Engineering of a quantum state by time-dependent decoherence-free subspaces

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We apply the time-dependent decoherence-free subspace theory to a Markovian open quantum system in order to present a proposal for a quantum-state engineering program. By quantifying the purity of the quantum state, we verify that the quantum-state engineering process designed via our method is completely unitary within any total engineering time. Even though the controls on the open quantum system are not perfect, the asymptotic purity is still robust. Owing to its ability to completely resist decoherence and the lack of restraint in terms of the total engineering time, our proposal is suitable for multitask quantum-state engineering program. Therefore, this proposal is not only useful for achieving the quantum-state engineering program experimentally, it also helps us build both a quantum simulation and quantum information equipment in reality.

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I. INTRODUCTION

Controlled manipulation by atoms and molecules using external controls, known as quantum-state engineering (QSE), has become an active field of modern research, which is a fundamental step in quantum computation [1] and quantum measurement tasks [2]. The adiabatic theorem of quantum mechanics provides a reliable method of controlling the quantum state of an isolated system [3,4]. Indeed, for the scheme of QSE, there are two unique advantages for the adiabatic method. First, the adiabatic method of QSE is robust when there is fluctuation in the coherent control fields. Second, since the parameters in the Hamiltonian vary adiabatically, the engineering timing does not need to be strictly controlled in order to be manipulated precisely. If the QSE process is accomplished, the quantum state will be steadied on the target state. Because of the advantages mentioned above, the adiabatic method has been chosen as an important part of the QSE program and experimentally realized through a number of techniques, such as nuclear magnetic resonance [5,6], superconducting qubits [7], trapped ions [8], and optical lattices [9].

When the quantum system is coupled to its surroundings, the adiabatic QSE process will experience considerable loss of fidelity, which limits the application of the adiabatic method. Actually, for open quantum systems, there is competition between the time required for adiabaticity and the decoherence time scales [10,11]. Therefore, identifying a protocol that is both fast and fault tolerant is an important research direction for quantum information processing and quantum control. Many enlightening proposals have also been put forward and evaluated for nonadiabatically engineered quantum states, such as inverse engineering control [12], optimal control [13], the fast quench dynamics method [14], and a method of combining incoherent and coherent controls [15,16]. The fundamental idea of these methods is to decrease the time needed to manipulate the quantum state so as to reduce the effect of decoherence on fidelity. Obviously, this is not enough to realize quantum information processes for real applications. On the one hand, the quantum state will lose its quantum character (the coherence between two quantum states or the entanglement between two quantum systems and decay into the steady state over time or through repeated operation on this quantum system. Thus it limits the QSE scheme in terms of achieving a multitask QSE program. On the other hand, the success of ultrafast QSE is determined by the fact that control of the quantum system must be ultraprecise and ultrafast, which strongly depends on the development of experimental technology.

In this paper, we propose a method to engineer the quantum state of an open system. With this innovative method, there is no limit to the total engineering time nor any loss of fidelity. Our QSE method was designed based on the time-dependent decoherence-free subspace (TDFS) scheme [15,17] in which the basic vectors are time dependent. In other words, such a DFS evolves smoothly in the total Hilbert space of the open quantum system by reservoir engineering technology [18,19]. If we manipulate the quantum system state properly, the quantum state will strictly follow the evolution of the TDFS, so as to protect it from the effect of decoherence. In comparison with existing works on QSE, our method is very effective and promising. Because the DFS scheme can act against decoherence completely, the quantum state in the DFS scheme does not lose any quantum character. More importantly, when the quantum state involves following the evolution of the TDFS strictly, the QSE process is unitary, as if the environment does not exist. Therefore, no matter how much time is spent on the QSE process, the target state will be reached with no loss of fidelity. Owing to the distinguishing features of our method, it offers a reliable path to implementing a multitask QSE process on identical open quantum systems one after another. Moreover, the robust QSE program can also be realized in a time-independent DFS. A DSF of at least two dimensions is required for the simplest QSE task, which means that we have to control several quantum systems at the same time. For our method, since the basic vectors of the TDFS are time dependent, a one-dimensional TDFS is sufficient to accomplish all QSE tasks in principle.

To illustrate the practical application of our method, a QSE program of a two-level open quantum system was designed according to the TDFS scheme. As shown in the results, the QSE process was completely unitary even over a long period of time. An analytical expression of the coherent control field was derived, by which we could check the QSE process in

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detail. As presented in the analytic expression, there was a singular point in the coherent control field that could not be reached in actual experiments. Therefore, we introduced some adjustments to the coherent control field, and the results remained satisfactory. The QSE process is always robust, even over the long term. Thus we can affirmatively conclude that our method is powerful and reliable in both its theoretical preciseness and its experimental feasibility.

