Double Compton scattering in a constant crossed field

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Two-photon emission of an electron in an electromagnetic plane wave of vanishing frequency is calculated. The unpolarized probability is split into a two-step process, which is shown to be exactly equal to an integration over polarized subprocesses, and a one-step process, which is found to be dominant over the formation length. The assumptions of neglecting spin and simultaneous emission, commonly used in numerical simulations, are discussed in light of these results.

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I. INTRODUCTION

It is well known that when an electron is accelerated by an electromagnetic field, it radiates [1]. When the wavelength of the radiation in the rest frame of the electron is much larger than the Compton wavelength, it is well described by classical electrodynamics. By photon emission we are referring to those situations in which shorter wavelengths are generated and a quantum electrodynamical description is necessary. Single-photon emission of electrons in a plane-wave background, commonly referred to as (nonlinear) Compton scattering, was first calculated five decades ago in monochromatic waves [2,3] and some time later, the effects of finite pulse shapes [4–9], electron spin [10], and photon [11] and external-field polarization [12,13] were investigated. Single Compton scattering has been experimentally observed in the weakly nonlinear regime [14,15] and advances in laser technology have motivated the study of two-photon emission in a pulsed-plane-wave background with possible experimental signatures having been discussed in the literature [16,17] (a review of strong-field QED effects can be found in [18–20]).

In the present paper, we will calculate two-photon emission of an electron in an electromagnetic plane wave of vanishing frequency, the so-called constant crossed field. On the one hand, this allows an analysis of two-photon emission in the nonperturbative region of large quantum nonlinearity parameter. On the other hand, the constant-crossed-field background is the one used overwhelmingly in current numerical simulations that combine particle-in-cell propagation of charged particles with Monte Carlo generation of quantum events such as photon emission [21–27]. These simulations iterate single-vertex processes to approximate higher-order ones and our double-vertex calculation can assess the faithfulness of this approximation. In their calculation of the two-loop electron mass operator in a constant crossed field [28], Morozov and Ritus produced expressions for the total probability of polarized two-photon emission. However, their restriction to total probabilities, brevity of exposition, and emphasis on infrared behavior differ significantly from the intention of the present article to study the role of spin and virtual processes in more depth, which are currently neglected by computational simulations.

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II. POLARIZED SINGLE COMPTON SCATTERING

We begin by calculating the probability for single Compton scattering, taking into account the polarization of all incoming and outgoing particles. Although our analysis is for electrons, analogous arguments apply to positrons. We consider the process

$$e^- \to e^- + \gamma \tag{1}$$

in a constant-field background where e^- refers to an electron and γ to a photon. The corresponding Feynman diagram is given in Fig. 1, where we note that p and q are the incoming and outgoing electron's four-momentum, respectively, and s_p and s_q the corresponding spin four-vectors, with k and ε_k the emitted photon's four-vector and polarization four-vector where $s_p^2 = s_q^2 = \varepsilon_k^2 = -1$.

The scattering matrix for single-photon emission can be written as [29]

$$S_{fi} = ie\int d^4x\, \overline{\psi}_{q,\sigma_q}(x)\frac{\sqrt{4\pi}\,\not{\varepsilon}_k^* e^{i\mathbf{k}\cdot\mathbf{x}}}{\sqrt{2k^0V}}\psi_{p,\sigma_p}(x), \tag{2}$$

where $\not{v} = \gamma\cdot v$ for a four-vector v, γ are the gamma matrices [29], σ is a spin index, and $e > 0$ is the elemental positron charge and m its mass, with V the normalization volume (unless otherwise stated, we work with $\hbar = c = 1$). The wave function for an electron in a plane-wave electromagnetic background is given by the Volkov solution to the Dirac equation [30]

$$\psi_{p,\sigma_p}(x) = \left[1 + \frac{e\not{\varkappa}\not{A}[\varphi(x)]}{2(\varkappa\cdot p)}\right]\frac{u_{p,\sigma_p}}{\sqrt{2p^0V}}e^{iS[p,\varphi(x)]}, \tag{3}$$

$$S(p,\varphi) = -p\cdot x - \int_0^{\varphi} d\phi\left[\frac{ep\cdot A(\phi)}{\varkappa\cdot p} - \frac{e^2A^2(\phi)}{2(\varkappa\cdot p)}\right], \tag{4}$$

where the external-field phase $\varphi = \varkappa\cdot x$ for external-field wave vector $\varkappa$ and vector potential $A^\mu(\varphi)$ and the normalization of the electron spinor u_p will be explained shortly. The probability for polarized single-photon emission P_γ is then given by

$$P_\gamma = V^2\int \frac{d^3k}{(2\pi)^3}\frac{d^3q}{(2\pi)^3}\mathrm{tr}|S_{fi}|^2. \tag{5}$$

When squaring the trace of $|S_{fi}|^2$, the electron spin is introduced using a standard method of writing the spin-density

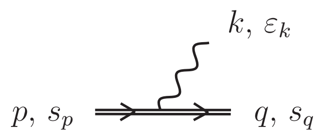

FIG. 1. Feynman diagram for single Compton scattering.

matrix for electron momentum p as [31]

$$u_{p,\sigma_p}\overline{u}_{p,\sigma_p'} = \tfrac{1}{2}[(\not{p}+m)(1-\gamma^5\not{s}_p)]_{\sigma_p\sigma_p'}. \qquad (6)$$

