR(x)L(x), \quad (3)$$

where R and L are real chiral fermions, and v is the velocity. This theory describes the scaling limit of the transverse field Ising chain, where the mass term is a measure of the distance to the quantum critical point. The initial state $|\Psi(0)\rangle$ of the quench process could be, for example, the ground state at a particular, but different, value m_0 of the mass [39,40]. The Hamiltonian is diagonalized through a mode expansion and takes the form

$$H=\int\frac{dk}{2\pi}\sqrt{m^2+v^2k^2}\;Z^\dagger(k)Z(k), \quad (4)$$

where $\{Z^\dagger(k),Z(q)\}=2\pi\delta(k-q)$. Clearly the mode occupation operators $N(k)=Z^\dagger(k)Z(k)$ commute with H and are therefore conserved. In cases like this, the GGE density matrix in a large, finite volume L is most conveniently constructed in terms of the charges $N(k)$ [9],

$$\rho_{\mathrm{GGE}}=\frac{1}{Z}\exp\left(-\sum_{n\in\mathbb{Z}}\lambda(k_n)N(k_n)\right),\quad k_n=\frac{2\pi n}{L}. \quad (5)$$

The Lagrange multipliers $\lambda(k)$ are related to the mode occupation numbers $n_\Psi(k)=\langle\Psi(0)|N(k)|\Psi(0)\rangle$ by $\lambda(k)=\ln[n(k)]-\ln[1-n(k)]$. In practice, it is more convenient to work with the "microcanonical" version of the GGE [41,42].

This is defined by the density matrix $\rho_{\rm GMC} = |\Phi\rangle\langle\Phi|$, where the state $|\Phi\rangle$ is an eigenstate of all $N(k_n)$ with eigenvalues equal to $n_\Psi(k_n)$. By construction, the knowledge of the eigenvalues $n_\Psi(k_n)$ of the conserved charges $N(k_n)$ is sufficient to construct $\rho_{\rm GMC}$.

The existence of conserved mode occupation operators in a large, finite volume is a particular property of free theories and does not generalize to the interacting case (see below). In contrast, no such problem arises for local conservation laws, which are therefore the appropriate charges to consider in the general case. Following the standard approach in a relativistic QFT (which we recall in the Supplemental Material [43]), one can construct the following set of ultralocal conserved charges for the free Majorana theory:

$$\begin{aligned} I_n^- &= \frac{iv}{2}\int dx\big[R(x)\partial_x^{2n+1}R(x) + L(x)\partial_x^{2n+1}L(x)\big],\\ I_n^+ &= \frac{i}{2}\int dx\big[R(x)v\partial_x^{2n+1}R(x) - L(x)v\partial_x^{2n+1}L(x)\\ &\quad + 2mR(x)\partial_x^{2n}L(x)\big]. \end{aligned} \tag{6}$$

A widely held belief is that the GGE (2) constructed from these charges is the same as the one built from the mode occupation operators (5). However, in the infinite volume, this cannot be generally the case, simply because there is a mismatch between the *countable* number of conserved charges and the *continuum* number of degrees of freedom in the field theory. To see this, we express the charges (6) in momentum space. This gives

$$I_n^\pm = (-1)^n \int_{-\infty}^{\infty} \frac{dk}{2\pi}\epsilon_n^\pm(k)\, N(k), \tag{7}$$

where $\epsilon_n^+(k) = \sqrt{m^2+v^2k^2}k^{2n}$ and $\epsilon_n^-(k) = vk^{2n+1}$. The question is then whether the knowledge of $i_n^\pm = \lim_{L\to\infty}\langle\Psi|I_n^\pm|\Psi\rangle/L$ is sufficient to reconstruct the function $n_\Psi(k)$ and hence the density matrix $\rho_{\rm GMC}$. The answer is negative: as is shown in the Supplemental Material [43], one can explicitly construct functions $f(k)$ such that $i_n^\pm = (-1)^n\int\frac{dk}{2\pi}\epsilon_n^\pm(k)[n_\Psi(k)+f(k)]$ are independent of f. This suggests that there are additional local conservation laws that need to be taken into account in the construction of the GGE. How do we find such charges? We recall that (3) is obtained as the scaling limit of a model of lattice Majorana fermions a_n, for which a complete set of local conservation laws is [44]

$$\begin{aligned} \mathcal{I}_n^+ &= \frac{iJ}{2}\sum_{j,\sigma=\pm1} a_{2j}[a_{2j+2n\sigma+1} - ha_{2j+2n\sigma-1}],\\ \mathcal{I}_{n-1}^- &= -\frac{iJ}{2}\sum_j [a_{2j}a_{2j+2n} + a_{2j-1}a_{2j+2n-1}]. \end{aligned}$$

