Correction to the geometric phase by structured environments: The onset of non-Markovian effects

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We study the geometric phase of a two-level system under the presence of a structured environment, particularly analyzing its correction with the ohmicity parameter s and the onset of non-Markovianity. We first examine the system coupled to a set of harmonic oscillators and study the decoherence factor as function of the environment's ohmicity parameter. Second, we propose the two-level system coupled to a nonequilibrium environment, and show that these environments display non-Markovian effects for all values of the ohmicity parameter. The geometric phase of the two-level system is therefore computed under the presence of both types of environment. The correction to the unitary geometric phase is analyzed in both the Markovian and non-Markovian regimes. Under Markovian environments, the correction induced on the system's phase is mainly ruled by the coupling constant between the system and the environment, while in the non-Markovian regime, memory effects seem to trigger a significant correction to the unitary geometric phase. The result is significant to the quantum information processing based on the geometric phase in quantum open systems.

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I. INTRODUCTION

A system can retain the information of its motion when it undergoes a cyclic evolution, in the form of a geometric phase; this was first put forward by Pancharatnam in optics [1] and later studied explicitly by Berry in a general quantal system [2]. Since then, great progress has been achieved in this field. For example, the application of the geometric phase has been proposed in many fields, such as geometric quantum computation. In this line of work, many physical systems have been investigated to realize geometric quantum computation, such as nuclear magnetic resonance (NMR) [3], Josephson junction [4], ion trap [5], and semiconductor quantum dots [6]. The quantum computation scheme for the geometric phase has been proposed based on Abelian or non-Abelian geometric concepts, and the geometric phase has been shown to be robust against faults in the presence of some kind of external noise due to the geometric nature of the Berry phase [7–10]. Then, for isolated quantum systems, the geometric phase is theoretically perfectly understood and experimentally verified. However, it has been shown that the interactions play an important role for the realization of some specific operations. As the gates operate slowly compared to the dynamical time scale, they become vulnerable to open system effects and parameter fluctuations that may lead to a loss of coherence. Consequently, the study of the geometric phase was soon extended to open quantum systems. Following this idea, many authors have analyzed the correction to the geometric phase under the influence of an external thermal or nonequilibrium environment, using different approaches (see [11–16]). In all cases, the purely dephasing model considered was a spin-1/2 particle coupled to the environment's degrees of freedom through a σ_z coupling. The interest on the geometric phase in open systems has also been extended to some experimental setups [17]. Lately, it has also been observed in a variety of superconducting systems [18,19], and the importance of quantifying decoherence when geometric operations are carried out in the presence of low-frequency noise has been shown. All real experiments generally imply the presence of an external environment which induces noise and dissipation on the subsystem depending on the strength of the coupling among them. Memory effects are also considered a relevant source of noise that can affect the dynamics of the system of interest. Thus, a detailed mechanism of decoherence due to external noise sources is still required in order to overcome the effect of a destructive decoherence in measurements of the geometric phase.

Within a microscopic approach, quantum Markovian master equations are usually obtained by means of the Born-Markov approximation, which assumes a weak system-environment coupling and several further, mostly rather drastic approximations [20]. However, these approximations are not applicable in many processes occurring in nature, such as strong environment couplings, structured and finite reservoirs, low temperatures, and large initial system-environment correlations [21]. In the case of any substantial deviation from the dynamics of a quantum Markov process, one often speaks of a non-Markovian process, implying that the dynamics is governed by significant memory effects. Unfortunately, a consistent general theory of quantum non-Markovianity does not exist and even the very definition of non-Markovianity is currently an issue. Very recently, important steps towards a general theory of non-Markovianity have been made. There has been a great effort to rigorously define the border between Markovian and non-Markovian quantum dynamics and to develop quantitative measures for the degree of memory effects in open systems [22–24]. The quantification of non-Markovianity is justified by the fact that there is increasing evidence of its crucial role as a resource for quantum technologies [24,25]. Non-Markovianity evolution is often characterized by decoherence phenomena and information trapping, hence leading to longer coherence times in comparison to the Markovian case. As reservoir engineering techniques become experimentally feasible, it is crucial to establish relations between the occurrence of non-Markovianity and the form of the environmental spectrum. Recently, there have been many studies of systems whose reduced dynamics are characterized by memory effects. Non-Markovian features play an important role in systems where the frequency spectrum of the environment is structured. In Ref. [26], the authors established a connection between the general form of the spectrum and the memory effects in the reduced system dynamics.

