Self-trapping triggered by losses in cavity QED

Raul Coto* and Miguel Orszag
Departamento de Física, Pontificia Universidad Católica de Chile, Casilla 306, Santiago 7820436, Chile

Vitalie Eremeev
Facultad de Ingeniería, Universidad Diego Portales, Santiago 8370191, Chile
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In a coupled cavity QED network model, we study the transition from a delocalized superfluidlike state to a localized Mott-insulator-like state, triggered by losses. Without cavity losses, the transition never takes place. Further, if we measure the quantum correlations between the polaritons via the negativity, we find a critical cavity damping constant, above which the negativity displays a single peak in the same time region where the transition takes place. Additionally, we identify two regions in the parameter space, where, below the critical damping, oscillations of the initial localized state are observed along with a multipeaked negativity, while, above the critical value, the oscillations die out and the transition is witnessed by a neat single peaked negativity.

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I. INTRODUCTION

Quantum phase transitions (QPTs), as opposed to classical transitions, driven by quantum rather than thermal fluctuations, occur at absolute zero and are induced by the change of some coupling constant or physical parameter. These phenomena have captured the attention of many researchers during the last decade [1–8].

In cavity QED networks, the tradeoff between the Mott insulator (MI) and superfluid (SF) phases has been especially studied. In the present context, these are not truly "phases" in the thermodynamic sense, since in the present model we deal only with a few cavities. However, it has been shown that even small systems containing a reduced number of cavities display the superfluid-Mott transition [9–11]. The importance of this transition is that it enables the system to go from a localized state (MI), with the excitations equally distributed throughout the system, to the state where the excitations move freely (SF), and more importantly all the excitations could be found in just one cavity.

In the literature, e.g., [9–11], there persists a common conviction that there is a key parameter that controls this transition, which is the atom-cavity detuning, Δ. In this paper we demonstrate the importance of other parameters like damping rate, hopping rate, and quantum correlation quantifiers (e.g., negativity) in detecting and controlling the mentioned phases.

In many problems of matter-light interactions, it is more convenient to model the atom and cavity field together as a quasiparticle called a polariton. These polaritons are the dressed states of the Jaynes-Cummings model. However, in the polariton basis, the variation of Δ leads to a change in the nature of this basis; e.g., in the limit of very large Δ, the polaritonic state goes to a purely photonic state, with the atoms in the ground state. Therefore, it is meaningless to talk about the SF polaritonic phase in that limit. This problem was addressed by Irish *et al.* [10]. Nevertheless, a QPT is still possible for small variations of Δ.

Recently, a similar QPT was found in [12–14], by controlling the atom-cavity coupling constant, g. For a fixed detuning, a variation of g leads also to a change in the polaritonic basis, but not for $\Delta = 0$, in which case, the states remain always maximally entangled between light and matter. All these transitions require an external control parameter, e.g., an external laser acting on each cavity and splitting the atomic levels by Stark shift, which increases Δ.

On the other hand, quantum correlations (QCs) and their role in quantum information are already well known [6,15,16] and a great effort has been devoted to the understanding of the connection between QCs and QPTs; see [16] and the references therein. In particular, QCs play an important role detecting quantum phase transitions [17,18]. In our case, since we are dealing with qudits, our best candidate for computing the quantum correlations is the negativity [19,20], which can be defined for a density matrix ρ as

$$\mathcal{N}(\rho) = \sum_i \frac{|\lambda_i| - \lambda_i}{2}, \tag{1}$$

where λ_i are the eigenvalues of the partial transpose of the density matrix $\rho^{T_{A(B)}}$, with respect to one of the subsystems. It essentially measures the degree to which $\rho^{T_{A(B)}}$ fails to be positive, and therefore it can be considered as a quantitative version of Peres's criterion for separability [21].

In this paper, we show a critical phenomenon, which is closely related to a QPT, with the losses playing a crucial role. We consider as initial a superfluidlike state, and show that it evolves to a Mott-insulator-like state during the time evolution of the system, only when the interaction with a reservoir is on. As in many models, we consider that the main source of dissipation originates from the leakage of the cavity photons due to imperfect reflectivity of the cavity mirrors. On the other hand, spontaneous emission will be neglected, assuming long atomic lifetimes, when dealing with the conditions imposed in our cavity QED system.