The lattice Hamiltonian itself is $\mathcal{I}_0^+$. The $\mathcal{I}_n^\pm$ have the important property that their densities have strictly finite ranges: the density of $\mathcal{I}_n^\pm$ involves only $n+2$ neighboring sites. The scaling limit is defined as $J\to\infty$, $h\to 1$, $a_0\to 0$ while keeping $J|h-1| = m$ and $Ja_0 = v$ fixed. In this limit, upon taking appropriate linear combinations of the lattice charges $\mathcal{I}_n^\pm$, one recovers the QFT charges (6). However, in the process of taking the scaling limit, we can also scale the index n in such a way that the combination $na_0 = \alpha$ is kept fixed, obtaining in this way conserved charges of the form (see Fig. 1)

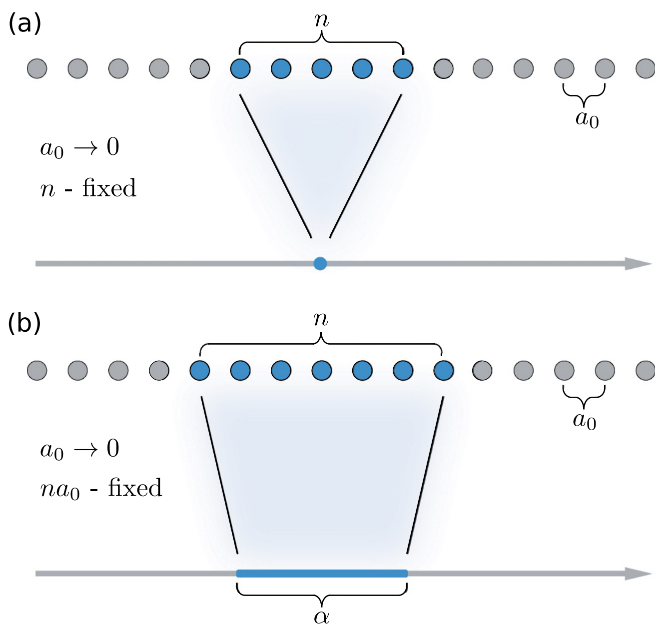

FIG. 1. (Color online) Construction of ultralocal and quasilocal charges by taking the continuum limit of an integrable lattice model with conservation laws $\mathcal{I}_n$, whose densities act on n consecutive lattice sites. (a) Ultralocal charges are obtained by taking the lattice spacing a_0 to zero, while keeping the index n fixed. (b) Quasilocal charges are obtained by taking the double scaling limit $a_0\to 0$, $n\to\infty$, while keeping $na_0 = \alpha$ fixed.

$$\begin{aligned} I^+(\alpha) &= \frac{i}{4}\int_0^L dx\,[R(x)+L(x)]\,(v\partial_x - m)\\ &\quad\times[R(x+\alpha) - L(x+\alpha) + (\alpha\to-\alpha)]\,,\\ I^-(\alpha) &= \frac{iv}{2}\int_0^L dx[R(x)R(x+\alpha) + L(x)L(x+\alpha)]\,. \end{aligned} \tag{8}$$

Here the index α is by construction a real positive number such that $0<\alpha<L$, where L is the system size and we have imposed periodic boundary conditions on the fields. The charges $I^\pm(\alpha)$ are no longer local quantities in the usual QFT sense, but they have densities with support on a finite interval of size α. We will call such operators *quasilocal*. In momentum space, we have $I^\pm(\alpha) = \int_{-\infty}^{\infty}\frac{dk}{2\pi}\epsilon^\pm(k,\alpha)N(k)$, where $\epsilon^+(k,\alpha) = \sqrt{m^2+v^2k^2}\cos(\alpha k)$ and $\epsilon^-(k,\alpha) = \sin(\alpha k)$. This establishes that $\{I^\pm(\alpha)\}$ of conserved charges is complete in the sense that the initial data $\langle\Psi|I^\pm(\alpha)|\Psi\rangle$ suffice to fix any given occupation number distribution $n_\Psi(k)$. Hence the appropriate GGE for the free Majorana theory is

$$\rho_{\rm GGE} = \frac{1}{Z}\exp\left(-\sum_{\sigma=\pm}\int_0^\infty d\alpha\,\lambda^\sigma(\alpha)I^\sigma(\alpha)\right). \tag{9}$$