In this framework, this study has a twofold motivation: on the one hand, the need to have a better understanding of the geometric phase in open quantum systems in order to achieve fault tolerance quantum computation, and, on the other hand, the need to understand memory effects as a source of noise for the system of interest. In this manuscript, we shall study the correction to the geometric phase of a two-level system coupled to an external environment. We shall use the spin-boson model since it has the advantage of having an analytical solution which can be useful to have a better insight into the onset of non-Markovian effects. By the use of the established relation between the form of the reservoir spectrum and the onset of non-Markovianity, we shall study the geometric phase of the open system as a function of the "ohmicity" of the reservoir, which allows the description of subohmic, ohmic, and superohmic spectra. The paper is organized as follows: in Sec. II, we briefly describe the model and present both types of environments to be analyzed: thermal equilibrium and nonequilibrium. We study the diffusion coefficients and the corresponding decoherence factors. Following Ref. [26], we study the onset of non-Markovianity in the environments considered. In Sec. III, we compute the geometric phase acquired by the system for different forms of the reservoir spectrum, as a function of the ohmicity parameter, and study how the memory effects of the environment affect the geometric phase. Finally, in Sec. IV, we make our final remarks.

II. THE MODEL

The spin-boson model is studied in a variety of fields, such as condensed-matter physics, quantum optics, physical chemistry, and quantum information science [27], in order to describe nonunitary effects induced in quantum systems due to a coupling with an external environment. For a quantum system, the influence of the surroundings plays a role at a fundamental level. When the environment is taken into consideration, the system dynamics can no longer be described in terms of pure quantum states and unitary evolution. From a practical point of view, all real systems interact with an environment, which means that we expect their quantum evolution to be plagued by nonunitary effects, namely, dissipation and decoherence. Most theoretical investigations of how the system is affected by the presence of an environment have been done using a thermal reservoir, usually assuming Markovian statistical properties and defined bath correlations [28,29] (there are also works on non-Markovian models such as, for example, Ref. [30]).

In the following, we shall consider a paradigmatic spin-boson model consisting of a two-level system coupled to an external environment. We shall consider two different types of environments and see the non-Markovian effects: thermal equilibrium environment and nonequilibrium environment. In both cases, we shall compute the diffusion and decoherence factor derived by the definition of the corresponding bath correlations and the spectral density $I(\omega)$, which can be defined for a general environment as

$$I(\omega) = \gamma_0 \frac{\omega^s}{\Lambda^{s-1}} \exp(-\omega/\Lambda). \tag{1}$$

In Eq. (1), Λ is the reservoir frequency cutoff and γ_0 is the coupling constant (which has different units for the different environment considered). By changing the value of the s parameter, one goes from subohmic reservoirs ($s < 1$) to ohmic ($s = 1$) and superohmic ($s > 1$) reservoirs, respectively. The ohmic spectrum is generally used to describe charged interstitials in metals. The supraohmic environment commonly describes the effect of the interaction between a charged particle and its electromagnetic field. The subohmic environment is used to model the type of noise occurring in solid-state devices due to low-frequency modes, similar to the "$1/f$" noise in Josephson junctions. Thus, the description of the spectral density function in terms of a continuous parameter s allows one to simulate paradigmatic models of open quantum systems by the variation of s. We stress that such engineering of the ohmicity of the spectrum is possible when simulating the dephasing model in trapped ultracold atoms [31].

A. Thermal equilibrium environments and the onset of non-Markovianity

We shall start by studying the decoherence effects induced by a thermal equilibrium environment, generally modeled by a set of harmonic oscillators. The interaction between the two-state system and the environment is entirely represented by a Hamiltonian in which the coupling is only through σ_z. In this particular case, $[\sigma_z, H_{\rm int}] = 0$ and the corresponding master equation for the reduced density matrix is much simplified, with neither frequency renormalization nor dissipation effects. The Hamiltonian of the complete system is written as

$$H_{\rm SB} = \frac{1}{2}\hbar\Omega\sigma_z + \frac{1}{2}\sigma_z \sum_k \lambda_k (g_k a_k^\dagger + g_k^* a_k) + \sum_k \hbar\omega_k a_k^\dagger a_k. \tag{2}$$

The interaction Hamiltonian is only proportional to σ_z, which means that the populations of the eigenstates remain constant while the off-diagonal terms of the reduced density matrix decay due to the existence of the environment, as

$$\rho_{r_{10}}(t) = \rho_{r_{01}}^*(t) = \rho_{r_{10}}(0) e^{-\mathcal{F}(t)} e^{-i\Omega t}, \tag{3}$$

where $\mathcal{F}$ is the decoherence factor defined as (see Ref. [13] for details)

$$\begin{aligned} \mathcal{F}(t) &= 2\int_0^\infty d\omega I(\omega) \coth\left(\frac{\omega}{2k_B T}\right) \frac{[1-\cos(\omega t)]}{\omega^2} \\ &= 2\int_0^t ds D(s), \end{aligned} \tag{4}$$

where $D(s)$ is the diffusion coefficient, present in the master equation. It is clear that $\mathcal{F}(t)$ not only depends on the temperature of the bath but also on the spectral density, particularly on the ohmicity parameter s.

