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Life cycle of superfluid vortices and quantum turbulence in the unitary Fermi gas

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The unitary Fermi gas (UFG) offers a unique opportunity to study quantum turbulence both experimentally and theoretically in a strongly interacting fermionic superfluid with the highest vortex line density of any known superfluid. It yields to accurate and controlled experiments and admits the only dynamical microscopic description via time-dependent density-functional theory, apart from dilute bosonic gases, of the crossing and reconnection of superfluid vortex lines conjectured by Feynman [R. P. Feynman, Prog. Low Temp. Phys. **1**, 17 (1955)] to be at the origin of quantum turbulence in superfluids at zero temperature. We demonstrate how various vortex configurations can be generated by using well-established experimental techniques: laser stirring and phase imprinting. New imaging techniques demonstrated by Ku *et al.* [M. J. H. Ku *et al.*, Phys. Rev. Lett. **113**, 065301 (2014)] should be able to directly visualize these crossings and reconnections in greater detail than performed so far in liquid helium. We demonstrate the critical role played by the geometry of the trap in the formation and dynamics of a vortex in the UFG and how laser stirring and phase imprint can be used to create vortex tangles with clear signatures of the onset of quantum turbulence.

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Quantized vortices are a hallmark of superfluids. Their generation, dynamics, evolution, and eventual decay have been studied experimentally and theoretically for some six decades in liquid ^{4}He and ^{3}He, Bose and Fermi cold-atom systems, neutron stars, condensed-matter systems, cosmology, and particle physics. The two-component unitary Fermi gas (UFG) is of particular interest due to precise experimental control in cold-atom traps and almost direct applicability to dilute matter in neutron star crusts where experiments and direct observations are not possible. The experimental control and universality of the system make it one of enormous interest for those studying phenomena in relativistic heavy-ion collision, nuclear physics, nuclear astrophysics, atomic physics, and condensed-matter physics.

In a recent Letter [1] we showed that the initial conditions as described in the experiment in [2], which claimed to have an axially symmetric trap, lead to the production of superfluid vortex rings. The properties of these rings provided a natural explanation of several puzzling characteristics displayed by the objects observed in [2], where they were called heavy solitons. In particular, the vortex ring scenario explained successfully the long oscillation periods, the unusually large apparent effective mass of the objects, and details of the imaging procedure that resulted in objects appearing much larger than the natural width of a vortex.

The experiment [2] was recently updated [3] with an improved method of imaging slices from the original experiment (tomography), conclusively demonstrating that the heavy solitons are ultimately vortex segments that consistently align themselves across the vertical imaging axis. The authors of [3] suggest that this systematic alignment is due to asymmetries in the trap arising from gravitational distortions of the optical trapping potential in the vertical direction.

The optical trapping potential in the x and y directions is an axially symmetric Gaussian altered by gravity in the vertical direction y:

$$V(x,y,z) = \frac{m\omega_z^2 z^2}{2} + O(z^4) + V_0\left[1 - \exp\left(-\frac{m\omega_x^2(x^2+y^2)}{2V_0}\right)\right] + mgy. \quad (1)$$

Shifting $y \to y + y_0$, where y_0 is the new minimum, gives the effective trapping potential

$$V(x,y+y_0,z) \approx \frac{m\omega_z^2 z^2}{2} + O(z^4) + \frac{m\omega_x^2 x^2}{2} + \frac{m\omega_y^2 y^2}{2} + Cy^3 + O(\delta^2) + \text{const}, \quad (2)$$

where $\delta = 3mg^2/4\omega_x^2 V_0$ is treated perturbatively and

$$\omega_y \approx \omega_x(1-\delta), \quad C \approx \frac{2m\omega_x^4}{3g}\delta. \quad (3)$$

According to [3], $\delta = 1 - \omega_y/\omega_x \approx 5\%$ is small, justifying this expansion. The axial symmetry is thus broken by two effects: an anisotropy δ and an anharmonicity Cy^3, which we characterize in terms of $C_0 = m\omega_y^2/2R$, where $C = C_0$ would give equal quadratic and cubic terms at the Thomas-Fermi radius R where $V(0,R+y_0,0) = \mu$. With the experimental parameters [2,3], $C/C_0 \approx \delta \approx 5\%$.