To calculate P_γ, we employ the method developed by Nikishov and Ritus (see [2,18] and a more detailed application to single Compton scattering in [11]). A key part of this method is that after momentum conservation has been applied to the integration of one outgoing particle's momentum in Eq. (5), the remaining integrand is independent of the projection of remaining outgoing particle momenta on the vector potential. This divergent integral is crucially reinterpreted as an integral over phase in the following way. Using the preceding equations, Eq. (2) can be written in the form

$$S_{fi} = ie\int d^4x\, K(\varphi)e^{i(q+k-p)\cdot x}, \qquad (7)$$

where the net contribution to the integral from $K(\varphi)$ occurs for phases around the stationary phase

$$\varphi_* = \frac{p\cdot a}{e\,a^2}\left(-1+\frac{\varkappa\cdot p}{\varkappa\cdot k}\frac{k\cdot a}{p\cdot a}\right). \qquad (8)$$

The divergent remaining integral can then be rewritten

$$\int d(k\cdot a)\Big|_{\text{on shell}} = \int\frac{d\varphi_*}{J}, \qquad (9)$$

where the prescription on shell refers to momentum conservation having been applied to the integral in q and $J = |\partial\varphi_*/\partial(k\cdot a)|$ is the Jacobian. We have in mind a calculation in a constant background. In this case, the integrand is independent of both φ_* and φ and since they are members of the same interval, an integration over one is equivalent to an integration over the other and the asterisk is hereafter dropped. When selecting a basis for the electron spin and photon polarization vectors, it is advantageous to maintain this symmetry so that the Nikishov-Ritus method can be consistently applied. For polarized photons but unpolarized electrons, a basis in which $\varepsilon_k\varkappa = 0$ is sufficient to preserve the symmetry in momentum. When electron spin is also included, a basis in which $s_p\varkappa = 0$ greatly simplifies expressions, but is insufficient alone to preserve the momentum symmetry. A key difference between electron spin and photon polarization is that the precession of the former due to the external field is already included to all orders by using the dressed Volkov propagator, whereas the evolution of the latter, described by a dressed photon propagator, is a higher-order effect in $\alpha = e^2 \approx 1/137$, the fine-structure constant, the first corrections of which enter at $O(\alpha^2)$ [32–35]. Since single Compton scattering is to first order in α, the evolution of photon polarization does not enter the calculation. Conversely, the evolution of the electron spin *can* be described by invoking the Bargmann-Telegedi-Michel equation [36]

$$\frac{ds_p}{d\tau} = \frac{e}{m}\left[\frac{g}{2}F\cdot s_p + \left(\frac{g}{2}-1\right)\frac{(s_p\cdot F\cdot p)p}{m^2}\right], \qquad (10)$$

where F is the Faraday tensor [1], $g\approx 2$ is the electron's gyromagnetic ratio [37,38], and τ is the proper time. By choosing a spin basis such that $ds_p/d\tau = 0$ as well as $s_p\varkappa = 0$, the momentum symmetry is preserved and ensures that the Nikishov-Ritus method can be applied without obstacle.

Our choices of vector potential, polarization basis, and spin basis that allow the Nikishov-Ritus method to be straightforwardly applied are

$$A(\varphi) = a^{(1)}g^{(1)}(\varphi) + a^{(2)}g^{(2)}(\varphi), \qquad (11)$$

$$\varepsilon_k^{(1,2)} = a^{(1,2)} - \frac{k\cdot a^{(1,2)}}{k\cdot\varkappa}\varkappa, \qquad (12)$$

$$\zeta_p = a^{(2)} - \frac{p\cdot a^{(2)}}{p\cdot\varkappa}\varkappa, \qquad (13)$$

where $a^{(1)}\cdot a^{(2)} = 0$, $a^{(1)}\cdot a^{(1)} = a^{(2)}\cdot a^{(2)} = -1$, and $g^{(2)}(x) = 0$. That these choices ensure the spin basis ζ_p does not precess can be seen from $d\zeta_p/d\tau = 0$ or in the rest frame of the electron $\boldsymbol{\zeta}_p\wedge\mathbf{B} = \mathbf{0}$, where $\mathbf{B}$ is the external-magnetic-field vector.

We now specify the calculation to a constant-crossed-field background by choosing $g^{(1)}(\varphi) = (m\xi/e)\varphi$, where $\xi = e|p\cdot F|/m|\varkappa\cdot p|$ is the classical nonlinearity parameter [39], which can be written as $\xi = m\mathcal{E}/\varkappa^0$, with $\mathcal{E} = E/E_{\text{cr}}$ the ratio of the electric-field amplitude E to the critical field $E_{\text{cr}} = m^2/e$. In a constant field, ξ is formally infinite as the limit $\varkappa^0\to 0$ has been taken to arrive at expressions for observables. In a constant-crossed-field background, total rates are functions of another gauge and relativistic invariant referred to as each particle's quantum nonlinearity parameter, which for a momentum p is given by $\chi_p = e|p\cdot F|/m^3$. Recognizing $\xi\varphi$ as being independent of the limit $\varkappa^0\to 0$, let us define the rate of single Compton scattering per unit normalized external-field phase $R_\gamma = P_\gamma/\xi\int d\varphi$. Suppose we expand the polarization and spins as

$$\varepsilon_k = c_1\varepsilon_k^{(1)} + c_2\varepsilon_k^{(2)}, \qquad (14)$$

$$s_p = \sigma_p\zeta_p, \quad s_q = \sigma_q\zeta_q, \qquad (15)$$

where $c_{1,2}\in\{0,1\}$ and $\sigma_{p,q}\in\{-1,0,1\}$; we then find

$$R_\gamma = -\alpha\int_0^{\chi_p} d\chi_k\,[C^{\cdot}\mathrm{Ai}(z) + C'\mathrm{Ai}'(z) + C_1\mathrm{Ai}_1(z)], \qquad (16)$$

where $\mathrm{Ai}(x) = \frac{1}{\pi}\int_0^\infty dk\cos(kx + k^3/3)$ is the Airy function, $\mathrm{Ai}'(x)$ its derivative, $\mathrm{Ai}_1(x) = \int_x^\infty dk\,\mathrm{Ai}(k)$, and

$$z = \left[\frac{\chi_k}{\chi_p(\chi_p-\chi_k)}\right]^{2/3},$$

$$C^{\cdot} = \frac{z}{\chi_p^2}\left[(\sigma_p+\sigma_q)\left(\chi_k - 2c_k^2\chi_p\right) - 2\left(1-c_k^2\right)\sigma_q\chi_k\right],$$