We stress for the lattice model itself, the conservation laws that give rise to the quasilocal charges in the scaling limit are both unnatural and unimportant: the goal is to describe finite subsystems of arbitrary size in the thermodynamic limit, and here truncated GGEs [44] involving only $\mathcal{I}_n^\pm$ with fixed n in the $L\to\infty$ limit are required.

III. INTERACTING INTEGRABLE QFTs

We next turn to the case of integrable QFTs (IQFTs) with nontrivial S-matrices. The scattering in IQFTs is purely elastic [45,46,47], and concomitantly a convenient way to describe their Hilbert spaces in the infinite volume is in terms of the Faddeev-Zamolodchikov algebra [46]. In the scalar case, the latter reads

$$\begin{aligned} Z(\theta_1)Z(\theta_2) &= S(\theta_1-\theta_2)Z(\theta_2)Z(\theta_1),\\ Z(\theta_1)Z^\dagger(\theta_2) &= 2\pi\delta(\theta_1-\theta_2)\\ &\quad + S(\theta_2-\theta_1)Z^\dagger(\theta_2)Z(\theta_1), \end{aligned} \tag{10}$$

where $Z^\dagger(\theta)$ and $Z(\theta)$ are creation and annihilation operators of elementary excitations with rapidity θ (related to momentum by $vq = M\sinh\theta$), and $S(\theta)$ is the two-particle S-matrix. In the infinite volume, the quantities $N(\theta) = Z^\dagger(\theta)Z(\theta)$ are integrals of motion and can be viewed as appropriate generalizations of the mode occupation numbers in free-field theories. Unfortunately, in contrast to the special case of free fields, the occupation numbers $N(\theta)$ cannot be used for the construction of the GGE [48]. The reason is that while for free fields the possible values for rapidities are simply given by $m\sinh\theta_n = 2\pi n/L$ and can be independently occupied, in the IQFT case the quantization conditions are given by the Bethe ansatz equations

$$e^{iLm\sinh\theta_n} = \prod_{m\neq n} S(\theta_n-\theta_m),\quad n=1,\ldots,N. \tag{11}$$

Hence the allowed values of θ_n depend on the entire set $\{\theta_m\}$ specifying the particular eigenstate under consideration. Due to this complication, it is not clear how to define a finite volume version of $N(\theta)$ in an operator sense [48]. We therefore want to construct the GGE using local conservation laws. The standard *ultralocal* conserved charges are related to conserved currents $\partial_\mu j_n^\mu(t,x)$ by $I_n = \int dx\, j_n^0(t,x)$. In relativistic IQFTs, there is a standard method for constructing I_n [45,47,49]. There, the index n is related to the Lorentz spin of the operator I_n. As n can take only discrete values, ultralocal conserved charges are insufficient for constructing GGEs for general initial states. To see this, we recall that their action on eigenstates can be represented in the form $I_n^\pm = \int d\theta\,\varepsilon_n^\pm(\theta)N(\theta)$ with $\varepsilon_n^+(\theta) = \cosh n\theta$ and $\varepsilon_n^-(\theta) = \sinh(n\theta)$ [45]. It is again convenient to consider the microcanonical version $\rho_{\rm GMC} = |\Phi\rangle\langle\Phi|$, which describes the saddle point of the GGE [42]. In the infinite volume limit, we require knowledge of the function $n(\theta) = \langle\Phi|N(\theta)|\Phi\rangle$ [50] in order to specify $\rho_{\rm GMC}$. The knowledge of the countable set $\{\langle\Phi|I_m|\Phi\rangle\}$ does not suffice to uniquely determine $n(\theta)$. Indeed, let us consider the family of states $|\Phi_f\rangle$ characterized by the macroscopically distinct mode occupations $n(\theta)+f(\theta)$, where $f(\theta)$ is an analytic function whose Fourier transform has an infinite number of zeros at $z_m = im$. Using the explicit expression for the eigenvalues of I_m given above, one finds that $\langle\Phi_f|I_m|\Phi_f\rangle = \langle\Phi|I_m|\Phi\rangle$. This establishes that the countable set $\{I_m\}$ of charges is in general insufficient to fully characterize $\rho_{\rm GMC}$.

