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Decoherence speed limit in the spin-deformed boson model

Sh. Dehdashti,[1,2,3,*] M. Bagheri Harouni,[4,†] B. Mirza,[3,‡] and H. Chen[1,2,§]

[1]*State Key Laboratory of Modern Optical Instrumentations, Zhejiang University, Hangzhou 310027, China*
[2]*The Electromagnetics Academy at Zhejiang University, Zhejiang University, Hangzhou 310027, China*
[3]*Department of Physics, Isfahan University of Technology, Isfahan 84156-83111, Iran*
[4]*Department of Physics, Faculty of Science, University of Isfahan, Hezar Jerib Street, Isfahan, 81746-73441, Iran*
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In this paper, we study the role of the nonlinear environment on the bound passage time of dynamical quantum spin systems, which is of great interest in quantum control and has been applied to quantum metrology, quantum computation, and quantum chemical dynamics. We consider the decoherence speed limit for the spin-deformed bosonic model and the impacts of the nonlinear environment and its temperature on the decoherence speed limit. Moreover, we show that, at an early enough time, the parameters associated with the nonlinear environment exhibit important roles in controlling the decoherence process. In addition our results reveal that, in long times, these parameters do not affect the decoherence process.

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I. INTRODUCTION

Quantum mechanics as a fundamental law of nature imposes limits to the evolution speed of quantum systems. Nowadays, these limits have found remarkable roles in different scenarios including quantum communication, identification of precision bounds in quantum metrology, formulation of computational limits of physical systems, and development of quantum optimal control algorithms [1–5]. In fact, quantum mechanics acts as a legislative body that imposes speed limits, on the one hand, as a fundamental problem and, on the other hand, when considering the effects of environment on the evolution of quantum systems.

In the first instance, as a fundamental problem, the quantum speed limit is imposed as a bound on the speed of evolution which is intimately related to the concept of passage time, τ_{min}. This is the time required for a given pure state $|\chi\rangle$ to become orthogonal to itself under unitary dynamics [6]. Also, earlier studies have indicated that the passage time, τ_{min}, can be the lower bound by the inverse of the variance in the energy of the system, i.e.,

$$\tau_{\text{min}} \geqslant \frac{\pi}{2}\frac{\hbar}{\Delta H}, \tag{1}$$

where $\Delta H = \left(\langle H^2\rangle - \langle H\rangle^2\right)^{1/2}$, whenever the dynamics under study is governed by a Hermitian Hamiltonian, H [7–13]. If the passage time problem is considered as a quantum brachistochrone problem, it has been shown that, whenever the Hamiltonian is non-Hermitian $\mathcal{PT}$-symmetric, the passage time can be made arbitrarily small without violating the time-energy uncertainty principle [14–17].

On the other hand, an analogous bound has been considered for some open quantum systems [18–29], since all systems are ultimately coupled to an environment [30–34]. In these cases, such a bound on the evolution of an open system would help to address the robustness of the quantum system which is applied, for example, to simulators and computers against decoherence [35]. Therefore, this quantity (bound of speed limit) requires the effects associated with the environment to be quantified. In this case, the role of nonlinearity, such as confinement and curvature as well as temperature, on the bound passage time is desirable. This motivates one to investigate the decoherence mediated by a structured environment through the passage time. In fact, the present contribution studies the impacts of temperature and nonlinearity of environment on the bound passage time for the spin system in contact with these environments.

Along these lines we study, in this paper, the bound passage time τ_ζ in a spin system, as a quantum system interacting with the deformed harmonic oscillators, as a nonlinear boson environment. In fact, the spin-boson model is one of the most important physical systems for both its theoretical aspects and its applications. With respect to theoretical aspects, the spin-boson model exhibits features characteristic of the decoherence process. Thus, it is an ideal candidate for the study of decoherence in two-level systems. The spin-boson model describes a single two-level system interacting with a large reservoir of boson field modes [36–39], i.e., a spin-$1/2$ particle coupled to an environment, which can be formulated by harmonic oscillators because of the central limit theorem [40,41]. This model has been widely studied in the context of decoherence and the dissipation process in quantum systems [31,42]. Also, the role of two-level (qubit) systems in quantum computing [31] and in experiments dealing with macroscopic quantum coherence has led to additional interest in the spin-boson model [43]. Two-level systems are also believed to be found in many amorphous materials [43,44] while the spin-boson model has been employed for some kinds of chemical reaction and the motion of defects in some crystalline solids and for analyzing the role of quantum decoherence in biological systems [45]. In addition, the effect of the nonlinear environment on the decoherence rate of a spin-boson model has been recently studied [46].