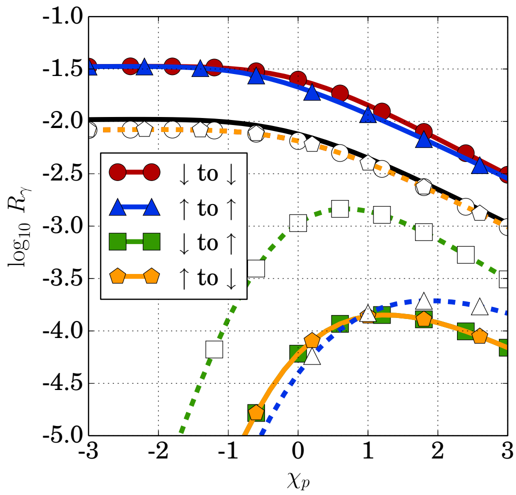

FIG. 2. (Color online) Total rate of polarized single Compton scattering R_γ. The closed (open) markers on solid (dashed) lines refer to a photon scattered into a polarization state parallel (perpendicular) to the external field where the marker-free solid line refers to the unpolarized rate.

$$C' = \frac{\chi_k z^{1/2}}{\chi_p^2}(1 - c_\sigma c_\delta \sigma_p \sigma_q) + \frac{2}{z\chi_p^2}(1+\sigma_p\sigma_q)(1 - c_\sigma c_\delta),$$
$$C_1 = \frac{1}{\chi_p^2}\left[1 + \sigma_p\sigma_q - \frac{c_\sigma c_\delta \sigma_p \sigma_q \chi_k^2}{\chi_p(\chi_p - \chi_k)}\right], \tag{17}$$

where the single polarization parameter $c_k = c_1 = \sqrt{1 - c_2^2}$ has been introduced with $2c_\sigma = c_1 + c_2$ and $c_\delta = c_2 - c_1$ simplifying notation. The rate for single Compton scattering with unpolarized electrons but a polarized photon [11] can be recovered if one takes the limit in Eq. (16) of zero spin $\sigma_p, \sigma_q \to 0$. If one then averages over photon polarizations, the totally unpolarized rate [2] is acquired. We note the appearance of an Ai($\cdot$) function in the integrand of R_γ, which is only present if electron spin is taken into account and is completely absent in standard unpolarized calculations. This term will appear again in the double Compton scattering calculation.

The rate of single Compton scattering for different combinations of initial and final polarizations of particles is illustrated in Fig. 2, where solid (dashed) lines correspond to emitted photons in a polarization state $\varepsilon_k = \varepsilon_k^{(1)}$ ($\varepsilon_k = \varepsilon_k^{(2)}$) and ↑ (↓) to spin states $\sigma = 1$ ($\sigma = -1$). We notice that in this nonprecessing spin basis, the no-flip polarization channels in which the spin of the electron is unchanged after photon emission are in general favored more than the spin-flip channels, which are suppressed, particularly for small χ_p.

III. DOUBLE COMPTON SCATTERING

Let us now turn to the calculation of double photon emission by an electron in a plane-wave field. We are considering the process

$$e^- \to e^- + \gamma + \gamma' \tag{18}$$

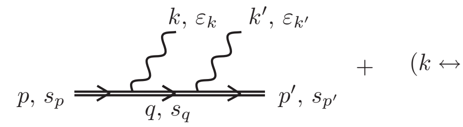

FIG. 3. Feynman diagram for double Compton scattering. The prescription on the exchange term ($k \leftrightarrow k'$) also applies to swapping the polarization vectors.

and the corresponding Feynman diagram is given in Fig. 3. The transition matrix element for this process is

$$S_{fi} = \overrightarrow{S_{fi}} + \overleftarrow{S_{fi}}, \tag{19}$$

$$\overrightarrow{S_{fi}} = -e^2 \int d^4x' d^4x\, \bar{\psi}_{p',\sigma_{p'}}(x') \frac{\sqrt{4\pi}\,\not{\varepsilon}_{k'}'^*}{\sqrt{2Vk'^0}} e^{ik'x'} G(x',x)$$
$$\times\, e^{ikx} \frac{\sqrt{4\pi}\,\not{\varepsilon}_k^*}{\sqrt{2Vk^0}} \psi_{p,\sigma_p}(x), \tag{20}$$

$$G(x',x) = \int \frac{d^4q}{(2\pi)^4}\left[1 + \frac{e\not{\varkappa}\not{A}(\varphi')}{2\varkappa\cdot q}\right] e^{iS(q,\varphi')}$$
$$\times \frac{\not{q} + m}{q^2 - m^2 + i0} e^{-iS(q,\varphi)}\left[1 + \frac{e\not{A}(\varphi)\not{\varkappa}}{2\varkappa\cdot q}\right], \tag{21}$$

