Steady-state one-way Einstein-Podolsky-Rosen steering in optomechanical interfaces

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Einstein-Podolsky-Rosen (EPR) steering is a form of quantum correlation and its intrinsic asymmetry makes it distinct from entanglement and Bell nonlocality. We propose here a scheme for realizing one-way Gaussian steering of two electromagnetic fields mediated by a mechanical oscillator. We reveal that the steady-state one-way steering of the intracavity and output fields is obtainable with different cavity losses or strong mechanical damping. The conditions for achieving this asymmetric steering are found, and it shows that the steering is robust against thermal mechanical fluctuations. The present scheme can realize hybrid microwave-optical asymmetric steering by optoelectromechanics. In addition, our results are generic and can also be applied to other three-mode parametrically coupled bosonic systems.

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I. INTRODUCTION

Steering was initially introduced by Schrödinger [1] in response to the famous EPR paradox proposed by Einstein, Podolsky, and Rosen in 1935 [2]. The paradox describes that two remote observers Alice and Bob share a pair of entangled particles and one observer, say Alice, can prepare the state of Bob's particle via different types of measurements on her own particle. Steering was termed as Alice's ability to nonlocally control Bob's state via local measurements.

Recently, steering has been revisted and rigorously formulated in Refs. [3–5]. The violation of a local hidden-state model for sceptical Bob demonstrates steering from Alice to Bob. It shows that Bell-nonlocal states violating Bell inequality [6,7] are a subset of the steerable states which in turn are a subset of the inseparable states. Steering embodies a kind of quantum correlation intermediate between entanglement and Bell nonlocality. Any demonstration of the EPR paradox is also a demonstration of steering and vice versa [5]. The EPR paradox was first realized by Ou *et al.* [8] and steering has recently been experimentally realized in different systems [9–11]. Besides being of fundamental interest, quantum steering is useful for quantum information such as quantum cryptography [12].

Inherently distinct from entanglement and Bell nonlocality, steering is intrinsically asymmetric between the two observers. That is, the roles played in steering by Alice and Bob are not exchangeable. Very interestingly, recent theoretical and experimental works have verified there exists asymmetric steering, i.e., one-way steering, which allows Alice's steering the state of Bob's particle but the reverse Bob-to-Alice steering is impossible. This one-way steering reflects the asymmetry of quantum correlations. One-way Gaussian steering has been experimentally achieved by controlling unequal losses of two entangled beams [13], and theoretical studies have also revealed this asymmetric steering in several systems of continuous and discrete variables [14–19].

In this paper, we propose a scheme for realizing one-way Gaussian steering of two electromagnetic fields by optomechanics with continuous pumps. In the past decade, considerable progress has been made in the field of quantum optomechanics [20]. Quantum ground-state cooling of mechanical oscillators [21], mechanically induced squeezing [22], and optomechanical entanglement [23] have been achieved. It makes optomechanical interfaces a very promising platform for demonstrating various quantum phenomena. Our system consists of two driven electromagnetic cavities mediated by a mechanical oscillator. Photon entanglement in such optomechanical interfaces has been studied in detail [24–31]. Here we focus on the steerability and asymmetry of the photon correlations. The conditions for achieving one-way steering in different directions for the cases of weak and strong mechanical damping are identified. Our scheme can realize hybrid microwave-optical steering in an optoelectromechanical interface.

This paper is arranged as follows. In Sec. II, the model is introduced. In Sec. III, the criteria of quantum steering is reviewed. In Secs. VI and V, the properties of the intracavity and output steering are investigated in detail. In Sec. V, we give the main summary.

II. MODEL

We consider a double cavity optomechanical system in which two separate cavity fields of frequencies $\omega_{cj} (j = 1,2)$ are mediated by a mechanical oscillator at frequency ω_m. The cavity fields, driven by coherent fields of frequencies ω_{lj}, can be optical modes [32], microwave modes [33], or both [34–36] [see Fig. 1(a)]. In particular, a recent experiment has realized the reversible transfer between microwave and optical photons with a mechanical element [34,35]. This optoelectromechanical interface may allow for quantum information processing with light at different wavelengths by exploiting microwave-optical quantum correlations [26–28]. Strong photon nonlinearity can also be achieved in such a setup, as studied in Ref. [37]. In the rotating frame of the driving laser frequencies ω_{lj}, the full Hamiltonian of the three-mode optomechanical system is given by

$$\hat{H}_0 = \omega_m \hat{B}^\dagger \hat{B} + \sum_j [\delta_j \hat{A}_j^\dagger \hat{A}_j + g_{0j} \hat{A}_j^\dagger \hat{A}_j (\hat{B}_j + \hat{B}_j^\dagger) + (\varepsilon_j \hat{A}_j + \varepsilon_j^* \hat{A}_j^\dagger)], \tag{1}$$

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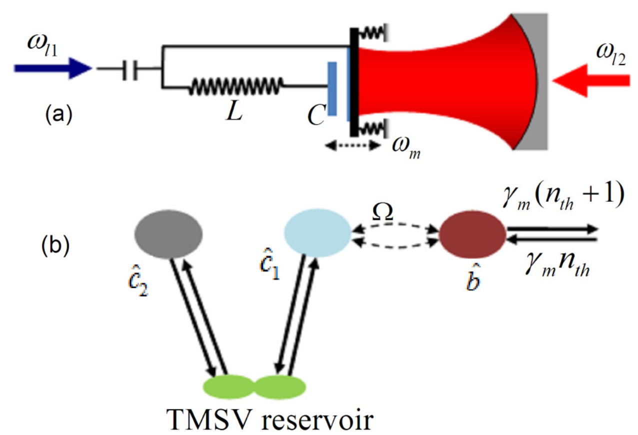

FIG. 1. (Color online) (a) The schematic plot of double-cavity optomechanics in which two separate electromagnetic fields (e.g., microwave and optical fields) are mediated by a mechanical oscillator vibrating at frequency ω_m. The two cavity fields are driven, respectively, on the red and blue sidebands of the driving fields. (b) After the squeezing transformation of Eq. (17), the composite mode $\hat{c}_2$ is decoupled to the mechanical mode $\hat{b}$ which interacts with the composite mode $\hat{c}_1$ via a beam-splitter-like interaction and meanwhile the two composite modes are coupled to an effective reservoir in a two-mode squeezed vacuum. The mechanical damping drives the steady two-mode cavity field states to be asymmetric.

where the bosonic operators $\hat{A}_j$ and $\hat{B}$ describe, respectively, the cavity and mechanical modes, the detunings $\delta_j = \omega_{cj} - \omega_{lj}$, g_{0j} represent the single-photon optomechanical couplings, and ε_j are the driving amplitudes.