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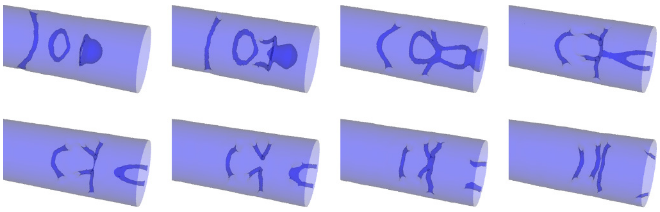

FIG. 1. (Color online) (Video available via Ref. [7].) Demonstration of off-axis vortices in the UFG generated by firing a bullet along an axially symmetric trap in a movie from the simulations included with Ref. [6]. Off-axis vortex rings attach to the walls and turn into vortex lines that cross and reconnect. The movie also demonstrates that vortex rings and lines move at similar velocities.

Here we show that accounting for gravity indeed induces the initially produced vortex rings [1,4,5] to convert into vortex lines, which oscillate along the long axis of the trap. The conversion of off-axis vortex rings into vortex lines on the boundary of a trap was first demonstrated in the UFG in the simulations of [6] (reproduced in Fig. 1). In the context of [2], the possibility of this conversion was also suggested in Ref. [8], where asymmetries were induced manually through stochastic noise. Without a systematic asymmetry in the trap, however, stochastic variations lead to randomly orientated vortex lines inconsistent with the experimental observations. The simulations [8] model only the condensate, but lack averaging over stochastic trajectories or a density matrix as required to properly describe thermal fluctuations. The role of these fluctuations also diminishes at low temperatures, becoming irrelevant at zero temperature. Our zero-temperature approximation is supported by [9], which finds thermal effects to be negligible for vortex reconnection.

Gravity provides a systematic breaking of the axial symmetry, allowing for a consistent conversion of the initial vortex rings into a horizontally oriented vortex line, though the exact nature of this conversion seems to be quite sensitive to geometry. A similar sensitivity to geometry was noted in both experiment [10] and theory [6], where a high degree of axial symmetry was found to be critical in the formation and stability of a vortex lattice. As can be seen in [6] (see the movie in [7]), [8], and the simulations presented here, vortex rings and vortices move with almost the same velocity. This is to be expected since small segments of both vortex rings or vortex lines move according to the well-established Magnus relationship in response to a similar buoyant force directed out from the trap (see [11] for a discussion and references therein). Both large rings and vortices have the same structure and core depletion and will therefore move with comparable velocities and periods up to small corrections due to geometric (curvature) effects.

Likewise, the expansion of a large vortex ring and a vortex segment will behave similarly, so the subtle dependence of the final image on the imaging procedure as discussed in [1] remains valid, explaining how the vortex core of the interparticle scale expands to appear as a much larger object. The only potentially noticeable difference between the motion of vortex rings and vortices is a possible asymmetry in the oscillations for vortex rings that return as a small ring down the center of the trap. It turns out, however, that the asymmetry in this motion is quite small except in the case of very large amplitude oscillations [1,12]. As a result, distinguishing between the two scenarios is best performed by imaging from different directions or through tomography [3].

Here we extend the fermionic superfluid local-density-approximation (SLDA) simulations of [1] to include an anisotropy δ and anharmonicity C/C_0 to model the dynamics of an imprinted domain wall in a cloud of $\sim$560 particles in the UFG. We use the same formalism and initial-state preparation detailed in [1] on a $32^2 \times 128$ lattice, but now include gravitational effects (2) comparable to those describing the trap in [3].

While our simulations only contain about 1000 particles, they provide a more accurate representation of the initial experimental conditions than available through any other technique. In particular, the extended Thomas-Fermi (ETF) model, a bosonic Gross-Pitaevskii (GPE)-like density-functional theory, used in [1,4,5] lacks a mechanisms for the superfluid to relax. Thus, while suitable for studying the qualitative dynamics of vortex motion in large traps, the ETF is not suitable for the period shortly after the imprint where the system exhibits significant relaxation (see Ref. [7] for a comparison between the SLDA and the ETF). An *ad hoc* relaxation was included in Ref. [8], but this model does not reproduce UFG equation of state and lacks the quantitative validation of the SLDA where the functional is fully determined by fitting quantum Monte Carlo and experimental results (see [13] and references therein). Despite the absence of an explicit collision integral, the large number of degrees of freedom in the SLDA permits many mechanisms for superfluid relaxation including various phonon processes, Cooper pair breaking, and Landau damping. In the UFG, the bulk viscosity vanishes [14] and below the critical temperature the shear viscosity is dominated almost exclusively by interacting phonons [15], which are included in the SLDA.

