

# Proliferation of effective interactions: Decoherence-induced equilibration in a closed many-body system

Pablo R. Zangara, Denise Bendersky, and Horacio M. Pastawski\*

*Instituto de Física Enrique Gaviola (IFEG), CONICET-UNC, and Facultad de Matemática, Astronomía y Física, Universidad Nacional de Córdoba, 5000, Córdoba, Argentina*

(Received 3 January 2015; revised manuscript received 22 February 2015; published 13 April 2015)

We address the question of how weak perturbations, which are quite ineffective in small many-body systems, can lead to decoherence and hence to irreversibility when they proliferate as the system size increases. This question is at the heart of solid-state NMR. There, an initially local polarization spreads all over due to spin-spin interactions that conserve the total spin projection, leading to an equilibration of the polarization. In principle, this quantum dynamics can be reversed by changing the sign of the Hamiltonian. However, the reversal is usually perturbed by nonreversible interactions that act as a decoherence source. The fraction of the local excitation recovered defines the Loschmidt echo (LE), here evaluated in a series of closed  $N$  spin systems with all-to-all interactions. The most remarkable regime of the LE decay occurs when the perturbation induces proliferated effective interactions. We show that if this perturbation exceeds some lower bound, the decay is ruled by an effective Fermi golden rule (FGR). Such a lower bound shrinks as  $N$  increases, becoming the leading mechanism for LE decay in the thermodynamic limit. Once the polarization stayed equilibrated longer than the FGR time, it remains equilibrated in spite of the reversal procedure.

DOI: [10.1103/PhysRevA.91.042112](https://doi.org/10.1103/PhysRevA.91.042112)

PACS number(s): 03.65.Yz, 05.70.Ln, 75.10.Jm, 74.25.nj

## I. INTRODUCTION

Within the usual wisdom it is quite intuitive to accept that a complex many-body dynamics could lead to a homogeneous spreading of an initially localized excitation. Such a process, which in the context of spin systems has long been known as *spin diffusion*, would lead to system equilibration. However, this naïve concept soon encounters limitations. On one hand, Anderson discovered that certain conditions preclude the spreading [1]. This problem still generates controversy, as the question of when closed many-body quantum systems equilibrate remains open [2–5]. On the other hand, even in conditions where the spin diffusion seems irreversible, nuclear magnetic resonance (NMR) experiments revealed that the apparently equilibrated state contains correlations encoding a memory of the initial state [6]. The pioneer in this field was Hahn. In his spin echo [7], the precession dynamics of each independent spin is reversed by changing the sign of the local magnetic fields. In those experiments, the many-spin interaction is not reversed and consistently it degrades the echo signal in a characteristic time  $T_2$ . Two decades later, Rhim, Pines, and Waugh exploited the fact that the spin-spin dipolar interaction can also be reversed [8]. This allows for the reversal of a global polarization state in the form of a “magic echo.” More specific was the development by Ernst and collaborators. There, a local spin excitation diffuses through a lattice much as an ink drop diffuses in a pond. A pulse sequence produces its refocusing, followed by the local detection as a polarization echo [9]. Again, the attenuation of the observed echo can be tentatively attributed to the noninverted terms in the Hamiltonian, as well as imperfections in the pulse sequence and interactions with some environment.

In a followup to those experiments, a quest to quantify the sources that degrade the echo signal in crystalline samples

was initiated [10–12]. As the sources of irreversibility can be progressively reduced, one might think that there are no limits in the experimental improvement of the echo. Nevertheless, in systems where a local excitation equilibrates [13,14], experiments show that even weak perturbations are highly effective in producing the echo degradation. Moreover, there are cases where the time scale of the decay is intrinsic to the reversed dynamics, i.e., a perturbation-independent decay (PID) [11,12]. If confirmed, this observation could have deep implications for the degree of controllability of quantum devices as it evidences a fragility of quantum dynamics towards minuscule perturbations. In fact, the sensitivity to perturbations or fragility of quantum systems [15–17] is a major problem that transversally affects several fields, e.g., chaos in quantum computers [18,19], NMR quantum information processing [20–22], quantum criticality [23,24], and, more recently, quantum control theory [25].

In order to capture the essentials of the described experiments, the Loschmidt echo (LE) is defined as the revival that occurs when an imperfect time-reversal procedure is implemented [26–28]. If the unperturbed evolution is given by a classically chaotic Hamiltonian, there exists a regime in which the decay rate of the LE corresponds to the classical Lyapunov exponent [29,30]. Such a particular PID holds for a semiclassical initial state built from a dense spectrum and a perturbation above certain threshold. Under weaker perturbations, the LE decay depends on their strength following a Fermi golden rule (FGR) [31]. Additionally, the LE semiclassical expansion showed that the PID regime results from the phase fluctuations along the unperturbed classical trajectories [29]. This represents a first identification of *irreversibility*, as measured by the LE, with *decoherence*.

In addressing actual many-spin dynamics, the situation is less clear. On one side, there is no classical Hamiltonian that serves as reference. On the other side, the numerical evaluation of the LE in a weakly perturbed *finite* spin system

\*horacio@famaf.unc.edu.ar

could not justify the experimental observations [10]. Since experiments involve almost *infinitely* large systems, one is left with the question of whether the mechanisms of LE decay and system equilibration could eventually emerge from a progressive increase in the system size towards the thermodynamic limit (TL). Here, we tackle such a question by considering extensive calculations with  $N = 12, 14$ , and 16 interacting spins, whose dynamics involve the complete  $2^N$ -dimensional Hilbert space [32]. We adopt a model with all-to-all dipolar interactions, which allows for small statistical fluctuations facilitating the analysis of the TL. As in the case of the polarization echo [9,10], our initial state is given by a local excitation in a single spin, and the detection is also a local measurement of the polarization [13,33,34].

We show that, in the presence of a small Hamiltonian perturbation, the decay of the LE follows a FGR, much as if the system were interacting with a continuum. This indicates that the system itself would indeed behave as its own environment. In addition, we observe that the excitation remains homogeneously distributed in spite of the time reversal. In other words, the equilibration produced by the unperturbed Hamiltonian becomes fully irreversible in the presence of an arbitrary small perturbation. The physical mechanism responsible for the mentioned FGR corresponds to a *proliferation* of two- and four-body effective interactions mediated by virtual processes. Remarkably, we show that the realm of this description is wider as the system size increases. Such an observation hints that, in the TL, the proliferation of effective interactions is the mechanism that rules irreversibility.