For the strong driving fields, one can linearize the above Hamiltonian of Eq. (1) by expressing the operators $\hat{A}_j = \alpha_j^{ss} + \hat{a}_j$ and $\hat{B} = \beta^{ss} + \hat{b}_j$, where $\alpha_j(\beta)^{ss}$ denote the steady-state amplitudes of the cavity (mechanical) modes, and the operators $\hat{a}_j$ and $\hat{b}$ describe the corresponding quantum fluctuations of the fields. Then, the linearized Hamiltonian of the system can be obtained as

$$\hat{H}_1 = \omega_m \hat{b}^\dagger \hat{b} + \sum_{j=1}^{2}[\Delta_j \hat{a}_j^\dagger \hat{a}_j + g_j(\hat{a}_j^\dagger + \hat{a}_j)(\hat{b} + \hat{b}^\dagger)], \quad (2)$$

where $\Delta_j = \delta_j + 2g_{0j}\mathrm{Re}(\beta^{ss})$ and the effective optomechnanical couplings $g_j = g_{0j}\alpha_j^{ss}$. The steady-state amplitudes $\alpha_j^{ss} = \frac{|\varepsilon_j|}{\sqrt{\kappa_j^2+\Delta_j^2}}$ and $\beta^{ss} = \frac{-i\sum_j g_{0j}|\alpha_j|^2}{\gamma_m + i\omega_m}$, where κ_j and γ_m are the loss rates of the cavity fields and the mechanical mode. The couplings g_j are thus controllable via changing the drive strengths.

We consider that cavity fields 1 and 2 are resonant with the red and blue sidebands of the driving fields, respectively, i.e., $\Delta_1 = -\omega_m$ and $\Delta_2 = \omega_m$. In an interaction picture with respect to $\hat{H}_0 = \sum_j \Delta_j \hat{a}_j^\dagger \hat{a}_j$ and under the rotation-wave approximation (RWA), the Hamiltonian of Eq. (2) becomes

$$\hat{H}_2 = g_1(\hat{a}_1\hat{b} + \hat{a}_1^\dagger \hat{b}^\dagger) + g_2(\hat{a}_2\hat{b}^\dagger + \hat{a}_2^\dagger \hat{b}). \quad (3)$$

The above Hamiltonian describes two kinds of three-wave mixing processes. The first part (related to g_1) characterizes that a blue-detuned pumping photon is absorbed (emitted) and a photon of cavity field 1 and a phonon are simultaneously emitted (absorbed). In such a parametric conversion, the entanglement between cavity field 1 and the mechanical mode can be formed. While the second part characterizes that the system absorbs (emits) a red-detuned pumping photon and a phonon and emits (absorbs) a photon of cavity field 2, which induces induces quantum-state transfer between cavity field 2 and the mechanical mode (i.e., upconversion). Consequently, with the mediation of the mechanical mode, the optomechanical entanglement is transferred to the two cavity fields, leading to the cavity-field entanglement. The Hamiltonian of Eq. (3) represents a typical three-mode parametric interaction and can be realized in some other bosonic systems [38–40], e.g., atomic ensembles coupled to optical fields [41], apart from the optomechanical interfaces. Our results are therefore generic and applicable to these systems.

Taking into account the cavity dissipation and mechanical damping, the equations of motion for the system's operators are given by

$$\frac{d}{dt}\hat{a}_1^\dagger = -\kappa_1 \hat{a}_1^\dagger + ig_1\hat{b} + \sqrt{2\kappa_1}\hat{a}_1^{\mathrm{in}\dagger}(t), \quad (4a)$$

$$\frac{d}{dt}\hat{a}_2 = -\kappa_2 \hat{a}_2 - ig_2\hat{b} + \sqrt{2\kappa_2}\hat{a}_2^{\mathrm{in}}(t), \quad (4b)$$

$$\frac{d}{dt}\hat{b} = -\gamma_m \hat{b} - ig_1\hat{a}_1^\dagger - ig_2\hat{a}_2 + \sqrt{2\gamma_m}\hat{b}^{\mathrm{in}}(t), \quad (4c)$$

where the noise operators $\hat{a}_j^{\mathrm{in}}(t)$ and $\hat{b}^{\mathrm{in}}(t)$ satisfy the nonzero correlations $\langle \hat{a}_j^{\mathrm{in}}(t)\hat{a}_{j'}^{\mathrm{in}\dagger}(t)\rangle = \delta_{jj'}\delta(t-t')$, $\langle \hat{b}^{\mathrm{in}\dagger}(t)\hat{b}^{\mathrm{in}}(t')\rangle = \bar{n}_{\mathrm{th}}\delta(t-t')$, and $\langle \hat{b}^{\mathrm{in}}(t)\hat{b}^{\mathrm{in}\dagger}(t')\rangle = (\bar{n}_{\mathrm{th}}+1)\delta(t-t')$, where $\bar{n}_{\mathrm{th}} = (e^{\hbar\omega_m/k_B T}-1)^{-1}$, T is the temperature, and k_B is the Boltzmann constant.

Note that the Hamiltonian of Eq. (3) under the RWA is only valid under the condition

$$\omega_m \gg \{g_j, \kappa_j, \gamma_m \bar{n}_{\mathrm{th}}\}. \quad (5)$$

In addition, with the Routh-Hurwitz criterion, the stability condition of Eq. (4) can be found to be

$$(\kappa_2+\gamma_m)\left[(\kappa_1+\kappa_2)(\kappa_1+\gamma_m)+g_2^2\right] > (\kappa_1+\gamma_m)g_1^2,$$
$$\kappa_1 g_2^2 - \kappa_2 g_1^2 + \gamma_m\kappa_1\kappa_2 > 0. \quad (6)$$

In the presence of the decoherence, to obtain the substantial entanglement between the cavity fields requires that the coupling rates g_j^2/κ_j exceed the mechanical decoherence rate $\gamma_m \bar{n}_{\mathrm{th}}$ [23,28].