As already mentioned above, we study the effects of nonlinearity and temperature of the environment on the bound passage time of a spin system. We show that the bound

*shdehdashti@zju.edu.cn
†m-bagheri@phys.ui.ac.ir
‡b.mirza@cc.iut.ac.ir
§hansomchen@zju.edu.cn

passage time τ_ζ depends on the environmental characteristics. The nonlinearity of an environment is illustrated by using f-deformed harmonic oscillators [47–57], instead of nondeformed ones. Also, two kinds of confinement, namely, the harmonic oscillator in an infinite well and that in the well with a finite depth, are considered as examples of f-deformed harmonic oscillators [58–61]. It is, therefore, clear that one can control the bound passage time τ_ζ of the coherence property of the quantum system by manipulating environmental characteristics.

The paper is organized as follows. In Sec. II, we briefly review the dynamics of spin systems which interact with a nonlinear environment and we introduce, as examples, two kinds of confinement which characterize the nonlinear environment, that is, a harmonic oscillator in the infinite well and one in the well with a finite depth constituting the environment. The pure relativity of the spin-deformed boson system is presented in this same section. We draw a comparison between the fidelities of spin systems which interact with these environments in Sec. III by obtaining the bound of passage time. Also in Sec. III, we draw a comparison between these two environments' effects on the bound passage time. Finally, Sec. IV is devoted to some conclusions and remarks.

II. PURE RELATIVITY OF SPIN-DEFORMED BOSON SYSTEM

We start with the spin-deformed boson model expressed by the following Hamiltonian [43]:

$$\hat{H} = \hat{H}_S + \hat{H}_\varepsilon + \hat{H}_{\text{int}}, \tag{2}$$

where, $\hat{H}_S = \frac{1}{2}\hbar\omega_0\hat{\sigma}_z$ is the self-Hamiltonian of the two-level system and $\hat{\sigma}_z$ is the usual z component of the Pauli matrix. Eigenstates of σ_z are denoted by $|+\rangle$ and $|-\rangle$. H_ε describes the familiar self-Hamiltonian of the environment which is assumed to be modeled by the nonlinear harmonic oscillators as follows:

$$\hat{H}_\varepsilon = \sum_i \hbar\omega_i(\hat{A}_i\hat{A}_i^\dagger + \hat{A}_i^\dagger\hat{A}_i), \tag{3}$$

where $\hat{A}$ and $\hat{A}^\dagger$ are generalized deformed operators which are introduced through the following definitions

$$\begin{aligned} \hat{A} &= \hat{a}\sqrt{f(\hat{n})} = \sqrt{f(\hat{n}+1)}\hat{a}, \\ \hat{A}^\dagger &= \sqrt{f(\hat{n})}\hat{a}^\dagger = \hat{a}^\dagger\sqrt{f(\hat{n}+1)}, \end{aligned} \tag{4}$$

where $f(\hat{n})$ is called the deformation function which governs the nonlinear properties of the system. The interaction Hamiltonian of the two-level system and the nonlinear environment can be written as

$$\hat{H}_{\text{int}} = \hat{\sigma}_z \otimes \sum_i (g_i\hat{A}_i^\dagger + g_i^*\hat{A}_i), \tag{5}$$

where g_i's are coupling coefficients. It is easy to see that in the limiting case $f(\hat{n}) \to 1$, or equivalently $\hat{A} \to \hat{a}$, we obtain the Hamiltonian of the spin-boson model [43]. To solve this model, we write this Hamiltonian in the interaction picture, $\hat{H}_{\text{int}}(t)$:

$$\begin{aligned} \hat{H}_{\text{int}}(t) &= e^{i\hat{H}_0 t}\hat{H}_{\text{int}}e^{-i\hat{H}_0 t} \\ &= \hat{\sigma}_z \otimes \sum_i \left(g_i\hat{A}_i^\dagger e^{i\omega_i\hat{G}(n_i)t} + g_i^* e^{-i\omega_i\hat{G}(n_i)t}\hat{A}_i\right), \end{aligned} \tag{6}$$

where $H_0 = H_S + H_\varepsilon$ and $G(\hat{n})$ is given by $G(\hat{n}) = \frac{1}{2}[(\hat{n}+2)f(\hat{n}+2) - \hat{n}f(\hat{n})]$. Therefore, the time evolution operator of this system, in the limit of weak interactions, can be written as

$$\begin{aligned} U(t) &= \mathcal{T}_{\leftarrow}\exp\left[-i\int_0^t dt'\hat{H}_{\text{int}}(t')\right] \\ &= \exp\left[\frac{1}{2}\sigma_z \otimes \sum_i (g_i\hat{\mathbb{A}}_i^\dagger + g_i^*\hat{\mathbb{A}}_i)\right], \end{aligned} \tag{7}$$

where the operators $\hat{\mathbb{A}}_i$ can be defined as the new deformed operators:

$$\hat{\mathbb{A}}_i = 2\frac{(1 - e^{-i\omega_i\hat{G}(\hat{n}_i)t})}{\omega_i\hat{G}(\hat{n}_i)}\sqrt{f(\hat{n}_i+1)}\hat{a}_i. \tag{8}$$