where $\overrightarrow{S_{fi}}$ corresponds to the first diagram in Fig. 3 and $\overleftarrow{S_{fi}}$ to exchanging $(k,\varepsilon_k) \leftrightarrow (k',\varepsilon_{k'})$. The method of calculation is similar to that in the previous section, with the added complication of having an extra diagram due to bosonic exchange symmetry of identical outgoing photons as well as a fermionic propagator (a detailed example of the Nikishov-Ritus approach being applied to a two-vertex process in a constant crossed background is given in the calculation of electron-seeded pair creation in [40]). We will confine ourselves to calculating the unpolarized double Compton scattering probability $P_{\gamma\gamma}$,

$$P_{\gamma\gamma} = \frac{1}{4}V^3 \int \frac{d^3p'}{(2\pi)^3}\frac{d^3k}{(2\pi)^3}\frac{d^3k'}{(2\pi)^3}\mathrm{tr}|S_{fi}|^2, \tag{22}$$

with the prefactor including an average over initial electron spins and symmetry factor due to identical diagrams. The modulus-squared amplitude contains each exchange term modulus squared plus interference terms

$$|S_{fi}|^2 = |\overrightarrow{S_{fi}}|^2 + |\overleftarrow{S_{fi}}|^2 + \overleftarrow{S_{fi}}\overrightarrow{S_{fi}}^* + \overrightarrow{S_{fi}}\overleftarrow{S_{fi}}^*. \tag{23}$$

Now suppose that the p' integral in Eq. (22) is performed by evaluating the standard total momentum-conserving δ function that arises in scattering matrix calculations and that the integrals over k and k' remain. In the standard fashion, these integrals can be reinterpreted as

$$\int d(k\cdot a)\, d(k'\cdot a)\bigg|_{\text{on shell}} = \int \frac{d\varphi_+ d\varphi_-}{J}. \tag{24}$$

Here $\varphi_+ = (\varphi_{x'} + \varphi_x)/2$ is the average of the stationary phases in the function describing the probability of photon emission at space-time points x and x' and hence corresponds to the center in phase between two emissions and $\varphi_- = \varphi_{x'} - \varphi_x > 0$ is the phase the electron travels between emissions, where

$J = |\partial(\varphi_+,\varphi_-)/\partial(k\cdot a,\, k'\cdot a)|$ is the corresponding Jacobian. Part of the integrand is completely independent of both these phases and we term this the two-step process. Part of the integrand depends only on φ_- and we term this the one-step process. The remaining part of the integrand depends on both phases. Since these phase integrations are formally infinite in a constant crossed field, the part that depends on both phases and gives a finite answer will be dropped from the calculation. It can be shown [40] that this neglected part corresponds to the interference terms in Eq. (23). As the total probability includes an integration over both photon momenta, one can then replace $|S_{fi}|^2 \to 2|\overrightarrow{S_{fi}}|^2$ in the total probability integrand. If we again define the rate $R_{\gamma\gamma} = P_{\gamma\gamma}/\xi\int d\varphi_+$ where $\varphi = \varphi_x$ is the phase of the first emission the total rate becomes

$$R_{\gamma\gamma} = R_{\gamma\gamma}^{(2)} + R_{\gamma\gamma}^{(1)}, \tag{25}$$

where the superscripts indicate the two- and one-step rates accordingly and

$$R_{\gamma\gamma}^{(2)} = \mathcal{I}_{\gamma\gamma}^{(2)}\,\xi\int d\varphi_-, \tag{26}$$

$$R_{\gamma\gamma}^{(1)} = \mathcal{I}_{\gamma\gamma}^{(1)}, \tag{27}$$

where the quantities $\mathcal{I}_{\gamma\gamma}$ are free of divergences associated with the infinite expanse of the background and will be referred to as the dynamical part of the rate in contrast to the space-time factors, such as integrations over the external-field phase.

A. Two-step double Compton scattering

The two-step process actually comprises terms from both the on- and off-shell parts of the fermion propagator, where off-shell terms are essential to preserve causality [40]. Therefore, it is incorrect to refer to the two-step process as the on-shell part. We find

$$\begin{aligned}\mathcal{I}_{\gamma\gamma}^{(2)} = -\alpha^2\int d\chi_k d\chi_{k'}[&C^{\cdot\cdot}\mathrm{Ai}(z)\mathrm{Ai}(z')\\ &+C''\mathrm{Ai}'(z)\mathrm{Ai}'(z')+C'^{1}\mathrm{Ai}'(z)\mathrm{Ai}_1(z')\\ &+C^{1'}\mathrm{Ai}_1(z)\mathrm{Ai}'(z')+C^{11}\mathrm{Ai}_1(z)\mathrm{Ai}_1(z')],\end{aligned} \tag{28}$$

where we have defined

$$z = \left[\frac{\chi_k}{\chi_p\chi_q}\right]^{2/3}, \quad z' = \left[\frac{\chi_{k'}}{\chi_{p'}\chi_q}\right]^{2/3}; \tag{29}$$

$$C^{\cdot\cdot} = -\frac{zz'\chi_p\chi_{p'}}{(\chi_p\chi_q)^2}, \tag{30}$$

$$C'' = \frac{1}{(\chi_p\chi_q)^2}\left(\frac{2}{z'}+\chi_{k'}z'^{1/2}\right)\left(\frac{2}{z}+\chi_k z^{1/2}\right), \tag{31}$$

$$C'^{1} = \frac{1}{(\chi_p\chi_q)^2}\left(\frac{2}{z}+\chi_k z^{1/2}\right), \tag{32}$$

$$C^{1'} = \frac{1}{(\chi_p\chi_q)^2}\left(\frac{2}{z'}+\chi_{k'}z'^{1/2}\right), \tag{33}$$