In Ref. [26], the authors have shown that there exists a link between the onset of non-Markovianity and the form of the reservoir spectrum for thermal equilibrium environments such as the ones considered here. They have stated that the crossover is signalled by the onset of periods during which

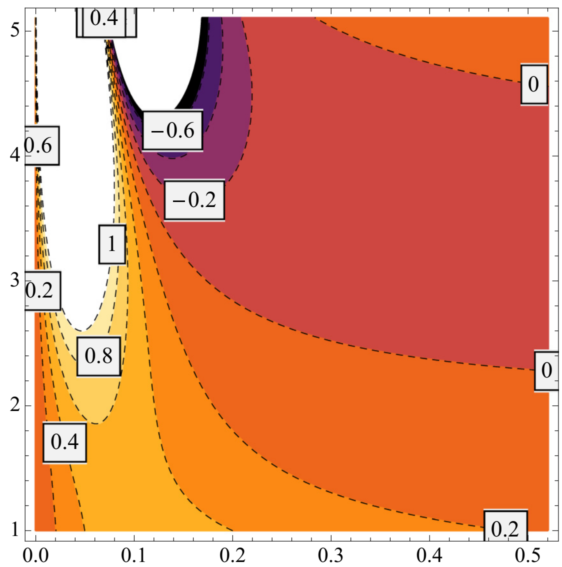

FIG. 1. (Color online) Density plot of the diffusion coefficient of thermal environments. The horizontal axis represents the time evolution as Ωt and the vertical axis is the ohmicity s. The darkest areas are negative values of the diffusion coefficient which has been shown to be related to the onset of non-Markovian effects as stated in [26]. Parameters used: $\gamma_0 = 0.1$ and $\Lambda = 10\Omega$.

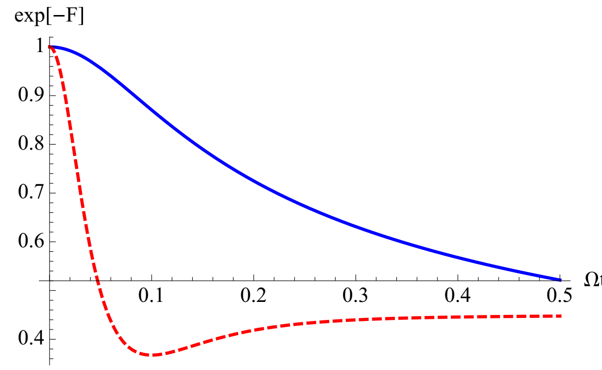

FIG. 2. (Color online) Decoherence factor as a function of the time for different values of the ohmicity parameter s. It can be seen that areas with negative diffusion coefficient in Fig. 1 present a nonmonotonic behavior in the decoherence factor leading to recoherence effects. The blue line corresponds to $s = 1$ and the red dashed line corresponds to $s = 4$. Parameters used: $\gamma_0 = 0.1$ and $\Lambda = 10\Omega$.

the diffusion rate is negative. A common feature of all non-Markovianity measures is that they are based on the nonmonotonic evolution in time of certain coefficients which signal the information backflow from the environment back to the main system. Without memory effects, decoherence rates (as a quantum channel) decrease in time monotonically. However, environment memory effects may produce a nonmonotonic behavior of the quantum channel capacity. This is related to the convex to nonconvex changes of the spectrum. The condition on the nonconvexity of the environmental spectrum is a necessary and sufficient condition for non-Markovianity at all temperatures. This has been set as $s > 2$ for zero-T environments. Hence, memory effects leading to information backflow and recoherence occur only if the reservoir spectrum is superohmic with $s > 2$. This means that even if the reduced dynamics is exact, and hence no Markovian approximation has been performed, the time evolution of the qubit does not present any memory effects for ohmic and subohmic spectra. This argument has been derived based on the nonmonotonic behavior of the decoherence factor $\mathcal{F}(t)$, and basically it is equivalent to the fact that the diffusion coefficient $D(s)$ becomes negative.

In Fig. 1, we present a contour plot for the diffusion coefficient for different equilibrium environments at zero temperature, to show the onset of non-Markovianity as derived in [26]. We can see the regions where the diffusion coefficient has a negative value, which are related to the onset of non-Markovian effects. For example, in Fig. 1, we see that for ohmic thermal environments, i.e., $s = 1$ in the vertical axis, we always have positive values for the diffusion coefficient. This means that the ohmic environment is always Markovian. For other environments, such as $s = 4$ in the vertical axis, we can find regions where the diffusion coefficient gets positive values and areas where the value is negative. This means that the memory effects become important after some time evolution.

