Wheeler's delayed-choice experiment: A proposal for the Bragg-regime cavity-QED implementation

Manzoor Ikram,[1] Muhammad Imran,[1,2] Tasawar Abbas,[3,*] and Rameez-ul-Islam [1]

[1]*National Institute of Lasers and Optronics, Nilore 45650, Islamabad, Pakistan*

[2]*Department of Physics and Applied Mathematics, PIEAS, Nilore 45650, Islamabad, Pakistan*

[3]*Department of Physics, COMSATS Institute of Information Technology, 45550 Islamabad, Pakistan*

(Received 1 November 2014; published 30 April 2015)

Wheeler's delayed-choice experiment highlights strange features of quantum theory such as pre-sensing of the experimental setup by the quantum object and the role of time. A recent proposal for such an experiment with an interferometer having a quantum beam splitter (QBS) [R. Ionicioiu and D. R. Terno, Phys. Rev. Lett. **107**, 230406 (2011)] and its subsequent experimental implementations through photonics and NMR have produced results including the modification in the concept of complementarity. Here we propose a matter-wave Mach-Zehnder-Bragg cavity-QED interferometric setup with final QBS engineered through a cavity field that is taken initially in the superposition of zero and one photon. The setup operates through first-order off-resonant Bragg diffraction of the neutral atoms from the cavity fields with the matter wave's particle (wave) nature marked through the absence (presence) of a photon in the final cavity. The proposal, addressing the issue through atomic de Broglie waves, can be executed within the present cavity-QED experimental scenario with appreciable success probability and fidelity.

DOI: 10.1103/PhysRevA.91.043636 PACS number(s): 03.75.Be, 03.67.Mn, 42.50.Pq, 42.50.Dv

I. INTRODUCTION

Complementarity is one of the fundamental principles underlying quantum theory and a central theme in the famous Einstein-Bohr debate [1,2]. Historically, Bohr generalized wave-particle duality into the governing concept of complementarity which asserts that, in quantum theory, the two conjugate or complementary quantum variables always stand mutually exclusive; i.e., the observation of one completely eradicates even the possibility of any meaningful measurement of the other [3,4]. The idea further dictates that, in principle, the conjugate quantum variables cannot even be ascertained through a single experimental arrangement and rather each complementary variable requires a unique setup for its measurement because the interaction between the measuring device and the quantum system under consideration form a holistic scenario that defines the phenomenon [5]. However, initially Bohr expounded the complementarity principle on a broad philosophical basis with no adequate mathematical description. Such a quantitative narrative was later furnished through many investigations including the one by Greenberger *et al.* [6–9]. Moreover, a comparison as well as quantitative link between complementarity and quantum uncertainty have also been debated recently [10,11].

Another very fascinating idea, initially quite independent of the complementarity scenario, was proposed by Wheeler and is now termed Wheeler's delayed-choice experiment (DCE) [8,12]. Wheeler's thought-provoking DCE has been extensively discussed both theoretically and experimentally and was aimed to highlight the role played by time in the quantum unitary evolution of a system because it hints that a quantum entity while evolving in its "present" can somehow sense the yet undecided futuristic setup that it will encounter later on. In a Mach-Zehnder interferometer, this can be done through a random and delayed-choice decision of inserting or removing the second beam splitter when the photon has already assumed its flight through the setup after entering from the first beam splitter. The decision concerning the insertion or removal of second beam splitter is implemented classically through coupling it with a random number generator. Many such experiments were conducted in line with the suggested classical delayed-choice framework in conformity with Wheeler's original proposal mentioned above, and most of them may be taken as state-of-the-art proofs showing no notable difference between real time and DCEs [13]. However, as suggested recently, the quantum version of Wheeler's delayed-choice experiment shows fascinating results with direct repercussion for the concept of the complementarity [14]. In the quantum delayed-choice experiment (QDCE) based on the Mach-Zehnder interferometer, the delayed decision for insertion or removal of the final beam splitter is carried out quantum mechanically by coupling it with an ancilla qubit. Such a beam splitter is termed the quantum beam splitter (QBS). The article suggests that, in the QDCE scenario, we can measure both the conjugate variables in the same setup and two mutually exclusive experimental arrangements are not needed in any way as affirmed by Bohr's elucidation of the complementarity. The proposal illustrates that particle and wave feature can be observed in a single experimental setup because the ancilla-qubit-forming QBS, i.e., $1/\sqrt{2}(|0\rangle + |1\rangle)$, gets entangled with the system under investigation to yield a state of the type [14]

$$\frac{1}{\sqrt{2}}(|0\rangle \otimes |\mathrm{BS}_2\ \mathrm{absent}\rangle \otimes |\mathrm{particle}\rangle + |1\rangle \otimes |\mathrm{BS}_2\ \mathrm{present}\rangle \otimes |\mathrm{wave}\rangle). \quad (1)$$

Here the wave functions $|\mathrm{particle}\rangle = 1/\sqrt{2}(|0\rangle + e^{i\phi}|1\rangle)$ and $|\mathrm{wave}\rangle = e^{i\phi/2}[\cos(\phi/2)|0\rangle - i\sin(\phi/2)|1\rangle]$ describe particle and wave behavior, respectively, with ϕ being the phase associated with each wave function. These two wave functions are not orthogonal in general, except for the phase $\phi = \pm\pi/2$.