This paper is organized as follows. We first briefly discuss the mapping of the Jaynes-Cummings-Hubbard model to the polaritons. Then we study the transition from the superfluidlike state to the Mott-insulator-like state or self-trapping effect

*rcoto@uc.cl

triggered by dissipation using the quantum correlations as a witness. Finally, we analyze the critical damping and discuss the results.

II. MAPPING TO THE POLARITON BASIS

In our model we consider a linear array of N coupled cavities, where each cavity supports a single field mode and contains a single two-level atom. Photons are allowed to hop between neighboring cavities.

The Hamiltonian of such a system, found in [22] and references therein, is given by

$$H = H^S + H^{\text{hopp}},$$
$$H^S = \sum_{j=1}^{N} \left[\omega_j^a |e\rangle_j\langle e| + \omega_j^c a_j^\dagger a_j + g_j(a_j^\dagger |g\rangle_j\langle e| + a_j|e\rangle_j\langle g|)\right], \tag{2}$$

$$H^{\text{hopp}} = \sum_{j=1}^{N-1} J_j[a_j^\dagger a_{j+1} + a_{j+1}^\dagger a_j], \tag{3}$$

where $|g\rangle$ and $|e\rangle$ are the ground and excited states of the two-level atom with transition frequency ω^a, $a^\dagger(a)$ is the creation(annihilation) operator of the cavity mode ω^c, g is the coupling strength between the atom and the cavity mode, J_j is the coupling strength (hopping) between the neighboring cavities, and N represents the number of cavities. For each cavity, the eigenstates of the Hamiltonian H^S (2) are the dressed states, which are known as polaritons [9,23]. These states are given by

$$|n-\rangle = \cos(\theta_n)|n,g\rangle - \sin(\theta_n)|n-1,e\rangle,$$
$$|n+\rangle = \sin(\theta_n)|n,g\rangle + \cos(\theta_n)|n-1,e\rangle,$$
$$E_{n\pm} = \omega^c n + \frac{\Delta}{2} \pm \frac{\sqrt{\Delta^2 + 4g^2 n}}{2}, \tag{4}$$

with $\Delta = \omega^a - \omega^c$, $\theta_n = \frac{1}{2}\arctan(\frac{g\sqrt{n}}{\Delta/2})$, and n corresponds to the number of photons inside each cavity.

One would expect that there is a map to rewrite the hopping Hamiltonian H^{hopp} (3), in the polariton basis too. We consider here a different way to rewrite this operator, which is equivalent to the one in [24], and a generalization of the one proposed in [9]. Following the Appendix, we found that the creation operator in the polariton basis can be written as

$$a^\dagger = \sum_{n=1}^{\infty} c_{n+} L_{n+}^\dagger + \sum_{n=1}^{\infty} c_{n-} L_{n-}^\dagger + \sum_{n=2}^{\infty} k_{n\pm} L_{n\pm}^\dagger + \sum_{n=2}^{\infty} k_{n\mp} L_{n\mp}^\dagger. \tag{5}$$

The lowering operator $L_{n+} = |(n-1)+\rangle\langle n+|$ destroys the state $|n+\rangle$. On the other hand, the raising operator $L_{n+}^\dagger$ creates the state $|n+\rangle$. It is easy to see that with this definition all the transitions relative to a state $|n\rangle$ will correspond to the jumping upwards or downwards by unity. The first two terms in Eq. (5) correspond to the transitions between subspaces of the same sign, e.g., from $|n-\rangle$ to $|(n-1)-\rangle$. The last two terms are related to interconverting the subspace, i.e., from the positive subspace to the negative one, and vice versa. We notice that the sum for these last terms starts at 2, since there is no interconverting transition from one to zero. It will be convenient, for the sake of simplicity, to define $P_+^\dagger = \sum_{n=1}^{\infty} c_{n+} L_{n+}^\dagger$, $P_-^\dagger = \sum_{n=1}^{\infty} c_{n-} L_{n-}^\dagger$, $P_\pm^\dagger = \sum_{n=2}^{\infty} k_{n\pm} L_{n\pm}^\dagger$, and $P_\mp^\dagger = \sum_{n=2}^{\infty} k_{n\mp} L_{n\mp}^\dagger$. The coefficients are found to be