Using Eqs. (1) and (4), the exact decoherence factor as a function of time and the ohmicity parameter s can be written as

$$\mathcal{F}(t) = 4\gamma_0 \frac{\Gamma[s]}{s-1}\big[1 - (1+\Lambda^2 t^2)^{-\frac{s}{2}} \times \{\cos[s \arctan(\Lambda t)] + \Lambda t \sin[s \arctan(\Lambda t)]\}\big], \quad (5)$$

where $\Gamma[x]$ is the gamma function. In Fig. 2, we present the evolution of the decoherence factor as time evolves. In ohmic ($s = 1$) thermal environments, the decoherence factor is known to be a monotonic decreasing function in time (blue line in Fig. 2). However, when the memory effects of the environment become relevant (for example, $s = 4$), the decoherence function does not have a monotonic behavior, leading to recoherence phenomena (red dashed line in Fig. 2). This is related to the negative regions of the diffusion coefficient, as can be seen in Fig. 1.

B. Nonequilibrium environments and the onset of non-Markovianity

In this section, we shall deal with nonequilibrium environments, represented by random perturbations with nonstationary statistics. The motivation to study these type of environments is twofold: on one hand, the diffusion coefficients computed for different environment spectra have negative parts for all values of the ohmicity parameter. This means they can be considered as non-Markovian environments for all values of the parameter s and, contrary to the environments studied above, there is not a critical value of s which determines the onset of non-Markovianity. On the other hand, these nonequilibrium baths may represent a proposal for engineering reservoirs in a manner reminiscent of a coherent control experiment using shaped pulses [32]. In this model, the control parameter λ is derived not from a laser pulse, but rather from well-defined phase relations between the modes of the bath.

The modeling of these nonequilibrium environments implies that the two-level quantum system presents an energy gap, $E_2(t) - E_1(t) = \hbar\omega(t)$, which fluctuates due to the influence of the environment, where $E_j(t)$, with $j = 1,2$, is the instantaneous energy of state j as perturbed by the surroundings. Following the idea proposed in [33], the bath is represented by a random function of time corresponding to the transition frequency of the two-state quantum system. In contrast to the usual treatment, the statistical properties of this random function are nonstationary, corresponding physically to, for example, impulsively excited phonons of the environment with initial phases that are not random, but which have defined values at $t = 0$. Due to these assumptions, this environment is not at thermal equilibrium. The time-dependent frequency is written in the form $\omega(t) = \Omega + \delta\omega(t)$.

Following this approach, it is easy to check that the off-diagonal terms of the reduced density matrix can be written as

$$\rho_{\mathrm{r}_{01}}(t) = e^{-i\Omega t}\left\langle e^{-i\int_0^t \delta\omega(s)ds}\right\rangle \rho_{\mathrm{r}_{01}}(0) \equiv e^{-i\Omega t} e^{-\tilde{\mathcal{F}}(t)} \rho_{\mathrm{r}_{01}}(0). \quad (6)$$

Here, we denote with $\langle\cdot\rangle$ the nonequilibrium average over the nonstationary random bath and $\tilde{\mathcal{F}}(t)$ is defined as the decoherence factor for the nonequilibrium environments, which reads

$$\tilde{\mathcal{F}}(t) = \exp\big(-\{\gamma_0 \exp(-4dt)[-1 + \exp(2dt) \times\Gamma(1+s)\{[1+4(t-\lambda)^2\Lambda^2]^{\frac{-(1+s)}{2}}\} \times \cos\{(1+s)\arctan[2\Lambda(t-\lambda)]\} + \cosh(2dt) + \sinh(2dt)]\}\big), \quad (7)$$

where d and λ are parameters of the environment model. By knowing the decoherence factor, we can have a better insight into the decoherence process induced in the system by the presence of a nonequilibrium environment.

In Fig. 3, we show a contour plot for the diffusion coefficients for different forms of the ohmicity parameter s in the vertical axis and different values of Ωt in the horizontal one. We can note that there is always a period when the coefficient becomes negative, even for $s = 1$. This means that memory effects are always present for these types of environments and the decoherence factors are always nonmonotonic decreasing functions, allowing recoherence effects on the system of interest. We can consider these environments to be non-Markovian for all values of s. We can also note a clear hierarchy in the coefficients: the bigger the value of s, the more negative is the diffusion coefficient and the more decoherence (and recoherence) induced by the environment. This fact can be observed in Fig. 4, where we show the decoherence factor for different nonequilibrium environments. It can be seen that even though we have $s = 1$, the behavior is nonmonotonic, which is quite different from the decoherence factor of a thermal equilibrium environment. In Fig. 4, we present the decoherence factor for different values of the ohmicity parameter s as a function of time. The red dashed line corresponds to $s = 1$, the black solid line is for $s = 2$, and the blue dotted line shows the case $s = 3$. In all cases, these nonequilibrium environments present a "dip," which makes the behavior of the decoherence factor nonmonotonic. The strength and location of the dip is determined by the other parameters of the model λ and d.