III. EPR STEERING CRITERIA

For the quadrature operators $\hat{X}_j = \hat{a}_j + \hat{a}_j^\dagger$ and $\hat{Y}_j = -i(\hat{a}_j - \hat{a}_j^\dagger)$, the Heisenberg uncertainty relations are $V(\hat{X}_j)V(\hat{Y}_j) \geqslant 1$, where the variances $V(\hat{O}) = \langle \hat{O}^2\rangle - \langle \hat{O}\rangle^2$ for $\hat{O} = (\hat{X}_j, \hat{Y}_j)$. According to Refs. [5,42], the EPR paradox and steering of bipartite Gaussian states are achievable on Gaussian measurements when

$$S_{12} = V_{\mathrm{inf}}(\hat{X}_1)V_{\mathrm{inf}}(\hat{Y}_1) < 1 \quad (7)$$

or

$$S_{21} = V_{\mathrm{inf}}(\hat{X}_2)V_{\mathrm{inf}}(\hat{Y}_2) < 1. \quad (8)$$

The inferred variances $V_{\text{inf}}(\hat{O}_j)$ are $V_{\text{inf}}[\hat{O}_{1(2)}] = V[\hat{O}_{1(2)}] - V^2(\hat{O}_1,\hat{O}_2)/V[\hat{O}_{2(1)}]$. The condition $S_{12} < 1$ ($S_{21} < 1$) means the steerability from cavity field 2 (1) to cavity field 1 (2). One-way steering occurs when only one of the above two inequalities holds.

Specifically, for our system, we have the steady average values $\langle \hat{a}_j^2 \rangle_{ss} = 0$ and $\langle \hat{a}_1 \hat{a}_2^\dagger \rangle_{ss} = 0$ (see Appendix A). The steering criteria of Eqs. (7) and (8) then reduce, respectively, to

$$|\langle \hat{a}_1 \hat{a}_2 \rangle_{ss}| > \sqrt{\langle \hat{a}_1^\dagger \hat{a}_1 \rangle_{ss}(\langle \hat{a}_2^\dagger \hat{a}_2 \rangle_{ss} + 1/2)} \tag{9}$$

and

$$|\langle \hat{a}_1 \hat{a}_2 \rangle_{ss}| > \sqrt{\langle \hat{a}_2^\dagger \hat{a}_2 \rangle_{ss}(\langle \hat{a}_1^\dagger \hat{a}_1 \rangle_{ss} + 1/2)}, \tag{10}$$

while the entanglement between the cavity fields measured by logarithmic negativity [43] requires (see Appendix B)

$$|\langle \hat{a}_1 \hat{a}_2 \rangle_{ss}| > \sqrt{\langle \hat{a}_1^\dagger \hat{a}_1 \rangle_{ss} \langle \hat{a}_2^\dagger \hat{a}_2 \rangle_{ss}}. \tag{11}$$

We see that nonclassical correlations between the two fields are necessary for the entanglement and steering. It is clearly shown from Eqs. (9)–(11) that steerable states are strictly inseparable but not necessarily vice versa.

IV. STEERING OF INTRACAVITY FIELDS

A. Weak mechanical damping regime ($\gamma_m \ll \kappa_j$)

First, we consider the case that $g_j > \kappa_j \gg \gamma_m$ such that we can temporarily neglect the mechanical damping at zero temperature for simplicity. Then, the steering conditions $S_{12} < 1$ and $S_{21} < 1$ for the steady cavity states reduce, respectively, to

$$(\kappa_1 - \kappa_2)\left(\kappa_2 g_2^2 - \kappa_1 g_1^2\right) > \kappa_1 \kappa_2 (\kappa_1 + \kappa_2)^2 \tag{12}$$

and

$$(\kappa_2 - \kappa_1)\left(\kappa_2 g_2^2 - \kappa_1 g_1^2\right) > \kappa_1 \kappa_2 (\kappa_1 + \kappa_2)^2, \tag{13}$$

which are incompatible. The condition for the steady-state entanglement reduces to

$$\left(\kappa_2 g_2^2 - \kappa_1 g_1^2\right) > 0. \tag{14}$$

For the same cavity loss rates ($\kappa_1 = \kappa_2$), both Eqs. (12) and (13) can not hold and therefore the steady entangled cavity-field states are definitely not steerable at zero mechanical damping. For different cavity loss rates, the direction of one-way steering depends on the ratio of κ_2/κ_1. We see that the one-way steering from cavity field 2 to cavity field 1 ($S_{12} < 1$ and $S_{21} > 1$) may be achieved when $\kappa_2 > \kappa_1$, whereas the reverse one-way steering may occur when $\kappa_2 < \kappa_1$. This result is plotted in Figs. 2 and 3 by considering $\gamma_m = 10^{-2}\kappa_1$. It shows that steady-state one-way steering can be obtained in two ways, apart from the transient two-way steering ($S_{12} < 1$ and $S_{21} < 1$). We thus conclude that in the steady-state regime only the cavity field with the larger dissipation rate can be steered by the other one. This is because in the absence of mechanical damping, the field under larger dissipation has a smaller steady-state mean photon number and also smaller quantum fluctuations $[V(\hat{O}_j) = \langle \hat{a}_j^\dagger \hat{a}_j \rangle + 1/2]$. This field is therefore more easily

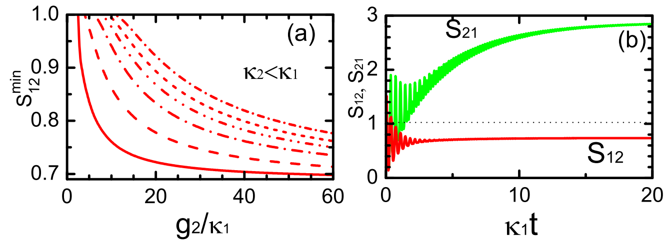

FIG. 2. (Color online) (a) Steady-state S_{12}, which is minimized with respect to g_1/κ_1 and κ_2/κ_1, versus g_2/κ_1 for $\kappa_2 < \kappa_1$ and $\gamma_m = 0.01\kappa_1$. From top to bottom, we have $\bar{n}_{\text{th}} = 1000$, 700, 500, 300, 100, and 0. The corresponding values of S_{21} are larger than one and not plotted. (b) S_{12} and S_{21} versus time for $\kappa_2 = 0.4\kappa_1$, $g_1 = 10\kappa_1$, $g_2 = 20\kappa_1$, $\gamma_m = 0.01\kappa_1$, and $\bar{n}_{\text{th}} = 0$. The values of κ_2 and g_1 in panel (b) are chosen such that S_{12} is minimized.

steered by the other one (since it has larger fluctuations and thus smaller inferred variances of the steered field).