*t.abbas.malik@gmail.com

The proposal for the QDCE has been demonstrated experimentally by many groups recently using photonics and NMR setups [15–19]. These theoretical and experimental explorations of QDCE, along with the modification to the old concept of complementarity, have also uncovered some very important and novel ideas related to quantum theory. They demonstrated, for example, the falsehood of local hidden-variable theories that treat particles and waves as real physical attributes of a quantum entity [15], deeper connections between complementarity and the superposition principle [17], and the interference between the particle and wave attributes of a quantum object [18]. More recently Almeida *et al.* proposed a setup for the implementation of QDCE in the cavity-QED scenario [20]. The proposal deals with Ramsey-type interference in the atomic internal degrees of freedom and invokes the concept of ability (inability) of interference fringes to mark wave (particle) aspects of the system instead of employing the usual concept of spatial modes. The experimental setup envisioned is, however, quite tedious comprising three atoms, Ramsey zones, and a high-Q cavity yielding the desired result after many quantum operations, controlled or otherwise. We, in the present article, suggest a very simple scheme based on off-resonant atom-field interactions of two-level atoms in the Mach-Zehnder-Bragg interferometric scenario to implement QDCE through spatially separated atomic de Broglie waves that explicitly highlights the wave-particle dilemma in its conventional and more realistic sense. It is worth noting here that atomic Bragg diffraction has already been utilized to address various quantum-information tasks [21–25] as well as the foundational issues related to the concept of complementarity [10].

The layout of the present work is as follows. Section II describes briefly the physics of the Bragg diffraction of the neutral two-level atoms from the cavity field. The section also elaborates on the engineering aspects of the Bragg-regime matter-wave interferometry and discusses the schematics for atomic beam splitters and mirrors under off-resonant interactions of the neutral atoms with the cavity fields. Section III furnishes the proposal of QDCE in full mathematical details and explains comprehensively how the particle aspect (PA) and wave aspect (WA), in the real sense of the words, can be seen in a single experimental setup. Finally Sec. IV summarizes the results with a brief discussion on the feasibility of the proposed scheme.

II. ATOM OPTICS BASED ON CAVITY-QED-ASSISTED BRAGG DIFFRACTION OF NEUTRAL ATOMS

Matter-wave Bragg diffraction is an energy- and momentum-conserving, elastic Raman scattering process, which follows the usual diffraction conditions. In conventional Bragg diffraction, light is scattered from the atomic lattice whereas in the case of matter waves, atoms are diffracted from the optical lattice, i.e., standing-wave field [26,27]. The momentum transfer results only in the discrete initial values of atomic momentum along the $\mathbf{k}$ vector of the field. This scattering process conserves both momentum and energy. The condition for the conservation of momentum is given by $Q_{\text{out}} = Q_0 + j\hbar\mathbf{k}$, where $Q_0 = j_0\hbar\mathbf{k}/\mathbf{2}$ is the initial momentum of the incoming atom with j_0 an even integer and

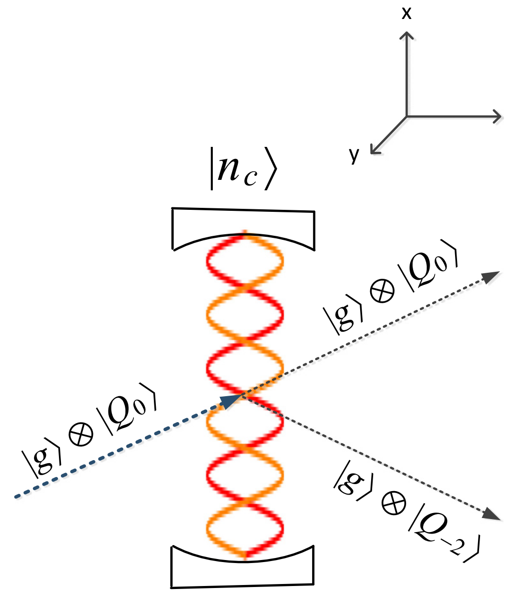

FIG. 1. (Color online) Bragg diffraction: Interaction of atom with the cavity-field standing wave.

Q_{out} the momentum after j interactions. The energy conservation requires that $|Q_{in}|^2/2M = |Q_{\text{out}}|^2/2M$, where M is the atomic mass [22,23]. Energy and momentum conservations, taken together, imply that $j(j + j_0)\hbar^2\mathbf{k}^2/2M = 0$. This yields two solutions: $j = 0$ corresponds to the undeflected atom and $j = -j_0$ corresponds to the deflected atom [28]. Thus for $j_0 = 2,4$, and 6 one obtains first-, second-, and third-order Bragg diffraction, respectively.

We consider a two-level atom initially in its ground state, $|g\rangle$, moving along the z axis with its quantized transverse momentum along the x axis, i.e., along the wave propagation of the cavity field, $|Q_0\rangle$, interacting with the cavity field in Fock state $|n_c\rangle$, as shown in Fig. 1. The order of the Bragg diffraction j_0 describes the number of interactions of the atom with the cavity field and is always an even integer because during one Rabi cycle the momentum imparted to the atom, along the cavity axis, through its interaction with the field is either zero or $2\hbar\mathbf{k}$ [24,25]. Thus the momenta acquired by the atom in first-, second-, and third-order Bragg diffraction from the cavity field will be $(|Q_0\rangle = |\hbar\mathbf{k}\rangle, |Q_{-2}\rangle = |-\hbar\mathbf{k}\rangle)$, $(|Q_0\rangle = |2\hbar\mathbf{k}\rangle, |Q_{-4}\rangle = |-2\hbar\mathbf{k}\rangle)$, and $(|Q_0\rangle = |3\hbar\mathbf{k}\rangle, |Q_{-6}\rangle = |-3\hbar\mathbf{k}\rangle)$, respectively, as a consequence of the so-called Bragg resonances [27]. Such an interaction of a two-level atom with quantized center-of-mass motion is governed by the interaction picture Hamiltonian, written under the dipole and rotating wave approximations as [22,23,25]

$$\hat{H}_I = \frac{Q_x^2}{2M} + \frac{\hbar\delta_c}{2}\sigma_z + \hbar\mu\cos(k\hat{x})(\hat{a}\sigma_+ + \hat{a}^\dagger\sigma_-). \quad (2)$$