The paper is organized as follows. In Sec. II we describe the many-spin model employed to simulate an ideal NMR experiment. Section III A encloses the LE formulation as an autocorrelation function. In Sec. III B we discuss the standard FGR description of the LE. In Sec. III C we introduce the effective interactions and we use them to evaluate an effective FGR. In Sec. IV we show the results obtained for the numerical evaluation of the LE time dependence, including the time scales and the asymptotic behavior as a function of the perturbation strength and the system size. Concluding remarks are made in Sec. V.

## II. SPIN MODEL FOR MANY-BODY DYNAMICS

As in the experimental systems, we consider  $N$  spin-1/2 particles, whose state at  $t = 0$  is given by the density matrix

$$\hat{\rho}_0 = \frac{1}{2^N} (\hat{\mathbf{1}} + 2\hat{S}_1^z). \quad (1)$$

Here,  $\hat{\rho}_0$  stands for a local excitation as  $\text{tr}[\hat{S}_1^z \hat{\rho}_0] = \frac{1}{2}$  and  $\text{tr}[\hat{S}_i^z \hat{\rho}_0] = 0 \ \forall i \neq 1$ . The initial polarization is oriented along the laboratory frame, where the overwhelming Zeeman field of a superconducting magnet splits the states according to their total spin projection. Thus, even though the spins would interact through the complete dipole-dipole interaction, the evolution is ruled by the truncated dipolar

Hamiltonian [35],

$$\begin{aligned} \hat{H}_{\text{dip}} &= \sum_{i,j}^N J_{ij}^{\text{dip}}(N) [2\hat{S}_i^z \hat{S}_j^z - (\hat{S}_i^x \hat{S}_j^x + \hat{S}_i^y \hat{S}_j^y)] \quad (2) \\ &= \sum_{i,j}^N J_{ij}^{\text{dip}}(N) \left[ 2\hat{S}_i^z \hat{S}_j^z - \frac{1}{2}(\hat{S}_i^+ \hat{S}_j^- + \hat{S}_i^- \hat{S}_j^+) \right]. \quad (3) \end{aligned}$$

This interaction conserves spin projection and is called *secular*. Indeed, the symmetry  $[\hat{H}_{\text{dip}}, \sum_{i=1}^N \hat{S}_i^z] = 0$  provides the relevant structure of subspaces given by specific  $z$  projections:  $\nu = \sum_{i=1}^N S_i^z = \frac{N}{2}, (\frac{N}{2} - 1), \dots, -\frac{N}{2}$ . In a system of  $N$  spins, there are  $N + 1$  subspaces of definite  $\nu$ , and therefore dynamics induced by  $\hat{H}_{\text{dip}}$  would be strictly confined to each of them.

We choose the coupling strength  $J_{ij}^{\text{dip}}(N)$  corresponding to an infinite range or all-to-all interaction model,

$$J_{ij}^{\text{dip}}(N) = J_{ji}^{\text{dip}}(N) = (1 + \chi)(-1)^k \frac{J_0}{\sqrt{N}}. \quad (4)$$

Here  $\chi$  is a random number taken from a uniform distribution in  $[-0.1, 0.1]$  that ensures the lifting of degeneracies while keeping the fluctuations of the second moment small. Since the sign of the dipolar interactions in a crystal depends on the spatial orientation of the interspin vector, we take  $k$  as a random number from a binary distribution  $\{0, 1\}$ . The price to be paid for an all-to-all network is the absence of the dynamically hierarchical structure of the experimental systems.

The factor  $1/\sqrt{N}$  ensures that the *local* second moment of the dipolar interaction  $\sigma_{\text{dip}}^2$  remains constant as  $N$  changes:

$$\sigma_{\text{dip}}^2 \simeq \sigma_i^2 = \sum_{j(\neq i)}^N \left( \frac{J_{ij}^{\text{dip}}(N)}{2} \right)^2 \simeq \frac{J_0^2}{4}. \quad (5)$$

Therefore, in spite of different cluster sizes,  $\hbar/\sqrt{\sigma_{\text{dip}}^2}$  recovers the characteristic spin-spin interaction time  $T_2$  and consequently  $J_0$  provides the natural energy unit.

A forward evolution ruled by many-body interactions according to  $\hat{H}_{\text{dip}}$  can be experimentally reversed by an appropriate pulse sequence, as reported in Ref. [12]. In order to perform the inversion  $\hat{H}_{\text{dip}} \rightarrow -\hat{H}_{\text{dip}}$ , the full spin state has to be tumbled down along the direction of a radio frequency (rf) field that is turned on immediately afterwards. The rf field rotates perpendicularly to the field of the superconducting magnet and hence it provides the rotating frame. We redefine the  $z$  direction in such a frame, and thus the rf irradiation yields a Zeeman Hamiltonian:

$$\hat{H}_Z = \sum_{i=1}^N \hbar\omega_1 \hat{S}_i^z. \quad (6)$$

Notice that  $\hat{H}_Z$  creates finite-energy gaps of magnitude  $\hbar\omega_1$  which separate the subspaces, but they are not as effective as the ‘‘infinite’’ splittings generated by the magnet (laboratory frame). As a consequence, the Hamiltonian terms that do not conserve polarization, called *nonsecular*, become relevant. The sign of the nonsecular contribution cannot be changed experimentally. Then, they constitute the perturbation  $\hat{\Sigma}$ , here embodied by a double quantum (DQ)

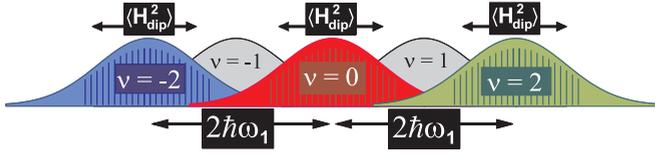


FIG. 1. (Color online) Pictorial representation of the LDOS of  $\hat{H}_{\text{dip}} + \hat{H}_Z$  respective to the state given in Eq. (1).

Hamiltonian

$$\hat{\Sigma} = \hat{H}_{dq} = \sum_{i,j}^N J_{ij}^{dq}(N) [\hat{S}_i^x \hat{S}_j^x - \hat{S}_i^y \hat{S}_j^y] \quad (7)$$

$$= \sum_{i,j}^N J_{ij}^{dq}(N) [\hat{S}_i^+ \hat{S}_j^+ + \hat{S}_i^- \hat{S}_j^-]. \quad (8)$$

Here, the coupling strength  $J_{ij}^{dq}(N)$  satisfies an analog definition as in Eq. (4). Notice that  $[\hat{H}_{dq}, \sum_{i=1}^N \hat{S}_i^z] \neq 0$  since  $\hat{H}_{dq}$  mixes subspaces whose projections differ in  $\delta v = \pm 2$  [36,37]. Experimentally, these intersubspace transitions are partially suppressed by increasing the rf power, i.e.,  $\hbar\omega_1$  [11,38].