Figures 2(a) and 3(a), respectively, plot the dependence of the minimized $S_{12}^{\min}$ and $S_{21}^{\min}$ (maximized steering), with respect to the couplings g_1/κ_1 and κ_2/κ_1, on g_2/κ_1 in the steady-state regime. We see that the one-way steering from the field 2 to the field 1 is much more robust against thermal fluctuations than the reverse one. This is because the thermal input leads to much greater enhancement of the mean photon number $\langle \hat{a}_2^\dagger \hat{a}_2 \rangle_{ss}$ than that of $\langle \hat{a}_1^\dagger \hat{a}_1 \rangle_{ss}$ via the beam-splitter interaction of the cavity field $\hat{a}_2$ with the mechanical mode coupled to the thermal reservoir. Therefore, even for the environment with a large thermal phonon number, the one-way field 2-to-1 steering can also be achievable. This shows that the present scheme can realize asymmetric steering without precooling the mechanical oscillator to its ground state.

B. Strong mechanical damping regime ($\gamma_m \gg \kappa_j$)

We next study the role played by strong mechanical damping in steering. For simplicity, we assume the cavity loss rates $\kappa_j = \kappa$. In this case and at zero temperature, the steady-state entanglement is always present and the steering condition $S_{21} < 1$ reduces to

$$\gamma_m/\kappa > 1/[(\Omega/2\kappa)^2 - 1], \tag{15}$$

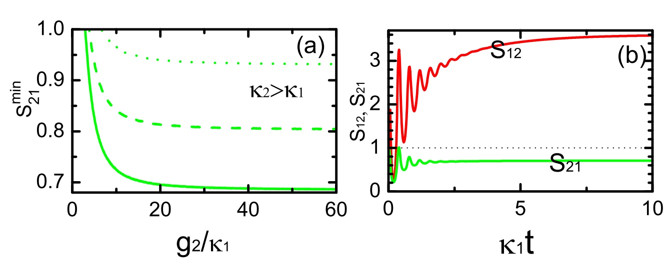

FIG. 3. (Color online) (a) Steady-state S_{21}, which is minimized with respect to g_1/κ_1 and κ_2/κ_1, versus g_2/κ_1 for $\kappa_2 > \kappa_1$ and $\gamma_m = 0.01\kappa_1$. From top to bottom, we have $\bar{n}_{\text{th}} = 40$, 20, and 0. The corresponding values of S_{12} are larger than one and not plotted. (b) S_{12} and S_{21} versus time for $\kappa_2 = 2.4\kappa_1$, $g_1 = 12\kappa_1$, $g_2 = 20\kappa_1$, $\gamma_m = 0.01\kappa_1$, and $\bar{n}_{\text{th}} = 0$. In panel (b), the values of κ_2 and g_1 are chosen such that S_{21} is minimized.

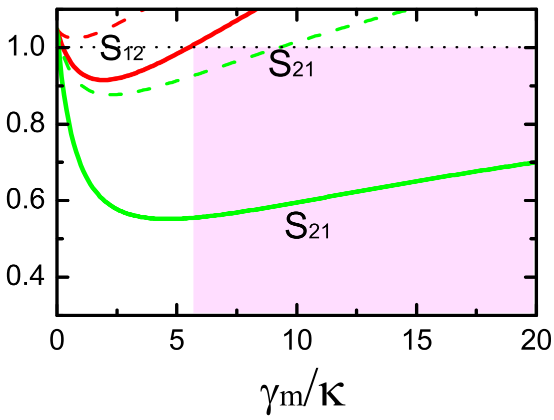

FIG. 4. (Color online) Steady-state S_{12} and S_{21} versus γ_m/κ for $\bar{n}_{\rm th}=0$ (solid curves) and $\bar{n}_{\rm th}=0.3$ (dashed curves). The other parameters are $g_1=6\kappa$ and $g_2=10\kappa$.

with $\Omega=\sqrt{g_2^2-g_1^2}$. This shows that for $\Omega\gg 2\kappa$, weak mechanical damping can still lead to the cavity field 1-to-2 steering. Meanwhile, the steering condition $S_{12}<1$ becomes approximately

$$\gamma_m/\kappa < \left(g_2\sqrt{\Omega^2-8\kappa^2}-\Omega^2\right)/2\kappa^2, \tag{16}$$

for $\gamma_m\gg\kappa$. When $\Omega\gg 2\kappa$, the right-hand side of Eq. (16) is larger than that of Eq. (15) and therefore the one-way steering from cavity field 1 to cavity field 2 can be obtained for γ_m violating Eq. (16). This is shown in Fig. 4. We see that strong mechanical damping in vacuum leads to the steady-state cavity field 1-to-2 one-way steering and its strength is impaired by thermal mechanical noise. The maximum obtainable steering is larger than that in the case of weak mechanical damping shown in Figs. 2 and 3. In addition, we find that in this regime of strong mechanical damping, the reverse one-way steering from cavity field 2 to cavity field 1 is unobtainable (see below for reason) and strong two-way steering can be achieved by minimizing S_{12} and S_{21} with respect to g_j/κ (see Fig. 5).

To show the mechanical damping can lead to the asymmetric cavity states, we perform the transformation

$$\hat{c}_j=\hat{S}(r)\hat{a}_j\hat{S}(-r), \tag{17}$$

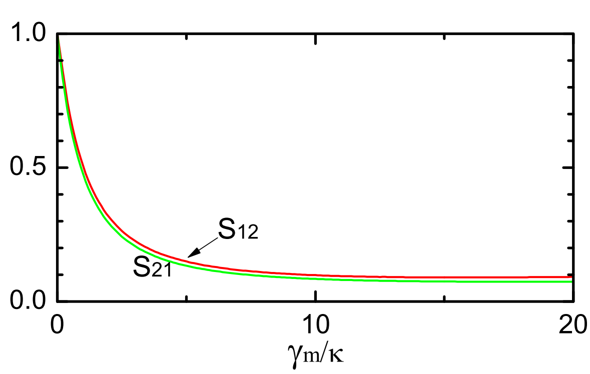

FIG. 5. (Color online) The dependence of the minimized S_{12} and S_{21}, with respect to g_1/κ and g_2/κ, in the steady-state regime on the γ_m/κ, for $\bar{n}_{\rm th}=0$.

