Improving the lifetime of the nitrogen-vacancy-center ensemble coupled with a superconducting flux qubit by applying magnetic fields

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One of the promising systems to realize quantum computation is a hybrid system where a superconducting flux qubit plays the role of a quantum processor and the nitrogen-vacancy- (NV^-) center ensemble is used as a quantum memory. We have theoretically and experimentally studied the effect of magnetic fields on this hybrid system, and found that the lifetime of the vacuum Rabi oscillation is improved by applying a few mT magnetic field to the NV^- center ensemble. Here, we construct a theoretical model to reproduce the vacuum Rabi oscillations with and without magnetic fields applied to the NV^- centers, and we determine the reason why magnetic fields can affect the coherent properties of the NV^- center ensemble. From our theoretical analysis, we quantitatively show that the magnetic fields actually suppress the inhomogeneous broadening from the strain in the NV^- centers.

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I. INTRODUCTION

Hybridization is a promising approach for quantum computation [1,2]. Each system has characteristics with its own advantages and disadvantages. To couple different systems, we hope to pick up the advantages of each system. One of the candidates for such hybrid systems is a superconducting circuit such as a superconducting flux qubit (FQ) and an electron spin ensemble such as nitrogen-vacancy (NV^-) centers [3–20], as described in Fig. 1. High controllability of superconducting FQs has already been achieved with existing technology [21]. Reliable gate operations have been already demonstrated [22]. Quantum nondemolition measurements can be performed by a Josephson bifurcation amplifier [21]. However, despite significant effort, the coherence time of the FQ is of the order of 10 μs [22,23]. On the other hand, the NV^- center has a long coherence time [24–30]. With dynamical decoupling, the coherence time of electrons of the NV^- center is 0.6 s [29], which is much longer than the FQ. So, coupling the FQ with the NV^- centers is a promising way to obtain both controllability and long coherence time [3–20].

It is often useful to transfer the state between the FQ and NV^- centers for quantum information processing. We keep quantum states in the quantum memory (NV^- centers) when gate operations are not required. On the other hand, to perform gate operations, we need to transfer the quantum states from the quantum memory to the quantum processor (FQ), which can be realized by using vacuum Rabi oscillation (VRO). However, the error rate of the state transfer in the current technology is an order of 10% [9,10,16], which is too large to perform quantum computation [31,32]. The noise mainly comes from the inhomogeneous broadening of the NV^- centers. Therefore, it is crucial to suppress the decoherence of the NV^- centers for computational tasks.

Improving the coherence time of the NV^- center ensemble also has a fundamental importance in the area of quantum metrology [33,34], quantum walk [33], and quantum simulation [35]. In these applications, the efficiency strongly depends on the coherence time of the ensemble of NV^- centers. So it is essential in these areas to find a way to improve the coherence time of the NV^- centers.

A cavity protection [18,36,37] is a promising way to improve the coherence time of the NV^- center ensemble. If the coupling strength between the ensemble of NV^- centers and a superconducting flux qubit (or microwave cavity) is larger than the inhomogeneous width of the NV^- centers, the collective mode of the NV^- centers could be well decoupled from the other subradiant states of the NV^- so that inhomogeneous broadening would be suppressed. However, it has been shown that, if the spectral density of the inhomogeneous broadening is described by a Lorentzian, the noise cannot be suppressed by the cavity protection [18,30]. Moreover, for the applications of quantum memory and quantum field sensing, it is sometimes necessary to turn off the interaction between the NV centers and superconducting circuit. So it is better to have an alternative scheme that will work even for such cases.

In this paper, we report an improvement of the lifetime of the VRO by applying an in-plne magnetic field to this hybrid system. We have observed VRO with and without the magnetic field, and the lifetime of the VRO with magnetic field is nearly twice that without the magnetic field. We have constructed a theoretical model to reproduce these results, and have found that the magnetic field suppresses the inhomogeneous strain effect.

