Scattering of two distinguishable photons by a Ξ-type three-level atom in a one-dimensional waveguide

T. Y. Li,[1] J. F. Huang,[1,2] and C. K. Law[1]

[1]*Department of Physics and Institute of Theoretical Physics, The Chinese University of Hong Kong, Shatin, Hong Kong Special Administrative Region, China*

[2]*Key Laboratory of Low-Dimensional Quantum Structures and Quantum Control of Ministry of Education, and Department of Physics, Hunan Normal University, Changsha 410081, China*

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We investigate theoretically quantum scattering of two distinguishable photon wave packets from a Ξ-type three-level atom in a one-dimensional waveguide. The quantum state of scattered photons is solved analytically, and the solution indicates how the two photons become strongly correlated after the scattering. We determine the two-photon reflection and transmission properties, and analyze the quantum entanglement between the scattered photons. In particular, we show that the degree of entanglement can be enhanced by decreasing the spectral width of the incident photon wave packets.

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I. INTRODUCTION

The transport properties of single photons controlled by interaction with atoms (or artificial atoms) inside a one-dimensional waveguide have been a subject of considerable interest in recent years [1–14]. In waveguide quantum electrodynamics (QED) systems, atoms can strongly interact with a continuum of field modes compared with those in free space, and the propagation directions of photons can be well monitored. By employing various interaction schemes with two- or three-level atoms, it is possible to control single photon propagation and to implement quantum gates for quantum information processing [15–17]. The investigation is also motivated by recent experimental progress in a wide variety of related systems, including quantum dot embedded in a nanowire [18,19], superconducting quantum circuit coupled to transmission lines [20–25], and photonic crystal waveguide with quantum dots [26–28] or natural atoms [29].

One of the interesting topics in waveguide QED is correlated transport of photons when two or more photons interact with each other inside the waveguide [30–39]. The photon-photon interaction is mediated by atoms. Physically, an atom with three effective internal levels serves as a basic scatterer that can be coupled to photons of distinct polarizations or frequencies, and hence inducing interaction between distinguishable photons. Some studies have reported interaction effects such as electromagnetic induced transparency [21] and conditional photon phase shifts in V-type and Ξ-type three-level systems [40,41]. Recently, Xu *et al.* has pointed out that frequency mixing and quantum entanglement are inevitable features occurring in general two-photon scattering problems [42]. However, these interesting features have not been further explored in Ξ-type systems in the context of waveguide QED.

In this paper we investigate a two-photon scattering problem involving a Ξ-type three-level scatterer in a one-dimensional (1D) waveguide. The study of a Ξ configuration is also important because it is a fundamental model for realizing cross-Kerr nonlinearity [43,44]. We shall present an analytic solution of the quantum state of scattered photons, and then discuss the transport properties as well as the frequency correlations. In addition, we investigate the quantum entanglement generated in the scattering process. As we shall see below, quantum entanglement arises from frequency mixing in the scattering process, i.e., the frequency of individual incident photon is no longer conserved after the scattering. However, because of the conservation of energy, the fluctuations of frequencies of each photon are correlated. We will analyze the entanglement by performing Schmidt decomposition of the two-photon state, which is known as a useful approach to characterize bi-partite pure-state entanglement [45–47]. Such an approach has been applied to various atomic and optical systems [48–55].

In our study, we consider the incident photons are in forms of wave packets, with their spectral widths as control parameters. We determine analytically how the widths of incident photons affect two-photon transmission and reflection probabilities. In addition, we quantity the entanglement by calculating the entanglement entropy and Schmidt number. The effects of photon loss due to atomic decay to nonwaveguide modes are also discussed.

II. THE PHYSICAL MODEL

The physical system under investigation consists of a Ξ-type three-level atom confined in an infinite long one -dimensional (1D) waveguide as shown in Fig. 1. We assume that the atom interacts with two distinct sets of waveguide modes (A and B), so that the field modes in A are responsible for the atomic transitions between $|3\rangle$ and $|2\rangle$, and the field modes in B are for the atomic transition between $|1\rangle$ and $|2\rangle$. This could be achieved, for example, by exploiting dipole selection rules so that modes in A and B are of orthogonal polarizations. Alternatively, if the upper and lower transition frequencies in the atom differ significantly, then A and B are distinguished by their frequencies. For optical fields, the Ξ-type scatterer considered in this paper may be realized by exploiting the level structure and selection rules of a InAs/GaAs quantum dot [56]. For microwave photons, the Ξ-type atom can be realized by superconducting quantum circuits [57–59].

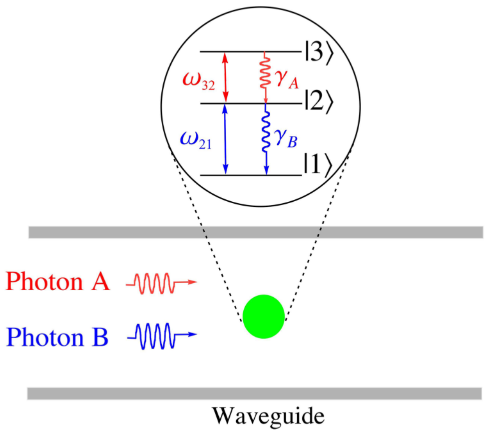

FIG. 1. (Color online) Schematic diagram of the system. A Ξ-type three-level atom confined at the center of a 1D waveguide interacts with a photon A (red) and a photon B (blue).

Specifically, the Hamiltonian of the system under rotating wave approximation is given by (with $\hbar = 1$)

$$H = H_A + H_F + H_{AF}, \tag{1}$$

with

$$H_A = \sum_{i=1}^{3} \omega_i |i\rangle\langle i|, \tag{2}$$

$$H_F = \int_0^\infty dk \sum_{n=L,R} \omega_k (a_{nk}^\dagger a_{nk} + b_{nk}^\dagger b_{nk}), \tag{3}$$

$$H_{AF} = \int_0^\infty dk \sum_{n=L,R} \left[\sqrt{\frac{\gamma_A}{2\pi}} a_{nk} |3\rangle\langle 2| + \text{H.c.} + \sqrt{\frac{\gamma_B}{2\pi}} b_{nk} |2\rangle\langle 1| + \text{H.c.} \right], \tag{4}$$

where $|i\rangle$ $(i = 1,2,3)$ are bare energy kets of the three-level atom with the corresponding energy ω_i, and a_{nk} (b_{nk}) and $a_{nk}^\dagger$ $(b_{nk}^\dagger)$ are annihilation and creation operators associated with the A (B) waveguide modes with frequency ω_k and wave number k. The subscripts $n = L$ and $n = R$ stand for the left- and right-propagating modes, respectively. These mode operators satisfy the commutation relations:

$$[a_{nk}, a_{n'k'}^\dagger] = [b_{nk}, b_{n'k'}^\dagger] = \delta(k - k')\delta_{nn'}, \tag{5}$$

$$[a_{nk}, b_{n'k'}] = [a_{nk}, b_{n'k'}^\dagger] = 0. \tag{6}$$

In writing the Hamiltonian, we have set the atom's position at the origin, and H_{AF} describes the atom-field interaction in which γ_A and γ_B are coupling constants. These coupling constants are also understood as decay rates: If the atom is initially prepared in the state $|3\rangle$ $(|2\rangle)$ in vacuum, then $2\gamma_A$ $(2\gamma_B)$ is the spontaneous emission rate to the corresponding waveguide modes.