where the two-mode squeezing operator $\hat{S}(r)=\exp\left[r(\hat{a}_1^\dagger\hat{a}_2^\dagger - a_1\hat{a}_2)\right]$ and $r=\tanh^{-1}(g_2/g_1)$, giving $\hat{c}_1=\sinh r\hat{a}_1^\dagger+\cosh r\hat{a}_2$ and $\hat{c}_2=\cosh r\hat{a}_1+\sinh r\hat{a}_2^\dagger$. In terms of the operators $\hat{c}_j$, the equations of Eq. (4) become $\partial_t\hat{c}_1=-\kappa\hat{c}_1-i\Omega\hat{b}+\sqrt{2\kappa}\hat{c}_1^{\rm in}(t)$, $\partial_t\hat{c}_2=-\kappa\hat{c}_2+\sqrt{2\kappa}\hat{c}_2^{\rm in}(t)$, and $\partial_t\hat{b}=-\gamma_m\hat{b}-i\Omega\hat{c}_1+\sqrt{2\gamma_m}\hat{b}^{\rm in}(t)$, where the noise operators $\hat{c}_j^{\rm in}(t)$ satisfy the nonzero correlations $\langle\hat{c}_j^{\rm in\dagger}(t)\hat{c}_j^{\rm in}(t')\rangle=\sinh^2 r\delta(t-t')$, $\langle\hat{c}_j^{\rm in}(t)\hat{c}_j^{\rm in\dagger}(t')\rangle=\cosh^2 r\delta(t-t')$, and $\langle\hat{c}_1^{\rm in}(t)\hat{c}_2^{\rm in}(t')\rangle=\sinh r\cosh r\delta(t-t')$. We see that the composite mode $\hat{c}_2$ is decoupled to the mechanical oscillator interacting with the mode $\hat{c}_1$ via $\hat{H}_{c1b}=\Omega\hat{c}_1\hat{b}^\dagger+\text{H.c.}$ and the two modes $\hat{c}_j$ are coupled to a two-mode squeezed vacuum reservoir [see Fig. 1(b)]. The symmetry of the cavity field states depends on that of the $\hat{c}_1$ and $\hat{c}_2$ modes. For zero mechanical damping, the two composite modes reduce to the reservoir's state and thus are symmetric. However, at finite mechanical damping in vacuum the mode $\hat{c}_1$ is cooled down and we have $\langle\hat{a}_1^\dagger\hat{a}_1\rangle_{ss}-\langle\hat{a}_2^\dagger\hat{a}_2\rangle_{ss}=\langle\hat{c}_2^\dagger\hat{c}_2\rangle_{ss}-\langle\hat{c}_1^\dagger\hat{c}_1\rangle_{ss}>0$, which results in the violation of Eq. (9) more easily than in that of Eq. (10) and thus the one-way steering of the field 2 by the field 1.

We note that a large mechanical damping rate can be obtained by using a low-quality mechanical oscillator with high resonant frequency. Alternately, it can also be achieved by weakly coupling a high-quality mechanical oscillator to a bad electromagnetic cavity to induce an optical heating for the mechanical oscillator, as analyzed in detail in Ref. [44].

V. OUTPUT STEERING SPECTRA

In this section, we consider the steering spectra of the output fields which are utilized for measurements and applications. By performing the Fourier transformation $\hat{O}(t)=\int_{-\infty}^{\infty}e^{-i\omega t}\hat{O}[\omega]d\omega/\sqrt{2\pi}$ on Eq. (4) and using the input-output relations $\hat{a}_j^{\rm out}=\sqrt{2\kappa_j}\hat{a}_j-\hat{a}_j^{\rm in}$, we have

$$\begin{aligned}\hat{a}_1^{\rm out}[\omega]=&M_{11}\hat{a}_1^{\rm in}[\omega]+M_{12}\hat{a}_2^{\rm in\dagger}[-\omega]+M_{1b}\hat{b}^{\rm in\dagger}[-\omega],\\ \hat{a}_2^{\rm out}[\omega]=&-M_{12}\hat{a}_1^{\rm in\dagger}[-\omega]+M_{22}\hat{a}_2^{\rm in}[\omega]+M_{2b}\hat{b}^{\rm in}[\omega].\end{aligned} \tag{18}$$

The expressions for $M_{jj'}$ and M_{jb} and the calculation of the steering spectra $S_{12}[\omega]$ and $S_{21}[\omega]$ are given in Appendix C.

At zero mechanical damping, we have $\langle\hat{a}_1^{\rm out\dagger}[\omega]a_1^{\rm out}[\omega]\rangle=\langle\hat{a}_2^{\rm out\dagger}[\omega]a_2^{\rm out}[\omega]\rangle=|m_{12}^2|$, which means that the output states at frequencies $\omega_{cj}+\omega$ are symmetric, independent of the ratio κ_2/κ_1. Therefore, unlike the intracavity situation, one-way spectral steering can impossibly be achieved when $\gamma_m=0$,

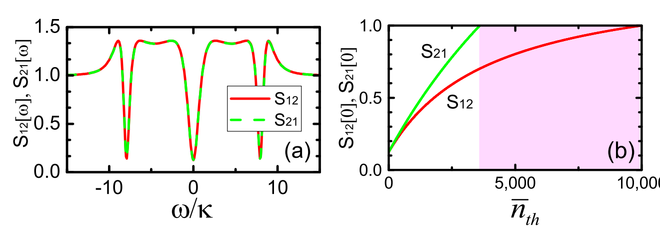

FIG. 6. (Color online) (a) The spectra of $S_{12}[\omega]$ and $S_{21}[\omega]$ for $g_1=6\kappa$, $g_2=10\kappa$, $\gamma_m=0.01\kappa$, and $\bar{n}_{\rm th}=0$. (b) $S_{12}[0]$ and $S_{21}[0]$ versus $\bar{n}_{\rm th}$.

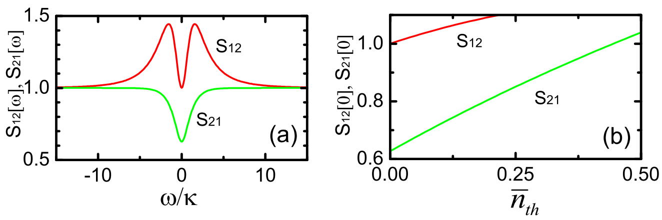

FIG. 7. (Color online) (a) The spectra of $S_{12}[\omega]$ and $S_{21}[\omega]$ for $g_1 = 2\kappa$, $g_2 = 3\kappa$, $\gamma_m = 9\kappa$, and $\bar{n}_{\mathrm{th}} = 0$. (b) $S_{12}[0]$ and $S_{21}[0]$ versus $\bar{n}_{\mathrm{th}}$.

even for the unbalanced cavity losses. The symmetric steering spectra are plotted in Fig. 6(a). It shows that strong two-way steering can be obtained around the frequencies

$$\omega = 0 \text{ and } \omega = \pm\sqrt{\Omega^2 - \kappa^2}, \tag{19}$$

at which the cavity fields are strongly excited. Nevertheless, for weak mechanical damping the one-way steering from the output field 2 to the field 1 at cavity resonances is obtainable via thermalizing the mechanical oscillator, as shown in Fig. 6(b), since the steering in this direction is more robust against thermal noise than the reverse steering, as is similarly shown in Figs. 2 and 3. This one-way steering at cavity resonances ($S_{12}[0] < 1$ and $S_{21}[0] > 1$) requires

$$g_1^2/\kappa\gamma_m < \bar{n}_{\mathrm{th}} < g_2^2/\kappa\gamma_m - 1. \tag{20}$$

Hence, the one-way steering from the output field 2 to the field 1 can still be achieved even for the large number of thermal phonons when $g_j^2/\kappa\gamma_m \gg 1$. It shows again that this directional one-way steering is much more robust against thermal noise than the reverse one, as exhibited in Figs. 2 and 3.