Here μ is the atom-field coupling constant, $\hat{a}^\dagger(\hat{a})$ is the field raising (lowering) operator, $\sigma_+ = |e\rangle\langle g|$ $(\sigma_- = |g\rangle\langle e|)$ and $\sigma_z = |e\rangle\langle e| - |g\rangle\langle g|$ are the corresponding atomic raising (lowering) and inversion operators. Atom-field detuning is denoted by δ_c whereas $Q_x^2(\hat{x})$ is the momentum (position) operator for the center-of-mass motion of the atom. The state vector depicting such an interaction for an arbitrary time t may be expressed as follows:

$$\left|\Psi_{A-C}^{Q}(t)\right\rangle = e^{-i(\frac{Q_0^2}{2M\hbar}-\frac{\delta_c}{2})t}\sum_{j=-\infty}^{\infty}\left[C_{n_c,g}^{A^{Q_j}}(t)\left|n_c,g,A^{Q_j}\right\rangle + C_{(n-1)_c,e}^{A^{Q_j}}(t)\left|(n-1)_c,e,A^{Q_j}\right\rangle\right], \quad (3)$$

where $|A^{Q_j}\rangle$ represents an atom in momentum space having quantized transverse momentum Q_j and carrying both the particle as well as the wave characteristics. $C_{n_c,g}^{A^{Q_j}}(t)[C_{(n-1)_c,e}^{A^{Q_j}}(t)]$ is the corresponding probability amplitude marking the atom in momentum state $|Q_j\rangle$ with the cavity-field state $|n_c\rangle[|(n-1)_c\rangle]$ and atomic internal state being in ground $|g\rangle$ (excited $|e\rangle$) after j interactions. The summation over j here designates the accumulative nature of transverse momenta acquired through a Bragg's order limited number of Rabi's interactional cycles. Schrödinger's equation yields [29]

$$\frac{\partial C_{n_c,g}^{A^{Q_j}}(t)}{\partial t} = -i\left[\left(\frac{j(j_0+j)\hbar k^2}{2M}\right)C_{n_c,g}^{A^{Q_j}}(t) + \frac{\mu\sqrt{n_c}}{2}\left(C_{(n-1)_c,e}^{A^{Q_{j+1}}}(t) + C_{(n-1)_c,e}^{A^{Q_{j-1}}}(t)\right)\right], \quad (4)$$

$$\frac{\partial C_{(n-1)_c,e}^{A^{Q_j}}(t)}{\partial t} = -i\left[\left(\frac{j(j_0+j)\hbar k^2}{2M}+\delta_c\right)C_{(n-1)_c,e}^{A^{Q_j}}(t) + \frac{\mu\sqrt{n_c}}{2}\left(C_{n_c,g}^{A^{Q_{j+1}}}(t) + C_{n_c,g}^{A^{Q_{j-1}}}(t)\right)\right]. \quad (5)$$

Expressions (4) and (5) stand for an infinite set of coupled first-order differential equations (ICDE) for all possible values of j, covering both resonant and off-resonant interactions and describe the atom's dynamics in momentum space up to a selected Bragg's diffraction order j_0. Here we opt for the off-resonant Bragg diffraction of ground-state atoms so that the Rabi excitational-deexcitational cycles become virtual and therefore suppresses the decoherence threat linked with the spontaneous emission of an excited atom. Now in order to tune out and make the interactions of-resonant, we have to assure that detuning δ_c is much larger than the recoil frequency $\omega_r = \hbar k^2/2M$, i.e., $\delta_c \gg \omega_r$. Furthermore, as clear from the above expressions, the adiabatic approximation for off-resonant Bragg diffraction implies that $\omega_r + \delta_c \gg \mu\sqrt{n_c}/2$. Here we opt for the first-order Bragg diffraction with $j_0 = 2$ and therefore the above set of ICDE subsequently reduces to only five significant expressions related to $j \in \{-3,-2,\dots,1\}$ [22,23,26,27,30]. Now under the large detuning limit we can ignore $j(j_0+j)\hbar k^2/2M$ in comparison with δ_c. Similarly, adiabatic approximation suggests, for an initially ground-state atom, we have $\partial C_{(n-1)_c,e}^{A^{Q_1}}(t)/\partial t = \partial C_{(n-1)_c,e}^{A^{Q_{-1}}}(t)/\partial t = \partial C_{(n-1)_c,e}^{A^{Q_{-3}}}(t)/\partial t = 0$. Further, under the same approximation, we can also ignore the probability amplitudes $C_{n_c,g}^{A^{Q_2}A}(t)$ and $C_{n_c,g}^{A^{Q_{-4}}}(t)$ as being negligibly small [22,28,31]. Finally a little algebraic simplification, along with the application of the above approximations, yields us with the following two coupled equations,

$$\frac{\partial C_{n_c,g}^{A^{Q_0}}(t)}{\partial t} = i\left[\frac{\mu^2 n_c}{2\delta_c}C_{n_c,g}^{A^{Q_0}}(t) + \frac{\mu^2 n_c}{4\delta_c}C_{n_c,g}^{A^{Q_{-2}}}(t)\right], \quad (6)$$

$$\frac{\partial C_{n_c,g}^{A^{Q_{-2}}}(t)}{\partial t} = i\left[\frac{\mu^2 n_c}{2\delta_c}C_{n_c,g}^{A^{Q_{-2}}}(t) + \frac{\mu^2 n_c}{4\delta_c}C_{n_c,g}^{A^{Q_0}}(t)\right]. \quad (7)$$

Probability amplitudes under first-order, off-resonant Bragg diffraction follow through the solution as

$$C_{n_c,g}^{A^{Q_0}}(t) = e^{2i\alpha n_c t}\left[C_{n_c,g}^{A^{Q_0}}(0)\cos(\alpha n_c t) + iC_{n_c,g}^{A^{Q_{-2}}}(0)\sin(\alpha n_c t)\right], \quad (8)$$

$$C_{n_c,g}^{A^{Q_{-2}}}(t) = e^{2i\alpha n_c t}\left[C_{n_c,g}^{A^{Q_{-2}}}(0)\cos(\alpha n_c t) + iC_{n_c,g}^{A^{Q_0}}(0)\sin(\alpha n_c t)\right], \quad (9)$$

where $\alpha = \mu^2/4\delta_c$ is effective Rabi frequency. Expressions (8) and (9) serve as our master equations for engineering the atom optics tools, i.e., atomic de Broglie wave beam splitters and mirrors designed under first-order off-resonant Bragg interactions of atoms with cavity fields.