This paper is structured as follows. In Sec. II, we briefly review the TDFS scheme and discuss how to engineer a quantum state within such a TDFS. In Sec. III, we manipulate a two-level open system to the target state by means of the TDFS QSE method. Both population engineering and phase engineering are discussed step by step. An adjustment to the nonphysical coherent control field is considered in Sec. IV in order to show that even when the coherent control field is defective, the TDFS QSE method is still unconditionally robust. We conclude with Sec. V.

II. THE TIME-DEPENDENT DECOHERENCE-FREE SUBSPACES AND THE QUANTUM-STATE ENGINEERING PROGRAM

Let us start with the TDFS scheme. A DFS is a subspace of the Hilbert space of the open quantum system, in which the dynamics of the quantum system is still unitary [20]. It has been shown that the principle behind the useful appearance of the DFS is symmetry of the interaction between the open quantum system and the environment. The existence of the DFS has been demonstrated experimentally in many physical systems [21–23], and many enlightening designs have also been proposed based on DFSs in order to realize quantum key distribution [24], quantum computation [25], and so on.

Although the DFS with fixed basic vectors (traditional DFS) is a promising candidate for quantum information processes, it is more suitable to storing and protecting the information coded in quantum systems; however, in a QSE field, the DSF is unable to manipulate quantum states precisely. For instance, at least three physical qubits are needed to construct one logical qubit against the effect of a dephasing environment, and the quantum computation program on such a logical qubit needs to accurately control the interactions between the physical qubits [26], i.e., two interactions have to be controlled simultaneously. However, it is difficult to manage the couplings between the physical qubits at the same time. Moreover, the decoherence becomes more complicated when the number of physical qubits and energy levels increases. In order to conquer these difficulties, the TDFS scheme is introduced [15,17].The TDFS is still a DFS, but its basic vectors depend on time, which means that the TDFS will evolve in the Hilbert space of the quantum system.

In the following, we restrict our discussion to an N-dimensional open quantum system and consider its dynamics as Markovian. In the interaction picture, the evolution of the quantum system must obey the Lindblad-Markovian master equation, given as

$$\begin{aligned}\dot{\rho}(t) &= -i[H,\rho(t)] + \mathcal{L}\rho(t),\\ \mathcal{L}\rho(t) &= \sum_\alpha \left[F_\alpha \rho(t) F_\alpha^\dagger - \frac{1}{2}\{F_\alpha^\dagger F_\alpha, \rho(t)\}\right],\end{aligned} \tag{1}$$

where F_α is the Lindblad operator. which describes the decoherence caused by the coupling to the environment, and H is the Hamiltonian, which consists of the coherent control field on the open quantum system. It has been shown in Ref. [27] that if some of the environmental parameters can be continuously varied as a function of time by means of reservoir engineering technology, the Lindblad operators in Eq. (1) will be time dependent. In other words, when the environment varies with time, the symmetry of the interaction between the open quantum system and its environment is time dependent [28,29]. It is also a way of engineering the state of the open quantum system, known as the incoherent control method.

In the context of the Linblad-Markovian master equation, the DFS is defined as a collection of quantum states in which the dynamics is unitary and the purity is constant during the evolution of the quantum states $\rho(t)$, i.e., $\partial \mathrm{Tr}[\rho^2(t)]/\partial t = 0$, leading to the following conditions on the TDFS [17]:

Theorem. Let the time evolution of an open quantum system in a finite-dimensional Hilbert space be governed by Eq. (1) with time-dependent Hamiltonian $H(t)$ and time-dependent Lindblad operators $F_\alpha(t)$. The subspace

$$\mathcal{H}_{\mathrm{DFS}}(t) = \mathrm{Span}\{|\Phi_1(t)\rangle, |\Phi_2(t)\rangle, \ldots, |\Phi_M(t)\rangle\} \tag{2}$$

is a TDFS if and only if each basis vector of $\mathcal{H}_{\mathrm{DFS}}(t)$ satisfies

$$F_\alpha(t)|\Phi_j(t)\rangle = c_\alpha(t)|\Phi_j(t)\rangle, \quad j = 1,\ldots,M; \alpha = 1,\ldots,K, \tag{3}$$

and $\mathcal{H}_{\mathrm{DFS}}(t)$ is invariant under

$$H_{\mathrm{eff}}(t) = G(t) + H(t) + \frac{i}{2}\sum_\alpha [c_\alpha^*(t) F_\alpha(t) - c_\alpha(t) F_\alpha^\dagger(t)]. \tag{4}$$

Here $G(t) = iU^\dagger(t)\dot{U}(t)$ and $U(t)$ is a unitary operator

$$U(t) = \sum_{j=1}^{M} |\Phi_j(0)\rangle\langle\Phi_j(t)| + \sum_{n=1}^{N-M} |\Phi_n^\perp(0)\rangle\langle\Phi_n^\perp(t)|. \tag{5}$$

In this theorem, the time-dependent Hamiltonian describes the coherent control on the open quantum system, which can be rewritten as $H(t) = \sum_n \Omega_n(t) H_n$ with the control Hamiltonian H_n and the coherent control field $\Omega_n(t)$. In our previous work [17], we have proved that the theorem mentioned here is a sufficient and necessary condition for existing the TDFS to exist. In the following, we will apply this theorem to design our QSE program and investigate the coherent and incoherent control projects in detail.