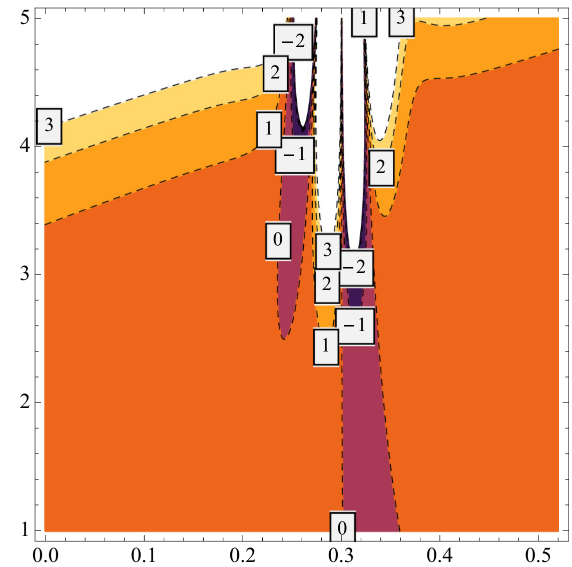

FIG. 3. (Color online) Density plot of the diffusion coefficient for the nonequilibrium environments. The horizontal axis represents the time evolution as Ωt and the vertical axis is the ohmicity s. The darkest areas are negative values of the diffusion coefficient. Parameters used: $\Lambda = 10\Omega$, $\gamma_0 = 0.1$, $\Omega\lambda = 0.3$, and $d = 2\Omega$.

III. CORRECTION TO THE GEOMETRIC PHASE

In order to compute the geometric phase (GP) and note how it is corrected by the presence of the environment, we shall briefly review the way the GP can be computed for a system under the influence of external conditions such as an external bath. In Ref. [11], a quantum kinematic approach was proposed and the GP for a mixed state under nonunitary

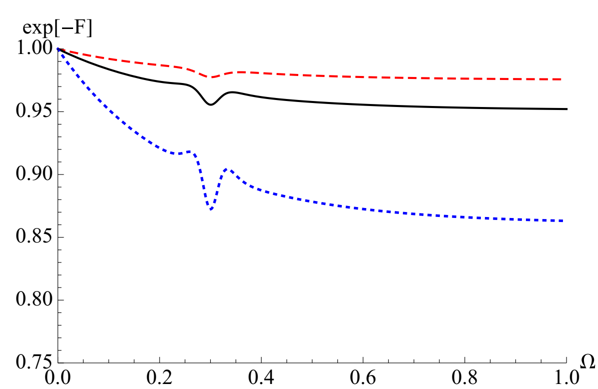

FIG. 4. (Color online) Decoherence factor for different values of the ohmicity parameter s as a function of time. We can see that the dip becomes more relevant and drastic for bigger values of s. The red dashed line is for $s = 1$, the black solid line is for $s = 2$, and the blue dotted line is for $s = 3$. Parameters used: $\Lambda = 10\Omega$, $\gamma_0 = 0.1$, $\Omega\lambda = 0.3$, and $d = 2\Omega$.

evolution has been defined as

$$\phi_G = \arg\left\{\sum_k \sqrt{\varepsilon_k(0)\varepsilon_k(\tau)}\langle\Psi_k(0)|\Psi_k(\tau)\rangle e^{-\int_0^\tau dt\langle\Psi_k|\frac{\partial}{\partial t}|\Psi_k\rangle}\right\}, \quad (8)$$

where $\varepsilon_k(t)$ are the eigenvalues and $|\Psi_k\rangle$ are the eigenstates of the reduced density matrix ρ_r (obtained after tracing over the reservoir degrees of freedom). In the last definition, τ denotes a time after the total system completes a cyclic evolution when it is isolated from the environment. Taking into account the effect of the environment, the system no longer undergoes a cyclic evolution. However, we shall consider a quasicyclic path $\mathcal{P}: t\epsilon[0,\tau]$, with $\tau = 2\pi/\Omega$ (Ω is the system's characteristic frequency). When the system is open, the original GP, i.e., the one that would have been obtained if the system had been closed ϕ_u, is modified. This means, in a general case, that the phase can be interpreted as $\phi_G = \phi_u + \delta\phi$, where $\delta\phi$ depends on the kind of environment coupled to the main system [13–15,17,34,35].

We want to compute the GP acquired by the system for different forms of the reservoir spectrum, as a function of the ohmicity parameter, which allows the description of subohmic, ohmic, and superohmic spectra. Particularly, we want to see how the unitary geometric phase for a two-level system, $\phi_u = \pi(1-\cos\theta)$, is corrected as a function of the ohmicity, i.e., the parameter s of the spectral density $I(\omega)$. Assuming an initial quantum state of the system as

$$|\psi(0)\rangle = \cos(\theta/2)|0\rangle + \sin(\theta/2)|1\rangle, \quad (9)$$

its evolution at a later time t is

$$|\psi(t)\rangle = e^{-i\Omega t}\cos[\theta_+(t)]|0\rangle + \sin[\theta_+(t)]|1\rangle, \quad (10)$$

where $\cos[\theta_+(t)]$ (and $\sin[\theta_+(t)]$) encodes diffusion induced on the subsystem due to the presence of the environment. As explained in Ref. [11], the GP is obtained by computing eigenvectors and eigenvalues of the reduced density matrix derived by using the state vector $|\psi(t)\rangle$ and using Eq. (8).