To construct the GGE, we therefore follow the procedure used for free fields: (i) find an integrable lattice discretization of the field theory (with lattice spacing a_0); (ii) follow the standard procedure [49] for constructing local integrals of motion $\mathcal{I}_n$ for integrable lattice models. Here the index n, roughly speaking, sets a number of lattice sites upon which the density of $\mathcal{I}_n$ acts; (iii) take a *double scaling limit* $a_0\to 0, n\to\infty$, while keeping $\alpha = na_0$ fixed. This procedure generates a continuous family of conserved charges $I(\alpha)$ (labeled by a real positive number α), which are quasilocal. In cases like the one considered below, it is known that the $\mathcal{I}_n$ form a complete set of integrals of motion on the lattice. Concomitantly, the set $\{I(\alpha)\}$ is sufficient to construct the GGE in a large finite volume, and hence in the thermodynamic limit.

We now illustrate this programme for the example of the nonlinear Schrödinger model, also known as the Lieb-Liniger δ-function Bose gas [51], which is a key theory for the description of ultracold quantum gases [52]. In particular, it underlies seminal experiments probing thermalization in such systems [2,3].

IV. NONLINEAR SCHRÖDINGER MODEL

The Hamiltonian density of the NLS is [53]

$$\mathcal{H} = \varphi^\dagger(x)\left[-\frac{\partial_x^2}{2m}-\mu\right]\varphi(x) + \lambda|\varphi^2(x)|^2, \tag{12}$$

where $\varphi(x,t)$ is a complex bosonic field and μ is a chemical potential. Quenches to the NLS have been previously considered by several groups [25,26,54–63]. A key issue in many of these works has been how to construct the appropriate GGE describing the stationary state at late times after the quench. Let us now address this question using the framework introduced above. The ultralocal integrals of motion for the NLS can be constructed by the quantum inverse-scattering method [49,64] through an appropriate expansion of the quantum transfer matrix. This provides a countable number of $\mathcal{I}_n$, which by the above argument are insufficient for constructing the GGE describing the stationary behavior after a quench from a general initial state. Moreover, as was discussed in detail in Ref. [63], the expectation values i_n in fact do not exist for many initial states due to ultraviolet divergences. These problems can be overcome by using quasilocal charges. To construct them, we utilize an integrable lattice regularization [65–67] of the NLS in terms of so-called q-boson operators fulfilling commutation relations,

$$B_j^\dagger B_k - q^2 B_k B_j^\dagger = \delta_{jk}. \tag{13}$$

The q-bosons are related to canonical lattice bosons b_j by the relation $B_j = \sqrt{\frac{[N_j+1]_q}{N_j+1}}\,b_j$, where $[x]_q = \frac{1-q^{-2x}}{1-q^{-2}}$. The Hamiltonian of the lattice model is

$$H_q = -\frac{1}{a_0^2}\sum_{j=1}^{L}\left(B_j^\dagger B_{j+1} + B_{j+1}^\dagger B_j - 2N_j\right), \tag{14}$$

where $N_j = b_j^\dagger b_j$. The lattice conserved charges $\mathcal{I}_n^\pm$ are known and their eigenvalues are [68]

$$i_n^\pm(p_1,\ldots,p_N) = \frac{1-q^{-2|n|}}{|n|a_0}\sum_{j=1}^{N} f^\pm(np_j), \tag{15}$$

where n is an integer, $f^+(x) = \cos(x)$, $f^-(x) = \sin(x)$, and $\{p_1,\ldots,p_N\}$ are solutions to the Bethe ansatz equations for the q-boson model. The NLS is recovered taking the scaling limit as $a_0 \to 0$ and $q \to 1$ with $c = 2\ln(q)/a_0$ fixed. The continuum field $\varphi(x)$ is related to the canonical lattice bosons by $\varphi(ja_0) = a_0^{-1/2} b_j$. In this limit, the appropriate rapidity variables are $\lambda_j = p_j/a_0$. The ultralocal conserved charges of the NLS are obtained by considering appropriate linear combinations of the $I_n^{\pm}$ and then taking the continuum limit; see, e.g., [63]. In contrast, the quasilocal charges $I^{\pm}(\alpha)$ are constructed by keeping $na_0 = \alpha$ fixed in the scaling limit. Their eigenvalues on Bethe ansatz states are then found to be

$$i^{\pm}(\alpha;\lambda_1,\ldots,\lambda_N) = \frac{1-e^{-c|\alpha|}}{|\alpha|}\sum_{j=1}^{N} f^{\pm}(\alpha\lambda_j). \tag{16}$$