A. Atomic de Broglie beam splitter with cavity field $|n_c\rangle$

Consider an atom, initially in ground state $|g\rangle$ with transverse momentum $|Q_0\rangle$, interacting with the cavity field $|n_c\rangle$ for a time t. These initial conditions then simplify equations (8) and (9) to furnish atomic de Broglie beam splitter (AdB-BS) transforms,

$$C_{n_c,g}^{A^{Q_0}}(t) = e^{2i\alpha n_c t}\cos(\alpha n_c t), \quad (10)$$

$$C_{n_c,g}^{A^{Q_{-2}}}(t) = ie^{2i\alpha n_c t}\sin(\alpha n_c t). \quad (11)$$

For a symmetric AdB-BS, we have to take $\alpha n_c t = \pi/4$. This furnishes the beam splitter transforms with $C_{n_c,g}^{A^{Q_0}}(t = \pi/4\alpha n_c) = i/\sqrt{2}$ and $C_{n_c,g}^{A^{Q_{-2}}}(t = \pi/4\alpha n_c) = -i/\sqrt{2}$. Similarly, for the cavity field comprising a single photon, i.e., $|1_c\rangle$ with initial conditions $C_{1_c,g}^{A^{Q_0}}(0) = 1/\sqrt{2}$ and $C_{1_c,g}^{A^{Q_{-2}}}(0) = 1/\sqrt{2}$, this yields general AdB-BS transforms, symmetric or otherwise, as follows:

$$C_{1_c,g}^{A^{Q_0}}(t) = \frac{e^{2i\alpha t}}{\sqrt{2}}[\cos(\alpha t) + i\sin(\alpha t)], \quad (12)$$

$$C_{1_c,g}^{A^{Q_{-2}}}(t) = \frac{e^{2i\alpha t}}{\sqrt{2}}[\cos(\alpha t) + i\sin(\alpha t)]. \quad (13)$$

B. Atomic de Broglie mirror with cavity field $|n_c\rangle$

An atomic mirror enacts a NOT gate transformation; i.e., it transforms the atomic momenta states: $|Q_0\rangle \to |Q_{-2}\rangle$ and $|Q_{-2}\rangle \to |Q_0\rangle$. Thus, for a ground-state atom, initially coming with momentum $|Q_0\rangle$, the interaction time corresponding to a half Rabi cycle, i.e., $\alpha n_c t = \pi/2$, gives mirror transforms $C_{n_c,g}^{A^{Q_0}}(t = \pi/2\alpha n_c) = 0$ and $C_{n_c,g}^{A^{Q_{-2}}}(t = \pi/2\alpha n_c) = -i$. Similarly, if the atom is initially in the ground state with momentum $|Q_{-2}\rangle$ then for an interaction time corresponding to a Rabi cycle, we get the alternative atomic de Broglie mirror (AdB-M) transforms $C_{n_c,g}^{A^{Q_0}}(t=\pi/2\alpha n_c)=-i$ and $C_{n_c,g}^{A^{Q_{-2}}}(t=\pi/2\alpha n_c)=0$. Now with this atom optics gadget available, we can proceed to develop QDCE based on the Mach-Zehnder-Bragg interferometer.

III. QDCE: MACH-ZEHNDER-BRAGG INTERFEROMETRIC IMPLEMENTATION WITH NEUTRAL ATOMS

The schematics for the proposed experimental setup are sketched in Fig. 2. The high-Q cavities C_1 and (C_2,C_3) contain Fock fields $|n_c\rangle$ and $|m_c\rangle$ acting as atomic de Broglie beam splitter (AdB-BS) and atomic de Broglie mirrors (AdB-M), respectively. However, the fourth cavity, i.e., C_4, that acts as the final quantum beam splitter, i.e., AdB-BS or QAdB-BS, is initially prepared in the superposition state $(\cos\theta|0_c\rangle + i\sin\theta|1_c\rangle)$, with θ being the interaction parameter of the atom with the field inside the cavity. Such a cavity field superposition can be prepared either by passing an auxiliary atom, initially in superposition of its internal states $(\cos\theta|g\rangle + i\sin\theta|e\rangle)$, through an initially vacuum-state cavity for a time corresponding to a half Rabi cycle. This corresponds to the swapping of the quantum information between the atom and the high-Q cavity. The subsequent detection of the atom in its ground state $|g\rangle$ guarantees the generation of the desired field superposition inside the cavity C_4. Alternatively, when an auxiliary atom initially in its excited state $|e\rangle$ transverses the vacuum-state cavity for a preselected time matching the desired θ value and then goes through a Ramsey zone earlier to its state-selective detection, this also culminates in the generation of the required field superposition state into the cavity [32].