For 1D waveguide QED systems, the loss of photons is mainly caused by scattering into the nonwaveguide modes. However, such a photon loss has been significantly reduced in recent experiments [18,24,28], so that photon emitted or scattered by the atom can be mostly captured by the waveguide mode. For the sake of clarity, we shall first neglect the photon loss in our calculation. A detailed calculation of photon loss will be presented later in this paper.

We remark that the one-dimensional treatment of the field modes is based on the assumption that the cross section of the waveguide is sufficiently small, typically of the size close to the wavelength, so that the atom only interacts with photons in the lowest transverse mode, and other transverse modes with higher cutoff frequencies are far off resonance from the atom. Such a one-dimensional configuration has been employed in most previous waveguide QED studies. The problem of waveguides with more than one transverse modes has been discussed recently [11,12], but this is beyond the scope of this paper. For simplicity, we assume $\omega_k = v_g k$ and the speed of light in the waveguide $v_g = 1$.

Next we introduce even- and odd-parity mode operators of the waveguide,

$$a_{ek} = \frac{1}{\sqrt{2}}(a_{Rk} + a_{Lk}), \quad a_{ok} = \frac{1}{\sqrt{2}}(a_{Rk} - a_{Lk}), \tag{7}$$

$$b_{ek} = \frac{1}{\sqrt{2}}(b_{Rk} + b_{Lk}), \quad b_{ok} = \frac{1}{\sqrt{2}}(b_{Rk} - b_{Lk}), \tag{8}$$

so that the Hamiltonian (1) can be separated into odd and even parts:

$$H = H^{(e)} + H^{(o)}, \tag{9}$$

where

$$H^{(o)} = \int_0^\infty dk \omega_k (a_{ok}^\dagger a_{ok} + b_{ok}^\dagger b_{ok}), \tag{10a}$$

$$H^{(e)} = \sum_{i=1}^{3} \omega_i |i\rangle\langle i| + \int_0^\infty dk \omega_k (a_{ek}^\dagger a_{ek} + b_{ek}^\dagger b_{ek}) + \sqrt{\frac{\gamma_A}{\pi}} \int_0^\infty dk\, (a_{ek}|3\rangle\langle 2| + \text{H.c.}) + \sqrt{\frac{\gamma_B}{\pi}} \int_0^\infty dk\, (b_{ek}|2\rangle\langle 1| + \text{H.c.}). \tag{10b}$$

Since the interaction involves only even modes, photons in the odd modes evolve freely.

In the rotating frame with respect to $H_0^{(e)} = \sum_{i=1}^3 \omega_i |i\rangle\langle i| + \int_0^\infty dk (\omega_{32} a_{ek}^\dagger a_{ek} + \omega_{21} b_{ek}^\dagger b_{ek})$, the Hamiltonian $H^{(e)}$ can be simplified to

$$H_I^{(e)} = \int_0^\infty dk \big(\Delta_k^{(A)} a_{ek}^\dagger a_{ek} + \Delta_k^{(B)} b_{ek}^\dagger b_{ek}\big) + \sqrt{\frac{\gamma_A}{\pi}} \int_0^\infty dk\, (a_{ek}|3\rangle\langle 2| + \text{H.c.}) + \sqrt{\frac{\gamma_B}{\pi}} \int_0^\infty dk\, (b_{ek}|2\rangle\langle 1| + \text{H.c.}). \tag{11}$$

where $\Delta_k^{(A)} = \omega_k - \omega_{32}$ and $\Delta_k^{(B)} = \omega_k - \omega_{21}$ (with $\omega_{ij} = \omega_i - \omega_j$) are defined.

III. SCATTERING WITH LORENTZIAN PHOTON WAVE PACKETS

Initially, an A photon and a B photon are injected from the left side of the waveguide (i.e., R mode) and the atom is in the lowest state $|1\rangle$. The initial state is denoted by

$$|\psi(0)\rangle = \int_0^\infty \int_0^\infty dpdq C_{pq}(0) a_{Rp}^\dagger b_{Rq}^\dagger |\emptyset\rangle |1\rangle, \tag{12}$$

where $|\emptyset\rangle$ is the common vacuum state of the waveguide, and $C_{pq}(0)$ are amplitudes. In terms of the odd and even modes, the initial state is rewritten as

$$|\psi(0)\rangle = \frac{1}{2}\int_0^\infty \int_0^\infty dpdq C_{pq}(0)\big(a_{op}^\dagger b_{oq}^\dagger + a_{ep}^\dagger b_{oq}^\dagger + a_{op}^\dagger b_{eq}^\dagger + a_{ep}^\dagger b_{eq}^\dagger\big)|\emptyset\rangle|1\rangle. \tag{13}$$

Specifically, we consider that the incident photons are disentangled and they are both prepared in a Lorentzian shape so that

$$C_{pq}(0) = \frac{\sqrt{\epsilon_A\epsilon_B}}{\pi}\frac{1}{\omega_p - \bar{\omega}_A + i\epsilon_A}\frac{1}{\omega_q - \bar{\omega}_B + i\epsilon_B} = \frac{\sqrt{\epsilon_A\epsilon_B}}{\pi}\frac{1}{\Delta_p^{(A)} - \delta_A + i\epsilon_A}\frac{1}{\Delta_q^{(B)} - \delta_B + i\epsilon_B}. \tag{14}$$

Here, $\bar{\omega}_j$ and ϵ_j are, respectively, the peak frequency and spectral width of the incident photon j ($j = A,B$), and $\delta_A = \bar{\omega}_A - \omega_{32}$ and $\delta_B = \bar{\omega}_B - \omega_{21}$ are detunings. Since the two photons are distinguishable, $C_{pq}(0)$ does not process the bosonic symmetry. In the limit $\epsilon_A,\epsilon_B \to 0$, the incident packets become two monochromatic waves.