$$\begin{aligned}
c_{n+} &= \sqrt{n}\sin(\theta_n)\sin(\theta_{n-1}) + \sqrt{n-1}\cos(\theta_n)\cos(\theta_{n-1}) n \geqslant 2,\\
c_{n-} &= \sqrt{n}\cos(\theta_n)\cos(\theta_{n-1}) + \sqrt{n-1}\sin(\theta_n)\sin(\theta_{n-1}) n \geqslant 2,\\
c_{1+} &= \sin(\theta_1), c_{1-} = \cos(\theta_1),\\
k_{n\pm} &= \sqrt{n}\sin(\theta_n)\cos(\theta_{n-1}) - \sqrt{n-1}\cos(\theta_n)\sin(\theta_{n-1}) n \geqslant 2,\\
k_{n\mp} &= \sqrt{n}\cos(\theta_n)\sin(\theta_{n-1}) - \sqrt{n-1}\sin(\theta_n)\cos(\theta_{n-1}) n \geqslant 2.
\end{aligned} \tag{6}$$

We now discuss which terms in Eq. (5) can be neglected and under what conditions. The interconverting operators, $P_\pm$ and $P_\mp$, are the first candidates to be neglected. For only one excitation, these operators vanish, since $k_\pm$ start from two excitations. Also, for a higher subspace, it is easy to see that the factor k_i decreases considerably. The worst setting is for the second subspace ($n = 2$): if we chose $\theta = \pi/4$, then $k_2 \approx 0.21$. If comparing it with other coefficients, i.e., $c_{2+} = c_{2-} \approx 1.21$, we realize that the interconverting operators are not so important during the evolution, and in most of the cases they are completely negligible.

As a result,the hopping Hamiltonian in the polaritonic basis reads

$$H^{\text{hopp}} = \sum_{j=1}^{N-1} J_j[(P_{(+)j}^\dagger + P_{(-)j}^\dagger) \otimes (P_{(+)j+1} + P_{(-)j+1}) + \text{H.c.}]. \tag{7}$$

However, there is still an interconversion between the two polariton types in terms like $P_{(+)j}^\dagger P_{(-)j+1}$. Angelakis *et al.* pointed out that such terms may be neglected within a rotating wave approximation [9], under the condition $J \ll (E_{(n-1)+} - E_{(n-1)-})$ [24].

Finally,

$$H^{\text{hopp}} = \sum_{j=1}^{N-1} J_j[P_{(+)j}^\dagger P_{(+)j+1} + P_{(-)j}^\dagger P_{(-)j+1}) + \text{H.c.}]. \tag{8}$$

Next, by choosing the initial condition of a particular type, for example, $|n-\rangle$, then the state $|n+\rangle$ will never show up, and we are allowed to consider, throughout the paper, the lower branch only. It is worth noting that for zero detuning ($\Delta = 0$) and restricting the cavities to only the first excited state $|1-\rangle$ we recover our previous results [22,25]. However, for finite detuning and allowing the cavities to have more than one excitation, the dynamics becomes more involved, resulting in new and interesting results.

III. RESULTS

A. Self-trapping effect witnessed by the negativity

To fix the ideas, we imagine starting our system in a superfluidlike state, where all excitations are in a single cavity.

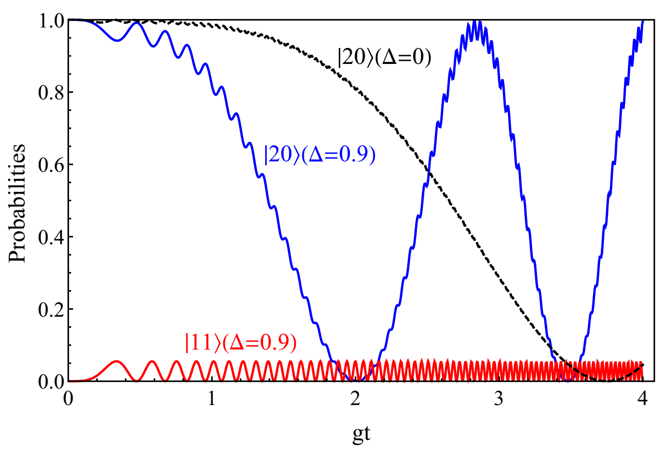

FIG. 1. (Color online) For $\Delta = 0$ the probability of finding the state $|11\rangle$ is zero, while for $\Delta = 0.9g$ it is different from zero. Also, the delay in the propagation of the initial state ($|20\rangle$) can be shortened by increasing the detuning. Here $\gamma = 0$ and $J = 0.03g$.