$$C^{11} = \frac{1}{(\chi_p\chi_q)^2}, \tag{34}$$

with $\chi_q = \chi_p - \chi_k$ and $\chi_{p'} = \chi_p - \chi_k - \chi_{k'}$. Written in this way, upon comparison with the probability for unpolarized single-photon scattering [averaging over photon polarization and setting $\sigma_p,\sigma_q \to 0$ in Eq. (17)], one can see that the two-step process is the integral over the product of single Compton scattering. We find that the unpolarized two-step rate can indeed be exactly factorized in terms of single Compton scattering processes, when the intermediate electron's spin is taken into account and assumed in an unchanged state when the second photon is emitted (we note here the relevance of choosing a nonprecessing spin basis)

$$R_{\gamma\gamma}^{(2)} = \frac{1}{2}\sum_{\sigma_q}\int d\chi_q\frac{\partial R_\gamma(\chi_p)}{\partial\chi_q}R_\gamma(\chi_q)\frac{\xi}{2}\int d\varphi_-. \tag{35}$$

B. One-step double Compton scattering

The one-step process involves an extra integration over a variable related to the virtuality of the propagating electron

$$\mathcal{I}_{\gamma\gamma}^{(1)} = -\frac{\alpha^2}{4\pi}\int d\chi_k d\chi_{k'}dt\,\frac{\mathcal{A}(t)+\mathcal{A}(-t)-2\mathcal{A}(0)}{t^2}, \tag{36}$$

$$\begin{aligned}\mathcal{A}(t) = {}&C_t^{\cdot\cdot}\mathrm{Ai}(z_t)\mathrm{Ai}(z_t')\\ &+C_t''\mathrm{Ai}'(z_t)\mathrm{Ai}'(z_t')+C_t'^{1}\mathrm{Ai}'(z_t)\mathrm{Ai}_1(z_t')\\ &+C_t^{1'}\mathrm{Ai}_1(z_t)\mathrm{Ai}'(z_t')+C_t^{11}\mathrm{Ai}_1(z_t)\mathrm{Ai}_1(z_t'),\end{aligned} \tag{37}$$

$$z_t = z+\frac{t}{z^{1/2}}, \quad z_t' = z'-\frac{t}{z'^{1/2}}, \tag{38}$$

where we have defined

$$\frac{C_t^{\cdot\cdot}}{C^{\cdot\cdot}} = 1-\frac{t}{2}\frac{\chi_q(\chi_p+\chi_q)(\chi_q+\chi_{p'})}{\chi_k\chi_{k'}}, \tag{39}$$

$$\frac{C_t''}{C''} = 1, \tag{40}$$

$$\frac{C_t'^{1}}{C'^{1}} = 1+\frac{t\chi_k}{2}, \tag{41}$$

$$\frac{C_t^{1'}}{C^{1'}} = 1-\frac{t\chi_{k'}}{2}, \tag{42}$$

$$\frac{C_t^{11}}{C^{11}} = 1+t\frac{\chi_q^2-\chi_k\chi_{k'}}{2\chi_q}+\frac{t^2}{4}(\chi_p\chi_{p'}-\chi_k\chi_{k'}). \tag{43}$$

After integrating over the propagator variable t and over $\chi_{k'}$, the remaining integral in χ_k diverges $\sim 1/\chi_k$ for $\chi_k \to 0$. This well-known infrared divergence was reported in other calculations in double Compton scattering [16,41,42] and should be canceled when self-energy corrections are included [28].

It can be shown [40] that the one-step process can be written as a term originating from the interference between two- and one-step parts of the amplitude plus a term originating solely from the one-step part of the amplitude. Here, as in electron-seeded pair creation in a constant crossed field, the one-step probability is negative. However, since the phase factor multiplying the two-step term is formally divergent, the total probability is non-negative. Unlike for electron-seeded pair creation, we find no threshold value of χ_p above which the total one-step process becomes positive.

IV. APPROXIMATIONS USED IN SIMULATION

We turn now to a comparison of the analytical results for double Compton scattering with approximations used in numerical simulations. In particular, we investigate two assumptions. First is the neglect of electron spin, which we found essential to correctly factorize the two-step part of double Compton scattering and which produced new terms in the integrand. Second is the neglect of the one-step process, also known as simultaneous two-photon emission by an electron.

A. Electron spin

As an example of employing the constant-crossed-field approximation, the unpolarized probability of the two-step process in a slowly varying external field can be written as a double iteration of single-photon emission

$$P^{(2)}_{\rm CCF} = \frac{1}{2}\sum_{\sigma_q}\int d\varphi_{\xi'}d\varphi_{\xi}d\chi_q\; P_\gamma[\chi_q]\frac{\partial P_\gamma[\chi_p,\chi_q]}{\partial\chi_q}, \qquad (44)$$

where we have allowed the external field to depend on the phase by defining $\varphi_\xi = \varphi\,\xi(\varphi)$, $\varphi_{\xi'} = \varphi'\,\xi(\varphi')$, $\varphi_{\xi'} > \varphi_\xi$, $\chi_p = \chi_p(\varphi_\xi)$, $\chi_q = \chi_q(\varphi_{\xi'})$, and $\chi_q < \chi_p$. If the external field is taken to be exactly constant $\xi(\varphi) = \xi$, Eq. (44) is synonymous with Eq. (35). The two-step process can therefore be consistently included in numerical simulation by allowing for single Compton scattering to occur multiple times. To investigate the importance of including electron spin, we plot the dynamical part of two-step double Compton scattering $\mathcal{I}^{(2)}_{\gamma\gamma}$ using Eq. (35) alongside the case when the propagating electron is forced to be unpolarized by setting $\sigma_q \to 0$ in Eq. (35), which we label $\overline{\mathcal{I}}^{(2)}_{\gamma\gamma}$. These are then compared with the $\chi_p \to 0$ asymptotic limit of Morozov and Ritus's polarized calculation and the $\chi_p \to \infty$ asymptotic limit derived from the current unpolarized analysis