A. Long time solution

We are interested in the system state in the long time limit, $t \gg \gamma_A^{-1},\gamma_B^{-1},\epsilon_A^{-1},\epsilon_B^{-1}$. To obtain the solution, we employ Laplace transform to solve the Schrödinger equation (see appendix). The long time solution is found to be

$$|\psi(t\to\infty)\rangle = \sum_{m=e,o}\sum_{n=e,o}|\psi_{mn}(t)\rangle, \tag{15}$$

where

$$|\psi_{oo}(t)\rangle = \frac{1}{2}\int_0^\infty\int_0^\infty dpdq C_{pq}(0) \times e^{-i(\Delta_p^{(A)}+\Delta_q^{(B)})t} a_{op}^\dagger b_{oq}^\dagger|\emptyset\rangle|1\rangle, \tag{16}$$

$$|\psi_{eo}(t)\rangle = \frac{1}{2}\int_0^\infty\int_0^\infty dpdq C_{pq}(0) \times e^{-i(\Delta_p^{(A)}+\Delta_q^{(B)})t} a_{ep}^\dagger b_{oq}^\dagger|\emptyset\rangle|1\rangle, \tag{17}$$

$$|\psi_{oe}(t)\rangle = \frac{1}{2}\int_0^\infty\int_0^\infty dpdq C_{pq}(0)e^{i\theta_q} \times e^{-i(\Delta_p^{(A)}+\Delta_q^{(B)})t} a_{op}^\dagger b_{eq}^\dagger|\emptyset\rangle|1\rangle, \tag{18}$$

$$|\psi_{ee}(t)\rangle = \frac{1}{2}\int_0^\infty\int_0^\infty dpdq\,\big(C_{pq}(0)e^{i\theta_q} + \Lambda_{pq}\big) \times e^{-i(\Delta_p^{(A)}+\Delta_q^{(B)})t} a_{ep}^\dagger b_{eq}^\dagger|\emptyset\rangle|1\rangle, \tag{19}$$

and

$$e^{i\theta_q} = \frac{\Delta_q^{(B)} - i\gamma_B}{\Delta_q^{(B)} + i\gamma_B}, \tag{20}$$

$$\Lambda_{pq} = \frac{-4\gamma_A\gamma_B\sqrt{\epsilon_A\epsilon_B}}{\pi\big(\Delta_q^{(B)} + i\gamma_B\big)\big(\Delta_p^{(A)} + \Delta_q^{(B)} + i\gamma_A\big)} \times \frac{1}{\big(\Delta_p^{(A)} + \Delta_q^{(B)} - \delta_A + i\epsilon_A + i\gamma_B\big)} \times \frac{1}{\big(\Delta_p^{(A)} + \Delta_q^{(B)} - \delta_A - \delta_B + i\epsilon_A + i\epsilon_B\big)}. \tag{21}$$

Equations (16) and (17) correspond to freely evolving kets since the atom in the state $|1\rangle$ does not couple to the B photon in odd modes. Equation (18) correspond to the case that the A photon is in odd modes whereas the B photon is in even modes. In this case the B photon interacts with the atom and picks up a single photon phase given by Eq. (20), while the A photon evolves freely. When both photons are in even mode, the two photons can interact indirectly through the atom, and the corresponding ket is given by Eq. (19) which has an interesting Λ_{pq} term. By Eq. (21), we note that Λ_{pq} has energy denominators that correlate the energies of the two scattered photons.

From above equations, the long time solution in the left- and right-mode basis is found to be

$$\begin{aligned}|\psi(t\to\infty)\rangle &= \int_0^\infty\int_0^\infty dpdq C_{pq}^{RR} a_{Rp}^\dagger b_{Rq}^\dagger|\emptyset\rangle|1\rangle + \int_0^\infty\int_0^\infty dpdq C_{pq}^{LR} a_{Lp}^\dagger b_{Rq}^\dagger|\emptyset\rangle|1\rangle \\ &+ \int_0^\infty\int_0^\infty dpdq C_{pq}^{RL} a_{Rp}^\dagger b_{Lq}^\dagger|\emptyset\rangle|1\rangle + \int_0^\infty\int_0^\infty dpdq C_{pq}^{LL} a_{Lp}^\dagger b_{Lq}^\dagger|\emptyset\rangle|1\rangle,\end{aligned} \tag{22}$$

where

$$C_{pq}^{RR} = \left(\frac{\Delta_q^{(B)}}{\Delta_q^{(B)} + i\gamma_B}C_{pq}(0) + \frac{1}{4}\Lambda_{pq}\right)e^{-i(\Delta_p^{(A)}+\Delta_q^{(B)})t}, \tag{23}$$

$$C_{pq}^{LR} = C_{pq}^{LL} = \frac{1}{4}\Lambda_{pq}e^{-i(\Delta_p^{(A)}+\Delta_q^{(B)})t}, \tag{24}$$

$$C_{pq}^{RL} = \left(\frac{-i\gamma_B}{\Delta_q^{(B)} + i\gamma_B}C_{pq}(0) + \frac{1}{4}\Lambda_{pq}\right)e^{-i(\Delta_p^{(A)}+\Delta_q^{(B)})t}, \tag{25}$$

are two-photon amplitudes associated with the four possible output channels after the scattering.

It is important to point out that both C_{pq}^{LR} and C_{pq}^{LL} in Eq. (24) only involve the Λ_{pq} term. This can be understood by the fact that the reflection of the A photon (described by C_{pq}^{LR} and C_{pq}^{LL}) is a two-photon process in which the photon A can interact with the atom only after the photon B is absorbed. Therefore the correlation between the two photons can be more clearly observed in the LR and LL channels. This is in contrast to scattering with indistinguishable photons in related systems where the corresponding two-photon correlation term cannot be separated out completely in the transmission and reflection amplitudes [32,38].