As the system evolves in time, there is a probability that these excitations will be uniformly distributed throughout the system, leading to a Mott-insulator-like state. This transition is strongly dependent on the detuning.

In Fig. 1 we show the time evolution of the probabilities of finding the initial state $|20\rangle$ and $|11\rangle$, when considering only two cavities without losses, for two different cases: with and without detuning. On one hand, we notice that for $\Delta = 0$ there is a delay in the propagation of the initial state to the second cavity, a result already discussed in some previous works [12–14], although in those papers the authors did not study the effect of the detuning to modify (shorten) this delay, as shown in Fig. 1, for $\Delta = 0.9g$. We can explain this improvement in the propagation time based on the fact that when using the polariton basis it is easy to see that an effective hopping shows up, which depends not only on J but also on the coefficient c_{n-} in Eq. (6), which increases with the detuning via θ_n. On the other hand, for the case of zero detuning, the probability of finding the state $|11\rangle$ is zero.

In what follows, we will describe a more realistic situation, with each cavity connected to its individual reservoir, and consider the same previous initial state, but this time for only the case of zero detuning. Rather than using the traditional master equation approach, we consider the system evolving with a non-Hermitian Hamiltonian, interrupted once in a while by instantaneous quantum jumps, a process usually referred to as the quantum trajectory or Monte Carlo wave-function method [26].

Figure 2 shows a self-trapping effect, where the SF-like state $|20\rangle$ goes to a MI-like state during the time evolution. So, our findings evidence a new important result: the presence of the cavity damping is sufficient for triggering the self-trapping effect, hence it is not necessary to control the detuning, Δ, or the atom-field coupling, g, as suggested in previous studies. We also show the time evolution of the negativity (1) and observe that it reaches its maximum value at the time region where the transition between the states $|20\rangle$ and $|11\rangle$ takes place.

B. Criticality with losses

The key feature in this work is the presence of losses. As we pointed out in the previous section, it leads to sudden transitions between two states. For a small loss rate, the polariton $|2\rangle$ oscillates, going back and forth from one cavity to the other, for a certain period of time until the probability associated to the state $|11\rangle$ starts growing and eventually reaches a slowly decaying plateau. Nevertheless, if we increase the losses to a critical value, the oscillations vanish and the state $|11\rangle$ appears faster. Furthermore, we are able to find a critical loss rate, γ_c, for which there is no reflection or oscillation of the probability associated to the polariton $|2\rangle$. This implies a break in the periodicity of the system, where the damping rate is large enough to eliminate the oscillation. In the following, we investigate the peculiarities of γ_c, and notice that it is a function of the coupling strength between the cavities as well as the detuning. In this subsection, we focus on the case $\Delta = 0$ with $\gamma_c(J)$ and postpone the discussion on the Δ dependence to the last section.