In this equation, the deformation functions $\mathbb{F}(\hat{n}_i,t)$ can be define by the following equation:

$$\hat{\mathbb{F}}(\hat{n}_i,t) = 2\frac{\left(1 - e^{-i\omega_i\hat{G}(\hat{n}_i-1)t}\right)}{\omega_i\hat{G}(\hat{n}_i-1)}\sqrt{f(\hat{n}_i)}. \tag{9}$$

Moreover, we assume that there is no correlation between the system and the environment at $t = 0$. In addition, we suppose that at the initial time, $t = 0$, the system state is a superposition of its two states:

$$|\Psi(0)\rangle = \left(\cos\frac{\theta}{2}|+\rangle + e^{-i\phi}\sin\frac{\theta}{2}|-\rangle\right)|\Phi_\varepsilon\rangle, \tag{10}$$

where $|\Phi_\varepsilon\rangle$ is the state of the environment at $t = 0$. θ and ϕ are two angles which take values in the intervals $0 \leqslant \theta \leqslant \pi$ and $0 \leqslant \phi \leqslant 2\pi$. By using Relation (7), the time evolution of the total system is obtained by

$$\begin{aligned} |\Psi(t)\rangle &= U(t)|\Psi(0)\rangle \\ &= \cos\frac{\theta}{2}|+\rangle|\varepsilon_+(t)\rangle + e^{-i\phi}\sin\frac{\theta}{2}|-\rangle|\varepsilon_-(t)\rangle, \end{aligned} \tag{11}$$

where

$$|\varepsilon_\pm(t)\rangle = \prod_i D_f\left(\frac{\mp g_i}{2}\right)|\Phi_\varepsilon\rangle \tag{12}$$

and

$$D_f(g) = \exp(g\hat{\mathbb{A}}^\dagger - g\hat{\mathbb{A}}) \tag{13}$$

is a deformed displacement operator. It is evident that in the limiting case of $f_i \to 1$, the above deformed displacement operator D_f reduces to the standard displacement operator, i.e.,

$$D_{f\to 1}\left(\frac{g_i}{2}\right) = \exp\left(\frac{(\lambda_i)}{2}\hat{a}_i^\dagger - \frac{(\lambda_i^*)}{2}\hat{a}_i\right), \tag{14}$$

where $\lambda_i = 2\frac{g_i}{\omega_i}(1 - e^{i\omega_i t})$. The interaction establishes a quantum correlation between the basic states $|+\rangle$ and $|-\rangle$ of the system and the corresponding associated states $|\varepsilon_+(t)\rangle$ and $|\varepsilon_-(t)\rangle$ of the environment. Moreover, the density matrix of the system may be written as

$$\rho_S = \begin{pmatrix} P_{11}(t) & P_{12}(t) \\ P_{12}^*(t) & P_{22}(t) \end{pmatrix}, \tag{15}$$

where $P_{11}(t) = \cos^2\frac{\theta}{2}\langle\varepsilon_+(t)|\varepsilon_+(t)\rangle$, $P_{22}(t) = \sin^2\frac{\theta}{2}\langle\varepsilon_-(t)|\varepsilon_-(t)\rangle$, and $P_{12}(t) = e^{i\phi}\cos\frac{\theta}{2}\sin\frac{\theta}{2}r^*(t)$. The quantity $r(t)$ is known as the decoherence factor and is defined by $r(t) = \langle\varepsilon_+(t)|\varepsilon_-(t)\rangle$.

In this paper, we suppose that the environment consists of deformed oscillators. These deformed oscillators require the associated effects of some physical parameters to be determined. In the present contribution, we investigate two deformation functions, the first being Eq. (5) below, whose relevant deformed algebra describes a harmonic oscillator in the center of a one-dimensional tiny (nanometre) rectangular infinite well [58]:

$$f(\hat{n}) = \gamma\hat{n} + \eta. \tag{16}$$

In this case, $\gamma = 2\pi/a$ is a scaling factor depending on the width of the well, $a = L/\sqrt{\hbar/\omega m}$ is a dimensionless parameter in which L is the width of the well, and $\eta = \sqrt{\gamma^2+1}$ [58]. Second, the deformation function $f(\hat{n})$ is selected as

$$f(\hat{n}) = -\gamma'\hat{n} + \eta'. \tag{17}$$

This deformation function describes a truncated harmonic oscillator (finite range potential) [59]. In this case, $\gamma' = 1/N$, where N is the total number of the bound states of the truncated harmonic oscillator potential possessed by the finite range potential, and $\eta' = \sqrt{\gamma'^2+1}$. It is worth noting that the total number of bound states in the present system is determined by the well depth D, i.e., $N = 4D/\hbar\omega$. In this case, also, dimensionless well depth D is defined by $D = L/\sqrt{\hbar/m\omega}$ [59].