II. EXPERIMENTAL SETUP

Let us now describe our experimental setup. Our system consists of a gap-tunable FQ [38,39] on which a diamond crystal with a NV^- density of approximately 5×10^{17} cm^{-3} is bonded [16]. The NV^- ensemble is created by the ion

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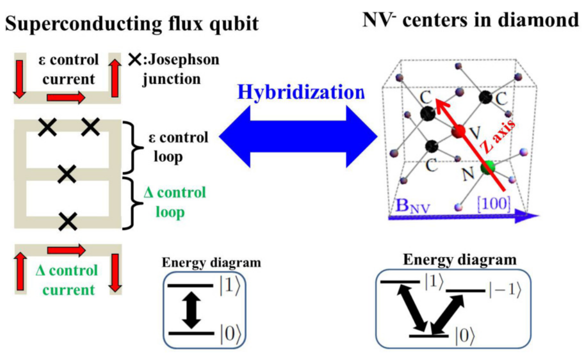

FIG. 1. (Color online) Illustration of the hybrid system composed of a superconducting flux qubit and an ensemble of NV$^-$ centers. The flux qubit has four Josephson junctions that form a two-level system. There are two control lines for the flux qubit. We use one of the control lines to change the energy bias ϵ, and use the other line to change the energy gap Δ. Diamond crystal is glued on top of a flux qubit, and this diamond contains NV$^-$ centers. The electron spins trapped in the NV$^-$ center form a three-level system, and so we have a V-type energy level structure for the NV$^-$ center.

implantation and annealing in vacuum [10,16]. The distance between the FQ and the surface of the diamond crystal is less than 1 μm. We can apply an external magnetic field of 2.6 mT along the [100] crystalline axis [16].

The gap-tunable FQ is fabricated with a superconducting loop containing four Josephson junctions. To control the FQ, a microwave line is fabricated around the FQ. The FQ is designed to couple with the superconducting quantum interference device (SQUID) structure via magnetic fields. The probability of the excited state of the FQ is measured by the SQUID.

To observe the VRO, we perform the following experiment. First, we excited the FQ by applying a microwave pulse where the FQ is decoupled from the NV$^-$ centers by the detuning. Second, we brought the FQ into the resonance of the NV$^-$ centers by applying a magnetic flux. Finally, after a time t, we can measure the excited probability of the FQ via the SQUID. The measurements were done in a dilution refrigerator at a temperature below 50 mK.

III. THEORETICAL MODEL

We describe our system by the Hamiltonian [9,13,18,19]

$$H = H_{\text{flux}} + H_{\text{int}} + H_{\text{ens}}, \quad (1)$$

$$H_{\text{flux}} = \frac{\hbar}{2}\Delta\hat{\sigma}_z + \frac{\hbar}{2}\epsilon\hat{\sigma}_x, \quad (2)$$

$$H_{\text{int}} = \hbar g_e \mu_B \hat{\sigma}_x \left(\sum_{k=1}^{N} \mathbf{B}_{\text{qb}}^{(k)} \cdot \mathbf{S}_k \right), \quad (3)$$

$$H_{\text{ens}} = \left(\sum_{k=1}^{N} \hbar D_k \hat{S}_{z,k}^2 + \hbar E_1^{(k)} (\hat{S}_{x,k}^2 - \hat{S}_{y,k}^2) + \hbar E_2^{(k)} (\hat{S}_{x,k}\hat{S}_{y,k} + \hat{S}_{y,k}\hat{S}_{x,k}) + \hbar g_e \mu_B \mathbf{B}_{\text{NV}}^{(k)} \cdot \mathbf{S}_{\mathbf{k}} \right), \quad (4)$$