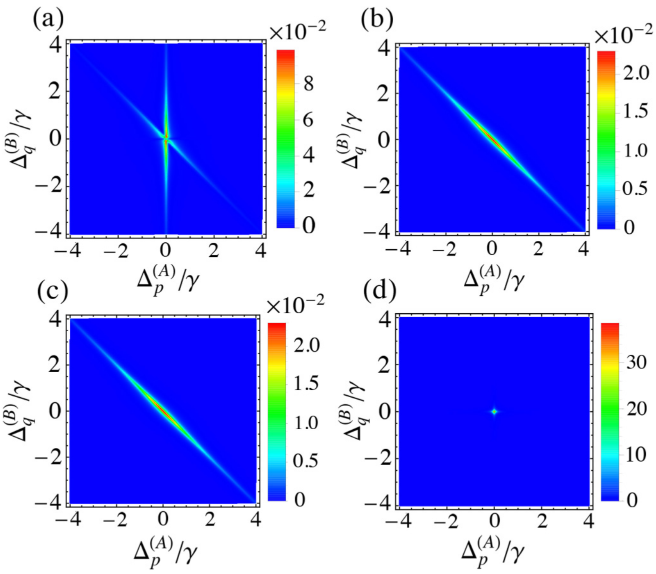

FIG. 2. (Color online) Plots of dimensionless quantities: (a) $|\gamma C_{pq}^{RR}|^2$, (b) $|\gamma C_{pq}^{LL}|^2$, (c) $|\gamma C_{pq}^{LR}|^2$, and (d) $|\gamma C_{pq}^{RL}|^2$ as a function of normalized detunings of photons, $\Delta_p^{(A)}/\gamma$ and $\Delta_q^{(B)}/\gamma$, when $\delta_A = \delta_B = 0$. Other parameters are $\gamma_A = \gamma_B \equiv \gamma$ and $\epsilon_A = \epsilon_B = 0.05\gamma$.

In Fig. 2 we illustrate the joint probability density in frequency space for the resonance case: $\delta_A = \delta_B = 0$. Among the four possible output amplitudes in Eqs. (23)–(25), the strongest frequency correlation appears in $|C_{pq}^{LR}|^2$ and $|C_{pq}^{LL}|^2$ [Figs. 2(b) and 2(c)]. Such a correlation is featured by a high concentration on the line $\Delta_p^{(A)} + \Delta_q^{(B)} = 0$, which is a constraint of energy conservation, and the width depends on the spectral widths of the incident photon wave packets and the coupling constants according to the energy denominators in Eq. (21). On the other hand, since $|C_{pq}^{RL}|^2$ is dominated by the $C_{pq}(0)$ term, Fig. 2(d) does not show any significant frequency correlation.

B. Two-photon transport probability

With the solution presented in the previous subsection, we can determine the transport probabilities. Specifically, we consider the resonant case $\delta_A = \delta_B = 0$. From Eq. (24), the probability of both photon A and B being reflected (P_{LL}) is the same as the probability for photon A being reflected and photon B being transmitted (P_{LR}). The expression is given by

$$\begin{aligned} P_{LL} = P_{LR} &= \iint dpdq \left|\frac{1}{4}\Lambda_{pq}\right|^2 \\ &= \frac{\gamma_A\gamma_B\epsilon_A\epsilon_B}{(\gamma_B+\epsilon_A)(\epsilon_B+\epsilon_A)(\gamma_A+\gamma_B+\epsilon_A)} \\ &\quad\times\frac{(\gamma_A+\gamma_B+2\epsilon_A+\epsilon_B)}{(\gamma_A+\epsilon_A+\epsilon_B)(\gamma_B+2\epsilon_A+\epsilon_B)} \\ &\equiv \frac{1}{16}\mathcal{N}, \end{aligned} \tag{26}$$

where $\mathcal{N} = \iint dpdq\,\Lambda_{pq}$ is the norm of Λ_{pq}.

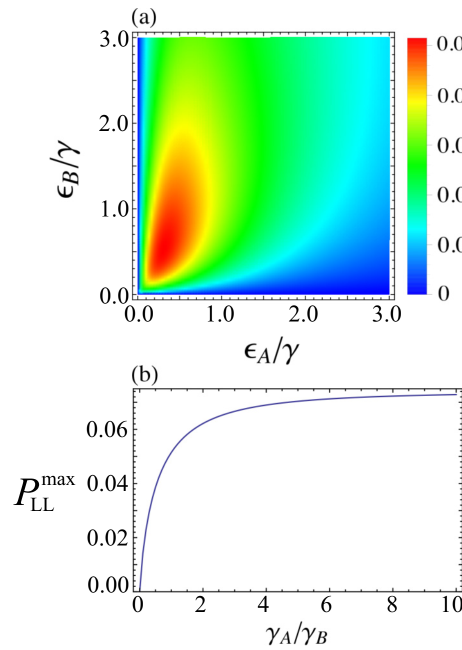

FIG. 3. (Color online) (a) Probability of two photons being reflected (P_{LL}) as a function of ϵ_A and ϵ_B for $\gamma_A = \gamma_B \equiv \gamma$. (b) Maximal probability $P_{LL}^{\max}$ as a function of coupling ratio γ_A/γ_B. $\delta_A = \delta_B = 0$ are used in both figures.

Furthermore, the probability of both photons being transmitted (P_{RR}) and the probability for photon B being reflected and photon A being transmitted (P_{RL}) are given by

$$\begin{aligned} P_{RR} &= \frac{\epsilon_B}{\gamma_B+\epsilon_B} - M_1 + \frac{1}{16}\mathcal{N}, \\ P_{RL} &= \frac{\gamma_B}{\gamma_B+\epsilon_B} - M_2 + \frac{1}{16}\mathcal{N}, \end{aligned} \tag{27}$$

with

$$\begin{aligned} M_1 &= \frac{2\gamma_A\gamma_B\epsilon_A\epsilon_B}{(\gamma_B+\epsilon_A)(\gamma_A+\gamma_B+\epsilon_A)(\gamma_B+\epsilon_B)} \\ &\quad\times\frac{\epsilon_B^2+[\gamma_A+2(\gamma_B+\epsilon_A)]\epsilon_B-\epsilon_A(\gamma_A+\epsilon_A)}{(\epsilon_A+\epsilon_B)(\gamma_A+\epsilon_A+\epsilon_B)(\gamma_B+2\epsilon_A+\epsilon_B)}, \\ M_2 &= \frac{2\gamma_A\gamma_B\epsilon_A\epsilon_B}{(\gamma_B+\epsilon_A)(\gamma_A+\gamma_B+\epsilon_A)(\gamma_B+\epsilon_B)} \\ &\quad\times\frac{1}{(\epsilon_A+\epsilon_B)(\gamma_A+\epsilon_A+\epsilon_B)(\gamma_B+2\epsilon_A+\epsilon_B)} \\ &\quad\times\big[2\gamma_B^2+(\epsilon_A+\epsilon_B)^2+\gamma_A(2\gamma_B+\epsilon_A+\epsilon_B) \\ &\qquad+2\gamma_B(2\epsilon_A+\epsilon_B)\big]. \end{aligned} \tag{28}$$

Note that $M_1 + M_2 = 4P_{LL}$.