$$\widetilde{\mathcal{I}}^{(2)}_{\gamma\gamma}(\chi_p) = \begin{cases} \frac{25\alpha^2}{12}, & \chi_p < 1 \\ \frac{196\alpha^2}{135}\,\frac{\pi^{3/2}}{3^{1/6}\Gamma(1/3)\Gamma(5/6)}\left(\frac{2}{\chi_p^2}\right)^{1/3}, & \chi_p > 1, \end{cases} \qquad (45)$$

where $\chi_p < 1$ ($\chi_p > 1$) is the $1/\chi_p \to \infty$ ($\chi_p \to \infty$) asymptotic limit. In Fig. 4 we note that the intermediate electron's spin seems to make very little difference to the total probability for double Compton scattering. We highlight the property of Compton scattering in a constant crossed field, that $\partial P_\gamma/\partial\chi_k \sim \chi_k^{-2/3}$ for $\chi_k \to 0$. Although the differential probability diverges as $\chi_k \to 0$, the total probability remains finite (the softening of this well-known infrared divergence has recently been studied in [43–45]). How to take into account this divergent number of photons is handled in a variety of ways by numerical simulations, but often a hard energy or χ_k cutoff is introduced, below which the effects of the emitted radiation are included using the classical equations of electrodynamics. By introducing a cutoff in our analysis, for example, neglecting photons with $\chi_k < 0.1$ (several orders of magnitude larger than what is usually considered [46]), the effect of the electron spin to the total probability is still only at the few percent level. Therefore, it seems that, within this approach, treating the propagating particle as a scalar in numerical simulations of multiphoton emission of electrons in intense laser pulses is a consistent approximation.

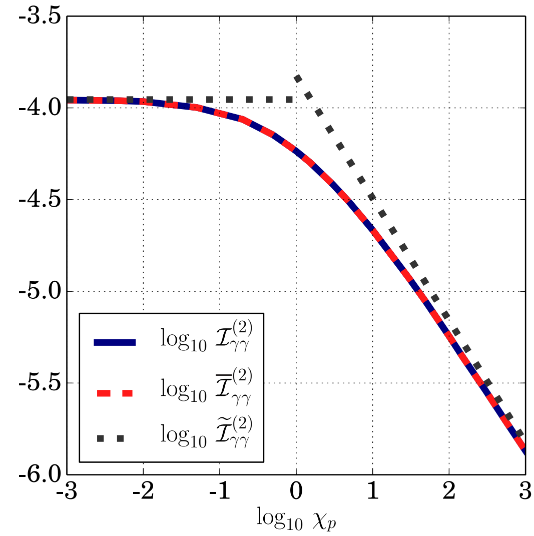

FIG. 4. (Color online) Dynamical part of the two-step probability $\mathcal{I}^{(2)}_{\gamma\gamma}$ with the approximation of using unpolarized electrons $\overline{\mathcal{I}}^{(2)}_{\gamma\gamma}$ and the asymptotic limits $\widetilde{\mathcal{I}}^{(2)}_{\gamma\gamma}$.

B. Simultaneous photon emission

With simultaneous photon emission, we are referring to the one-step process given by the integral in Eq. (36). It has already been commented that this is divergent and negative. Therefore, comparison of the factorized two-step process with the rest of the probability of two-photon emission is not possible at $O(\alpha^2)$ without including self-energy terms. Although it makes little sense to compare two- and one-step processes without including self-energy terms, as they may also appreciably affect photon emission for large χ values, we can assess how much of the one-step process is neglected in simulation codes when a cutoff in the emitted photons' χ parameter is used. For example, in the simulation of photon emission by a single electron in [46], a cutoff of $\chi_k = \Delta_k = 10^{-5}$ was chosen. In Fig. 5 we compare the total probability of the one- and two-step processes using χ_k and $\chi_{k'}$ cutoffs of $\Delta_k = \Delta_{k'} = \Delta = 10^{-5}$. For $\chi_p \gg 1$, $\mathcal{I}^{(1)}_{\gamma\gamma}$ agrees with the scaling and sign given by Morozov and Ritus for this limit, although we were unable to compare results in a quantitative way. We note that the absolute ratio of dynamical parts of the one- to the two-step probabilities becomes larger than unity already at $\chi_p \approx 0.1$ and linearly increases to more than an order of magnitude for $\chi_p \gtrsim 50$. After repeating the calculation for a range of cutoffs $\Delta \in [10^{-7}, 10^{-2}]$, although the exact ratio is weakly cutoff dependent, for $\chi_p \in [1,1000]$, the linear increase in the ratio appears cutoff independent. Also in Fig. 5 is the total rate

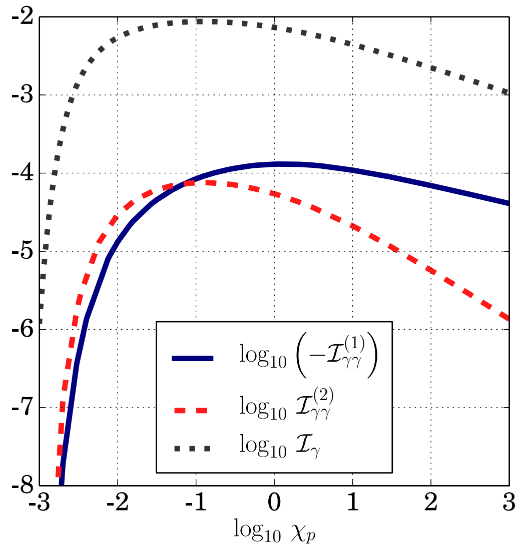

FIG. 5. (Color online) Absolute dynamical part of the one-step probability $\mathcal{I}^{(1)}_{\gamma\gamma}$ compared to equivalent two-step $\mathcal{I}^{(2)}_{\gamma\gamma}$ and single Compton scattering $\mathcal{I}^{(1)}_{\gamma} = R^{(1)}_{\gamma}$ quantities when an infrared cutoff $\Delta_k = \Delta_{k'} = 10^{-5}$ is included.