where $\hat{\sigma}_{x,y,z}$ denotes the Pauli matrix for FQ with $\hat{\sigma}_x$ whose eigenstates correspond to two persistent-current states. Also, we define $\hat{\sigma}_{\pm} = \frac{1}{2}(\hat{\sigma}_x \pm i\hat{\sigma}_y)$. The electron spin-1 operators of the NV$^-$ center are represented by $\hat{S}_{x,y,z}$. H_{flux} denotes the Hamiltonian of the FQ where Δ denotes the energy gap and ϵ denotes the magnetic energy bias. H_{ens} represents the ensemble composed of N individual NV$^-$ centers where $D/2\pi \simeq 2.878$ GHz denotes a zero-field splitting, $E_k = \sqrt{(E_1^{(k)})^2 + (E_2^{(k)})^2}$ denotes a strain-induced splitting, $g_e\mu_B \mathbf{B}_{\text{NV}} \cdot \mathbf{S}$ denotes a Zeeman splitting, and $\mathbf{B}_{\text{NV}}$ denotes a magnetic field with $g_e\mu_B/2\pi = 28\ \text{MHz}\,\text{mT}^{-1}$. A quantization axis ($z$ axis) to be the direction from the vacancy to the nitrogen is set by the zero-field splitting of the NV$^-$ center. For a small magnetic field $D \gg g_e\mu_B|\mathbf{B}_{\text{NV}}|$, the x and y components of the magnetic field are insignificant to change the quantized axis of the NV$^-$ center, and so we include only the effect of the z axis of the field. We consider three relevant types of the magnetic field $\mathbf{B}_{\text{NV}}$: an in-plane external magnetic field B_{ex}, an inhomogeneous magnetic field due to the $P1$ centers B_{inh}, and a hyperfine field from the nitrogen nuclear spins B_{HF}. The term H_{int} denotes the magnetic coupling between the FQ and the NV$^-$ centers where $\mathbf{B}_{\text{qb}}^{(k)}$ represented the magnetic field induced by the persistent current of the FQ. Since collective enhancement of the coupling strength between the NV$^-$ and FQ is not available along the z axis of the NV$^-$ center [13], we can ignore the coupling with $\hat{S}_z$. We can write H_{int} as $H_{\text{int}} = \hbar \sum_{k=1}^{N} g\hat{\sigma}_x \cdot (\hat{S}_{x,k}\cos\phi_k - \hat{S}_{y,k}\sin\phi_k)$ where $g = g_e\mu_B B_{\text{qb}}^{(xy)}$ denotes a Zeeman splitting of the NV$^-$ spin due to FQ magnetic field $B_{\text{qb}}^{(xy)}$ in the x-y plane and ϕ_k denotes the angle of the field in the plane.

For an ensemble of the NV$^-$ center with only a few excitations in it at most, we can use the Holstein-Primakoff approximation to treat NV$^-$ spins as an ensemble of harmonic oscillators [40]. We define creation (destruction) operators of a bright state and a dark state of the NV center [19] by $\hat{b}_k^\dagger = |A_+\rangle_k\langle 0|$ and $\hat{d}_k^\dagger = |A_-\rangle_k\langle 0|$, ($\hat{b}_k = |0\rangle_k\langle A_+|$ and $\hat{d}_k = |0\rangle_k\langle A_-|$) where $|A_+\rangle_k = \frac{\cos\phi_k}{\sqrt{2}}(|1\rangle_k + |-1\rangle_k) + \frac{i\sin\phi_k}{\sqrt{2}}(|1\rangle_k - |-1\rangle_k)$, $|A_-\rangle_k = \frac{i\sin\phi_k}{\sqrt{2}}(|1\rangle_k + |-1\rangle_k) + \frac{\cos\phi_k}{\sqrt{2}}(|1\rangle_k - |-1\rangle_k)$. Here, $|A_+\rangle_k$ denotes the state of the NV$^-$ center to be directly coupled with the FQ while $|A_-\rangle_k$ has no direct coupling with the FQ. Since we consider only one or zero excitation in a total system, we can also replace a ladder operator of the FQ with a creation operator of a harmonic oscillator as $\hat{\sigma}_+ \to \hat{c}^\dagger$.