We start in Fig. 2 with a moderate anisotropy $\delta \approx 10\%$ that elongates the trap vertically as in the experiment. As demonstrated in [1], the phase imprint rapidly seeds the formation of a vortex ring, but as this ring evolves, it is attracted to the surface and eventually hits the nearer boundaries on

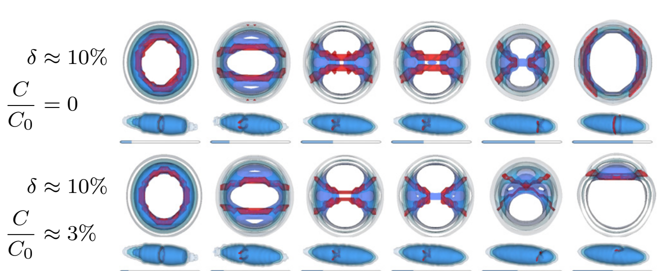

FIG. 2. (Color online) (Video available via Ref. [7].) Effects of moderate anisotropy $\delta = 1 - \omega_y/\omega_x \approx 10\%$ (the trap is taller than it is wide). The ring hits the narrow walls on the side, forming two parallel vortices. Without any anharmonicity $C = 0$ (top), these undergo several reconnection, oscillating between a horizontal and a vertical orientation. The presence of an anharmonicity $C/C_0 \approx 3\%$ breaks the symmetry, eventually expelling one vortex from the system, resulting in a long-lived vortex that oscillates back and forth.

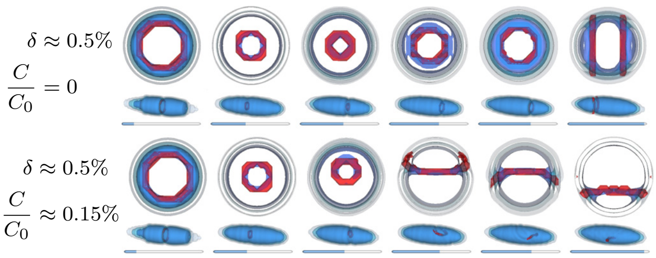

FIG. 3. (Color online) (Video available via Ref. [7].) Effects of weak anisotropy. The ring now exists long enough to return as a small ring. Without an anharmonicity (top) several oscillations occur before the ring expands and hits the walls forming two parallel vortices. Adding a weak anharmonicity (bottom) causes the small ring to rotate. Once twisted off axis, this ring rapidly converts to a single vortex as shown in [8]. The rapid production of a single vortex from a twisted defect was also described in [17] and provides a convenient way of constructing vortices.

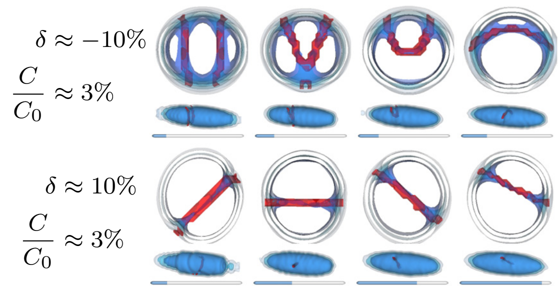

FIG. 4. (Color online) (Video available via Ref. [7].) The trap on the top is wider than it is tall, the energetically favored configuration would be a vertical vortex, but the initial conditions lead to a slightly more energetic vortex aligned along the longer of the two axes. On the bottom, we imprint an oblique vortex line in a trap with the same asymmetry as the experiment. As it oscillates long the trap, the vortex alignment oscillates about the favored horizontal position, but without significant damping, and continues past this alignment to another oblique position. The presence of significant damping, such as through phonons at finite T, might allow this to relax to a horizontal alignment. However, this relaxation would likely be comparable to the time it takes the vortex to oscillate out of the system; the true minimum energy state has no vortex at all.

the side, converting to a pair of vortex lines. These lines then oscillate along the trap undergoing a crossing and reconnection process similar to that seen using GPE simulations [16], the mechanism responsible for quantum turbulence, changing orientation from horizontal to vertical and back. The addition of a $C/C_0 \approx 3\%$ anharmonicity along the y direction, similar to that induced by gravity in the experiment, breaks the mirror symmetry and one of the two vortices is gradually ejected from the system, leaving a single vortex oriented horizontally along the shorter axis of the trap that oscillates through the cloud, consistent with the observations [3].