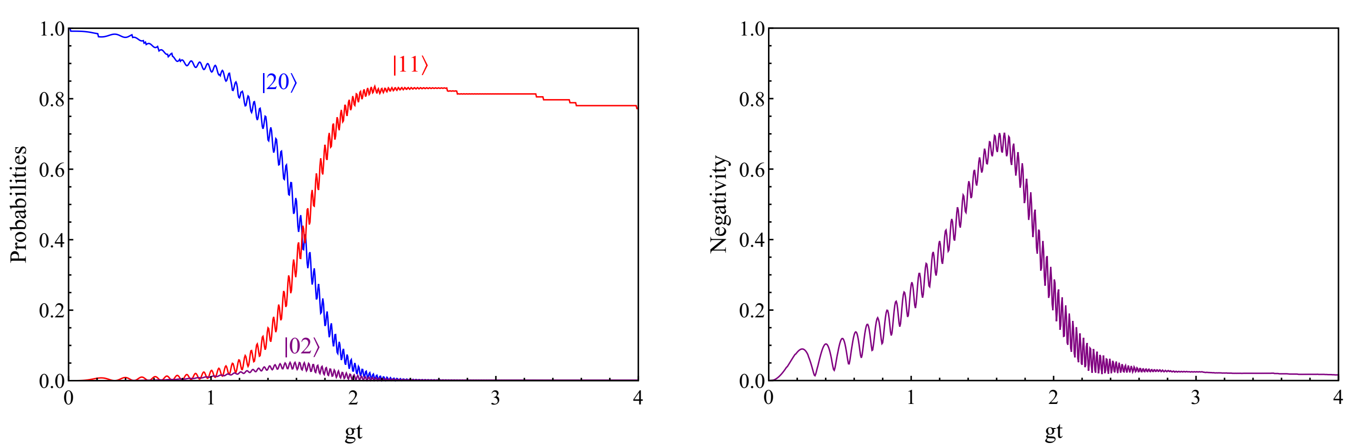

FIG. 2. (Color online) When turning on the interaction with the reservoir, a self-trapping behavior shows up, where the initial state $|20\rangle$ evolves to the delocalized state $|11\rangle$ and the polariton $|2\rangle$ disappears from the system, indicating that there is no more hopping of the excitations. The negativity reaches a single maximum in the time region where the transition takes place. Here $\gamma = 0.05g$, $J = 0.03g$, and $\Delta = 0$.