Moving to a rotating frame with angular frequency ω defined by $U = e^{-i[(1/2)\omega\hat{\sigma}_z + \omega\hat{S}_z^2]t}$ and making the rotating wave approximation, we obtain the simplified Hamiltonian $H \simeq \hbar\omega_c\hat{c}^\dagger\hat{c} + \sum_{k=1}^{N}[\hbar\omega_b^{(k)}\hat{b}_k^\dagger\hat{b}_k + \hbar\omega_d^{(k)}\hat{d}_k^\dagger\hat{d}_k + \hbar g'(\hat{c}^\dagger\hat{b}_k + \hat{c}\hat{b}_k^\dagger) + \hbar(J_k + iJ'_k)\hat{b}_k^\dagger\hat{d}_k + \hbar(J_k - iJ'_k)\hat{b}_k\hat{d}_k^\dagger]$ where $\omega_c = \sqrt{\epsilon^2 + \Delta^2}$, $\omega_b^{(k)} \simeq D_k - \omega - E_{y,k}$, $\omega_d^{(k)} \simeq D_k - \omega + E_{y,k}$, $J_k = g_e\mu_B B_z^{(k)}$, $J'_k = E_{x,k}$, $g' = g\Delta/\sqrt{\epsilon^2 + \Delta^2}$, $E_{x,k} = E_k\cos 2\phi_k$, and $E_{y,k} = E_k\sin 2\phi_k$.

IV. MAIN RESULTS

Now the dynamics of this hybrid system can be investigated using the Heisenberg equations of motion. We write

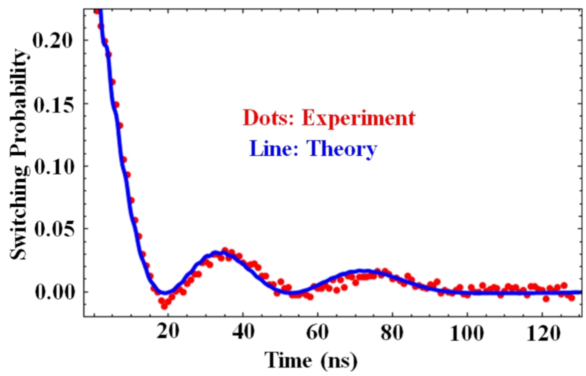

FIG. 2. (Color online) Vacuum Rabi oscillations between the FQ and NV$^-$ centers without an applied external magnetic field. Red dots show the experimental results, while the blue line shows the results of a numerical model where we use $N = 1200$, $\epsilon = 0$, $\Gamma_c/2\pi = 0.3$ MHz, $\Gamma_b/2\pi = \Gamma_d/2\pi = 0.44$ MHz, $\delta D_k/2\pi = 0.08$ MHz (FWHM), $\delta(g\mu_B B_z^{(k)})/2\pi = 3.1$ MHz (FWHM), $\delta E_{1,k}/2\pi = \delta E_{2,k}/2\pi = 4.4$ MHz (FWHM), $A_{\rm HF} = 2.3$ MHz, and $\sqrt{N}g/2\pi = 13$ MHz.

Heisenberg equations of motion as

$$\frac{d}{dt}\hat{c} = -i\omega_c\hat{c} - i\left(\sum_{k=1}^{N} g_k \sin\xi \cdot \hat{b}_k\right) - \Gamma_c\hat{c}, \quad (5)$$

$$\frac{d}{dt}\hat{b}_k = -i\omega_b^{(k)}\hat{b}_k - iJ_k\hat{d}_k + J_k'\hat{d}_k - ig\sin\xi_k \cdot \hat{c} - \Gamma_b\hat{b}_k, \quad (6)$$

$$\frac{d}{dt}\hat{d}_k = -i\omega_d^{(k)}\hat{d}_k - iJ_k\hat{b}_k - J_k'\hat{b}_k - \Gamma_d\hat{d}_k, \quad (7)$$

where Γ_c, Γ_b, and Γ_d denote the decay rate of $\hat{c}$, $\hat{b}$, and $\hat{d}$, respectively. We numerically solve these equations with an initial state of $|\psi(t=0)\rangle = \hat{c}^\dagger|\text{vac}\rangle$, and plot the renormalized excitation probability of the FQ (which corresponds to a switching probability of SQUID) in Figs. 2 and 3.