In Fig. 3 we compare similar systems, but now with asymmetries an order of magnitude smaller. In this case, the vortex ring persists much longer before converting into vortex lines, long enough to return as a small vortex down the center of the trap. This smaller vortex appears to be very sensitive to even a tiny anharmonicity, which causes it to tilt upward and collide with the upper wall of the trap, rapidly forming a single horizontally aligned vortex. This rapid formation of a single vortex from a tilted ring or imprint was also obtained in simulation [8] and mentioned in [3,17] and seems like a more reliable mechanism for consistently forming a single horizontally aligned vortex (see the movie tilted imprint in Ref. [7]).

In particular, as demonstrated on the top of Fig. 4, even when given a negative anisotropy $\delta \approx -10\%$ so that the trap is compressed vertically, a metastable horizontally aligned vortex may still result. It was suggested in [3] that the horizontal alignment results from the fact that this state has lower energy, but evolution in these systems is conservative to a high degree of accuracy, hence rapid progress toward a state with lower energy or larger statistical weight is not guaranteed *a priori*. Indeed, a lower-energy state exists, that with the vortex ejected from the system, and we expect the time scale for a vortex to relax into a stable horizontal configuration to be comparable to or longer than the time scale for a vortex to lose its energy and completely oscillate out of the system. To check, we imprinted a misaligned vortex in a trap with both asymmetries $\delta \approx 10\%$ and $C/C_0 \approx 3\%$. As shown on the bottom of Fig. 4, although this vortex does rotate toward the horizontal configuration, it quickly rotates past the configuration to an approximate mirror misalignment.

While here we have stressed the role of broken symmetries in the evolution of vortex rings, it is still a challenge for experimentalists to perform similar experiments in axially symmetric traps, similar to those used in [10], where vortex rings are likely to survive for long periods of time.

Almost six decades ago, Feynman [18] envisioned vortex crossing and reconnection as responsible for quantum turbulence in cold superfluids that lack dissipation at zero temperature. While many study turbulent phenomena [19–25], we are aware of a few recent experiments that have directly observed in experiment the vortex crossing and reconnection mechanism [26–29]. These dynamics and interactions play a crucial role in many fermionic superfluids with applications to conduced-matter physics, neutron stars, cosmology, and particle physics (see, e.g., [30]). For example, to explain pulsar glitches in neutron stars, one may need to quantitatively understand energy loss during crossing and reconnection as inputs to glitching models (see, e.g., [11]). The UFG provides an almost ideal laboratory to study these phenomena and benchmark the SLDA. Using multiple tilted imprints, for example, one can control the generation and arrangement of multiple vortices in order to study collisions, reconnection, and interactions. The UFG and SLDA thus provide a new microscopic framework to study aspects of quantum turbulence in a strongly interacting system, complementing weakly interacting dilute Bose gases [31,32] modeled with the GPE as the only microscopic frameworks presently available for studying superfluid dynamics.

The phase imprint technique can be utilized also to create turbulent states with many tangled vortices. Here we