We consider a two-level atom, prepared initially in its ground level $|g\rangle$ with transverse momentum $|Q_0\rangle$, that interacts off-resonantly with the cavity C_1 possessing field $|n_c\rangle$ under first-order Bragg diffraction. The initial state of the atom-field system, i.e., $|\Psi_{AF}(t_1=0)\rangle = |n_c,g,A^{Q_0}\rangle$, after interaction with the cavity C_1 forming the first AdB-BS of the Mach-Zehnder interferometer, gets transformed, in accordance with expressions (8) and (9) of Sec. II, to the following state for an interaction time $\pi/4\alpha n_c$:

$$|\Psi_{AF}(t_1=\pi/4\alpha n_c)\rangle = \frac{i}{\sqrt{2}}(|A^{Q_0}\rangle + i|A^{Q_{-2}}\rangle)\otimes|n_c,g\rangle. \tag{14}$$

We can trace out $|n_c\rangle$ from the above expression, leaving us with a superposition of split atomic de Broglie wave packets in momentum space traveling orthogonal to each other with momenta $|Q_0\rangle = |\hbar\mathbf{k}\rangle$ and $|Q_{-2}\rangle = |-\hbar\mathbf{k}\rangle$, respectively.

Next, this state passes through the two AdB-M mirror cavities C_2 and C_3 each having field $|m_c\rangle$. From expressions derived in previous section, we note that mirror action under first-order off-resonant Bragg interactions lasting for a complete Rabi cycle, i.e., $t_2=\pi/2\alpha m_c$, change the state of the split atomic wave packets emerging from the C_2 and C_3 mirror cavities as follows:

$$|\Psi_{AF}(t_2=\pi/2\alpha m_c)\rangle = \frac{i}{\sqrt{2}}(|A^{Q_0}\rangle - i|A^{Q_{-2}}\rangle)\otimes|g\rangle. \tag{15}$$

Here we have traced out the cavity field $|m_c\rangle$. Next, this two-level atom bearing both particle and wave attributes, and in superposition of its external transverse momenta states, faces the final beam splitter AdB-BS$_2$ which is a quantum beam splitter initially prepared in the state $[\cos(\theta)|0_c\rangle + i\sin(\theta)|1_c\rangle]$, as mentioned earlier. Now the initial composite state of the atom-field system for the final interaction of the Mach-Zehnder-Bragg interferometer, i.e., $|\Psi_{AF}(t_3=0)\rangle = i(|A^{Q_0}\rangle - i|A^{Q_{-2}}\rangle)/\sqrt{2}\otimes[\cos(\theta)|0_c\rangle + i\sin(\theta)|1_c\rangle]\otimes|g\rangle$, may be rearranged as follows:

$$\begin{aligned}|\Psi_{AF}(t_3=0)\rangle = \frac{1}{\sqrt{2}}\big[&i\cos(\theta)\big(\big|A_p^{Q_0}\big\rangle - i\big|A_p^{Q_{-2}}\big\rangle\big)\otimes|0_c\rangle \\ &- \sin(\theta)\big(\big|A_w^{Q_0}\big\rangle - i\big|A_w^{Q_{-2}}\big\rangle\big)\otimes|1_c\rangle\big]\otimes|g\rangle.\end{aligned} \tag{16}$$

This suggests that when the Bragg-diffracted atom faces vacuum-state cavity $|0_c\rangle$, i.e., the case marking the absence of AdB-BS$_2$, it sheds its wave characteristics but the AdB-BS$_2$

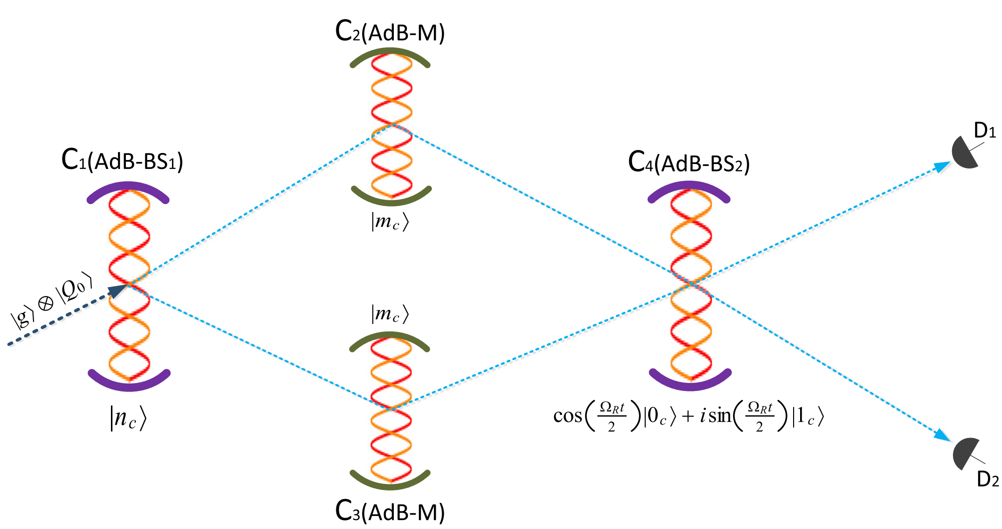

FIG. 2. (Color online) Proposed Mach-Zendher-Bragg cavity-QED interferometric experimental setup.

cavity (C_4) having a $|1_c\rangle$ field state hints at the presence of the quantum beam splitter that can potentially Bragg-diffract the matter waves resulting in the availability of an option for the de Broglie atomic matter wave interference fringes. Thus the atom in this case will exhibit its wave nature while suppressing the particle signatures. The atom with complementary particle and wave characteristics is therefore designated with $|A_p^{Q_j}\rangle$ and $|A_w^{Q_j}\rangle$, with $j = 0, -2$, respectively, in above expression.