Since the goal of the QSE program is to design a path in the Hilbert space to connect the initial state and the target state, the TDFS is the best candidate for implementing a fast and robust QSE program. In the rest of this section, we show how to engineer a state of an open quantum system in the target state. The design process is illustrated in Fig. 1. From the condition of the TDFS, one can conclude that the it can be constructed by combining the incoherent control project with the coherent control project on the open quantum system. The two sorts of controls perform different duties. For the incoherent control, since the basic vectors of the TDFS are

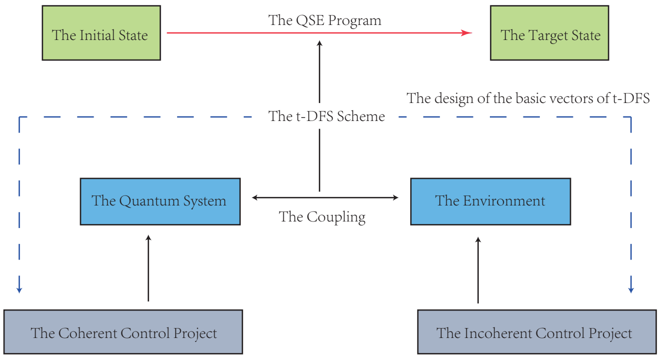

FIG. 1. (Color online) A schematic diagram for the TDFS QSE program. The solid lines are the control process of the TDFS, whereas the dashed lines are to illustrate the design principle of the TDFS. The goal of the QSE program is to design a path in the Hilbert space to connect the initial state with the target state, which is assisted by the TDFS scheme. According to the QSE program, the basic vectors of the TDFS can be used to determine the incoherent control program, and the coherent control project is also fixed by Eq. (6). Therefore, by elegantly combining two projects together, the quantum state will evolve strictly following the TDFS.

common eigenvectors of the Lindblad operators, the design is used to obtain time-dependent Lindblad operators whose common eigenvectors must connect the initial state with the target state. At the same time, the evolution of the quantum state must follow the TDFS strictly, which is the duty of the coherent control part. The coherent control field is not only determined by the incoherent control design, but also restricted by the condition mentioned above, that the TDFS must be invariant under the operator H_{eff}, i.e., $\langle\Phi_i^{\perp}(t)|H_{\text{eff}}|\Phi_j(t)\rangle = 0$ for $\forall\, i,j$, where $|\Phi_i^{\perp}(t)\rangle$ is one of the basic vectors of the component subspace of $\mathcal{H}_{\text{DFS}}(t)$. Considering the concrete structure of H_{eff} in Eq. (4), the condition mentioned above can be reduced to the following form:

$$\langle\Phi_k(t)|H(t)|\Phi_n^{\perp}(t)\rangle = -i\langle\dot{\Phi}_k(t)|\Phi_n^{\perp}(t)\rangle - \frac{i}{2}\sum_{\alpha}\gamma_\alpha c_\alpha^*(t)\langle\Phi_k(t)|F_\alpha(t)|\Phi_n^{\perp}(t)\rangle. \tag{6}$$

As shown in the above equation, the coherent control project (the left terms) and the incoherent control project (the right terms) restrict each other. When the design on the incoherent control project is confirmed, the basic vectors of the TDFS are determined at the same time, which also fixes the coherent control project via Eq. (6). On the other hand, any requirement on the coherent control field (e.g., the shape of the laser field) also limits the incoherent control project. Thus if both the coherent control and the incoherent control projects are manipulated synchronously, the state of the open quantum system will be locked in the TDFS. Therefore, the QSE process is protected completely by the TDFS within an arbitrary total engineering time.

Here we should make some remarks on the TDFS QSE program. (1) Although the TDFS is a scheme for combining the coherent controls with the incoherent ones, as reported in Ref. [15], the TDFS QSE scheme completely protects the quantum state by the symmetry of the interaction between the open quantum system and its environment. (2) Unlike traditional DFS QSE schemes, the basic vectors of the DFS are time dependent, which helps us coherently engineer the quantum state even in a one-dimensional TDFS. (3) The total engineering time of the QSE is not dependent upon the decay rate of the open system; rather, it is determined by the incoherent control project.

III. ENGINEERING QUANTUM STATES BY THE TDFS SCHEME

In the above section, we proposed a realizable method for engineering the quantum state of a single atom by means of the TDFS scheme. The interest in this topic is driven by fundamental connections to quantum physics, as well as by potential applications to quantum-state measurements [30] and quantum computing [31]. In the following, we will show how to engineer the quantum states of a two-level atom into target states.