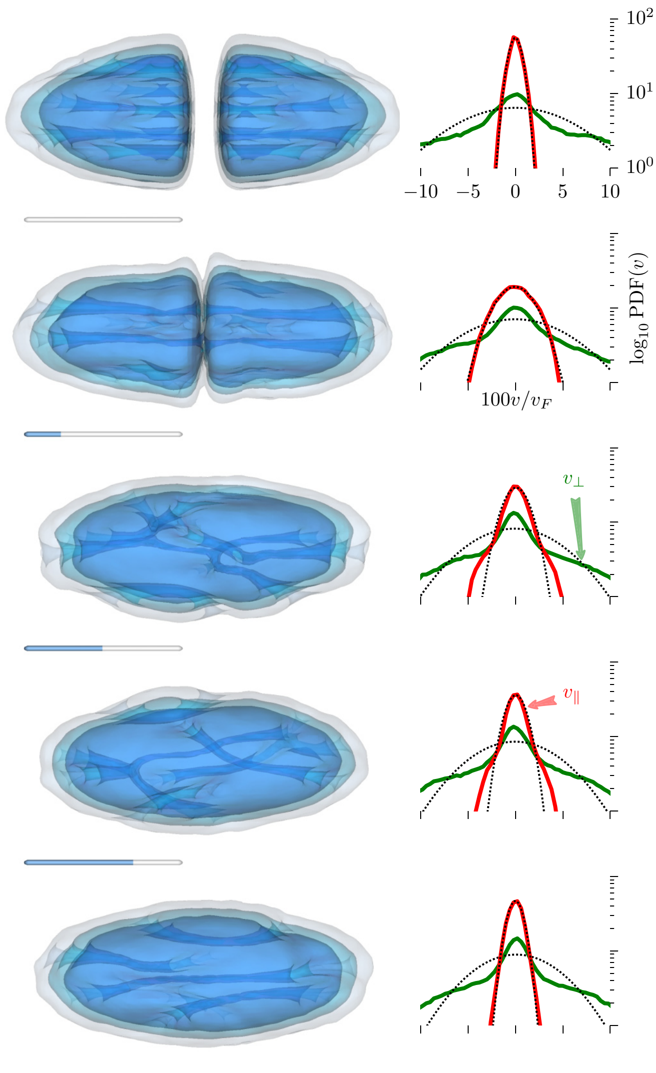

FIG. 5. (Color online) (Video available via Ref. [7].) Generation of quantum turbulence by phase imprint of the vortex lattice. In the left column consecutive frames show (a) a vortex lattice with a knife edge dividing the cloud, (b) just after phase imprint removal of the knife, and (c)–(e) decay of turbulent motion. In the right column we show the corresponding PDFs for longitudinal $v_\parallel$ and transverse $v_\perp$ components of collective velocity. Dotted lines show the Gaussian best fit to the data.

demonstrate one approach, adding a phase imprint [2,3] to a lattice of vortices that can be created experimentally by stirring using laser beams [10]. In Fig. 5 we show consecutive frames of turbulent motion exhibiting crossings and reconnection of quantized vortices in an elongated harmonic trap. The simulation was done in a $48^2 \times 128$ box comprising 1410 fermions (see Ref. [7] for a movie). We also show the corresponding probability distribution function (PDF) of the velocities for longitudinal $v_\parallel$ and transverse $v_\perp$ components of the velocity (with respect to long axis).

We start with the ground state of a cloud cut in half with a knife-edge potential. We then stir the system with two circulating laser beams parallel to the long axis of the trap. Once a vortex lattice is generated, we imprint a π phase shift between the halves. Just before removing the edge knife, we introduce a slight tilt to speed the formation of a vortex tangle. After the knife edge is removed, the vortex lines twist, cross, and reconnect. From the velocity PDFs one sees a clear departure from Gaussian behavior as the tangle evolves, a hallmark of quantum turbulence. Eventually the system relaxes to a vortex lattice and equilibrates in $v_\parallel$. Somewhat similar velocity PDFs are seen in theoretical studies of dilute Bose gases [33] and in the phenomenological filament model of the crossing-reconnection vortex line dynamics [34].

In conclusion, have shown the crucial role played by the trap geometry in the formation of a vortex line after a phase imprint. In particular we identified a few possible scenarios for the short-term evolution of the phase imprint in the experiments [2,3], showing that the details are highly sensitive to geometric factors. To precisely characterize the behavior realized in the experiments [2,3], the experiment will likely need to be simulated with precise values of the trapping asymmetries known and with realistic particle numbers that are currently beyond the capabilities of the most advanced implementations of the SLDA approach. Satisfactory agreement with the latest experiments by Ku *et al.* serves as the next step in validating the time-dependent SLDA, demonstrating that it is capable of qualitatively describing the complex dynamics of strongly interacting fermionic systems. We have demonstrated that reconnection is likely present in the early stages of the experiments [2,3] (see Figs. 2 and 3) and can be selected by reducing the anharmonicity of the trap. We have also presented that the phase imprint technique can be utilized to generate quantum turbulent state. Therefore, using improved imaging techniques [3] coupled with carefully designed initial conditions, cold-atom experiments have a great opportunity to directly probe and quantify the dynamics and interactions vortices and the potential to significantly advance our understanding of quantum turbulence. In this regard, the unitary Fermi gas is of particular interest as the results will have almost direct impact on superfluid dynamics and turbulent phenomena in strongly interacting Fermi superfluids, in particular neutron star phenomenology.

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