Now, for an arbitrary interaction time t_3 under first-order off-resonant Bragg diffraction, the second part of the above expression evolves in accordance with Eqs. (12) and (13) of the previous section and yields the final quantum state of the two-level atom emerging out of the q-AdB-BS_2 as

$$|\Psi_{AF}(t_3)\rangle = \frac{i\cos(\theta)}{\sqrt{2}}\left(\left|A_p^{Q_0}\right\rangle - i\left|A_p^{Q_{-2}}\right\rangle\right)\otimes|0_c\rangle - \frac{\sin(\theta)e^{i\chi}}{\sqrt{2}}\left[\left(\cos\frac{\chi}{2}+\sin\frac{\chi}{2}\right)\left|A_w^{Q_0}\right\rangle - i\left(\cos\frac{\chi}{2}-\sin\frac{\chi}{2}\right)\left|A_w^{Q_{-2}}\right\rangle\right]\otimes|1_c\rangle, \quad (17)$$

where we have taken $\chi = \mu^2 t_3/2\delta_c$. The atomic internal state $|g\rangle$, being in outer product form, is traced out from the above expression as it is no longer needed for further explorations. In a more abbreviated form, the above expression can be written as

$$|\Psi_{AF}(t_3)\rangle = \cos(\theta)|\text{particle}\rangle|0_c\rangle - \sin(\theta)|\text{wave}\rangle|1_c\rangle, \quad (18)$$

where

$$|\text{particle}\rangle = \frac{i}{\sqrt{2}}\left(\left|A_p^{Q_0}\right\rangle - i\left|A_p^{Q_{-2}}\right\rangle\right), \quad (19)$$

$$|\text{wave}\rangle = \frac{e^{i\chi}}{\sqrt{2}}\left\{\left(\cos\frac{\chi}{2}+\sin\frac{\chi}{2}\right)\left|A_w^{Q_0}\right\rangle + i\left(\cos\frac{\chi}{2}-\sin\frac{\chi}{2}\right)\left|A_w^{Q_{-2}}\right\rangle\right\}. \quad (20)$$

The orthogonality expression for these two states comes out to be $\langle\text{particle}|\text{wave}\rangle = ie^{i\chi}\cos(\chi/2)$ suggesting that, in general, these states are not orthogonal to each other except for $\chi = \pm\pi$. From this equation it is evident that if the cavity field in the quantum beam splitter AdB-BS_2 is found to be in vacuum, i.e., $|0_c\rangle$, then the interferometer will be open and the atom will subsequently behaves as a particle. However, for AdB-BS_2 field state $|1_c\rangle$, the interferometer will be correspondingly closed and we will witness the wave behavior of the Bragg-diffracted atom. In general, the trade-off between the particle and wave aspects of the atom is governed by the variable θ, i.e., the interaction parameters selected while preparing the field state in the AdB-BS_2. We note that for $\theta = \theta_P = 0$, the atom behaves completely like a particle whereas for $\theta = \theta_W = \pi/2$ it thoroughly exhibits its wave characteristics (Fig. 3). However, for $\theta \,\epsilon\,]\theta_P,\theta_W[$, the trade-off between particle and wave aspects highlights itself in a morphing pattern. Such a particle-wave morphology, keeping in line with Ref. [14], in our case comes to be

$$I_0(\theta,\chi) = \frac{\cos^2(\theta)}{2} + \frac{\sin^2(\theta)}{2}[1+\sin(\chi)], \quad (21)$$

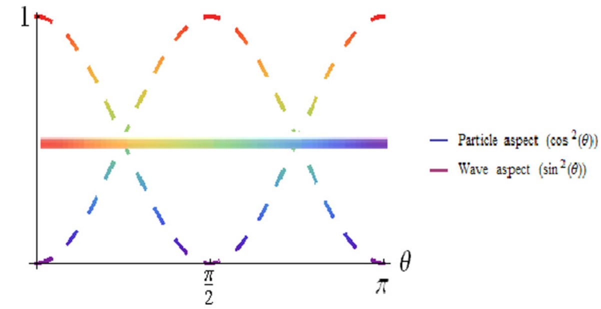

FIG. 3. (Color online) Particle-wave features of the atom versus $\theta = \Omega_R t'/2$.

if the trace is taken over $|A^{Q_0}\rangle$. Alternatively, if the trace is taken over $|A^{Q_{-2}}\rangle$ we get

$$I_0(\theta,\chi) = \frac{\cos^2(\theta)}{2} + \frac{\sin^2(\theta)}{2}[1-\sin(\chi)], \quad (22)$$

and this is plotted in Fig. 4. Concerning wave features, the interference fringe visibility or distinguishability is governed by the variable χ, i.e., interaction parameter of the atom with the AdB-BS_2 under first-order off-resonant Bragg diffraction, but we need not elaborate it further in the present context. We rather conclude the section by noting that here Bohr's stand about entirely different experimental setups needed to demonstrate complementary variables stays uniquely refuted if somehow we can incorporate a quantum atom-optical element prepared in the superposition state into the interferometer, as demonstrated by the final quantum beam splitter in the present schematics; then we can observe both particle and wave characteristics of a quantum entity in a single setup.

IV. SUMMARY AND DISCUSSION

Cavity QED is one of the pioneer discipline to demonstrate various protocols of quantum information [33]. High-Q cavities with a lifetime up to a fraction of a second are available now [34] with demonstrated feasibility of successive

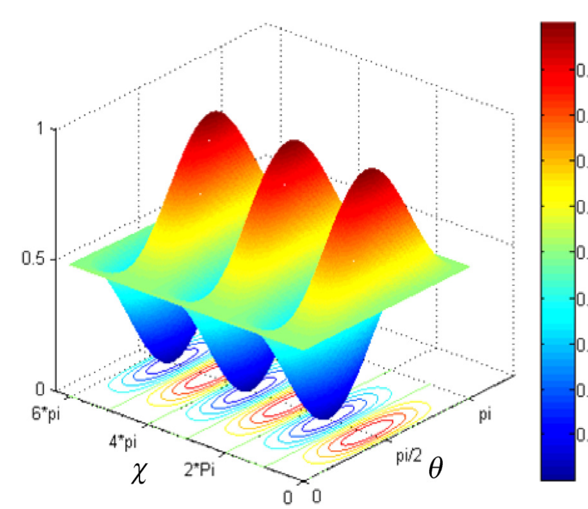

FIG. 4. (Color online) Morphing behavior and the trade-off between particle and wave aspects of the Bragg-diffracted atom.