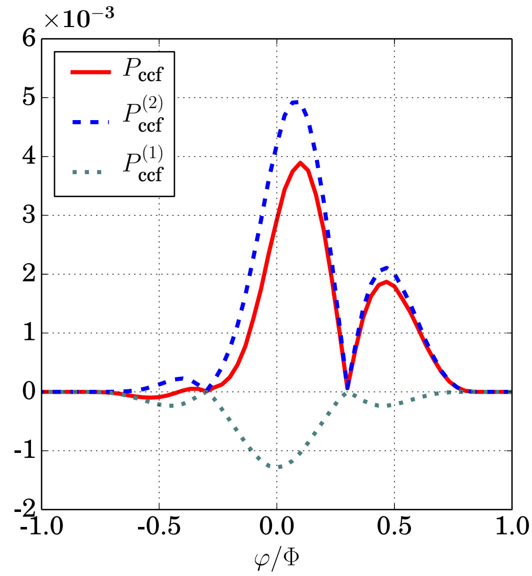

FIG. 6. (Color online) Few-cycle pulse with field strength equivalent to 500 TeV focused to a focal width of 10 μm with a 910-nm wavelength with peak $\xi = 10$ and pulse duration of 5 fs counterpropagating with 10-GeV seed electrons.

of single Compton scattering, which is dominant for all χ_p investigated.

Over the formation length $1/\xi\varkappa^0$, the one-step process can clearly be dominant compared to the two-step process, which implies that current numerical approaches are inappropriate for simulating double photon emission in this parameter regime. For an N-cycle pulse, with a wavelength on the order of the formation length, one would have to consider the integral over the entire pulse to determine when the two-step process is dominant. We recall that the total rate for two-photon emission in a constant crossed field is given by

$$R_{\gamma\gamma} = \mathcal{I}^{(2)}_{\gamma\gamma}\frac{\xi}{2}\int d\varphi_- + \mathcal{I}^{(1)}_{\gamma\gamma} + \frac{\mathcal{I}^{(0)}_{\gamma\gamma}}{\xi\int d\varphi_-}, \tag{46}$$

where $\mathcal{I}^{(0)}_{\gamma\gamma}$ is the interference between exchange terms, neglected as the phase factor is formally infinite, but reintroduced here when discussing approximating more complicated fields as constant crossed. The constant-crossed approximation to an arbitrary field is expected to be valid when $\xi \gg 1$ and χ_p is much larger than the two electromagnetic invariants e^2F^2/m^4 and e^2FF^*/m^4 for the Faraday tensor F and its dual F^* [18]. The present results imply that the further assumption made in simulations of the rate of generating n photons being due to the n-step process is questionable for $n = 2$ when $\xi \lesssim O(10^2)$ and $\chi_p \gtrsim 1$. To demonstrate this, we use the constant-crossed-field approximation in Eq. (44) and

$$P^{(1)}_{\mathrm{CCF}} = \int d\varphi_\xi\, R^{(1)}_{\gamma\gamma}[\chi_p(\varphi_\xi)] \tag{47}$$

to estimate double Compton scattering in the field of a laser pulse $E = E_0\cos^4(\pi\varphi/2\Phi)\cos\varphi$ for pulse width $\Phi = \varkappa^0\tau$. In Fig. 6 the relative difference in photon yield due to including the one-step process is -25%, however the main qualitative difference due to including the one-step process is the instant when two-photon emission starts to become significant, which is predicted to occur a half cycle later. One could speculate whether this has implications for the dynamics of QED cascades. In electron-seeded QED cascades, single-photon processes are initially dominant and after a length $1/\alpha\xi\varkappa^0$, two-photon processes are expected to dominate. However, this length is much larger than the formation length, so again the suppression due to including virtual processes will be negligible.

Although we have seen that only when the spatial extent of the external field is less than two orders of magnitude of the formation length ($100/\xi\varkappa^0$) and $\chi_p \gtrsim 1$, the one-step process can be comparable to the two-step one, the analysis raises the question of how accurate it is to simulate an n-photon emission including just the n-step process.

V. TRANSVERSE MOMENTUM DISTRIBUTION

In a calculation of electron-seeded pair creation in a constant crossed field, it was suggested that the significantly different transverse momentum distributions could be a method to experimentally separate the one- and two-step processes [40]. The same method is investigated here for double Compton scattering. The distribution in the emitted photons' dimensionless transverse momentum $k_y = k \cdot a^{(2)}/m$ and $k'_y = k' \cdot a^{(2)}/m$ can be related to the inclusive rate by the integral

$$R^{(j)}_{\gamma\gamma} = \int dk_y\, dk'_y \frac{\partial^2 R^{(j)}}{dk_y dk'_y}, \tag{48}$$

where $j \in \{1,2\}$ for the one- and two-step rates. Figure 5 suggests that at $\chi_p > 0.1$, the one- and two-step rates could