Let us now show that the set $\{I^{\pm}(\alpha)\}$ is sufficient for constructing the microcanonical version of the GGE, i.e., the density matrix $\rho_{\mathrm{GMC}} = |\Phi\rangle\langle\Phi|$. Here $|\Phi\rangle$ is a particular Bethe eigenstate [42]. In a large finite volume, L it is characterized by rapidities $\{\lambda_1,\ldots,\lambda_N\}$, and we are interested in the thermodynamic limit $N,L \to \infty$ with N/L fixed. In this limit, the state is described by a root density $\rho_\Phi(\lambda)$, which arises from the finite volume quantity $\rho_L(\lambda_j) = \frac{1}{L(\lambda_{j+1}-\lambda_j)}$. The expectation values of the quasilocal charges are then

$$\lim_{L\to\infty}\frac{1}{L}\frac{\langle\Phi|I^{\pm}(\alpha)|\Phi\rangle}{\langle\Phi|\Phi\rangle} = \frac{1-e^{-c|\alpha|}}{|\alpha|}\int_{-\infty}^{\infty} d\lambda\, f^{\pm}(\alpha\lambda)\,\rho_\Phi(\lambda). \tag{17}$$

This shows that $\rho_\Phi(\lambda)$ can be determined by Fourier transform from the expectation values of the $I^{\sigma}(\alpha)$. Inspection of (16) shows that in contrast to the ultralocal charges [63], there are no ultraviolet divergences in the expectation values (17) [the integral over $\rho_\Phi(\lambda)$ is equal to the density and must be finite].

V. DISCUSSION

The main lesson to be drawn from our work is that understanding the nonequilibrium evolution in QFTs requires one to go beyond the usual concept of locality. More precisely, we have shown that the construction of generalized Gibbs ensembles in QFTs requires integrals of motion $I^{\pm}(\alpha)$ that are not strictly local. In the cases we have considered, the densities of the $I^{\pm}(\alpha)$ act nontrivially only on intervals of length α, and they are different from known nonlocal conserved charges related to Yangian or quantum group symmetries [69,70]. We stress that the locality of the charges required to build a GEE is a different matter from the locality of the quantity $\lambda_n I_n$ entering the definition of the GGE density matrix [71]. We have presented a general argument showing that GGEs built from the usual local conservation laws I_m are generally insufficient for describing the stationary state at late times after quantum quenches (this does not preclude the possibility that they may do so in particular examples). In analogy to observations made for the transverse field Ising chain [44], we expect that in order to obtain an accurate description of the stationary values of local observables acting on a subsystem of size ℓ, only charges with $\alpha\ell + \xi$ will be required. Here ξ is a constant related to the correlation length in the stationary state.

Our work raises a number of open problems. First, our construction should be employed to determine the expectation values of local observables for particular quenches to the NLS model directly from the GGE. This requires the generalization of the method developed in Ref. [30] to the q-boson model. Second, it would be interesting to consider quantum quenches in other QFTs such as the sine-Gordon or SU(2) Thirring models. Here an additional complication arises, because the conservation laws obtained by standard methods for the corresponding lattice regularizations are no longer complete [30–34], and charges such as those constructed in [72–74] should be taken into account. Third, we expect quasilocal charges to be of importance for certain nonintegrable models in the context of prethermalization [75–83]. For a number of examples, it has been found that quenching to lattice models with weak integrability breaking terms, which includes the case of weakly interacting systems, leads to relaxation of local observables to nonthermal values at intermediate time scales. It has been suggested and substantiated in particular cases that almost conserved charges are the underlying cause of these prethermalization plateaus. It would be interesting to investigate this issue for QFTs in light of our findings. Finally, quasilocal charges may also be of importance for understanding the equilibration of QFTs in large-N limits [84].

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