In Fig. 3(a), P_{LL} is shown as a function of the widths of incident photon wave packets. We see that P_{LL} can be controlled by the incident widths, and a maximum value can be attained at a certain value of ϵ_A and ϵ_B. For the equal-coupling case $\gamma_A = \gamma_B = \gamma$ shown in Fig. 3(a), we find that $\epsilon_A \approx 0.31\gamma$ and $\epsilon_B \approx 0.53\gamma$ are optimal. For unequal coupling cases, the maximum value of P_{LL} (denoted by $P_{LL}^{\max}$) can be determined numerically. For example, at $\gamma_A = 5\gamma_B$, $P_{LL}^{\max} \approx 0.07$ can be reached when $\epsilon_A \approx 0.45\gamma_B$ and $\epsilon_B \approx 0.87\gamma_B$. For a general coupling ratio γ_A/γ_B, the optimal widths ϵ_A and ϵ_B are obtained numerically, and the corresponding $P_{LL}^{\max}$ is shown as a function of γ_A/γ_B in Fig. 3(b). We see that $P_{LL}^{\max}$ increases with γ_A/γ_B, and it saturates at around 7%.

C. Two-photon entanglement

The strong frequency correlation between the two photons scattered in LL and LR channels [Figs. 2(b) and 2(c)] suggests that photons are highly entangled in these two output channels. In order to examine the two-photon entanglement quantitatively, we consider a normalized form of Λ_{pq} defined by $\Lambda'_{pq} \equiv \mathcal{N}^{-\frac{1}{2}}\Lambda_{pq}$, and perform the Schmidt decomposition [45–47] so that

$$\Lambda'_{pq} = \sum_n \sqrt{\lambda_n}\psi_n(\omega_p)\phi_n(\omega_q). \tag{29}$$

Here λ_n are Schmidt eigenvalues, and ψ_n and ϕ_n are biorthogonal Schmidt modes. Note that we have confined our discussion to the two-photon amplitude C_{pq}^{LL} that captures two-photon entanglement most effectively.

To quantify the degree of quantum entanglement, a common measure for bipartite systems in the pure state is the entropy of entanglement defined by

$$S = -\sum_n \lambda_n \log_2 \lambda_n. \tag{30}$$

Alternatively, the Schmidt number K [60], defined by

$$K \equiv \frac{1}{\sum_n \lambda_n^2}, \tag{31}$$

is an indicator of the effective number of Schmidt modes and it has been employed to study entanglement in atomic and photonic systems [49,61–64].

We have calculated both S and K numerically for various incident widths. Specifically, the λ_n are obtained numerically by the eigenvalues of the reduced density matrix of photon A (or B) formed by C_{pq}^{LL}. For the case of equal coupling $\gamma_A = \gamma_B = \gamma$ and identical widths $\epsilon_A = \epsilon_B \equiv \epsilon$, the entanglement entropy is shown in Fig. 4(a). The figure indicates that two-photon entanglement can be significantly enhanced by using photon wave packets with narrow spectral widths $\epsilon \ll \gamma$. The increase of entanglement entropy is understood from the energy denominator $\Delta_p^{(A)} + \Delta_q^{(B)} + i\epsilon_A + i\epsilon_B$ in the last term in Eq. (21), where a small ϵ would limit the energy fluctuations and hence enforcing the correlation.

We also plot the values of K as a function of γ/ϵ in Fig 4(b). It is interesting that K increases approximately linearly with γ/ϵ when $\gamma/\epsilon > 1$. This means that the number of Schmidt modes increases roughly linearly with the inverse of incident widths. This feature is also observed numerically in Raman scattering with atom recoil [49].

D. Effects of nonwaveguide modes

In this subsection we discuss the photon loss when the atom interacts with nonwaveguide modes. Photons can escape the waveguide by scattering from the atom into nonwaveguide modes. To include nonwaveguide modes in the dynamics, the full Hamiltonian is modified by

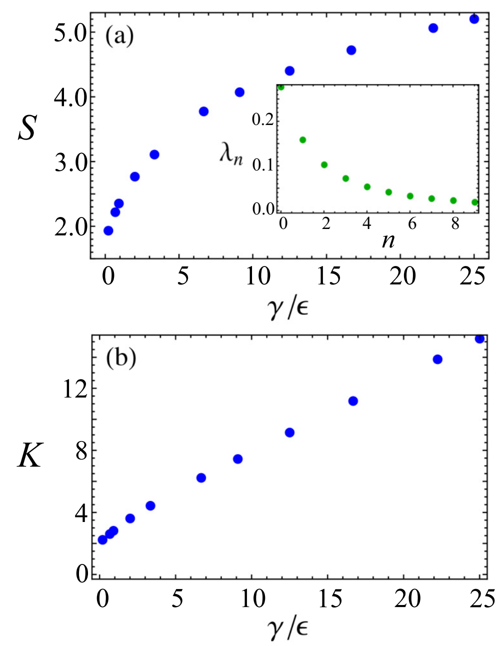

FIG. 4. (Color online) (a) Entanglement entropy of Λ'_{pq} as a function of γ/ϵ with $\gamma_A = \gamma_B \equiv \gamma$ and $\delta_A = \delta_B = 0$. The inset shows the first 10 Schmidt eigenvalues for $\epsilon = 0.1\gamma$. (b) Schmidt number K of Λ'_{pq} as a function of γ/ϵ; same parameters as in (a).

$$\mathcal{H} = H + \int_0^\infty dk\nu_k(c_k^\dagger c_k + d_k^\dagger d_k) + \int dk[g_{2,k}^* c_k|2\rangle\langle 1| + g_{3,k}^* d_k|3\rangle\langle 2| + \text{H.c.}]. \tag{32}$$

Here we have introduced two independent oscillator baths to model the nonwaveguide modes, one for the transition $|2\rangle$ and $|3\rangle$ and the other the transition $|1\rangle$ and $|2\rangle$. The $c_k^\dagger$ (c_k) and $d_k^\dagger$ (d_k) are creation (annihilation) operators of the nonwaveguide modes of frequency ν_k, and $g_{3,k}$ and $g_{2,k}$ are the corresponding strength. Both oscillator baths are assumed in the vacuum state.