Consider a two-level atom with ground state $|0\rangle$ and excited state $|1\rangle$ coupled to both a broadband squeezed vacuum field and a coherent control field $\Omega(t)$. In the Markov approximation, the influence of the reservoir on the system of atoms can be described by the dynamical semigroup with the generator

$$\mathcal{L} = -i[H,\cdot] + \mathcal{L}_D. \tag{7}$$

In the rotating frame, the Hamiltonian of the two-level atom can be written as

$$H = \Omega(t)|0\rangle\langle 1| + \text{H.c.} \tag{8}$$

The dissipator caused by the coupling to the squeezed vacuum is

$$\begin{aligned}\mathcal{L}_D\rho = {} & \gamma\cosh^2(r)\big(\sigma_+\rho(t)\sigma_- - \tfrac{1}{2}\{\sigma_+\sigma_-\rho(t)\}\big)\\ & + \gamma\sinh^2(r)\big(\sigma_-\rho(t)\sigma_+ - \tfrac{1}{2}\{\sigma_-\sigma_+\rho(t)\}\big)\\ & + \gamma\sinh(r)\cosh(r)\exp(-i\theta)\sigma_-\rho(t)\sigma_-\\ & + \gamma\sinh(r)\cosh(r)\exp(i\theta)\sigma_+\rho(t)\sigma_+,\end{aligned} \tag{9}$$

where r is the squeezing parameter and θ is the squeezing phase; σ_- (σ_+) is the lowering (raising) operator and γ is the spontaneous decay rate. In Eq. (9), we have assumed that the vacuum squeezing field is perfect. If we redefine the decoherence operator as follows:

$$L = \cosh(r)\exp(-i\theta/2)\sigma_- + \sinh(r)\exp(i\theta/2)\sigma_+, \tag{10}$$

the dissipator can be transformed into the Lindblad form,

$$\mathcal{L}_D\rho = \gamma/2(2L\rho L^\dagger - \{L^\dagger L,\rho\}). \tag{11}$$

By definition, the DFS is composed of states that undergo unitary evolution. Obviously, the one-dimensional (1D) DFS is inadequate for engineering the quantum states into the target state. But if the basic vectors of the DFS depend on time, the DFS will evolve in the Hilbert space of the two-level atom. So we need to find the 1D DFS, and then let it evolve to the target state. This is the main idea of the TDFS.

First, according to the necessary and sufficient condition of TDFSs, a subspace spanned by $\mathcal{H}_t = \{|\phi\rangle\}$ is a decoherence-free subspace if $|\phi\rangle$ is the eigenvector of the Lindblad operator L. It is obvious that the Lindblad operator L [Eq. (10)] gives two nonorthogonal eigenvectors,

$$|\phi_1\rangle = [\sqrt{\sinh(r)}\exp(i\theta/2)|0\rangle + \sqrt{\cosh(r)}|1\rangle]/p,$$
$$|\phi_2\rangle = [-\sqrt{\sinh(r)}\exp(i\theta/2)|0\rangle + \sqrt{\cosh(r)}|1\rangle]/p, \quad (12)$$

with eigenvalues $\lambda_1 = \sqrt{\sinh(r)\cosh(r)}$ and $\lambda_2 = -\sqrt{\sinh(r)\cosh(r)}$, in which $p = \sinh(r) + \cosh(r)$ is the normalizing factor. Any of the eigenvectors can be the basic vector of the subspace $\mathcal{H}_t$. To maintain generality, we choose $|\phi_1\rangle$ to construct the 1D subspace $\mathcal{H}_t$. At the same time, the basic vector of the orthogonal complementary space is also determined by

$$|\phi^{\perp}\rangle = [\sqrt{\cosh(r)}\exp(i\theta/2)|0\rangle - \sqrt{\sinh(r)}|1\rangle]/p. \quad (13)$$

The set of bases $\{|\phi_1\rangle, |\phi^{\perp}\rangle\}$ is a complete set of the Hilbert space of the two-level atom $\mathcal{H}$. Clearly, the eigenvector $|\phi_1\rangle$ depends on the parameters of the squeezed vacuum, i.e., the squeezed parameter r and the squeezed phase θ. Assume that there is a time-dependent squeezed parameter $r(t)$ and a squeezed phase $\theta(t)$, both of which ought to be reasonably chosen and realizable in the laboratory.