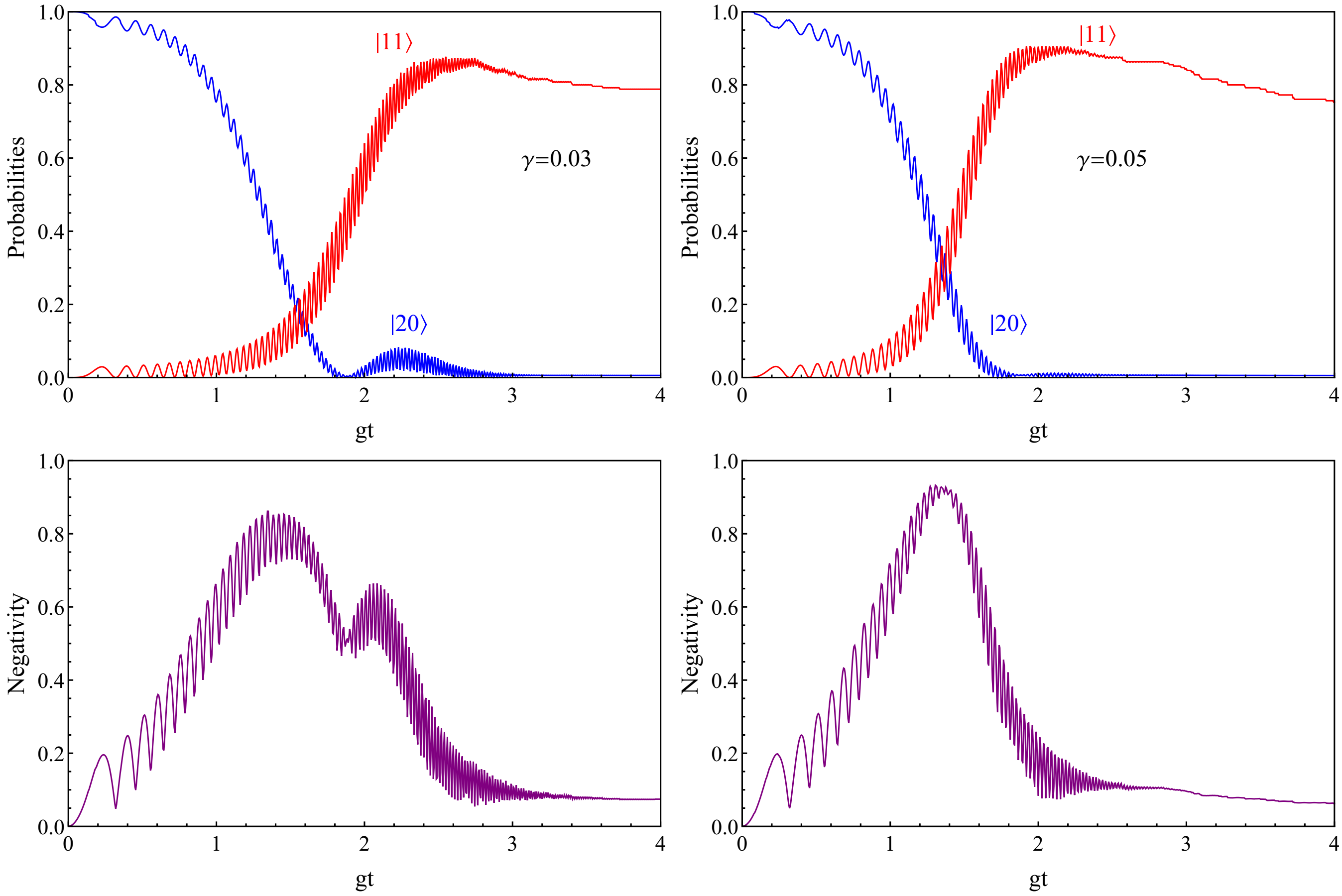

FIG. 3. (Color online) Time evolution of the states $|20\rangle$ (initial one) and $|11\rangle$. From top to bottom: $\gamma < \gamma_c$ and $\gamma \approx \gamma_c$. The negativity is a witness to determine γ_c. More than one peak implies a backwards motion of the polariton $|2\rangle$. Here $\Delta = 0$ and $J = 0.06g$.

As was mentioned before, the negativity can help us to throw some light on the search of criticality. In Fig. 3 we present two cases, with $\gamma < \gamma_c$ and $\gamma \approx \gamma_c$. On one hand, when we are below the critical value, there are oscillations of the initial state, leading to two or more peaks in the negativity. On the other hand, when going near or above γ_c, we find a single peak, but as we increase the damping constant it displaces to the left in time, with a decreasing maximum. Considering these two facts, we numerically estimate the critical damping to satisfy the relation $\gamma_c(J) \approx J$, that corresponds to a negativity with a single peak at its maximum value.

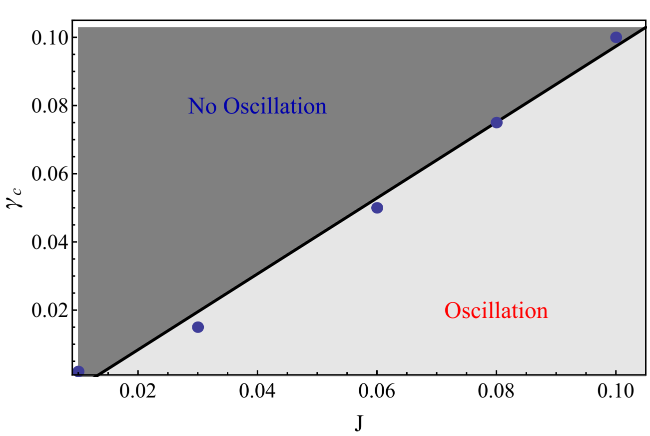

FIG. 4. (Color online) The dependence γ_c vs J (both in units of g) identifying two regions: one with a sudden transition from the state $|20\rangle$ to $|11\rangle$ and the other where a strong oscillation of the state $|20\rangle$ is observed and the transition takes a longer time. Here $\Delta = 0$.

Thus, we can clearly identify two regions as shown in Fig. 4, a lower area that corresponds to an oscillating probability associated to the initial state and the upper region with no oscillations and where the self-trapping takes place with a single peaked negativity.

IV. CONCLUSIONS AND DISCUSSION

In this work, we demonstrated for a system of two coupled cavities with dissipation to individual reservoirs the occurrence of the self-trapping effect, implying a transition from a superfluid to a Mott insulator phase.

Further, we found a critical damping rate, above which the initial superfluid state damps out in time while the Mott insulator phase is being created. We find the negativity to be a very good witness that shows, at the critical damping rate, a single peak at its maximum value in the same time region where the transition takes place.

We notice that in the lossless case the initial state oscillates back and forth; that is, the two excitations oscillate between the two cavities, and the MI phase $|11\rangle$ will never be reached.