The information contained in the time domain, embodied in the experimental  $T_2$  time scale, can be complemented with the spectral picture given by the local density of states (LDOS). This last shows how a particular state distributes among the eigenstates of a given Hamiltonian. There is extensive recent literature recognizing the LDOS as an indicator for the onset of chaos [39,40], relaxation time scales [41–43], and the size of the fluctuations around the steady state [44]. Even though our evaluation of the dynamics does not rely on diagonalization [32], one can infer the shape of the unperturbed LDOS ( $\hat{H}_{\text{dip}} + \hat{H}_Z$ ) respective to the initial state defined in Eq. (1). See Fig. 1. When  $\hbar\omega_1 = 0$ , the subspaces of spin projection are basically degenerate, and the unperturbed LDOS is a single Gaussian of width  $\langle H_{\text{dip}}^2 \rangle = \sum_i \sigma_i^2 / 4 \simeq N \sigma_{\text{dip}}^2 / 4$ , i.e., the *global* second moment of  $\hat{H}_{\text{dip}}$ . If  $\hbar\omega_1 \gtrsim \sqrt{\langle H_{\text{dip}}^2 \rangle}$ , the subspaces' LDOS separate from each other. A subspace with spin projection  $v$  has mean energy  $E_v \simeq v \hbar\omega_1$ , and therefore the unperturbed LDOS within each subspace is

$$P_v(\varepsilon) \simeq \frac{1}{\sqrt{2\pi \langle H_{\text{dip}}^2 \rangle}} \exp \left[ -\frac{(\varepsilon - E_v)^2}{2 \langle H_{\text{dip}}^2 \rangle} \right]. \quad (9)$$

The time domain can be explicitly recovered from the Fourier transform of  $P_v(\varepsilon)$  [43].

### III. THE LOSCHMIDT ECHO

#### A. The autocorrelation function

In order to simulate an ideal LE procedure, we assume that forward evolution occurs under the unperturbed Hamiltonian  $\hat{H}_0 = \hat{H}_{\text{dip}} + \hat{H}_Z$ . Even though this evolution would correspond to the laboratory frame, the addition of the  $\hat{H}_Z$  term stands for sake of a symmetrical time reversal. Besides, as  $[\hat{H}_{\text{dip}}, \hat{H}_Z] = 0$ , the inclusion of  $\hat{H}_Z$  does not introduce any nontrivial dynamics. At time  $t_R$ , a pulse sequence changes the sign of  $\hat{H}_{\text{dip}}$ , and a *backward* evolution occurs, affected by

the perturbation. Hence, the backward dynamics is described by  $-\hat{H}_0 + \hat{\Sigma} = -\hat{H}_{\text{dip}} - \hat{H}_Z + \hat{H}_{dq}$ , which in the experiment would correspond to the rotating frame. The evolution operators for each  $t_R$  period are  $\hat{U}_+(t_R) = \exp[-\frac{i}{\hbar} \hat{H}_0 t_R]$  and  $\hat{U}_-(t_R) = \exp[-\frac{i}{\hbar} (-\hat{H}_0 + \hat{\Sigma}) t_R]$  respectively. Then, the LE operator,

$$\hat{U}_{\text{LE}}(2t_R) = \hat{U}_-(t_R) \hat{U}_+(t_R), \quad (10)$$

produces an imperfect refocusing at time  $2t_R$  evaluated as

$$M_{1,1}(t = 2t_R) = 2tr[\hat{S}_1^z \hat{U}_{\text{LE}}(t) \hat{\rho}_0 \hat{U}_{\text{LE}}^\dagger(t)]. \quad (11)$$

Since  $\hat{S}_1^z$  is a local (“one-body”) operator, Eq. (11) is equivalent to the expectation value in a single superposition state [45],

$$M_{1,1}(t) = 2 \langle \Psi_{\text{neq}} | \hat{U}_{\text{LE}}^\dagger(t) \hat{S}_1^z \hat{U}_{\text{LE}}(t) | \Psi_{\text{neq}} \rangle, \quad (12)$$

where

$$|\Psi_{\text{neq}}\rangle = |\uparrow_1\rangle \otimes \sum_{r=1}^{2^{N-1}} \frac{1}{\sqrt{2^{N-1}}} e^{i\varphi_r} |\xi_r\rangle. \quad (13)$$

Here,  $\varphi_r$  is a random phase uniformly distributed in  $[0, 2\pi)$ , and  $\{|\xi_r\rangle\}$  stands for the computational basis states of the Hilbert space corresponding to  $N - 1$  spins.

#### B. The standard Fermi golden rule approach

Let us now introduce the regimes of the LE decay, following the dynamical paradigm from Refs. [29,31,46]. If the perturbation during the backward evolution is extremely small, the short time expansion of the LE operator yields a quadratic decay that extends until recurrences show up. This constitutes the perturbative regime

$$\begin{aligned} M_{1,1}(t) &= 2 \langle \Psi_{\text{neq}} | \hat{U}_{\text{LE}}^\dagger(t) \hat{S}_1^z \hat{U}_{\text{LE}}(t) | \Psi_{\text{neq}} \rangle \\ &\simeq 1 - \frac{1}{4} \langle \Psi_{\text{neq}} | [\hat{\Sigma}^2 - 2\hat{\Sigma} \hat{S}_1^z \hat{\Sigma}] | \Psi_{\text{neq}} \rangle t^2 \\ &\simeq 1 - [t/\tau_\phi]^2. \end{aligned} \quad (14)$$

Here,  $1/\tau_\phi$  scales up linearly with the strength of the perturbation through its local second moment. In fact,  $\Sigma_{\alpha\beta}$ , i.e., the matrix elements of  $\hat{\Sigma}$ , do not exceed the level spacing  $d_{\alpha\beta}$  associated with two directly connected states (DCS)  $\alpha$  and  $\beta$ .