In the following, we design an experimental process to create the 1D TDFS. First, the two-level atom is placed in the vacuum field. When it couples to the vacuum, the two-level atom decays into the ground state $|0\rangle$. Here we consider a more realistic case of an extremely small population in the excited state. So the initial state we use here is $|\varphi(0)\rangle = \sqrt{1-o^2}|0\rangle + o|1\rangle$, where o is an extremely small constant. After that, we engineer the surroundings of the two-level atom from the vacuum field to the squeezed vacuum field by means of engineering reservoir technology [32,33], which results in the time dependence of the squeezed parameters. The way in which the parameters depend on time is determined by the scheme of the reservoir engineering [34,35]. For simplicity, both the squeezed parameter and the squeezed phase are set to depend on time linearly,

$$r(t) = \mu t + o, \quad \theta = \nu t, \quad (14)$$

where μ and ν are constants related to the concrete method of reservoir engineering. With the evolution of the squeezed field parameters, the subspace $\mathcal{H}_t$ is a time-dependent 1D subspace in which the quantum state of a two-level atom is protected against decoherence. Thus the two-level atom is controlled to guarantee that the quantum state is bound to the subspace $\mathcal{H}_t$ at all times. In other words, the subspace $\mathcal{H}_t$ is a TDFS if and only if the two-level atom is controlled to make sure that the subspace $\mathcal{H}_t$ is invariant under $H_{\rm eff}(t)$, as shown in Eq. (4). When the effective Hamiltonian $H_{\rm eff}(t)$ acts on a quantum state $|\phi\rangle$ in the TDFS $\mathcal{H}_t$, the quantum state $|\varphi\rangle = H_{\rm eff}(t)|\phi\rangle$ is still within the TDFS $\mathcal{H}_t$, i.e., $\langle\phi^{\perp}|\varphi\rangle = 0$. Taking the Hamiltonian Eq. (8) into the effective Hamiltonian $H_{\rm eff}(t)$ and considering the above requirement, we are able to find an accurate function of the coherent control field $\Omega(t)$. The real part $\Omega_R(t)$ and the imaginary part $\Omega_I(t)$ of the coherent control field $[\Omega(t) = \Omega_R(t) + i\Omega_I(t)]$ can be written as

$$\Omega_R(t) = -\cos(\nu t) f_1(t) - \sin(\nu t) f_2(t),$$
$$\Omega_I(t) = \sin(\nu t) f_1(t) - \cos(\nu t) f_2(t), \quad (15)$$

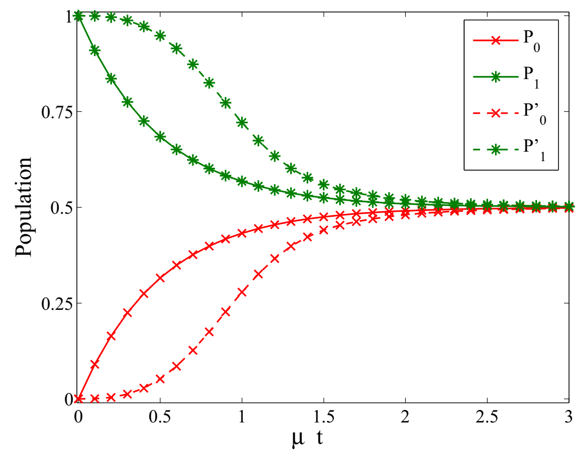

FIG. 2. (Color online) The population of the ground state $|0\rangle$ (red cross lines) and the excited state $|1\rangle$ (green star lines) versus the dimensionless parameter μt. The results were obtained by calculating the master equation with the coherence control field $\Omega(t)$ (solid lines) and without the coherence control field (dashed lines). The figure is evaluated for $\mu = \gamma$ and $\nu = 2\pi\gamma/3$.

with $f_1(t) = \nu\exp(-\mu t - o)\sqrt{\sinh(\mu t + o)\cosh(\mu t + o)}/2$ and $f_2(t) = \exp(-\mu t - o)[\mu/\sqrt{\sinh(\mu t + o)\cosh(\mu t + o)} + \gamma\sqrt{\sinh(\mu t + o)\cosh(\mu t + o)}]/2$. By combining the reservoir engineering scheme Eq. (10) with the coherence control field Eq. (15), the 1D TDFS is constructed and the quantum state of the two-level atom evolves from the ground state $|0\rangle$ to a superposition state $|\phi\rangle$ coherently. In the same way, the QSE with a different initial state can also be engineered.

To judge the validity of our scheme for population engineering, we studied both the population transferring from the ground state to the excited state and the purity of the quantum state. In Fig. 2, the populations in the ground state $|0\rangle$ (red cross lines) and the excited state $|1\rangle$ (green star lines) are both plotted. The solid lines in the figure are the populations in the ground state (P_0) and the excited state (P_1), which are plotted according to the master equation (7) with the coherence control field Eq. (15); the dashed lines are the populations in the ground state (P_0') and the excited state (P_1'), which are plotted based on the same master equation but without the coherence control field. Here we choose $\mu = \gamma$ and $\nu = 2\pi\gamma/3$. Fig. 2 shows that either the two-level state is manipulated by $\Omega(t)$ or not and that the population will definitely transfer from the ground state to the excited state; the populations in the ground state and the excited state are equal when the steady state is reached.