By employing Weisskopf-Wigner approximation, the quantum dynamics of the system (atom and photons in waveguide modes) is effectively governed by a non-Hermitian Hamiltonian H' as (see Appendix)

$$H' = H - i\gamma_2|2\rangle\langle 2| - i\gamma_3|3\rangle\langle 3|, \tag{33}$$

where γ_2 and γ_3 are the decay rates of the levels $|2\rangle$ and $|3\rangle$ to the nonwaveguide modes, respectively. In other words, the coupling to baths of nonwaveguide modes causes atomic decay. Such a theoretical treatment of damping has been employed in related waveguide QED systems [40].

The probabilities of losing photons via the decay of levels $|3\rangle$ and $|2\rangle$ are, respectively, given by

$$P_{\text{loss}}^{(3)} = \gamma_3 \int_0^\infty \langle\psi(t')|\sigma_{33}|\psi(t')\rangle dt', \tag{34}$$

$$P_{\text{loss}}^{(2)} = \gamma_2 \int_0^\infty \langle\psi(t')|\sigma_{22}|\psi(t')\rangle dt'. \tag{35}$$

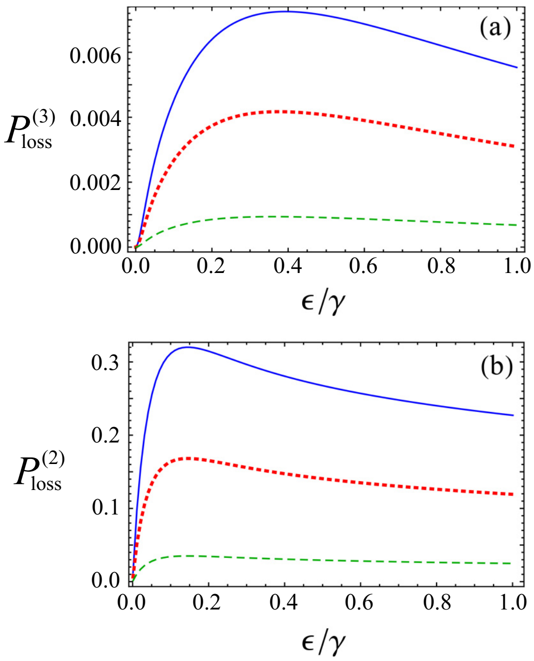

FIG. 5. (Color online) Photon loss probabilities (a) $P^{(3)}_{\rm loss}$ and (b) $P^{(2)}_{\rm loss}$ as functions of $\epsilon \equiv \epsilon_A = \epsilon_B$ for the resonance case with $\delta_A = \delta_B = 0$ and $\gamma_A = \gamma_B \equiv \gamma$. The decay rates are set to be $\gamma_3 = \gamma_2 = 0.1\gamma$ (solid line), $\gamma_3 = \gamma_2 = 0.05\gamma$ (dotted line), and $\gamma_3 = \gamma_2 = 0.01\gamma$ (dashed line).

Here $\sigma_{33} = |3\rangle\langle 3|$ and $\sigma_{22} = |2\rangle\langle 2|$ are atomic projection operators, and $|\psi(t')\rangle$ is the system state at time t' under the Hamiltonian (33).

Based on the Laplace transform solution in the Appendix, we can calculate $|\psi(t)\rangle$, and $P^{(3)}_{\rm loss}$ and $P^{(2)}_{\rm loss}$ are obtained. In Fig. 5, $P^{(3)}_{\rm loss}$ and $P^{(2)}_{\rm loss}$ are plotted for different values of widths, assuming $\epsilon_A = \epsilon_B \equiv \epsilon$. We notice that $P^{(3)}_{\rm loss}$ is much smaller than $P^{(2)}_{\rm loss}$. This is understood by the fact that the occupation probability of $|3\rangle$ is always much smaller than that of $|2\rangle$ because the excitation to $|3\rangle$ involves the absorption of two photons, which is a higher order process.

In Fig. 5, we see that when the incident widths are sufficiently narrow, $\epsilon \ll \gamma$, the values of $P^{(3)}_{\rm loss}$ and $P^{(2)}_{\rm loss}$ are close to zero. This is because when $\epsilon \ll \gamma$, the state $|3\rangle$ and $|2\rangle$ are almost unoccupied in the whole scattering process. On the other hand, when incident widths are large, $\epsilon \gg \gamma$, $P^{(3)}_{\rm loss}$ and $P^{(2)}_{\rm loss}$ decrease with γ, since photons with a broader spectrum are more difficult to excite the atom.

As a remark, in a recent experiment using a two-level quantum dot in a photonic crystal waveguide [28], the ratio of rate of decay into nonwaveguide modes to that into waveguide modes is about 0.02. Also, this ratio in another experiment using quantum dot in a photonic nanowire [18] is about 0.05. Furthermore, in an experiment using a superconducting artificial atom coupled to microwave photons in a 1D open transmission line [24], the ratio of intrinsic loss to the atom relaxing rate can be smaller than 0.05. In these experiments, the decay rate into nonwaveguide modes can be much smaller than that into the waveguide mode, allowing a small loss of photons to nonwaveguide modes.

IV. CONCLUSIONS

In this paper we have presented an exact analytic solution of a scattering problem in which two distinguishable photon wave packets are scattered by a Ξ-type scatterer in a one-dimensional waveguide. From the long time solution of the scattered photons, we determined the two-photon transmission and reflection properties. Since initially the atom has a zero population in the middle level $|2\rangle$, it is expected that the photon A would be transmitted with most of the probability. However, we find that there can be still an appreciable probability (about 14%) that photon A can be reflected. Once the photon A is reflected, it is strongly entangled with the photon B. The entanglement manifests in the frequency correlation in which the probability distribution concentrates on the line $\Delta_p^{(A)} + \Delta_q^{(B)} = 0$. Such an entanglement is quantified by the entanglement entropy and the Schmidt number. We discover that the degree of entanglement can be increased by decreasing the spectral width of the incident wave packets. In particular, we observe that the Schmidt number K grows almost linearly with the inverse of the spectral width of incident photons.

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APPENDIX: DERIVATION OF SOLUTION EQ. (19)

In this Appendix we provide a derivation for Eq. (19), namely the evolution for the part associated with two photons initially in even modes [the last term in Eq. (13)]. We start with the Hamiltonian (32) which includes the coupling to nonwaveguide modes. In the rotating frame respective to $H_0^{(e)} + \int_0^\infty dk\nu_k(c_k^\dagger c_k + d_k^\dagger d_k)$, the Hamiltonian $\mathcal{H}$ becomes $\mathcal{H}_I$ given by

$$\mathcal{H}_I = H^{(o)} + H_I^{(e)} + \mathcal{V}, \tag{A1}$$

where

$$\mathcal{V} = \int dk[g^*_{2,k}c_k|2\rangle\langle 1|e^{i(\omega_{21}-\nu_k)t} + \text{H.c.}] + \int dk[g^*_{3,k}d_k|3\rangle\langle 2|e^{i(\omega_{32}-\nu_k)t} + \text{H.c.}]. \tag{A2}$$

Note that the interactions terms only appear in $H_I^{(e)}$ and $\mathcal{V}$; the photons in the odd mode evolve freely.