As the perturbation increases, its second moment exceeds the typical level spacing among DCS. There, the onset of the FGR occurs. In such a case, the perturbed energies  $\tilde{E}_\alpha$  are obtained from the unperturbed  $E_\alpha$ , using second-order perturbative series, which must be evaluated in the TL:

$$\tilde{E}_\alpha \simeq E_\alpha + \lim_{\eta \rightarrow 0^+} \lim_{N \rightarrow \infty} \sum_{\beta} \frac{|\Sigma_{\alpha\beta}|^2}{d_{\alpha\beta} + i\eta} = E_\alpha + \Delta_\alpha - i\Gamma_\alpha,$$

where the real shift  $\Delta_\alpha$  and the imaginary correction  $\Gamma_\alpha$  are defined as

$$\Delta_\alpha = \mathcal{P} \sum_{\beta} \frac{|\Sigma_{\alpha\beta}|^2}{d_{\alpha\beta}}, \quad (15)$$

$$\Gamma_\alpha = 2\pi \sum_{\beta} |\Sigma_{\alpha\beta}|^2 \delta(E_\beta - E_\alpha). \quad (16)$$

Here  $\mathcal{P}$  stands for principal value. In most practical cases,  $\Delta_\alpha$  provides a small energy shift that can be neglected. Notice that the decay introduced by  $\Gamma_\alpha$  requires the *mixing of infinitely many quasidegenerate states*. Additionally,  $\Gamma_\alpha$  can be replaced by its local energy average,

$$\langle \Gamma \rangle = 2\pi \langle \Sigma^2 \rangle / d. \quad (17)$$

Here,  $d$  stands for the mean level spacing among the DCS. Therefore, within the standard FGR approximations, a single LE operator already contains a decay,

$$\begin{aligned} \hat{U}_{\text{LE}}(t) &\simeq \sum_{\alpha} e^{i\tilde{E}_\alpha t/\hbar} e^{-iE_\alpha t/\hbar} |\alpha\rangle \langle \alpha| \simeq \\ &\simeq \sum_{\alpha} e^{-\Gamma_\alpha t/\hbar} |\alpha\rangle \langle \alpha| \simeq e^{-(\Gamma)t/\hbar} \hat{\mathbf{I}}. \end{aligned} \quad (18)$$

This constitutes the standard random matrix theory (RMT) approach to the LE [31,46].

### C. From virtual interactions to an effective Fermi golden rule

As pointed above, the nonsecular DQ perturbation  $\hat{\Sigma}$  only mixes states from different Zeeman subspaces. Then, the previous FGR requirement of mixing quasidegenerate states is not fulfilled. Nevertheless, as hinted by the experiments [11], the DQ interaction could produce effective secular terms of major relevance in the TL. We now formalize these ideas, showing how a small nonsecular perturbation can connect quasidegenerate states through virtual processes.

Given a specific total spin projection  $\nu$ , its corresponding subspace  $\mathcal{S}_\nu$  is coupled to the subspaces  $\mathcal{S}_{\nu+2}$  and  $\mathcal{S}_{\nu-2}$  by the DQ interaction. In other words, the perturbation  $\hat{\Sigma}$  produces transitions with  $\delta\nu = \pm 2$  that involve an energy difference of  $2\hbar\omega_1$ . However, there are higher order transitions that avoid the energy mismatch. For instance, when state  $|\uparrow\downarrow\downarrow\rangle$  swaps to  $|\uparrow\uparrow\uparrow\rangle$  and then to  $|\downarrow\downarrow\uparrow\rangle$  (back to the initial subspace), one gets an effective flip flop between spins 1 and 3. This constitutes an intrasubspace effective coupling of order  $(J^{dq})^2/(\hbar\omega_1)$ . A more sophisticated process occurs when  $|\uparrow\uparrow\downarrow\downarrow\rangle$  swaps to  $|\uparrow\uparrow\uparrow\uparrow\rangle$  and then back to  $|\downarrow\downarrow\uparrow\uparrow\rangle$ . It provides for a four-body effective interaction. Therefore, if the energy gaps  $\hbar\omega_1$  are large enough, *intersubspace* transitions are in fact truncated, but then *intrasubspace* transitions mediated by satellite subspaces set in. These lead us to the corresponding effective Hamiltonian,

$$\hat{V}_{\text{eff}} \simeq \sum_{k,l} \sum_{i,j} \frac{J_{lk}^{dq} J_{ij}^{dq}}{8\hbar\omega_1} (\hat{S}_l^+ \hat{S}_k^+ \hat{S}_i^- \hat{S}_j^- + \hat{S}_l^- \hat{S}_k^- \hat{S}_i^+ \hat{S}_j^+). \quad (19)$$

Such a result finds a formal justification either on a Green's function approach to the effective Hamiltonian [47] or in the average Hamiltonian theory [48]. It is crucial to notice that

$\hat{V}_{\text{eff}}$  can indeed mix quasidegenerate states within a particular  $\mathcal{S}_\nu$ . Furthermore, it can couple states in  $\mathcal{S}_\nu$  that were not originally coupled by  $\hat{H}_{\text{dip}}$ . In practice, this means that effective matrix elements do appear in places where the original raw  $\hat{H}_{\text{dip}}$  had null entries, leading to a remarkable *proliferation* of interactions.

In principle, destructive interferences can take place. For instance, the transition from state  $|\uparrow\uparrow\downarrow\downarrow\rangle$  to  $|\uparrow\uparrow\uparrow\uparrow\rangle$  and then back to  $|\downarrow\downarrow\uparrow\uparrow\rangle$  would cancel out the transition from  $|\uparrow\uparrow\downarrow\downarrow\rangle$  to  $|\downarrow\downarrow\downarrow\downarrow\rangle$  and then back to  $|\downarrow\downarrow\uparrow\uparrow\rangle$ . Many of the destructive interferences enabled by a homogeneous all-to-all model, i.e.,  $J_{lk}^{dq} = J_{ij}^{dq}$  for any  $l, k, i, j$  indexes, are nevertheless removed by the randomization of parameters  $k$  and  $\chi$ . Other realistic spin models, in which the strength and sign of the spin-spin interaction depends on the relative positions of the spins, would not exhibit such an interference. Based on the same argument, the effective hopping corrections in Eq. (19) generate almost random entries in the Hamiltonian of each subspace. This proliferation may justify modeling the dynamics through standard RMT instead of the two-body random ensembles [49,50].

The natural step now consists in formulating an effective FGR description as in the RMT approach introduced in Sec. III B. Accordingly, we define the global second moment of the virtual interactions:

$$\langle V_{\text{eff}}^2 \rangle = \left\langle \sum_{\beta} |\langle \beta | \hat{V}_{\text{eff}} | \alpha \rangle|^2 \right\rangle_{\alpha} = \left| a \frac{(J^{dq})^2}{2\hbar\omega_1} \right|^2, \quad (20)$$

where  $a$  is a geometrical coefficient that counts the average number of states connected to a given state  $\alpha$ . Also,  $\langle \cdot \rangle_{\alpha}$  denotes the average over all unperturbed eigenstates  $\alpha$ . In analogy to Eq. (17),

$$\Gamma_{\text{eff}} \sim 2\pi \langle V_{\text{eff}}^2 \rangle d_{\text{eff}}^{-1} = 2\pi \left| a \frac{(J^{dq})^2}{2\hbar\omega_1} \right|^2 d_{\text{eff}}^{-1}, \quad (21)$$

where  $d_{\text{eff}}^{-1}$  is the density of DCS by the virtual interaction. It can be estimated as  $d_{\text{eff}} \sim bJ^{\text{dip}}$  for some geometrical coefficient  $b \ll 1$ . Both  $a$  and  $b$  stand for a subtle interplay between  $N$ , the coordination number of the lattice, the selection rules of the interaction, etc.