However, the principle behind the similarity mentioned above is different. On the one hand, when the two-level atom is not manipulated by the coherent control field Eq. (15), its quantum character will gradually be lost because of the coupling to the squeezed vacuum field. If we consider the purity (p') of the quantum state (see the green solid line in Fig. 3), we find it decays over time. As a consequence, the quantum state will become the maximally mixed state [2]. On the other hand, when the two-level atom is coherently controlled on the basis of Eq. (15), the quantum state of the two-level system stabilizes in the TDFS from the beginning to the end. The TDFS ensures that the evolution of the quantum

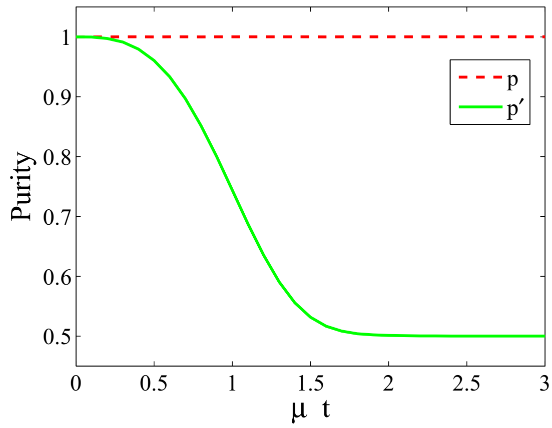

FIG. 3. (Color online) The purity versus the dimensionless parameter μt. The results were obtained by calculating the master equation with the coherence control field $\Omega(t)$ (red solid lines) and without the coherence control field (green dashed lines).The figure is evaluated for $\mu = \gamma$ and $\nu = 2\pi\gamma/3$.

state is unitary and that the purity (p) does not change over time (see the red dashed line in Fig. 3). The results obtained above coincide with our previous prediction. We should also mention that the choice of $\mu = \gamma$ is not a necessary requirement on our QSE scheme; it is used to compare our scheme with the decoherence process of a two-level atom. Theoretically speaking, the selections of μ and ν are quite arbitrary. The only limiting factor is the experimental technology for the reservoir engineering.

For illustrating the phase engineering more obviously, the Bloch vectors are plotted in Fig. 4. As shown by the red straight line in Fig. 4, the quantum state decays to the center of the Bloch sphere gradually in the case of absence of the coherent control field. Even though the squeezed vacuum field is engineered accordingly, there is no response of the phase to the reservoir engineering. However, when the two-level atom is manipulated by the coherent control field [Eq. (15)], the

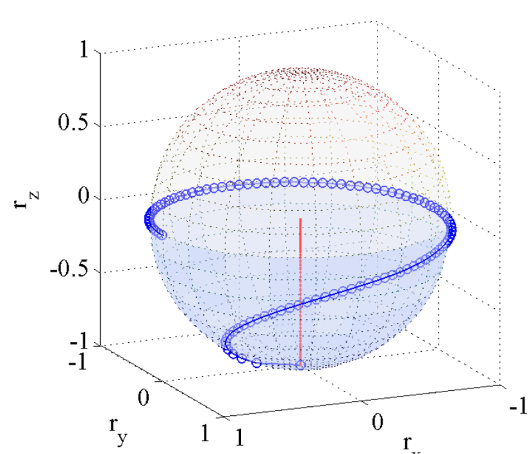

FIG. 4. (Color online) The evolution of the quantum state on the Bloch sphere. The results were obtained by calculating the master equation with the coherence control field $\Omega(t)$ (blue curved line) and without the coherence control field (red straight line). The figure is evaluated for $\mu = \gamma$ and $\nu = 2\pi\gamma/3$.

situation is changed. The phase between the ground state and the excited state (blue curved line in Fig. 4) varies following our prediction, which is useful in the quantum computation program [1].

The coherence control field is the key point for implementing the TDFS scheme for both the population and the phase engineering. When the atom is controlled opportunely, the Bloch vector of the quantum state is on the Bloch sphere's surface. Otherwise, the Bloch vector will enter the Bloch sphere towards the zero vector [2]. Considering the asymptotic behavior of the TDFS's basic vector, every single point on the surface of the lower part can be reached by means of the TDFS scheme.