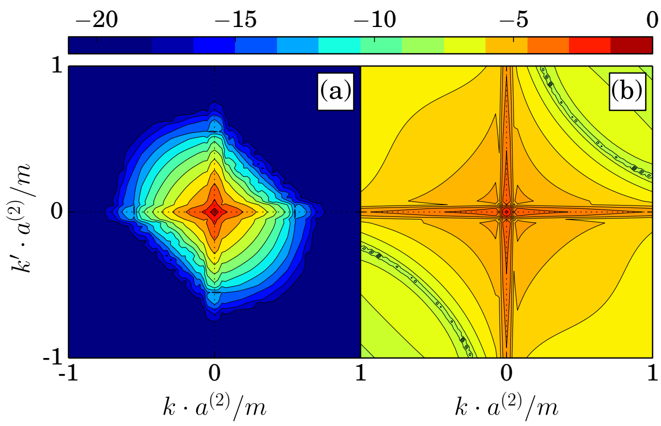

FIG. 7. (Color online) Logarithm to base 10 of the differential rate for the two-step process $\partial^2 R^{(2)}_{\gamma\gamma}/dk_y\, dk'_y$ when (a) $\chi_p = 0.1$ and (b) $\chi_p = 10$.

become distinguishable over a formation length. For this reason, we numerically calculate the transverse momentum distribution for $\chi_p = 0.1$ and 10 for the two-step and one-step processes, which are displayed in Figs. 7 and 8, respectively. We note the symmetry in the distributions under the substitution $(k_y,k'_y) \to (k'_y,k_y)$ is due to exchange symmetry. In [17] it was asserted that any photons measured outside the emission cone of single Compton scattering would reveal a double Compton scattering signal. A semiclassical explanation was given based on the modified trajectory in double Compton scattering. In Fig. 9 we have explored whether a similar approach can be used in a constant-crossed-field background to distinguish the one-step process from single Compton scattering. Although the ratio of single to double Compton scattering can be an order of magnitude larger than the factor of α one might naively expect, it appears that single Compton scattering dominates also in the transverse momentum plane.

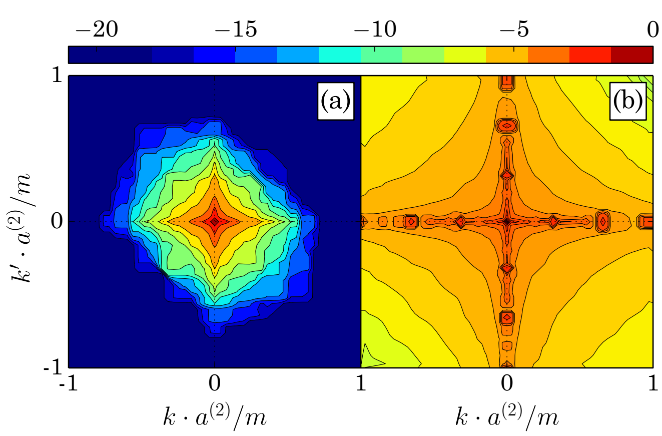

FIG. 8. (Color online) Logarithm to base 10 of the differential rate $\partial^2(-R^{(1)}_{\gamma\gamma})/dk_y\, dk'_y$ for the one-step process for (a) $\chi_p = 0.1$ and (b) $\chi_p = 10$.

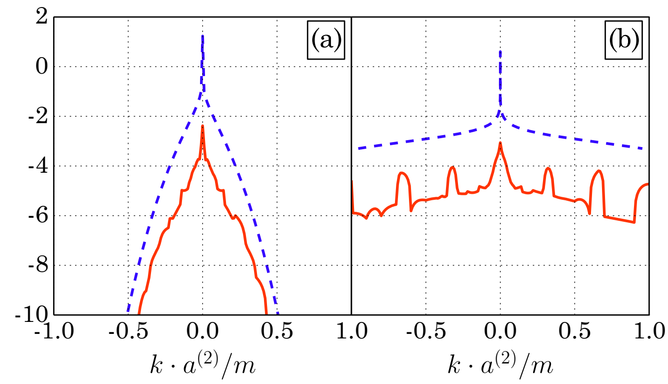

FIG. 9. (Color online) Comparison of the integrated differential rate $\int dk'_y[\partial^2(-R^{(1)}_{\gamma\gamma})/dk_y dk'_y]$ of the one-step process (solid line) compared with the differential rate for single Compton scattering $\partial R_\gamma/\partial k_y$ (dashed line) for (a) $\chi_p = 0.1$ and (b) $\chi_p = 10$.

VI. CONCLUSION

Using a nonprecessing spin basis to describe electron polarization, the probability of a spin flip following single Compton scattering in a constant crossed field was found to be suppressed. The unpolarized rate for double Compton scattering in a constant crossed field was written as a sum of a two-step process, which is exactly factorizable as single Compton scattering integrated over the longitudinal momentum of a polarized intermediate electron, and a one-step process, which is entirely negative and dominates the total probability over the formation length $1/\xi\varkappa^0 = \lambda E_{\mathrm{cr}}/E$. Regarding numerical simulation of double photon emission, we found the assumption that the intermediate electron is unpolarized, to be accurate to the few percent level, depending on the photon energy cutoff used. Numerical simulation in its current form appears unsuitable to describe higher-order processes when the quantum nonlinearity parameter χ is large and the external-field dimensions are less than a couple of orders of magnitude of the formation length, due to the effects of virtual processes. However, even when the emitted photons' transverse momentum distribution was calculated, an indication of when the virtual process of simultaneous double Compton scattering is more probable than single Compton scattering was not found. Moreover, when external-field dimensions are much larger than the single-vertex formation length, the sequential process dominates the simultaneous process.

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