By increasing the mechanical damping rate which enters the strong damping regime, we plot the steering in Fig. 7. Interestingly, we see that the one-way steering from the output field 1 to the field 2 can be achievable over all frequencies. At zero temperature the condition $S_{21}[0] < 1$ always holds while $S_{12}[0] > 1$ requires the mechanical damping rate

$$\gamma_m/\kappa > g_2^2/\kappa^2, \tag{21}$$

for which the one-way steering in this direction is achievable.

We finally discuss the effects of the internal losses of the cavities on the output steering. The detailed calculation is presented in the Appendix C. The results are shown in Fig. 8.

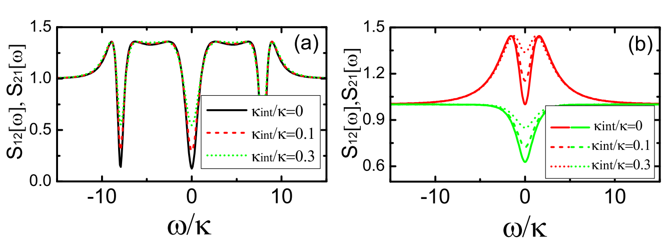

FIG. 8. (Color online) The spectra of $S_{12}[\omega]$ and $S_{21}[\omega]$ for different internal loss rates of the cavities. The same internal loss rates $\kappa_j^{\mathrm{int}} = \kappa^{\mathrm{int}}$ are assumed, the parameters in (a) are the same as those in Fig. 6 (a), and the parameters in (b) are the same as in Fig. 7 (a). In (a) we have $S_{12}[\omega] = S_{21}[\omega]$.

One can see that the internal losses can strongly suppress the strengths of the two-way and one-way steering of the output fields. It also shows that the directions of the one-way steering are not changed with the same rates of the internal losses.

VI. CONCLUSION

In summary, we propose a scheme for realizing one-way steering of two electromagnetic fields by double-cavity optomechanics. The two cavity fields are mediated by a mechanical oscillator and driven, respectively, by a red and a blue detuned strong coherent field. We show that asymmetric steering can be achieved for unequal cavity losses or strong mechanical damping in the regime of steady states. The conditions for achieving one-way steering of the intracavity and output fields are found. The effect of the internal losses of the cavities on the steering is also discussed. The asymmetric steering may be useful in quantum communication and information. Besides optoelectromechanical interfaces, our results are also applicable to other three-mode parametrically coupled bosonic systems. Further work will consider tripartite steering in the present system.

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APPENDIX A: THE STEADY-STATE SOLUTION OF THE SYSTEM

Equations (4a) to (4c) can be rewritten into the simple form

$$\frac{d}{dt}\psi = \mathbf{A}\psi + \sqrt{2\mathbf{K}}\psi_{\mathrm{in}}(t), \tag{A1}$$

where $\psi = (\hat{a}_1,\hat{a}_1^\dagger,\hat{a}_2,\hat{a}_2^\dagger,\hat{b},\hat{b}^\dagger)^T$, $\mathbf{K} = \mathrm{diag}(\kappa_1,\kappa_1,\kappa_2,\kappa_2,\gamma_m,\gamma_m)$, and $\psi_{\mathrm{in}} = (\hat{a}_1^{\mathrm{in}},\hat{a}_1^{\mathrm{in}\dagger},\hat{a}_2^{\mathrm{in}},\hat{a}_2^{\mathrm{in}\dagger},\hat{b}^{\mathrm{in}},\hat{b}^{\mathrm{in}\dagger})^T$. The matrix

$$\mathbf{A} = \begin{pmatrix} -\kappa_1 & 0 & 0 & 0 & 0 & -ig_1 \\ 0 & -\kappa_1 & 0 & 0 & ig_1 & 0 \\ 0 & 0 & -\kappa_2 & 0 & -ig_2 & 0 \\ 0 & 0 & 0 & -\kappa_2 & 0 & ig_2 \\ 0 & -ig_1 & -ig_2 & 0 & -\gamma_m & 0 \\ ig_1 & 0 & 0 & ig_2 & 0 & -\gamma_m \end{pmatrix}. \tag{A2}$$

From Eq. (A1) the second-order moments $\Phi = \langle\psi\psi^T\rangle$ satisfy

$$\frac{d}{dt}\Phi = \mathbf{A}\Phi + \Phi\mathbf{A}^T + 2\mathbf{KD}, \tag{A3}$$

where $\mathbf{D} = \mathrm{diag}(\mathbf{D}_1,\mathbf{D}_2,\mathbf{D}_3)$, with the entries $\mathbf{D}_{1,2} = \left(\begin{smallmatrix} 0 & 1 \\ 0 & 0 \end{smallmatrix}\right)$ and $\mathbf{D}_3 = \left(\begin{smallmatrix} 0 & \bar{n}_{\mathrm{th}}+1 \\ \bar{n}_{th} & 0 \end{smallmatrix}\right)$. In the steady-state regime, we have

$$\langle \hat{a}_1^\dagger \hat{a}_1 \rangle_{ss} = \frac{\kappa_2(\kappa_1+\kappa_2+\gamma_m)g_1^2g_2^2 + \gamma_m(\bar{n}_{\rm th}+1)\big[\kappa_1 g_2^2 - \kappa_2 g_1^2 + \kappa_2(\kappa_1+\kappa_2)(\kappa_2+\gamma_m)\big]g_1^2}{\big(\kappa_1 g_2^2 - \kappa_2 g_1^2 + \gamma_m\kappa_1\kappa_2\big)\big[(\kappa_2+\gamma_m)g_2^2 - (\kappa_1+\gamma_m)g_1^2 + (\kappa_1+\kappa_2)(\kappa_1+\gamma_m)(\kappa_2+\gamma_m)\big]}, \tag{A4a}$$