IV. ADJUSTMENT OF THE COHERENT CONTROL FIELD

If we neglect the extremely small constant o in Eq. (15), the coherence control field should have a singular point at $t = 0$, i.e., $\lim_{o\to 0} \Omega(0) = \infty$. It is difficult to achieve such a control function experimentally. In the following, we propose another coherence control function to avoid such a case. It can be observed that the single point is caused by the denominator $[\sqrt{2\sinh(\mu t)\cosh(\mu t)}]$ of the function $f_2(t)$, so that the control function of the coherent control field can be adjusted accordingly. The new control field $\Omega'(t)$ has the same structure of $\Omega(t)$ as Eq. (15); the only difference is that

$$f_2(t) = \frac{\exp(-\mu t)}{2}\left(\frac{\mu}{\sqrt{\sinh(\mu t+\epsilon)\cosh(\mu t)}} + \gamma\sqrt{\sinh(\mu t)\cosh(\mu t)}\right), \tag{16}$$

where ϵ is a small constant. When we use the control function $\Omega'(t)$ instead of $\Omega(t)$, the evolution of the quantum state is not unitary and the purity must decay. Here we intend to study the effect of this modification on both the coherence control field and the purity of the quantum state. On the one hand, with the increase of constant ϵ, the control field's strength $\Omega'(0)$ becomes weaker and weaker, which is advantageous in the realization of the TDFS scheme experimentally. On the other hand, the modification of the coherence control field cannot protect the quantum state perfectly. Therefore, we will concentrate on the asymptotic state of the two-level atom first. Since the adjustment to the coherent control field in this way does not affect the phase engineering and the manipulation of the phase has no effect on the asymptotic purity, it is convenient to choose $\nu = 0$ in the following discussion. As a consequence, the coherent control field is given by

$$\Omega'(t) = -i\frac{\exp(-\mu t)}{2}\frac{[\mu + \gamma\sinh(\mu t)\cosh(\mu t)]}{\sqrt{\sinh(\mu t+\epsilon)\cosh(\mu t)}}. \tag{17}$$

Taking the above coherent control function in Eq. (7), the matrix elements of the quantum state ρ with respect to the basis $|0\rangle$ and $|1\rangle$ satisfy the following differential equation set:

$$\begin{aligned}\dot{\rho}_{00} &= \sinh^2(\mu t) - 2i\Omega'(t)\rho_{01} - \cosh(2\mu t)\rho_{00},\\ \dot{\rho}_{01} &= i\Omega'(t)(1-2\rho_{00}) - \exp(-2\mu t)\rho_{01},\end{aligned} \tag{18}$$

in which $\Omega'(t) = -\Omega'(t)^*$ and $\rho_{00} + \rho_{11} = 1$ were considered. Direct calculations show that the quantum state is a unique

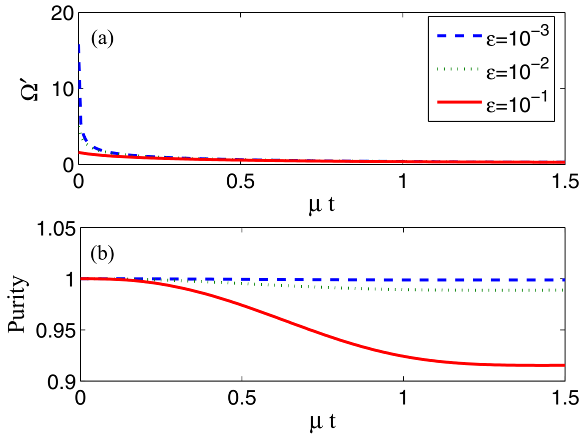

FIG. 5. (Color online) The evolution of (a) the control field $|\Omega'|$ and (b) the purity p. The results were obtained by calculating the master equation with the adjusted control function $\Omega'(\epsilon)$ with $\epsilon = 10^{-3}$ (blue dashed line), $\epsilon = 10^{-2}$ (green dotted line), and $\epsilon = 10^{-1}$ (red solid line). The figure is evaluated for $\mu = \gamma$.

stationary asymptotic state ρ^s that has nonvanishing matrix elements

$$\rho^s_{00} = 1/2, \quad \rho^s_{01} = 2\Omega'_s, \tag{19}$$

with the asymptotic strength of the coherence control field $\Omega'_s = \exp(\epsilon/2)/4$. When the coherent control field is absent, the asymptotic state is the maximally mixed state; when $\epsilon = 0$, the TDFS scheme can be achieved. Generally speaking, the asymptotic purity as a function of the parameter ϵ can be written as

$$p_s = \frac{1 + \exp(-\epsilon)}{2}, \tag{20}$$

In Fig. 5, the evolutions of both the purity and the coherence control field are plotted. With the increase of the parameter ϵ, the strength of the control field is evidently reduced. At the same time, the purity also decays. However, we can see that even the control field strength is so weak [red solid line in FIG. 5(a)] that the asymptotic purity is still high. In other words, although the coherence field cannot be reached as in Eq. (15), the coherence of the quantum state is also robust.