In what follows, we present a numerical study of the LE dynamics to show how it depends on the strength of the effective perturbation  $\Sigma_{\text{eff}} = (J^{dq})^2/(\hbar\omega_1)$ . One of the purposes consists in finding the applicability of the effective FGR.

## IV. LOSCHMIDT ECHO NUMERICAL EVALUATION

Figure 2 shows the typical LE dynamics for different perturbation strengths  $\Sigma_{\text{eff}}$ . In particular, Figs. 2(I) and 2(II) show a Gaussian to exponential transition as  $\Sigma_{\text{eff}}$  decreases. A similar transition has been reported for the survival probability of specific many-body states [39,51].

Figures 2(III) and 2(IV) show an asymptotic plateau for  $M_{1,1}(t)$  that sets in when the perturbation is small enough (i.e., large  $\hbar\omega_1$ ). In order to quantify such an observation, we plot in Fig. 3 the LE asymptotic plateau  $M_{1,1}(t \rightarrow \infty)$  as a function of  $\Sigma_{\text{eff}}$ . Below a perturbation threshold, say  $\Sigma_{\text{eff}} \lesssim 0.05J_0$  in Fig. 3, the LE equilibrates slightly above  $1/N$ . The asymptotic

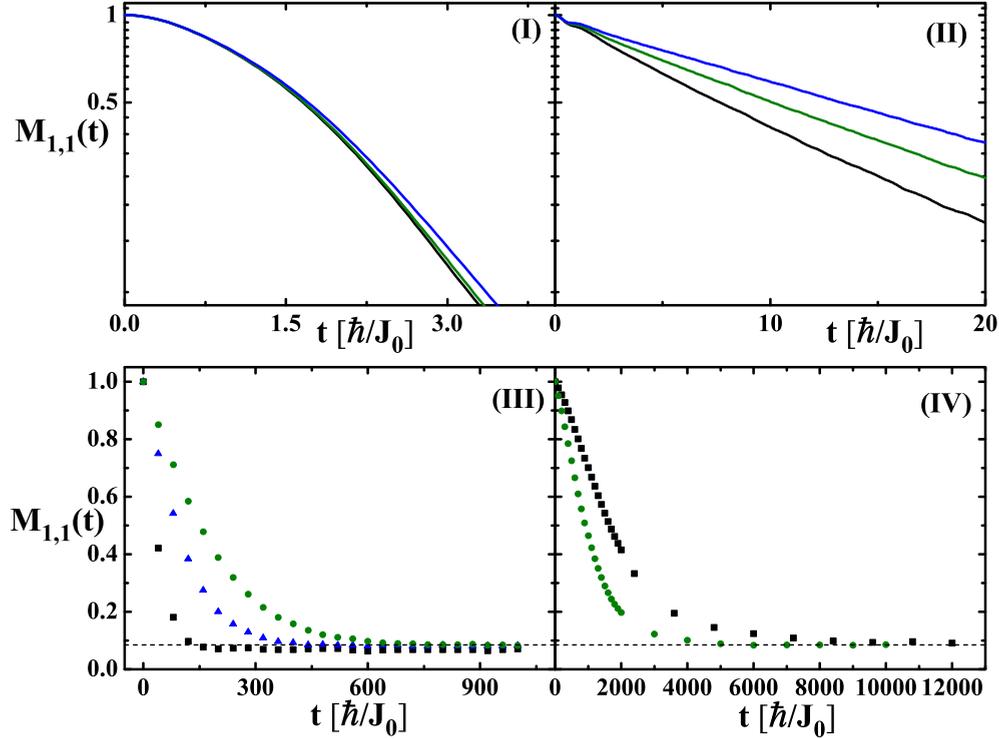


FIG. 2. (Color online) LE time dependence,  $N = 14$ . The magnitudes of  $(J^{dq})^2/(\hbar\omega_1)$  in units of  $J_0$  are, from top to bottom, (I) 0.67, 1.35,  $\infty$ ; (II) 0.19, 0.21, 0.23; (III) 0.071, 0.048, 0.038; and (IV) 0.013, 0.009. Plots (I) and (II) are in log-linear scale, while plots (III) and (IV) are in linear scale. Horizontal dashed line in plots (III) and (IV) stands for the  $1/N$  asymptote.

equidistribution  $1/N$  becomes very precise for  $N$  above 18 (data not shown). It is important to notice that this equilibration goes beyond the raw one that occurs in the forward evolution,

$$2\langle\Psi_{\text{neq}}|\hat{U}_+^\dagger(t)\hat{S}_1^z\hat{U}_+(t)|\Psi_{\text{neq}}\rangle \xrightarrow{(t\rightarrow\infty)} 1/N. \quad (22)$$

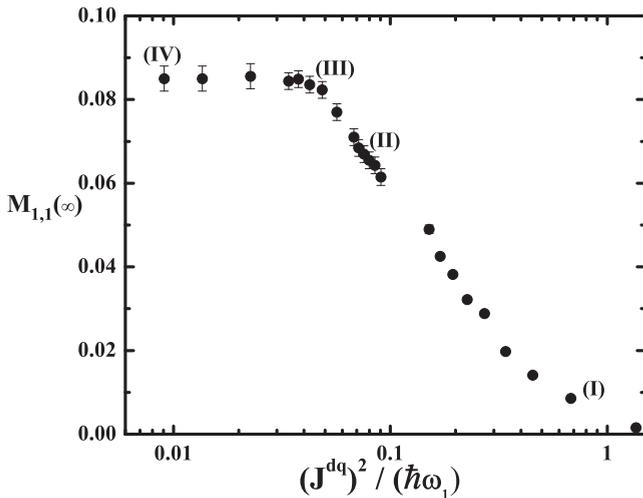


FIG. 3. LE asymptote  $M_{1,1}(t \rightarrow \infty)$  as a function of  $(J^{dq})^2/(\hbar\omega_1)$  (in units of  $J_0$ ). The labels I, II, III, and IV correspond to the representative cases shown in Fig. 2. Data set corresponds to  $N = 14$ .