$$\langle \hat{a}_2^\dagger \hat{a}_2 \rangle_{ss} = \frac{\kappa_1(\kappa_1+\kappa_2+\gamma_m)g_1^2g_2^2 + \gamma_m\bar{n}_{\rm th}\big[\kappa_1 g_2^2 - \kappa_2 g_1^2 + \kappa_1(\kappa_1+\kappa_2)(\kappa_1+\gamma_m)\big]g_2^2}{\big(\kappa_1 g_2^2 - \kappa_2 g_1^2 + \gamma_m\kappa_1\kappa_2\big)\big[(\kappa_2+\gamma_m)g_2^2 - (\kappa_1+\gamma_m)g_1^2 + (\kappa_1+\kappa_2)(\kappa_1+\gamma_m)(\kappa_2+\gamma_m)\big]}, \tag{A4b}$$

$$\langle \hat{a}_1 \hat{a}_2 \rangle_{ss} = -\frac{\kappa_1 g_1 g_2\big[\kappa_2 g_1^2 + (\kappa_2+\gamma_m)\big(g_2^2+\kappa_2\gamma_m\big)\big] + \gamma_m\bar{n}_{\rm th}\big[\kappa_1 g_2^2 - \kappa_2 g_1^2 + \kappa_1\kappa_2(\kappa_1+\kappa_2+2\gamma_m)\big]}{\big(\kappa_1 g_2^2 - \kappa_2 g_1^2 + \gamma_m\kappa_1\kappa_2\big)\big[(\kappa_2+\gamma_m)g_2^2 - (\kappa_1+\gamma_m)g_1^2 + (\kappa_1+\kappa_2)(\kappa_1+\gamma_m)(\kappa_2+\gamma_m)\big]}. \tag{A4c}$$

APPENDIX B: THE DERIVATION OF THE ENTANGLEMENT CONDITION IN EQ. (11)

The correlation matrix σ of the steady two-mode cavity field states, defined as $\sigma_{ij} = \langle \xi_i\xi_j + \xi_j\xi_i\rangle/2$ and $\xi = (\hat{X}_1,\hat{Y}_1,\hat{X}_2,\hat{Y}_2)$, can be obtained as

$$\sigma = \begin{pmatrix} n_1 & 0 & c & 0 \\ 0 & n_1 & 0 & -c \\ c & 0 & n_2 & 0 \\ 0 & -c & 0 & n_2 \end{pmatrix}, \tag{B1}$$

with $n_j = \langle \hat{a}_j^\dagger \hat{a}_j\rangle_{ss} + 1/2$ and $c = \langle \hat{a}_1\hat{a}_2\rangle_{ss}$. Here, the quadrature operators are scaled by $1/2$ with respect to their definition in the text. By expressing the correlation matrix σ in terms of three 2×2 matrices σ_1, σ_2, and σ_3 as $\sigma = \left(\begin{smallmatrix}\sigma_1 & \sigma_3 \\ \sigma_3^T & \sigma_2\end{smallmatrix}\right)$, the logarithmic negativity E_N is defined as

$$E_N = \max[0, -\ln(2\lambda)], \tag{B2}$$

where $\lambda = 2^{-1/2}\sqrt{\Sigma(\sigma) - \sqrt{\Sigma(\sigma) - 4\det\sigma}}$ and $\Sigma(\sigma) = \det\sigma_1 + \det\sigma_2 - 2\det\sigma_3$. The entanglement thus occurs for $\lambda < 1/2$, which is equivalent to

$$[c^2 - (n_1+1/2)(n_2+1/2)][c^2 - (n_1-1/2)(n_2-1/2)] < 0. \tag{B3}$$

Since the positivity of the two-mode cavity-field states requires that $c^2 < (n_1+1/2)(n_2+1/2)$, the above entanglement condition reduces to

$$|\langle \hat{a}_1\hat{a}_2\rangle_{ss}| > \sqrt{\langle \hat{a}_1^\dagger\hat{a}_1\rangle_{ss}\langle \hat{a}_2^\dagger\hat{a}_2\rangle_{ss}}. \tag{B4}$$

APPENDIX C: THE CALCULATION OF THE STEERING SPECTRA

When considering the internal losses of the cavities, the equations of motion in Eq. (4) can be generalized to

$$\frac{d}{dt}\hat{a}_1^\dagger = -\left(\kappa_1 + \kappa_1^{\rm int}\right)\hat{a}_1^\dagger + ig_1\hat{b} + \sqrt{2\kappa_1}\hat{a}_1^{\rm in\dagger}(t) + \sqrt{2\kappa_1^{\rm int}}\hat{a}_1^{\rm int\dagger}(t), \tag{C1a}$$

$$\frac{d}{dt}\hat{a}_2 = -\left(\kappa_2 + \kappa_2^{\rm int}\right)\hat{a}_2 - ig_2\hat{b} + \sqrt{2\kappa_2}\hat{a}_2^{\rm in}(t) + \sqrt{2\kappa_2^{\rm int}}\hat{a}_2^{\rm int}(t), \tag{C1b}$$

$$\frac{d}{dt}\hat{b} = -\gamma_m\hat{b} - ig_1\hat{a}_1^\dagger - ig_2\hat{a}_2 + \sqrt{2\gamma_m}\hat{b}^{\rm in}(t), \tag{C1c}$$

where $\kappa_j^{\rm int}$ denote the intrinsic loss rates of the cavities. The noise operators $\hat{a}_j^{\rm int}(t)$ satisfy nonzero correlations $\langle \hat{a}_j^{\rm int}(t)\hat{a}_{j'}^{\rm int\dagger}(t)\rangle = \delta_{jj'}\delta(t-t')$. Note that in the presence of the internal losses, the expressions of $\langle \hat{a}_j^\dagger\hat{a}_j\rangle$ and $\langle \hat{a}_1\hat{a}_2\rangle$ are still unchanged when replacing $\kappa_j \to \kappa_j + \kappa_j^{\rm int}$. So, the conditions for the intracavity steering (discussed in the main text) remain the same when κ_j represent the total loss rates.