We shall consider the thermal equilibrium and nonequilibrium environments considered above. With the decoherence factors and the reduced density matrix computed analytically for each case, we can compute the GP and see how it is affected by the memory effects present in the different environments considered.

A. Thermal equilibrium environments

By considering the thermal equilibrium environments in the zero-T limit and using the corresponding decoherence factor [Eq. (5)], we can find an expression for the correction to the GP in terms of a series expression in the coupling constant,

$$\phi_G = \phi_u + \gamma_0 \sin^2\theta\cos\theta\,\Gamma(s-2)\Bigg(2\pi(s-2) + \left(1+\frac{4\pi^2\Lambda^2}{\Omega^2}\right)^{-\frac{s}{2}}\Bigg\{4\pi\cos\left[s\arctan\left(\frac{2\pi\Lambda}{\Omega}\right)\right] + \left(\frac{4\pi^2\Lambda}{\Omega}-\frac{\Omega}{\Lambda}\right)\sin\left[s\arctan\left(\frac{2\pi\Lambda}{\Omega}\right)\right]\Bigg\}\Bigg). \quad (11)$$

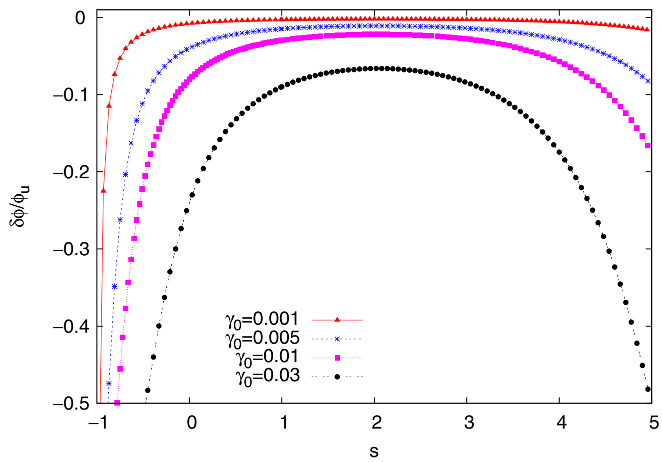

FIG. 5. (Color online) Correction to the geometric phase (in units of the unitary phase) as a function of the ohmicity parameter s. Solid-crossed red line is for $\gamma_0 = 0.001$, dotted-asterisk blue line is for $\gamma_0 = 0.005$, dotted-square magenta line is for $\gamma_0 = 0.01$, and double-dotted black-circled line is for $\gamma_0 = 0.03$ for zero-T environments. Parameters used: $\theta = \pi/3$, $\Lambda = 10\Omega$.

As expected, in the limit $s \to 1$, the correction to the phase approaches $\delta\phi \approx 4\pi\gamma_0\sin^2\theta\cos\theta\,(-1+\ln[2\pi\Lambda/\Omega])$. In the limit $s \to 3$, the correction is given by $\delta\phi \approx 4\pi\gamma_0\sin^2\theta\cos\theta$. Both expressions agree with the corrections to the ohmic ($s = 1$) and supraohmic ($s = 3$) geometric phases in the zero-temperature limit found in Ref. [13]. As this result is derived for small values of the coupling constant γ_0, in the following we shall compute the exact GP numerically.

In Fig. 5, we show the correction induced by different environmental types on the geometric phase (normalized by the value of the unitary geometric phase ϕ_u) as a function of the ohmicity for different values of γ_0. In this figure, we can note that the geometric phase is very much destroyed when $s \to -1$. In that case, we are considering the effect of a noise, similar to $1/f$, which is very harmful. This type of noise can be considered a subohmic environment due to the fact that low frequencies are predominant. The correction to the phase grows as the effect of low-frequency modes of the environment becomes more relevant. On the contrary, as s starts increasing, we obtain the correction to the phase similar to the one of an ohmic environment ($s = 1$) at zero T [13]. As can be expected, decoherence induced by this type of environment is low for a very weak coupling (less than 10% for smaller values of γ_0). However, it becomes significant for bigger values of γ_0; for example, for $\gamma_0 = 0.03$ in Fig. 5, the correction is bigger than 20%. This agrees with the results in Ref. [13]: more decoherence induced on the system implies a bigger correction to the unitary geometric phase. Near the *ohmic region* of the parameters set, the correction depends mainly on γ_0, as expected, being negligible when the small value of γ_0 is such that it is not able to destroy coherences in the system. For $1 < s < 2$, the correction to the phase is more insensitive to the ohmicity value, and it is evident that there is a change in behavior for $s > 2$ (it starts increasing), in agreement with the onset of non-Markovianity. This shows that non-Markovian environments induce a bigger correction to the