Consider an initial state corresponding to the last term in Eq. (13), i.e., two photons initially in even modes; the state at later time t is given by

$$|\Phi_e(t)\rangle = \alpha(t)|\emptyset\rangle|3\rangle + \int_0^\infty dp(\beta_{1p}(t)a^\dagger_{ep} + \beta_{2p}(t)d^\dagger_p)|\emptyset\rangle|2\rangle + \int_0^\infty dp\int_0^\infty dq(\zeta_{1pq}(t)a^\dagger_{ep}b^\dagger_{eq} + \zeta_{2pq}(t)a^\dagger_{ep}c^\dagger_q + \zeta_{3pq}(t)b^\dagger_{ep}d^\dagger_q + \zeta_{4pq}(t)d^\dagger_p c^\dagger_q)|\emptyset\rangle|1\rangle, \tag{A3}$$

where $\alpha(t)$, $\beta_{ip}(t)$, and $\zeta_{ipq}(t)$ probability amplitudes at time t. The initial amplitudes are $\alpha(0) = \beta_{ip}(0) = \zeta_{2pq}(0) = \zeta_{3pq}(0) = \zeta_{4pq}(0) = 0$ and $\zeta_{1pq}(0) = C_{pq}(0)/2$. By the Hamiltonian (A1), the Schrödinger equation gives

$$i\dot{\alpha}(t) = \int_0^\infty dp\,\sqrt{\frac{\gamma_A}{\pi}}\beta_{1p}(t) + \int_0^\infty dp\; g^*_{2,p}\beta_{2p}(t)e^{i(\omega_{32}-\nu_p)t}, \tag{A4a}$$

$$i\dot{\beta}_{1p}(t) = \Delta_p^{(A)}\beta_{1p}(t) + \sqrt{\frac{\gamma_A}{\pi}}\alpha(t) + \int_0^\infty dq\,\sqrt{\frac{\gamma_B}{\pi}}\zeta_{1pq}(t) + \int_0^\infty dq\; g^*_{2,q}\zeta_{2pq}(t)e^{i(\omega_{21}-\nu_q)t}, \tag{A4b}$$

$$i\dot{\beta}_{2p}(t) = g_{2,p}\alpha(t)e^{-i(\omega_{32}-\nu_p)t} + \int_0^\infty dq\,\sqrt{\frac{\gamma_B}{\pi}}\zeta_{3pq}(t) + \int_0^\infty dq\; g^*_{2,q}\zeta_{4pq}(t)e^{i(\omega_{21}-\nu_q)t}, \tag{A4c}$$

$$i\dot{\zeta}_{1pq}(t) = \left(\Delta_p^{(A)} + \Delta_q^{(B)}\right)\zeta_{1pq}(t) + \sqrt{\frac{\gamma_B}{\pi}}\beta_{1p}(t), \tag{A4d}$$

$$i\dot{\zeta}_{2pq}(t) = \Delta_p^{(A)}\zeta_{2pq}(t) + g_{2,q}\beta_{1p}(t)e^{-i(\omega_{21}-\nu_q)t}, \tag{A4e}$$

$$i\dot{\zeta}_{3pq}(t) = \Delta_q^{(B)}\zeta_{3pq}(t) + \sqrt{\frac{\gamma_B}{\pi}}\beta_{2p}(t), \tag{A4f}$$

$$i\dot{\zeta}_{4pq}(t) = g_{2,q}\beta_{2p}(t)e^{-i(\omega_{21}-\nu_q)t}. \tag{A4g}$$

Now $\zeta_{ipq}(t)$ $(i = 2,3,4)$ in (A4e) and (A4f) can be formally expressed in terms of integrals involving β_{1p} and β_{2p}. By substituting the expressions of $\zeta_{ipq}(t)$ into Eqs. (A4a)–(A4c), and making the Markrovian approximation,

$$\int_{-\infty}^\infty dk|g_{n,k}|^2 e^{-i(\omega_{nn-1}-\nu_k)(t-t')} = 2\gamma_n\delta(t-t'), \quad n = 2,3, \tag{A5}$$

(A4) becomes

$$i\dot{\alpha}(t) = \int_0^\infty dp\,\sqrt{\frac{\gamma_A}{\pi}}\beta_{1p}(t) - i\gamma_3\alpha(t), \tag{A6a}$$

$$i\dot{\beta}_{1p}(t) = \Delta_p^{(A)}\beta_{1p}(t) + \sqrt{\frac{\gamma_A}{\pi}}\alpha(t) + \int_0^\infty dq\,\sqrt{\frac{\gamma_B}{\pi}}\zeta_{1pq}(t) - i\gamma_2\beta_{1p}(t), \tag{A6b}$$

$$i\dot{\zeta}_{1pq}(t) = \left(\Delta_p^{(A)} + \Delta_q^{(B)}\right)\zeta_{1pq}(t) + \sqrt{\frac{\gamma_B}{\pi}}\beta_{1p}(t), \tag{A6c}$$

so that effects of nonwaveguide modes are captured by the γ_3 and γ_2 terms, which agree with the non-Hermitian Hamiltonian in Eq. (33).