Indeed, a perfect reversal of  $\hat{U}_+(t)$  would unravel the equilibration stated in Eq. (22). Nevertheless, the fact that  $M_{1,1}(t \rightarrow \infty)$  still keeps  $\sim 1/N$  means that the perturbation stabilizes the spreading of the spin polarization, turning such a process into an irreversible phenomenon. In addition, one should notice that the final state conserves the total spin projection despite the nonconserving nature of the DQ perturbation. In fact, this evidences the relevance of the effective interactions discussed in Sec. III C, since they provide a LE decay mechanism without compromising the conservation of spin projection.

In order to quantitatively assess the LE decay, we define its characteristic time  $\tau_\phi$  as  $M_{1,1}(\tau_\phi) = 2/3$ . We plot the rates  $1/\tau_\phi$  in Fig. 4 as a function of  $\Sigma_{\text{eff}}$  for  $N = 12, 14, 16$ . For each size, we identify the regimes in which the rate scales linearly and quadratically with  $\Sigma_{\text{eff}}$ . The former case can be understood as being strictly perturbative, i.e., Eq. (14). The latter is associated to the effective FGR, i.e., Eq. (21), as  $1/\tau_\phi - 1/\tau_{\phi,N}^0 \propto \Sigma_{\text{eff}}^2$  and  $1/\tau_{\phi,N}^0 \rightarrow 0$  as  $N \rightarrow \infty$ . The vanishing  $1/\tau_{\phi,N}^0$  in the present all-to-all model differs from the rate offsets observed in some hierarchical lattices [33]. The numerical observation that the effective FGR onset moves steadily towards weaker perturbations as  $N$  increases constitutes the main result of our paper.

The comparison between Figs. 3 and 4 for the  $N = 14$  case evidences that the regime where the effective FGR is valid coincides with the  $\sim 1/N$  equilibration of the spin polarization. This contrasts with the nonergodic behavior expected for the perturbative regime. In terms of time scales, given an arbitrarily small perturbation characterized by its corresponding FGR time  $\tau_\phi$ , if the forward evolution  $\hat{U}_+(t)$  occurs for a time  $t \gg$

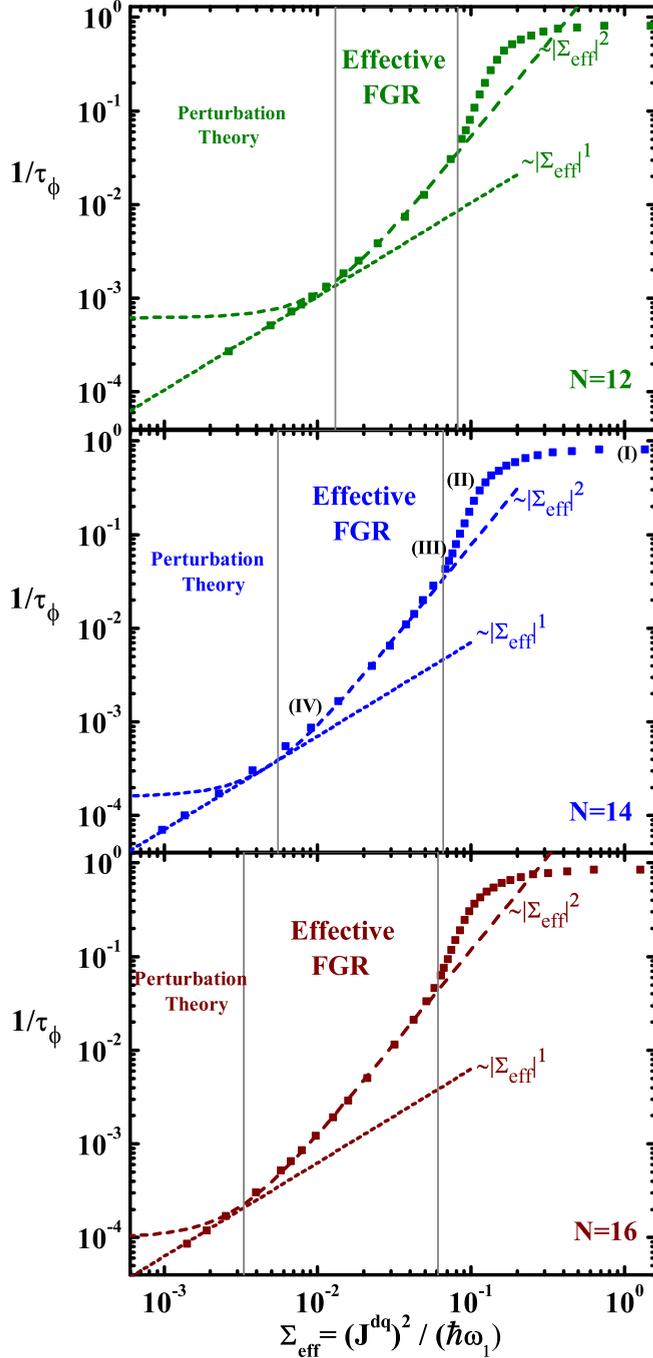


FIG. 4. (Color online) LE decay rates  $1/\tau_\phi$  (log scale, in units of  $J_0/\hbar$ ) as a function of the effective perturbation  $\Sigma_{\text{eff}} = (J^{\text{dq}})^2 / (\hbar\omega)$  (units of  $J_0$ ), for  $N = 12, 14, 16$ . The labels I, II, III, and IV from Figs. 2 and 3 are included in the case  $N = 14$ .

$\tau_\phi$ , then the equilibration in Eq. (22) becomes irreversible for any practical purpose.

As compared with equilibration, thermalization constitutes a much more specific process [4,5]. We can identify the initial condition of our system, i.e., Eq. (1), as an infinite temperature equilibrium state plus an excitation. Given the impossibility to revert the dynamics, the correlations are useless and the system ends up cooled down to a finite temperature state. We do not elaborate further along this line as it would not contribute to our central discussion.

## V. CONCLUSIONS

We have computed the LE, here defined as the local polarization recovered after a perturbed time-reversal procedure, showing a wealth of dynamical regimes. The dynamics of clusters of interacting spins has been evaluated employing their complete Hilbert space. In order to analyze the emergence of the TL as  $N$  increases up to 16 spins, we have adopted an all-to-all interaction model. Forward dynamics is generated by a reversible truncated dipolar Hamiltonian  $\hat{H}_0$  that provides a natural decomposition of the Hilbert space into subspaces of definite spin projection. As in the original experiments, a nonreversible perturbation  $\hat{\Sigma}$  couples subspaces which are separated by controllable energy gaps.