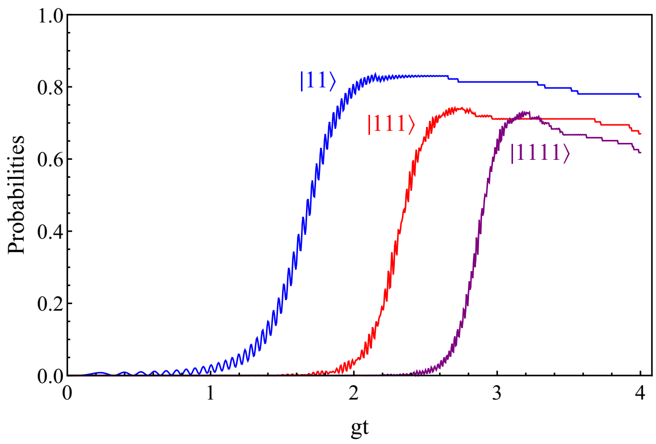

FIG. 5. (Color online) Probability of finding the Mott-insulator-like state $|11\rangle$, $|111\rangle$, and $|1111\rangle$, corresponding to $N=2$, 3, and 4, respectively. Notice that the state $|1111\rangle$ shows up later, but it reaches its maximum value in less time as compared to the other two cases, thus showing a more abrupt behavior, closer to a real phase transition. Here $\Delta=0$, $\gamma=0.05g$, and $J=0.03g$.

Hence, we can assert that the true triggering mechanism for this self-trapping effect is the presence of the dissipation in the cavities.

As was discussed above, increasing the detuning also increases the effective hopping, and therefore one would expect a similar dependence of the critical losses with Δ as with J. Even though this is only true for a small variation of Δ, as we increase it, the slope of the straight line shown in Fig. 4 becomes higher. Moreover, as we increase Δ further, a rapid oscillation of the negativity shows up, which makes it impossible to identify one single peak. For even larger detuning, say $\Delta=4g$, the state is almost photonic (there is no entanglement between light and matter), implying that there will be no self-trapping at all, since the atoms are in the ground state and photons can hop freely.

Furthermore, we studied the behavior of the transition when adding more cavities to the system, up to four. In that case, the MI-like state ($|111\rangle$ and $|1111\rangle$ for three and four cavities, respectively) shows up at a later time (as compared to the two cavity case), which is to be expected since the system is bigger and thus it takes more time to propagate through the array; see Fig. 5. Also, the slope of the state corresponding to $N=4$ is bigger than for $N=2$, indicating that we are getting closer to a real phase transition in the thermodynamic sense. This whole analysis was done at zero temperature. In the near future, we plan to investigate the thermal effects on these transitions [27,28].

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APPENDIX: MAPPING LADDER OPERATORS

For the sake of simplicity, we restrict our analysis to only two cavities and the first excited states, results that can be easily generalized for larger systems. Next, we calculate how the creation and destruction operators act on the polaritonic states:

$$\begin{aligned} a|n\pm\rangle &= \begin{pmatrix} \sqrt{n}\sin(\theta_n) & \sqrt{n-1}\cos(\theta_n) \\ \sqrt{n}\cos(\theta_n) & -\sqrt{n-1}\sin(\theta_n) \end{pmatrix} \begin{pmatrix} |n-1,g\rangle \\ |n-2,e\rangle \end{pmatrix}, \\ a^\dagger|n\pm\rangle &= \begin{pmatrix} \sqrt{n+1}\sin(\theta_n) & \sqrt{n}\cos(\theta_n) \\ \sqrt{n+1}\cos(\theta_n) & -\sqrt{n}\sin(\theta_n) \end{pmatrix} \begin{pmatrix} |n+1,g\rangle \\ |n,e\rangle \end{pmatrix}. \end{aligned} \tag{A1}$$

Using the spectral decomposition of the identity operator,

$$I = |0g\rangle\langle 0g| + \sum_{n=1}^{\infty} |n+\rangle\langle n+| + \sum_{n=1}^{\infty} |n-\rangle\langle n-|, \tag{A2}$$

we found the representation of the creation operator in terms of the polariton operators:

$$\begin{aligned} Ia^\dagger I = I\{&|1,g\rangle\langle 0,g| + [\sqrt{2}\sin(\theta_1)|2,g\rangle + \cos(\theta_1)|1,e\rangle]\langle 1+| + [\sqrt{3}\sin(\theta_2)|3,g\rangle \\ &+ \sqrt{2}\cos(\theta_2)|2,e\rangle]\langle 2+| + [\sqrt{4}\sin(\theta_3)|4,g\rangle + \sqrt{3}\cos(\theta_3)|3,e\rangle]\langle 3+| \\ &+ [\sqrt{2}\cos(\theta_1)|2,g\rangle - \sin(\theta_1)|1,e\rangle]\langle 1-| + [\sqrt{3}\cos(\theta_2)|3,g\rangle \\ &- \sqrt{2}\sin(\theta_2)|2,e\rangle]\langle 2-| + [\sqrt{4}\cos(\theta_3)|4,g\rangle - \sqrt{3}\sin(\theta_3)|3,e\rangle]\langle 3-| + \dots\} \\ = &\sin(\theta_1)|1+\rangle\langle 0,g| + \cos(\theta_1)|1-\rangle\langle 0,g| + [\sqrt{2}\sin(\theta_2)\sin(\theta_1)|2+\rangle \\ &+ \sqrt{2}\cos(\theta_2)\sin(\theta_1)|2-\rangle + \cos(\theta_2)\cos(\theta_1)|2+\rangle - \sin(\theta_2)\cos(\theta_1)|2-\rangle]\langle 1+| \\ &+ [\sqrt{3}\sin(\theta_3)\sin(\theta_2)|3+\rangle + \sqrt{3}\cos(\theta_3)\sin(\theta_2)|3-\rangle + \sqrt{2}\cos(\theta_3)\cos(\theta_2)|3+\rangle \\ &- \sqrt{2}\sin(\theta_3)\cos(\theta_2)|3-\rangle]\langle 2+| + [\sqrt{2}\cos(\theta_2)\cos(\theta_1)|2-\rangle + \sqrt{2}\sin(\theta_2)\cos(\theta_1)|2+\rangle \end{aligned}$$

$$+\sin(\theta_2)\sin(\theta_1)|2-\rangle-\cos(\theta_2)\sin(\theta_1)|2+\rangle]\langle 1-|+[\sqrt{3}\cos(\theta_3)\cos(\theta_2)|3-\rangle$$
$$+\sqrt{3}\sin(\theta_3)\cos(\theta_2)|3+\rangle+\sqrt{2}\sin(\theta_3)\sin(\theta_2)|3-\rangle-\sqrt{2}\cos(\theta_3)\sin(\theta_2)|3+\rangle]\langle 2-|]+\ldots. \quad \text{(A3)}$$

Rewriting the above expression, we readily found

$$\begin{aligned}Ia^\dagger I=&\sin(\theta_1)L^\dagger_{1+}+\cos(\theta_1)L^\dagger_{1-}+[\sqrt{2}\sin(\theta_2)\sin(\theta_1)+\cos(\theta_2)\cos(\theta_1)]L^\dagger_{2+}\\&+[\sqrt{2}\cos(\theta_2)\sin(\theta_1)-\sin(\theta_2)\cos(\theta_1)]L^\dagger_{2\mp}+[\sqrt{3}\sin(\theta_3)\sin(\theta_2)\\&+\sqrt{2}\cos(\theta_3)\cos(\theta_2)]L^\dagger_{3+}+[\sqrt{3}\cos(\theta_3)\sin(\theta_2)-\sqrt{2}\sin(\theta_3)\cos(\theta_2)]L^\dagger_{3\mp}\\&+[\sqrt{2}\sin(\theta_2)\cos(\theta_1)-\cos(\theta_2)\sin(\theta_1)]L^\dagger_{2\pm}+[\sqrt{2}\cos(\theta_2)\cos(\theta_1)\\&+\sin(\theta_2)\sin(\theta_1)]L^\dagger_{2-}+[\sqrt{3}\sin(\theta_3)\cos(\theta_2)-\sqrt{2}\cos(\theta_3)\sin(\theta_2)]L^\dagger_{3\pm}\\&+[\sqrt{3}\cos(\theta_3)\cos(\theta_2)+\sqrt{2}\sin(\theta_3)\sin(\theta_2)]L^\dagger_{3-}+\ldots,\end{aligned} \quad \text{(A4)}$$

with $L^\dagger_{1+}=|1+\rangle\langle 0,g|$, $L^\dagger_{2-}=|2-\rangle\langle 1-|$, and $L^\dagger_{2\mp}=|2-\rangle\langle 1+|$. Finally, we write the creation operator in the polaritonic basis as

$$a^\dagger=\sum_{n=1}^{\infty}c_{n+}L^\dagger_{n+}+\sum_{n=1}^{\infty}c_{n-}L^\dagger_{n-}+\sum_{n=2}^{\infty}k_{n\pm}L^\dagger_{n\pm}+\sum_{n=2}^{\infty}k_{n\mp}L^\dagger_{n\mp}. \quad \text{(A5)}$$

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