Now let us detail our simulation technique and the core elements of it. The Lorentzian distributions, which have been typically used to describe the inhomogeneous broadening of the NV$^-$ centers [9,10,12,19], are assumed for $D_0^{(k)}$, $E_{1,k}$, and $E_{2,k} (k = 1,2,\ldots,N)$ to include the effect of the inhomogeneous lattice distortion of the NV$^-$ centers. Next, due to the electron spin-half bath in the environment such as the $P1$ center, a randomized magnetic field on the NV center exists, and the nitrogen nuclear spin splits the electron-spin energy into three levels via a hyperfine coupling. To include both these effects, we use a random distribution of the magnetic fields with the form of the mixture of three Lorentzian functions that are separated with $2\pi \times 2.3$ MHz due to the hyperfine interaction with ^{14}N nuclear spin [9,16]. With these assumptions, we have reproduced the VRO with and without the applied magnetic field in Figs. 2 and 3. While the FQ can induce both transitions $|0\rangle_k \leftrightarrow |-1\rangle_k$ and $|0\rangle_k \leftrightarrow |1\rangle_k$ with zero applied magnetic field, the FQ induces only one of them with an applied magnetic field of a few mT due to the detuning effect [7,10,13,16,18]. This changes the effective coupling strength between the FQ and NV centers, and is the cause of the different time interval of the oscillations in Figs. 2 and 3. With an applied magnetic field of 2.6 mT, the VRO can be observed until around 170 ns, while we cannot observe a clear oscillation beyond 100 ns without an applied magnetic field. Thus, it shows the external magnetic field improves the lifetime of the VRO.

Next we explain how we determine the parameters for the numerical simulations. Γ_c can be determined by the T_1 measurement of the FQ, which was performed independent of the VRO experiment [16]. Since the frequency shift of E is 50 times larger than that of D when an electric field is applied [41], we use $\delta E \simeq 50\delta D$ in this paper. It is known that, from the spectroscopic measurements, a sharp peak located in the middle of the avoided crossing structure was observed in this hybrid system [19], and one can determine the $\Gamma_b (= \Gamma_d)$ from the width of this sharp peak [19]. The time period of the VRO let us specify the value of $\sqrt{N}g$. Moreover, as we describe later, the envelope of the VRO with (without) a magnetic field is mainly determined by δB_z (δB_z and δE). So, by fitting the spectroscopy and VRO with and without a magnetic field, one can specify the necessary parameters for our model.

We understand why the applied magnetic field actually improves the coherence time of the NV$^-$ centers. There are two relevant decoherence sources for the NV centers, inhomogeneous magnetic fields, and the strain distribution [9,10,16]. The magnetic-field noise comes from an effective Hamiltonian between the $P1$ center and the NV center as $H_{\rm eff} = \sum_{k,l}\lambda_{k,l}\hat{\sigma}_z^{(P1,l)}\hat{S}_z^{(k)}$ where a flip-flop term is negligible due to the large energy difference between them. However, this term commutes with the Zeeman term of the applied external magnetic field as $g_e\mu_B B_{\rm ex}\hat{S}_z$, and so an external magnetic field cannot affect this. On the other hand, the Hamiltonian of the inhomogeneous strain does not commute with the Zeeman Hamiltonian of the applied external magnetic field. Actually, the Hamiltonian of the kth NV$^-$ center is written as $H_{\rm NV}^{(k)} = \hbar\omega_b^{(k)}\hat{b}_k^\dagger\hat{b}_k + \hbar\omega_d^{(k)}\hat{d}_k^\dagger\hat{d}_k + [(J_k + iJ_k')\hat{b}_k^\dagger\hat{d}_k + \text{e.c}]$, and the eigenenergies are $E_0 = 0$ and $E_\pm/\hbar = D \pm \sqrt{E_{x,k}^2 + E_{y,k}^2 + (g_e\mu_B B_z^{(k)})^2}$. If the magnetic field is large, we can expand the eigenenergies of the excited states as $E_\pm/\hbar \simeq g_e\mu_B B_z^{(k)} \pm \frac{E_{x,k}^2 + E_{y,k}^2}{2g_e\mu_B B_z^{(k)}}$. So the effect of the