By performing the Fourier transformation $\hat{O}(t) = \int_{-\infty}^{\infty} e^{-i\omega t}\hat{O}[\omega]d\omega/\sqrt{2\pi}$ on Eq.(4), we have

$$-i\omega\hat{a}_1^\dagger[-\omega] = -\kappa_1\hat{a}_1^\dagger[-\omega] + ig_1\hat{b}[\omega] + \sqrt{2\kappa_1^{\rm in}}\hat{a}_1^{\rm in\dagger}[-\omega] + \sqrt{2\kappa_1^{\rm int}}\hat{a}_1^{\rm int\dagger}[-\omega], \tag{C2}$$

$$-i\omega\hat{a}_2[\omega] = -\kappa_2\hat{a}_1^\dagger[\omega] + ig_2\hat{b}[\omega] + \sqrt{2\kappa_2}\hat{a}_2^{\rm in\dagger}[\omega] + \sqrt{2\kappa_2^{\rm int}}\hat{a}_2^{\rm int\dagger}[\omega], \tag{C3}$$

$$-i\omega\hat{b}[\omega] = -\gamma_m\hat{b}[\omega] - ig_1\hat{a}_1^\dagger[-\omega] - ig_2\hat{a}_2[\omega] + \sqrt{2\gamma_m}\hat{b}^{\rm in}[\omega]. \tag{C4}$$

With the input-output relations $\hat{a}_j^{\rm out}[\omega] = \sqrt{2\kappa_j}\hat{a}_j[\omega] - \hat{a}_j^{\rm in}[\omega]$, we can express the output fields, in terms of the input ones, as

$$\hat{a}_1^{\rm out}[\omega] = M_{11}\hat{a}_1^{\rm in}[\omega] + M_{12}\hat{a}_2^{\rm in\dagger}[-\omega] + N_{11}\hat{a}_1^{\rm int}[\omega] + N_{12}\hat{a}_2^{\rm int\dagger}[-\omega] + M_{1b}\hat{b}^{\rm in\dagger}[-\omega], \tag{C5a}$$

$$\hat{a}_2^{\rm out}[\omega] = -M_{12}\hat{a}_1^{\rm in\dagger}[-\omega] + M_{22}\hat{a}_2^{\rm in}[\omega] - N_{12}\hat{a}_1^{\rm int\dagger}[-\omega] + N_{22}\hat{a}_2^{\rm int}[\omega] + M_{2b}\hat{b}^{\rm in}[\omega], \tag{C5b}$$

where

$$M_{11}[\omega] = \frac{(\kappa_2 - i\omega)g_1^2 + \left(\kappa_1 - \kappa_1^{\text{int}} + i\omega\right)\left[g_2^2 + (\kappa_2 - i\omega)(\gamma_m - i\omega)\right]}{(\kappa_1 - i\omega)g_2^2 - (\kappa_2 - i\omega)g_1^2 + (\kappa_1 - i\omega)(\kappa_2 - i\omega)(\gamma_m - i\omega)},$$

$$M_{12}[\omega] = \frac{2\sqrt{\kappa_1\kappa_2}g_1g_2}{(\kappa_1 - i\omega)g_2^2 - (\kappa_2 - i\omega)g_1^2 + (\kappa_1 - i\omega)(\kappa_2 - i\omega)(\gamma_m - i\omega)},$$

$$N_{11}[\omega] = \frac{2\sqrt{\kappa_1\kappa_1^{\text{int}}}\left[g_2^2 + (\kappa_2 - i\omega)(\gamma_m - i\omega)\right]}{(\kappa_1 - i\omega)g_2^2 - (\kappa_2 - i\omega)g_1^2 + (\kappa_1 - i\omega)(\kappa_2 - i\omega)(\gamma_m - i\omega)},$$

$$N_{12}[\omega] = \frac{2\sqrt{\kappa_1\kappa_2^{\text{int}}}g_1g_2}{(\kappa_1 - i\omega)g_2^2 - (\kappa_2 - i\omega)g_1^2 + (\kappa_1 - i\omega)(\kappa_2 - i\omega)(\gamma_m - i\omega)},$$

$$M_{1b}[\omega] = \frac{-2i\sqrt{\kappa_1\gamma_m}g_1(\kappa_2 - i\omega)}{(\kappa_1 - i\omega)g_2^2 - (\kappa_2 - i\omega)g_1^2 + (\kappa_1 - i\omega)(\kappa_2 - i\omega)(\gamma_m - i\omega)},$$

$$M_{22}[\omega] = -\frac{(\kappa_1 - i\omega)g_2^2 + \left(\kappa_2 - \kappa_2^{\text{int}} + i\omega\right)\left[g_1^2 - (\kappa_1 - i\omega)(\gamma_m - i\omega)\right]}{(\kappa_1 - i\omega)g_2^2 - (\kappa_2 - i\omega)g_1^2 + (\kappa_1 - i\omega)(\kappa_2 - i\omega)(\gamma_m - i\omega)},$$

$$M_{2b}[\omega] = \frac{-2i\sqrt{\kappa_2\gamma_m}g_2(\kappa_1 - i\omega)}{(\kappa_1 - i\omega)g_2^2 - (\kappa_2 - i\omega)g_1^2 + (\kappa_1 - i\omega)(\kappa_2 - i\omega)(\gamma_m - i\omega)}.$$

With the spectral definitions of the quadratures $\hat{X}_j^{\text{out}}[\omega] = \hat{a}_j^{\text{out}}[\omega] + \hat{a}_j^{\text{out}\dagger}[-\omega]$ and $\hat{Y}_j^{\text{out}}[\omega] = -i\hat{a}_j^{\text{out}}[\omega] + i\hat{a}_j^{\text{out}\dagger}[-\omega]$, we have

$$\left\langle \hat{X}_1^2[\omega]\right\rangle = \left\langle Y_1^2[\omega]\right\rangle = |M_{11}[\omega]|^2 + |M_{12}[-\omega]|^2 + |N_{11}[\omega]|^2 + |N_{12}[-\omega]|^2 + |M_{1b}[-\omega]|^2(\bar{n}_{\text{th}} + 1) + |M_{1b}[\omega]|^2\bar{n}_{\text{th}}, \tag{C6}$$

$$\left\langle \hat{X}_2^2[\omega]\right\rangle = \left\langle Y_2^2[\omega]\right\rangle = |M_{22}[\omega]|^2 + |M_{12}[-\omega]|^2 + |N_{22}[\omega]|^2 + |N_{12}[-\omega]|^2 + |M_{2b}[\omega]|^2(n_{\text{th}} + 1) + |M_{2b}[-\omega]|^2\bar{n}_{\text{th}}, \tag{C7}$$

$$\begin{aligned}\langle \hat{X}_1[\omega]\hat{X}_2[\omega]\rangle = -\langle \hat{Y}_1[\omega]\hat{Y}_2[\omega]\rangle &= -M_{11}[\omega]M_{12}[-\omega] + M_{12}^*[-\omega]M_{22}^*[\omega] - N_{11}[\omega]N_{12}[-\omega] + N_{12}^*[-\omega]N_{22}^*[\omega]\\ &+M_{1b}^*[-\omega]M_{2b}^*[\omega](\bar{n}_{\text{th}} + 1) + M_{1b}[\omega]M_{2b}[-\omega]\bar{n}_{\text{th}}.\end{aligned} \tag{C8}$$

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