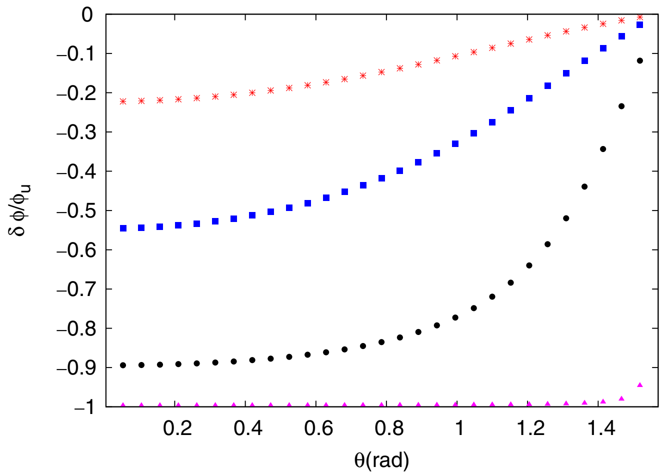

FIG. 6. (Color online) The correction to the GP as a function of the angle that defines the state vector in the Bloch sphere (θ) for ohmic and supraohmic environments in the zero-T limit. Red asterisks represent the ohmic environment ($s = 1$), while blue squares are for a supraohmic environment of $s = 2$ and black circles for one of $s = 2.5$. The triangles represent $s = 3$. Parameters used: $\gamma_0 = 0.01$ and $\Lambda = 10\Omega$.

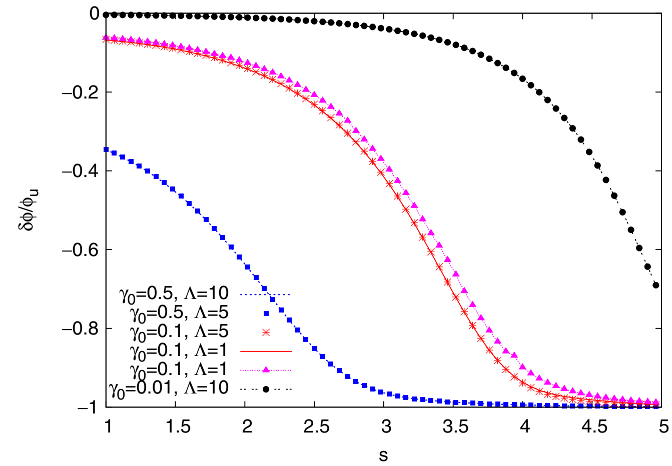

FIG. 7. (Color online) Correction to the unitary geometric phase for different values of the model's parameters. The blue dotted line is for $\gamma_0 = 0.5$ and $\Lambda = 10\Omega$, while the blue square dots are for $\gamma_0 = 0.5$ and $\Lambda = 5\Omega$. The red asterisks are for $\gamma_0 = 0.1$ and $\Lambda = 5\Omega$, while the solid red line is for $\gamma_0 = 0.1$ and $\Lambda = 1\Omega$, and the triangle dotted line is for $\gamma_0 = 0.1$ and $\Lambda = 1\Omega$ but changing the parameters of λ and d. Finally, the dotted black line is for $\gamma_0 = 0.01$ and $\Lambda = 10\Omega$. Parameters used: $\Omega\lambda = 0.5$, $d = 1\Omega$.

unitary geometric phase. As expected, this fact is enlarged for bigger couplings between system and environment, i.e., values of γ_0, since these imply a bigger decoherence effect. These corrections have not been studied in Ref. [13]. In conclusion, corrections to the phase are double. First, we have the common hierarchy in the induced correction ruled by the coupling to the environment: the bigger correction occurs with the bigger value of γ_0. Second, it is possible to see that the correction to the phase becomes more relevant when the ohmicity parameter surpasses its critical value. It becomes evident that the more non-Markovian the environment, the biggest correction to the geometric phase for the same coupling to the environment. This explains the behavior observed in Fig. 5.

In Fig. 6, the normalized correction is plotted for different environments at zero temperature. Red asterisks represent an ohmic environment ($s = 1$), while blue squares are for a supraohmic environment of $s = 2$ and black circles are for one of $s = 2.5$. The triangles represent $s = 3$. We can see that for $s > 2$, the correction to the geometric phase is bigger and its behavior is more drastic, in agreement with the onset of non-Markovianity expected to be for environments at zero temperature.