We address a regime in which the perturbation induces two- and four-body effective interactions that can mix quasidegenerate states. These states were not directly coupled by the dipolar Hamiltonian. Moreover, since the effective interactions have fewer restrictions to the selection rules, they proliferate within each subspace. In such a regime, the LE decay is characterized by an effective FGR whose realm of validity widens towards weaker perturbations as  $N$  increases. The analysis of this lower bound follows a specific sequence for the two limits: first  $N \rightarrow \infty$  and then  $\|\hat{\Sigma}\| \rightarrow 0^+$ . Then, in the TL, even a slight perturbation yields a LE decay ruled by an effective FGR, which is enhanced by the mechanism of proliferation of effective interactions.

In our model, forward many-spin dynamics can already yield an asymptotic equidistribution of the polarization. Remarkably, we observe that the excitation remains homogeneously distributed in spite of the time reversal. Therefore, while the equilibrated state indeed contains correlations that encode a full memory of the initial state, such correlations are useless in the presence of arbitrarily small perturbations. These would render the time reversal of the Hamiltonian completely ineffectual.

## ACKNOWLEDGMENTS

This work benefited from discussions with L. Buljubasich, A. K. Chattah, C. Cormick, L. Foa Torres, F. Pastawski, C. Pineda, C. Sánchez, L. F. Santos, and D. A. Wisniacki. We are grateful to C. Bederián for the technical support on GPU hardware. We acknowledge support from CONICET, ANPCyT, SeCyT-UNC, and MINCyT-Cor. This work used Mendieta Cluster from CCAD at UNC, which is part of SNCAD-MinCyT, Argentina.

[1] P. W. Anderson, Absence of diffusion in certain random lattices, *Phys. Rev.* **109**, 1492 (1958).

[2] A. Polkovnikov, K. Sengupta, A. Silva, and M. Vengalattore, Colloquium: Nonequilibrium dynamics of

- closed interacting quantum systems, *Rev. Mod. Phys.* **83**, 863 (2011).
- [3] S. Goldstein, J. L. Lebowitz, R. Tumulka, and N. Zanghì, Long-time behavior of macroscopic quantum systems. Commentary accompanying the English translation of John von Neumann's 1929 article on the quantum ergodic theorem, *Eur. Phys. J. H* **35**, 173 (2010).
- [4] J. Eisert, M. Friesdorf, and C. Gogolin, Quantum many-body systems out of equilibrium, *Nat. Phys.* **11**, 124 (2015).
- [5] R. Nandkishore and D. A. Huse, Many-body localization and thermalization in quantum statistical mechanics, *Annu. Rev. Condens. Matter Phys.* **6**, 15 (2015).
- [6] R. G. Brewer and E. L. Hahn, Atomic memory, *Sci. Am.* **251**, 50 (1984).
- [7] E. L. Hahn, Spin echoes, *Phys. Rev.* **80**, 580 (1950).
- [8] W.-K. Rhim, A. Pines, and J. S. Waugh, Time-reversal experiments in dipolar-coupled spin systems, *Phys. Rev. B* **3**, 684 (1971).
- [9] S. Zhang, B. H. Meier, and R. R. Ernst, Polarization echoes in NMR, *Phys. Rev. Lett.* **69**, 2149 (1992).
- [10] P. R. Levstein, G. Usaj, and H. M. Pastawski, Attenuation of polarization echoes in nuclear magnetic resonance: A study of the emergence of dynamical irreversibility in many-body quantum systems, *J. Chem. Phys.* **108**, 2718 (1998).
- [11] H. M. Pastawski, P. R. Levstein, G. Usaj, J. Raya, and J. Hirschinger, A nuclear magnetic resonance answer to the Boltzmann-Loschmidt controversy? *Phys. A (Amsterdam, Neth.)* **283**, 166 (2000).
- [12] G. Usaj, H. M. Pastawski, and P. R. Levstein, Gaussian to exponential crossover in the attenuation of polarization echoes in NMR, *Mol. Phys.* **95**, 1229 (1998).
- [13] H. M. Pastawski, P. R. Levstein, and G. Usaj, Quantum dynamical echoes in the spin diffusion in mesoscopic systems, *Phys. Rev. Lett.* **75**, 4310 (1995).
- [14] H. M. Pastawski, G. Usaj, and P. R. Levstein, Quantum interference phenomena in the local polarization dynamics of mesoscopic systems: an NMR observation, *Chem. Phys. Lett.* **261**, 329 (1996).
- [15] Ph. Jacquod, I. Adagideli, and C. W. J. Beenakker, Decay of the Loschmidt echo for quantum states with sub-Planck-scale structures, *Phys. Rev. Lett.* **89**, 154103 (2002).
- [16] Z. P. Karkuszewski, C. Jarzynski, and W. H. Zurek, Quantum chaotic environments, the butterfly effect, and decoherence, *Phys. Rev. Lett.* **89**, 170405 (2002).
- [17] D. Bendersky, P. R. Zangara, and Horacio M. Pastawski, Fragility of superposition states evaluated by the Loschmidt echo, *Phys. Rev. A* **88**, 032102 (2013).
- [18] B. Georgeot and D. L. Shepelyansky, Quantum chaos border for quantum computing, *Phys. Rev. E* **62**, 3504 (2000).
- [19] V. V. Flambaum, Time dynamics in chaotic many-body systems: Can chaos destroy a quantum computer? *Aust. J. Phys.* **53**, 489 (2000).
- [20] C. M. Sánchez, P. R. Levstein, R. H. Acosta, and A. K. Chattah, NMR Loschmidt echoes as quantifiers of decoherence in interacting spin systems, *Phys. Rev. A* **80**, 012328 (2009); C. M. Sánchez, R. H. Acosta, P. R. Levstein, H. M. Pastawski, and A. K. Chattah, Clustering and decoherence of correlated spins under double quantum dynamics, *ibid.* **90**, 042122 (2014).
- [21] S. W. Morgan, V. Oganessian, and G. S. Boutis, Multispin correlations and pseudothermalization of the transient density matrix in solid-state NMR: Free induction decay and magic echo, *Phys. Rev. B* **86**, 214410 (2012).
- [22] G. Kaur, A. Ajoy, and P. Cappellaro, Decay of spin coherences in one-dimensional spin systems, *New J. Phys.* **15**, 093035 (2013).
- [23] H. T. Quan, Z. Song, X. F. Liu, P. Zanardi, and C. P. Sun, Decay of Loschmidt echo enhanced by quantum criticality, *Phys. Rev. Lett.* **96**, 140604 (2006).
- [24] A. Silva, Statistics of the work done on a quantum critical system by quenching a control parameter, *Phys. Rev. Lett.* **101**, 120603 (2008).
- [25] T. Caneva, A. Silva, R. Fazio, S. Lloyd, T. Calarco, and S. Montangero, Complexity of controlling quantum many-body dynamics, *Phys. Rev. A* **89**, 042322 (2014).
- [26] T. Gorin, T. Prosen, T. H. Seligman, and M. Žnidarič, Dynamics of Loschmidt echoes and fidelity decay, *Phys. Rep.* **435**, 33 (2006).
- [27] Ph. Jacquod and C. Petitjean, Decoherence, entanglement, and irreversibility in quantum dynamical systems with few degrees of freedom, *Adv. Phys.* **58**, 67 (2009); C. Petitjean and Ph. Jacquod, Quantum reversibility and echoes in interacting systems, *Phys. Rev. Lett.* **97**, 124103 (2006).
- [28] A. Goussev, R. A. Jalabert, H. M. Pastawski, and D. A. Wisniacki, Loschmidt echo, *Scholarpedia* **7**, 11687 (2012).
- [29] R. A. Jalabert and H. M. Pastawski, Environment-independent decoherence rate in classically chaotic systems, *Phys. Rev. Lett.* **86**, 2490 (2001).
- [30] F. M. Cucchiatti, D. A. R. Dalvit, J. P. Paz, and W. H. Zurek, Decoherence and the Loschmidt echo, *Phys. Rev. Lett.* **91**, 210403 (2003).
- [31] Ph. Jacquod, P.G. Silvestrov, and C.W.J. Beenakker, Golden rule decay versus Lyapunov decay of the quantum Loschmidt echo, *Phys. Rev. E* **64**, 055203 (2001).
- [32] A. D. Dente, C. S. Bederián, P. R. Zangara, and H. M. Pastawski, GPU accelerated Trotter-Suzuki solver for quantum spin dynamics, [arXiv:1305.0036](https://arxiv.org/abs/1305.0036) [physics.comp-ph].
- [33] P. R. Zangara, A. D. Dente, P. R. Levstein, and H. M. Pastawski, Loschmidt echo as a robust decoherence quantifier for many-body systems, *Phys. Rev. A* **86**, 012322 (2012).
- [34] P. R. Zangara, A. D. Dente, A. Iucci, P. R. Levstein, and H. M. Pastawski, Interaction-disorder competition in a spin system evaluated through the Loschmidt echo, *Phys. Rev. B* **88**, 195106 (2013).
- [35] C. P. Slichter, *Principles of Magnetic Resonance*, 2nd ed., Springer Series in Solid State Sciences (Springer, Berlin, 1980).
- [36] J. Baum, M. Munowitz, A. N. Garroway, and A. Pines, Multiple-quantum dynamics in solid state NMR, *J. Chem. Phys.* **83**, 2015 (1985).
- [37] M. Munowitz, A. Pines, and M. Mehring, Multiple-quantum dynamics in NMR: A directed walk through Liouville space, *J. Chem. Phys.* **86**, 3172 (1987).
- [38] G. Usaj, Mesoscopic echoes and polarization echoes in NMR: a study about the spin excitation dynamics in interacting systems, Ph.D. thesis, Universidad Nacional de Cordoba, Argentina, 1999.
- [39] L. F. Santos, F. Borgonovi, and F. M. Izrailev, Chaos and statistical relaxation in quantum systems of interacting particles,