interactions of thousands of atoms [35]. Experimental Bragg diffraction of atoms from light fields has, by now, also proved its potential beyond any doubt and had been demonstrated up to 8th order with good results through classical as well as quantized fields [10,30,36–40]. Similarly Bragg-regime interferometry is also a standard tool to handle many quantum-informatics tasks [41]. Thus, keeping the cited experimental research scenario in view, we are quite optimistic about the laboratory execution of our proposal. Furthermore, the proposed scheme is inherently deterministic and is expected to yield results with good fidelities. This is because fidelity in cavity-QED-based schemes is mainly affected by the interaction time errors. However, Bragg diffraction being a long interaction time regime with two spatially well separated outputs is generally less perturbed by such errors. Moreover, the interaction time errors can be minimized using atoms delivered from an ensemble of ultracold atoms from magneto-optical traps [10,30,36–38]. Bragg diffraction of ^{85}Rb atoms by Rempe's group at the Max Planck Institute of Quantum Optics using an optical wavelength of 780 nm is experimentally more relevant to the presented scheme, and quite a matching experimental setup for Bragg diffraction of ^{85}Rb atoms utilizing counterpropagating laser beams has already been demonstrated within another context [10]. Therefore just the inclusion of a high-Q cavity, which is available and being employed in their experimental schematics for execution of various tasks [36,42], at an adjustable distance is sufficient to efficiently demonstrate our proposed scheme. The group works with cold Rb atoms with $M = 85$ amu and illuminated with field of wavelength $\lambda = 780$ nm. The corresponding vacuum Rabi coupling and recoil frequency are $\mu = 2\pi \times 16.4$ MHz and $\omega_r = \hbar k^2/2M = 2.4 \times 10^4$ rad s^{-1}, respectively. The high-Q cavity being utilized has a finesse 4.4×10^5 and the detuning usually induced for such experiments is around 1 GHz. These parameters nicely satisfy the criteria for off-resonant first-order Bragg diffraction, i.e., $\delta_c \gg \omega_r$ and $\omega_r + \delta_c \gg \mu\sqrt{n_c}/2$. The desired interaction time therefore comes to be 0.5 μs [36,42] which is much smaller than the cavity lifetime of tens of microseconds. Alternatively, one can also experimentally realize the present proposal using much lighter helium atoms ($M = 4$ amu), at operating parameters $\lambda = 543.5$ nm, $\omega_r = 1.06$ MHz, $\delta_c = 6.28$ GHz and single-photon effective Rabi frequency $\mu^2/4\delta_c = 120$ KHz. The interaction time therefore comes to be about 13 μs. High-Q cavities having finesse $\mathcal{F} = 7.85 \times 10^6$ with lifetimes of a few milliseconds are available in this regime [23,43].

In summary, we have proposed an experimentally executable protocol for the demonstration of QDCE that is very akin and relevant to the extensively debated concept of complementarity lasting for more than a century concerning wave-packet duality of the de Broglie waves. The proposal, as shown in previous section, explicitly elaborates that both particle and wave aspects can be handled in a single QDCE experimental arrangement in striking contrast with Bohr's point of view.

Moreover the presented scheme has many merits over its technical counterparts in other fields including the ones based on photonics, NMR, and atomic internal degrees of freedom. First, it explicitly exhibits the particle and wave aspects of the actual matter waves and hence highlights the modification of the complementarity concept in its true sense. Second, the dynamics of the ground-state atoms in momentum space has been shown to be decoherence resistant ensuring the coherent sustainability of the states over comparatively longer time spans [44]. Third, the state-selective atomic detection is thoroughly deterministic and yields sufficiently good, noise-free results with no dark counts or missing clicks.

ACKNOWLEDGMENT

M. Ikram and M. Imran gratefully acknowledge the support from Higher Education Commission (HEC) of Pakistan.

[1] N. Bohr, Naturwissenschaften **16**, 245 (1928).
[2] A. Whitaker, *Einstein, Bohr, and the Quantum Dilemma* (Cambridge University Press, New York, 1996).
[3] R. J. Glauber, Ann. NY Acad. Sci. **480**, 336 (1986).
[4] W. K. Wootters and W. H. Zurek, Phys. Rev. D **19**, 473 (1979).
[5] N. Bohr, *Essays 1958-1962 on Atomic Physics and Human Knowledge* (Wiley, London, 1963).
[6] D. M. Greenberger and A. Yasin, Phys. Lett. A **128**, 391 (1988).
[7] S. Durr and G. Rempe, Am. J. Phys. **68**, 1021 (2000).
[8] B. G. Englert, Phys. Rev. Lett. **77**, 2154 (1996).
[9] G. Jaeger, A. Shimony, and L. Vaidman, Phys. Rev. A **51**, 54 (1995).
[10] S. Durr, T. Nonn, and G. Rempe, Phys. Rev. Lett. **81**, 5705 (1998); Nature (London) **395**, 33 (1998).
[11] H.-Yu Liu, J.-H. Huang, J.-R. Gao, M. S. Zubairy, and S.-Y. Zhu, Phys. Rev. A **85**, 022106 (2012).
[12] For details please see J. A. Wheeler, in *Mathematical Foundations of Quantum Mechanics*, edited by A. R. Marlow (Academic Press, New York, 1978); J. A. Wheeler, in *Quantum Theory and Measurement*, edited by J. A. Wheeler and W. H. Zurek (Princeton University Press, Princeton, NJ, 1984); A. J. Leggett, in *Compendium of Quantum Physics*, edited by D. Greenberger, K. Hentschel, and F. Weinert (Springer, Berlin, 2009).
[13] T. Hellmuth, H. Walther, A. Zajonc, and W. Schleich, Phys. Rev. A **35**, 2532 (1987); J. Baldzuhn, E. Mohler, and W. Martienssen, Z. Phys. B: Condens. Matter **77**, 347 (1989); V. Jacques, E. Wu, F. Grosshans, F. Treussart, P. Grangier, A. Aspect, and J.-F. Roch, Science **315**, 966 (2007); Phys. Rev. Lett. **100**, 220402 (2008).
[14] R. Ionicioiu and D. R. Terno, Phys. Rev. Lett. **107**, 230406 (2011).
[15] A. Peruzzo, P. Shadbolt, N. Brunner, S. Popescu, and J. L. O'Brien, Science **338**, 634 (2012).
[16] F. Kaiser, T. Coudreau, P. Milman, D. B. Ostrowsky, and S. Tanzilli, Science **338**, 637 (2012).
[17] J.-S. Tang, Y.-L. Li, X.-Y. Xu, G.-Y. Xiang, C.-F. Li, and G.-C. Guo, Nat. Photon. **6**, 600 (2012).
[18] R. Auccaise, R. M. Serra, J. G. Filgueiras, R. S. Sarthour, I. S. Oliveira, and L. C. Celeri, Phys. Rev. A **85**, 032121 (2012).