What we have shown above is only an example of adjusting the control field Ω, but there are still many more methods to adjust it. Generally speaking, we can introduce the adjusted control field as

$$\Omega_g(\epsilon) = \Lambda(\epsilon)\Omega, \tag{21}$$

where $\Lambda(\epsilon)$ is an adjusted function, ϵ should not have to be a constant, and Ω is the coherent control field, as shown in Eq. (15) with $\nu = 0$. The function $\Lambda(\epsilon)$ needs to satisfy the following conditions: (1) The adjusted control field Ω_g must be analyzed at the singular point of the control field Ω; (2) Under the control of Ω_g, the purity must be as robust as possible. To give an example, we introduce a simple function $\Lambda(t) = \sqrt{\sinh(\mu t)/\sinh[\mu t + \epsilon_0 \exp(-\Gamma t)]}$, where Γ is the decay rate of the parameter ϵ_0. Such an adjusted control field certainly satisfies both of the conditions mentioned above. For $t = 0$,

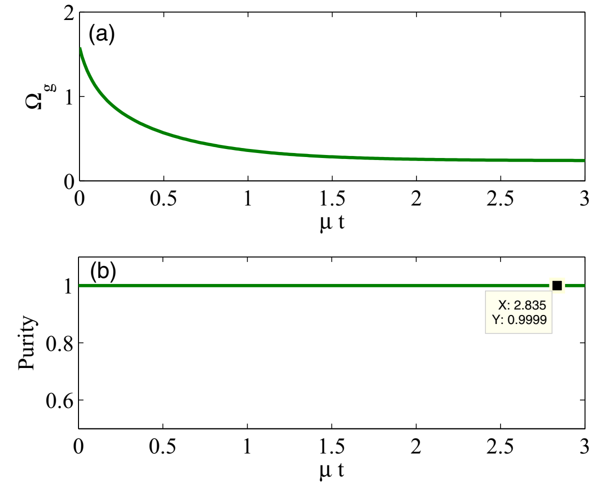

FIG. 6. (Color online) The evolution of (a) the control field $|\Omega_g|$ and (b) the purity p. The results were obtained by calculating the master equation with the control function Ω_g with $\epsilon_0 = 10^{-1}$ and $\Gamma = 10^3\gamma$. The figure is evaluated for $\mu = \gamma$.

the singular point of the control field vanishes. And since

$$\frac{d|\Omega_g|}{d\Gamma} = |\Omega|\sqrt{\frac{\sinh[\mu t + \epsilon(t)]}{\sinh \mu t}} \times \frac{\Gamma \exp(-\Gamma t)\cosh[\mu t + \epsilon(t)]\sinh(\mu t)}{2\sinh[\mu t + \epsilon(t)]} > 0,$$

the following inequality can be given:

$$\frac{dp_s}{d\Gamma} = 16|\Omega_g|\frac{d|\Omega_g|}{d\Gamma} > 0.$$

These results indicate that the more rapidly the parameter decays, the higher is the purity obtained. The purity of the quantum state controlled by $\Omega'(\epsilon_0)$ is the lower limit, which is controlled by Ω_g. This adjustment on the control function is so powerful that a simple and realizable control field can protect the quantum character of the two-level atom. In Fig. 6, the control field Ω_g and the purity p are presented, where the decay rate Γ is chosen as $\Gamma = 10^3\mu$ and the constant parameter is $\epsilon_0 = 10^{-1}$. The evolution of the quantum state is almost unitary and the purity is no less than 0.9999. It is important to emphasize that the TDFS scheme is universal and allows several ways of engineering the reservoir coupled to the main system. By engineering the reservoir and choosing the control field properly, the quantum state of the main system can be engineered as though the surrounding environment does not exist.

V. SUMMARY

We have presented a proposal employing a TDFS scheme to engineer the quantum state of an open quantum system. We showed that, although the quantum system couples to a decohering environment, the QSE process designed using our method is completely unitary within an arbitrary total engineering time. As shown in this paper, the TDFS QSE program is designed according to an elegant combination of incoherent control and coherent control projects, which play different roles in this program.

Such a method is powerful and reliable for realizing quantum information and quantum simulation equipment. First, a QSE task can be implemented in a one-dimensional TDFS with no fidelity loss. For the QSE program of an open quantum system, the previous proposals either need numerous physical qubits to construct a multidimensional DFS or require ultrafast operation on a single physical qubit in order to preserve the quantum character of the open quantum system. Moreover, every single common eigenvector of the Lindblad operators with any eigenvalue can be chosen as the basic vector of the TDFS. This provides various selections for implementing the QSE program, which is useful for finding the best scheme in realization of the QSE program. Second, a real quantum information process always involves numerous operations on a single qubit. A tiny loss in fidelity in one of the QSE programs will lead to the quantum information process failure after repeated operation on the same qubit. To avoid this, the QSE process must be unitary, or at least the asymptotic purity of the quantum state must be robust. Our method can achieve a unitary operation on the qubit to implement a multitask QSE program with no loss of fidelity. Even if the effect of decoherence excites the state, the asymptotic purity remains satisfactory. So the proposed scheme is not only a reliable QSE process experimentally, it is also a good choice when the goal is to construct a real quantum computer or quantum communication equipment.

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