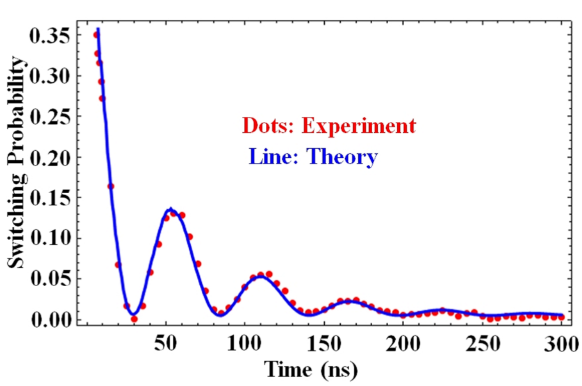

FIG. 3. (Color online) Vacuum Rabi oscillations between the FQ and NV$^-$ centers with an applied external magnetic field of 2.6 mT along the [100] crystalline axis. Except for the magnetic field, we use the same parameters as those in Fig. 2.

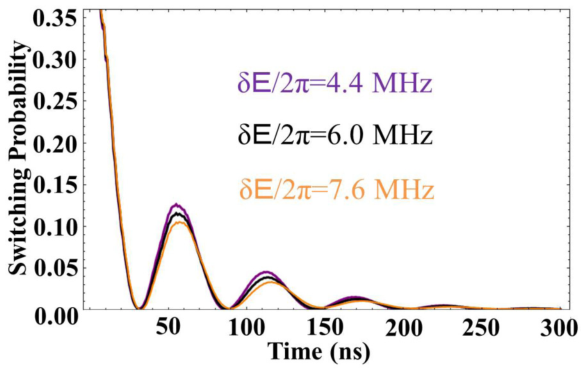

FIG. 4. (Color online) Numerical simulations of vacuum Rabi oscillations with the applied magnetic field for several values of the strains. Except for the values of the strains, we use the same parameters as those in Fig. 3.

variations of $E_{x,k}$ and $E_{y,k}$ become negligible, and this could improve the lifetime of the VRO [42].

To confirm this effect, we performed another numerical simulation of the VRO with an applied magnetic field for the strain values of $\delta E/2\pi = 4.4,6.0,7.6$ MHz in Fig. 4. Interestingly, the three VROs shown in Fig. 4 are almost the same. These results clearly show that the lifetime of the VRO is quite robust against the inhomogeneous strains. So we conclude that the external magnetic field suppress the effect of inhomogeneous strain so that the improvement of the lifetime has been observed in our experiment. It is worth mentioning that, although possible suppression of the strain distribution by the applied magnetic field is mentioned in [43], we first demonstrate such a mechanism by the experiment.

It is known that applying a transversal magnetic field can improve the coherence time of the NV$^-$ centers when the strain distribution is much larger than the decoherence rate due to the randomized magnetic fields [44]. Here, the application of the transversal magnetic field can suppress the decoherence due to the environmental magnetic field, while this cannot suppress the inhomogeneous broadening of the strain [44]. On the other hand, our scheme to apply a horizontal magnetic field is complemental to this, because we can suppress the strain inhomogeneous broadening. Our scheme has an advantage especially when we can decrease the broadening due to the magnetic field by using another technique.

Actually, it is possible to combine our scheme with another technique to reduce the inhomogeneous magnetic fields. For example, one way to suppress the magnetic noise for the NV$^-$ centers is to decrease the $P1$ centers (a nitrogen atom substituting a carbon atom) by using differently synthesized diamond crystals [16]. $P1$ centers are considered as an electron spin-half bath, and they cause randomized magnetic fields to decohere the NV$^-$ centers [9,10]. However, reduction of the $P1$ centers cannot suppress the noise due to the strain. Therefore, by applying a magnetic field with a diamond having less $P1$ centers, further improvement of the coherence time should be possible. This provides us with a sensitive diamond-based field sensor, and will be useful for a long-lived quantum memory during quantum computation in future applications.

V. CONCLUSION

In conclusion, we have observed an improvement of a lifetime of the vacuum Rabi oscillations between the FQ and NV centers by applying an in-plne magnetic field. By reproducing the experimental result from a theoretical model, we have found that the applied magnetic field can suppress the inhomogeneous broadening of the strain. This result is a relevant step toward the realization of the long-lived quantum memory for a superconducting flux qubit.

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