B. Nonequilibrium environments

Finally, we compute the correction to the GP for the nonequilibrium environments which have memory effects for all values of s. Therefore, we shall consider them as non-Markovian environments. In Fig. 7, we show the behavior of the normalized correction to the unitary geometric phase as a function of the ohmicity s. In this figure, we can see that for smaller values of s and particular values of λ and d parameters (which determine a small dip in the decoherence factor coefficient as shown in Fig. 4), the nonequilibrium environment does not have a big influence on the geometric phase, and the open geometric phase coincides with the unitary one. In Fig. 7, we present different sets of values for the parameters of the environment's model. It is easy to note that the correction is mainly ruled by the value of the coupling constant; see, for example, the blue dotted line and the blue dots. Both lines correspond to the same value of $\gamma_0 = 0.5$, but different frequency cutoff Λ. The same occurs with the red line and the red asterisks with $\gamma_0 = 0.1$. The magenta triangles share the same value of γ_0, but the values of λ and d are interchanged with respect to the red line and asterisks. Finally, the black lines indicate smaller values of γ_0. When s increases (even for the set of parameters γ_0, λ, and d, for which decoherence is negligible), the effect of the memory effects on the geometric phase of the nonequilibrium environment is stronger and the correction to the geometric phase is bigger. After this, the non-Markovian correction to the phase presents an abrupt slope, leading to bigger corrections compared to the equilibrium baths considered above. By an analytical comparison of the perturbative Eqs. (11) and (12), it can be seen that the correction induced by nonequilibrium environments is similar to that of the thermal ones for ohmicity parameters $-1 < s < 1$ (for equal coupling constant γ_0 and environment cutoff Λ). However, as s increases, the correction induced by nonequilibrium environments gets bigger in comparison, becoming particularly important for $s > 3$. This confirms the importance of the memory effects of the environment in the correction of the geometric phase, as already stated in Refs. [26,36,37], where the authors studied the memory effects of the environment for different two-level systems.

As in the previous section, we can estimate the correction to the unitary geometric phase in an expansion in powers of the coupling between system and environment γ_0. In this case, the geometric phase can be approximated analytically as

$$\phi_G \approx \phi_u + \gamma_0\, \Gamma[s+1] \sin^2\theta \cos\theta, \qquad (12)$$

where we have neglected $O(\gamma_0\Omega/\Lambda)$ terms. This approximate expression matches very well with the low-γ_0 corrections given in Fig. 7. This expression also gives the leading correction to the phase found in Ref. [16] for $s = 1$ and $s = 3$. On general grounds, we can see that the nonequilibrium bath is more harmful and induces a bigger correction on the geometric phase for all values of θ than the equilibrium environment.

IV. CONCLUSIONS

The geometric phase of quantum states could have a potential application in holonomic quantum computation since the study of spin systems effectively allows us to contemplate the design of a solid-state quantum computer. However, decoherence is the main obstacle to overcome. Furthermore, in most cases of practical interest, quantum systems are subjected to many noise sources with different amplitudes and correlation times, corresponding *de facto* to a nonequilibrium environment.

We have computed the correction of the geometric phase under the presence of structured reservoirs. We have defined the spectral density as a function of the ohmicity parameter s. This is advantageous because it allows one to study subohmic, ohmic, and supraohmic environments in the same approach. One could wonder if the correction to the geometric phase is due to the non-Markovianity of the environment or simply to a stronger effective interaction among the system and the environment.

First, we have considered the structured reservoirs to be composed of a set of harmonic oscillators at zero temperature. In the case of thermal equilibrium environments, we have shown that for small values of the ohmicity $s \leqslant 2$, the hierarchy on the correction of the unitary geometric phase is mainly due to the value of γ_0. This means that the stronger the coupling to the environment, the stronger the correction induced on the geometric phase as expected. However, for $s > 2$, even though the hierarchy is repeated, we can see that the non-Markovianity also has an important role in the correction. We have checked this argument by considering the decoherence factor for different values of s and observing that the correction to the geometric phase is strongly influenced by this coefficient.

Second, we have considered the structured environments to be modeled by a type of nonequilibrium environment. The nonequilibrium feature is represented by a nonstationary random function corresponding to the fluctuating transition frequency between two quantum states coupled to the surroundings. We have shown that the diffusion coefficients always have negative values for the same period of time for $s \geqslant 1$, which means a nonmonotonic behavior of the decoherence factors. This allows one to consider these nonequilibrium environments as non-Markovian ones for all values of s. In this framework, we have computed the nonunitary geometric phase for the qubit in a quasicyclic evolution under the presence of these particular nonequilibrium environments, both numerically and analytically. When comparing the correction of the geometric phase induced by these environments and the thermal ones, memory effects of the environments with random noise are more harmful and the correction induced is bigger, particularly for values of $s \geqslant 3$.

Finally, these kinds of environments could become a proper experimental setup for the observation of the geometric phase. Our work on the assessment of the environmental effect on the geometric phase, especially in the non-Markovian regime (either for equilibrium or nonequilibrium environments), could be of great importance in using the geometric phases in two-level systems to implement the quantum gates. The results presented in this paper can also provide a clue to observe the GP in a two-level system.

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