[19] S. S. Roy, A. Shukla, and T. S. Mahesh, Phys. Rev. A **85**, 022109 (2012).
[20] N. G. de Almeida, A. T. Avelar, and W. B. Cardoso, Phys. Lett. A **378**, 1254 (2014).
[21] A. A. Khan and M. S. Zubairy, Fortschr. Phys. **46**, 417 (1998); S. Qamar, S.-Y. Zhu, and M. S. Zubairy, Phys. Rev. A **67**, 042318 (2003); A. Khalique and F. Saif, Phys. Lett. A **314**, 37 (2003); R. Islam, M. Ikram, R. Ahmed, A. H. Khosa, and F. Saif, J. Mod. Opt. **56**, 875 (2009); F. Saif, R. Islam, and A. H. Khosa, J. Phys. B **43**, 015501 (2010); R. Islam, A. H. Khosa, F. Saif, and J. A. Bergou, Quantum Inf. Process. **12**, 129 (2013); T. Abbas, R. Islam, and M. Ikram, Int. J. Theor. Phys. **54**, 1193 (2015).
[22] A. A. Khan and M. S. Zubairy, Phys. Lett. A **254**, 301 (1999).
[23] A. H. Khosa, M. Ikram, and M. S. Zubairy, Phys. Rev. A **70**, 052312 (2004).
[24] R. Islam, M. Ikram, and F. Saif, J. Phys. B **40**, 1359 (2007).
[25] R. Islam, A. H. Khosa, and F. Saif, J. Phys. B **41**, 035505 (2008).
[26] R. J. Cook and A. F. Bernhardt, Phys. Rev. A **18**, 2533 (1978).
[27] A. F. Bernhardt and B. W. Shore, Phys. Rev. A **23**, 1290 (1981).
[28] M. Marte and S. Stenholm, Appl. Phys. B **54**, 443 (1992).
[29] For mathematical details, please see [22–25] and R. Islam, *Engineering Entanglement in Cavity Quantum Electrodynamical Systems* (Lambert Academic Publishing, Germany, 2010).
[30] D. M. Giltner, R. W. McGowan, and S. A. Lee, Phys. Rev. A **52**, 3966 (1995).
[31] J. Lawall and M. Prentiss, Phys. Rev. Lett. **72**, 993 (1994).
[32] M. O. Scully and M. S. Zubairy, *Quantum Optics* (Cambridge University Press, Cambridge, 1997).
[33] J. M. Raimond, M. Brune, and S. Haroche, Rev. Mod. Phys. **73**, 565 (2001).
[34] S. Gleyzes, S. Kuhr, C. Guerlin, J. Bernu, S. Deleglise, U. B. Hoff, M. Brune, J.-M. Raimond, and S. Haroche, Nature (London) **446**, 297 (2007).
[35] S. Deleglise, I. Dotsenko, C. Sayrin, J. Bernu, M. Brune, J.-M. Raimond, and S. Haroche, Nature (London) **455**, 510 (2008).
[36] S. Durr, S. Kunze, and G. Rempe, Quantum Semiclassical Opt. **8**, 531 (1996).
[37] S. Kunze, S. Durr, and G. Rempe, Europhys. Lett. **34**, 343 (1996).
[38] P. J. Martin, P. L. Gould, B. G. Oldaker, A. H. Miklich, and D. E. Pritchard, Phys. Rev. A **36**, 2495 (1987).
[39] D. W. Vernooy, V. S. Ilchenko, H. Mabuchi, E. W. Streed, and H. J. Kimble, Opt. Lett. **23**, 247 (1998).
[40] C. J. Hood, M. S. Chapman, T. W. Lynn, and H. J. Kimble, Phys. Rev. Lett. **80**, 4157 (1998).
[41] For a comprehensive review, please see A. D. Cronin, J. Schmiedmayer, and D. E. Pritchard, Rev. Mod. Phys. **81**, 1051 (2009); T. Abbas, R. Islam, and M. Ikram, J. Phys. B **47**, 245502 (2014).
[42] P. Münstermann, T. Fischer, P. W. H. Pinkse, and G. Rempe, Opt. Commun. **159**, 63 (1999); P. Münstermann, T. Fischer, P. Maunz, P. W. H. Pinkse, and G. Rempe, Phys. Rev. Lett. **82**, 3791 (1999); T. Puppe, P. Maunz, T. Fischer, P. W. H. Pinkse, and G. Rempe, Phys. Scr. **T112**, 7 (2004).
[43] C. J. Hood, H. J. Kimble, and J. Ye, Phys. Rev. A **64**, 033804 (2001).
[44] P. Ball, Nature (London) **453**, 22 (2008).