- [Phys. Rev. Lett. \*\*108\*\*, 094102 \(2012\)](#); Onset of chaos and relaxation in isolated systems of interacting spins: Energy shell approach, [Phys. Rev. E \*\*85\*\*, 036209 \(2012\)](#).
- [40] D. A. Wisniacki and A. J. Roncaglia, Sensitivity to perturbations and quantum phase transitions, [Phys. Rev. E \*\*87\*\*, 050902 \(2013\)](#).
- [41] A. D. Dente, Raúl A. Bustos-Marín, and H. M. Pastawski, Dynamical regimes of a quantum swap gate beyond the Fermi golden rule, [Phys. Rev. A \*\*78\*\*, 062116 \(2008\)](#); A. D. Dente, P. R. Zangara, and H. M. Pastawski, Non-Markovian decay and dynamics of decoherence in private and public environments, [ibid. \*\*84\*\*, 042104 \(2011\)](#).
- [42] N. Ares and D. A. Wisniacki, Loschmidt echo and the local density of states, [Phys. Rev. E \*\*80\*\*, 046216 \(2009\)](#); D. A. Wisniacki, N. Ares, and E. G. Vergini, Universal response of quantum systems with chaotic dynamics, [Phys. Rev. Lett. \*\*104\*\*, 254101 \(2010\)](#).
- [43] E. J. Torres-Herrera, Manan Vyas, and L. F. Santos, General features of the relaxation dynamics of interacting quantum systems, [New J. Phys. \*\*16\*\*, 063010 \(2014\)](#); E. J. Torres-Herrera and L. F. Santos, Quench dynamics of isolated many-body quantum systems, [Phys. Rev. A \*\*89\*\*, 043620 \(2014\)](#); Nonexponential fidelity decay in isolated interacting quantum systems, [90](#), 033623 (2014).
- [44] P. R. Zangara, A. D. Dente, E. J. Torres-Herrera, H. M. Pastawski, A. Iucci, and L. F. Santos, Time fluctuations in isolated quantum systems of interacting particles, [Phys. Rev. E \*\*88\*\*, 032913 \(2013\)](#).
- [45] G. A. Álvarez, E. P. Danieli, P. R. Levstein, and H. M. Pastawski, Quantum parallelism as a tool for ensemble spin dynamics calculations, [Phys. Rev. Lett. \*\*101\*\*, 120503 \(2008\)](#).
- [46] F. M. Cucchietti, C. H. Lewenkopf, E. R. Mucciolo, H. M. Pastawski, and R. O. Vallejos, Measuring the Lyapunov exponent using quantum mechanics, [Phys. Rev. E \*\*65\*\*, 046209 \(2002\)](#).
- [47] H. M. Pastawski, L.E.F. Foa Torres, and E. Medina, Electron phonon interaction and electronic decoherence in molecular conductors, [Chem. Phys. \*\*281\*\*, 257 \(2002\)](#).
- [48] U. Haeblerlen and J. S. Waugh, Coherent averaging effects in magnetic resonance, [Phys. Rev. \*\*175\*\*, 453 \(1968\)](#).
- [49] O. Bohigas and J. Flores, Two-body random hamiltonian and level density, [Phys. Lett. B \*\*34\*\*, 261 \(1971\)](#).
- [50] V. V. Flambaum, G. F. Gribakin, and F. M. Izrailev, Correlations within eigenvectors and transition amplitudes in the two-body random interaction model, [Phys. Rev. E \*\*53\*\*, 5729 \(1996\)](#).
- [51] V. V. Flambaum and F. M. Izrailev, Unconventional decay law for excited states in closed many-body systems, [Phys. Rev. E \*\*64\*\*, 026124 \(2001\)](#).