To solve (A6), we perform a Laplace transformation defined by $\tilde{f}(s) = \int_0^\infty f(t)e^{-st}dt$, then the transformed (A6) becomes

$$is\tilde{\alpha}(s) = \int_0^\infty dp\,\sqrt{\frac{\gamma_A}{\pi}}\tilde{\beta}_{1p}(s) - i\gamma_3\tilde{\alpha}(s), \tag{A7a}$$

$$is\tilde{\beta}_{1p}(s) = \Delta_p^{(A)}\tilde{\beta}_{1p}(s) + \sqrt{\frac{\gamma_A}{\pi}}\tilde{\alpha}(s) + \int_0^\infty dq\,\sqrt{\frac{\gamma_B}{\pi}}\tilde{\zeta}_{1pq}(s) - i\gamma_2\tilde{\beta}_{1p}(s), \tag{A7b}$$

$$is\tilde{\zeta}_{1pq}(s) = \left(\Delta_p^{(A)} + \Delta_q^{(B)}\right)\tilde{\zeta}_{1pq}(s) + \sqrt{\frac{\gamma_B}{\pi}}\tilde{\beta}_{1p}(s) + i\zeta_{1pq}(0). \tag{A7c}$$

By substituting Eq. (A7c) and Eq. (A7a) into Eq. (A7b), the equation of $\tilde{\beta}_{1p}(s)$ reads

$$\left(is - \Delta_p^{(A)} + i\gamma_B + i\gamma_2\right)\tilde{\beta}_{1p}(s) = \frac{\gamma_A}{\pi(is + i\gamma_3)}\int dp'\tilde{\beta}_{1p'}(s) + i\sqrt{\frac{\gamma_B}{\pi}}\int_0^\infty dq\frac{\zeta_{1pq}(0)}{is - \Delta_p^{(A)} - \Delta_q^{(B)}}. \tag{A8}$$

For the given Lorentzian packet (14), the solution of $\tilde{\beta}_{1k}(s)$ takes the form,

$$\begin{aligned}\tilde{\beta}_{1k}(s) = {}& \frac{i\sqrt{\gamma_B/\pi}}{is - \Delta_k^{(A)} + i\gamma_B + i\gamma_2}\int_0^\infty dq\frac{\zeta_{1kq}(0)}{is - \Delta_k^{(A)} - \Delta_q^{(B)}}\\ &+ \frac{\sqrt{\gamma_B/\pi}}{is - \Delta_k^{(A)} + i\gamma_B + i\gamma_2}\frac{\gamma_A}{s + \gamma_A + \gamma_3}\frac{1}{\pi}\int_0^\infty dp\int_0^\infty dq\frac{\zeta_{1pq}(0)}{\left(is - \Delta_p^{(A)} - \Delta_q^{(B)}\right)\left(is - \Delta_p^{(A)} + i\gamma_B + i\gamma_2\right)}.\end{aligned}$$

Then, from Eq. (A7c), we obtain the expression of $\tilde{\zeta}_{1pq}(s)$,

$$\tilde{\zeta}_{1pq}(s) = \frac{i\zeta_{1pq}(0)}{is - \Delta_p^{(A)} - \Delta_q^{(B)}} + \frac{1}{is - \Delta_p^{(A)} - \Delta_q^{(B)}} \frac{i\gamma_B}{is - \Delta_p^{(A)} + i\gamma_B + i\gamma_2} \frac{1}{\pi} \int_0^\infty dq' \frac{\zeta_{1pq'}(0)}{is - \Delta_p^{(A)} - \Delta_{q'}^{(B)}}$$
$$+ \frac{1}{is - \Delta_p^{(A)} - \Delta_q^{(B)}} \frac{\gamma_B}{is - \Delta_p^{(A)} + i\gamma_B + i\gamma_2} \frac{\gamma_A}{s + \gamma_A + \gamma_3} \frac{1}{\pi^2} \int_0^\infty dp' \int_0^\infty dq' \left[\frac{\zeta_{1p'q'}(0)}{\left(is - \Delta_{p'}^{(A)} - \Delta_{q'}^{(B)}\right)} \times \frac{1}{\left(is - \Delta_{p'}^{(A)} + i\gamma_B + i\gamma_2\right)} \right]. \tag{A9}$$

The right side of Eq. (A9) can be integrated explicitly for $\zeta_{1pq}(0) = C_{pq}(0)/2$ given in Eq. (14),

$$\tilde{\zeta}_{1pq}(s) = \frac{i\zeta_{1pq}(0)}{is - \Delta_p^{(A)} - \Delta_q^{(B)}} + \frac{\sqrt{\epsilon_A\epsilon_B}}{\pi} \frac{\gamma_B}{is - \Delta_p^{(A)} - \Delta_q^{(B)}} \frac{1}{is - \Delta_p^{(A)} + i\gamma_B + i\gamma_2} \frac{1}{is + i\epsilon_B - \delta_B - \Delta_p^{(A)}} \frac{1}{\Delta_p^{(A)} - \delta_A + i\epsilon_A}$$
$$- \frac{\sqrt{\epsilon_A\epsilon_B}}{\pi} \frac{2i\gamma_B\gamma_A}{is - \Delta_p^{(A)} - \Delta_q^{(B)}} \frac{1}{is + i\gamma_A + i\gamma_3} \frac{1}{is - \Delta_p^{(A)} + i\gamma_B + i\gamma_2} \times \frac{1}{(is + i\epsilon_A + i\epsilon_B - \delta_A - \delta_B)(is + i\epsilon_A + i\gamma_B + i\gamma_2 - \delta_A)}. \tag{A10}$$

Finally, by taking the inverse Laplace transformation of (A10) and keeping the terms with $e^{-i(\Delta_p^{(A)}+\Delta_q^{(B)})t}$, which survive in the long time limit $t \gg \gamma_A^{-1}, \gamma_B^{-1}, \gamma_2^{-1}, \gamma_3^{-1}, \epsilon_A^{-1}, \epsilon_B^{-1}$, the long time solution of $\zeta_{1pq}(t)$ is obtained:

$$\zeta_{1pq}(t \to \infty) = \tfrac{1}{2}[C_{pq}(0)e^{i\theta'_q} + \Lambda'_{pq}]e^{-i(\Delta_p^{(A)}+\Delta_q^{(B)})t}, \tag{A11}$$

where

$$e^{i\theta'_q} = \frac{\Delta_q^{(B)} + i\gamma_2 - i\gamma_B}{\Delta_q^{(B)} + i\gamma_2 + i\gamma_B}, \tag{A12}$$

$$\Lambda'_{pq} = \frac{-4\gamma_A\gamma_B\sqrt{\epsilon_A\epsilon_B}}{\pi\left(\Delta_q^{(B)} + i\gamma_B + i\gamma_2\right)\left(\Delta_p^{(A)} + \Delta_q^{(B)} + i\gamma_A + i\gamma_3\right)} \times \frac{1}{\left(\Delta_p^{(A)} + \Delta_q^{(B)} - \delta_A + i\epsilon_A + i\gamma_B + i\gamma_2\right)\left(\Delta_p^{(A)} + \Delta_q^{(B)} - \delta_A - \delta_B + i\epsilon_A + i\epsilon_B\right)}. \tag{A13}$$

For the ideal case where there is no coupling to nonwaveguide modes, i.e., $\gamma_2 = \gamma_3 = 0$, Eq. (A13) is reduced to Eq. (19) in the text.

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