As you know, there are different distinguishability measures for arbitrary pairs of density matrices like relative purity, trace norm distance, Uhlmann fidelity, etc. [62]. In fact, some of these measures have been applied to describe the quantum speed limit, which can be illustrated by the relative purity [18], the quantum Fisher information [19], and finally the Bures angle [20]. The first one, i.e., relative purity $\mathsf{F}(t)$, which is defined by the following relation [18],

$$\mathsf{F}(t) = \frac{\mathrm{tr}[\rho(0)\rho(t)]}{[\rho^2(0)]}, \tag{18}$$

is one of the best measures to determine to what extent the time evolution of the two-state quantum system preserves the coherence. Therefore, using Relation (15), we can obtain the dynamical behavior of the relative purity as follows

$$\mathsf{F}(t) = \cos^4\frac{\theta}{2} + \sin^4\frac{\theta}{2} + 2\cos^2\frac{\theta}{2}\sin^2\frac{\theta}{2}\mathrm{Re}[r(t)], \tag{19}$$

in which Re $[r(t)]$ is the real part of the decoherence rate. To investigate environmental effects, a particular initial state Φ_ε of the environment should be chosen. Here, we consider two types of initial states for the environment, the ground state and the thermal state.

We assume that each harmonic oscillator in the environment is initially in the ground state $|0\rangle$ (the environment in the vacuum state), $|\Phi_\varepsilon\rangle = \prod_i |0\rangle_i$, where the index i runs over all the environmental oscillators. Thus, Relation (12), in the vacuum state, is reduced to a nonlinear coherent state,

$$|\varepsilon_\pm(t)\rangle = \prod_i \exp\left(\left|\frac{g_i}{2}\right|^2 \mathbb{F}_i(1)\right)\sum_{m=0}^{M}\left(\frac{\mp g_i}{2}\right)^m \frac{[\mathbb{F}(n_i)]!}{\sqrt{n_i!}}|n_i\rangle, \tag{20}$$

where $[\mathbb{F}(n_i,t)]! = \mathbb{F}(n_i,t)[\mathbb{F}(n_i-1,t)]!$, with $[\mathbb{F}(0,t)]! = 1$. The M parameter corresponds to the Hilbert space dimension. It is worth noting that in the case of the deformation function given in Relation (16), the Hilbert space is of an infinite dimension, whereas in the one expressed by Eq. (17) the Hilbert space has a finite dimension.

Thus, the decoherence factor $r(t)$ is obtained as

$$r(t) = \prod_i \left[\exp\left(\frac{-|g_i|^2}{2}|\mathbb{F}(1)|^2\right)\exp_{\mathbb{F}}(-|g_i|^2)\right], \tag{21}$$

where

$$\exp_{\mathbb{F}}(|g_i|) = \sum_{n_i=0}^{M}\frac{\left(|\frac{g_i}{2}|^{2n_i}\{[|\mathbb{F}(n_i,t)|]!\}^2\right)}{n_i!}. \tag{22}$$

It is clear, after some calculation, that when f_i approaches 1 the relative purity F is reduced to

$$\mathsf{F}_{f_i\to 1} = \cos^4\frac{\theta}{2} + \sin^4\frac{\theta}{2} + 2\cos^2\frac{\theta}{2}\sin^2\frac{\theta}{2}\exp\left[-\frac{1}{2}\sum_i|\lambda_i|^2\right]. \tag{23}$$

However, in the general case, the relative purity depends on the deformation function which describes the environment structure. For a quantitative investigation of this parameter, we need to determine the related deformation function. For this purpose, we consider the two deformation functions introduced in previous section. It is worth noting that these two deformation functions are related to the confined properties of the environment.

In Fig. 1 is plotted the fidelity of the spin system which interacts with the deformed environment as a function of time and the infinite well width a in panel (a) and with the well depth D in panel (b). It is clear from Fig. 1(a) that any increase in the value of parameter a causes the quantum state to exhibit a greater robustness against the decoherence process. Also, the quantum state is more resistant against the decoherence process with decreasing well depth D. Moreover, in Figs. 1(b) and 1(d), we have shown the dynamics of the fidelity of the spin system for different values of a and D, respectively. These plots show that decoherence occurs as time elapses. This process is a consequence of the role of environment in the dynamics. Decoherence is an inevitable process because of the environmental effects. An important point in this process is the control of decoherence. It seems that the present model provides a theory to investigate the effects of certain parameters. Comparison of Figs. 1(b) and 1(d) reveals that the finite well exhibits more robustness against the decoherence process than the infinite one.

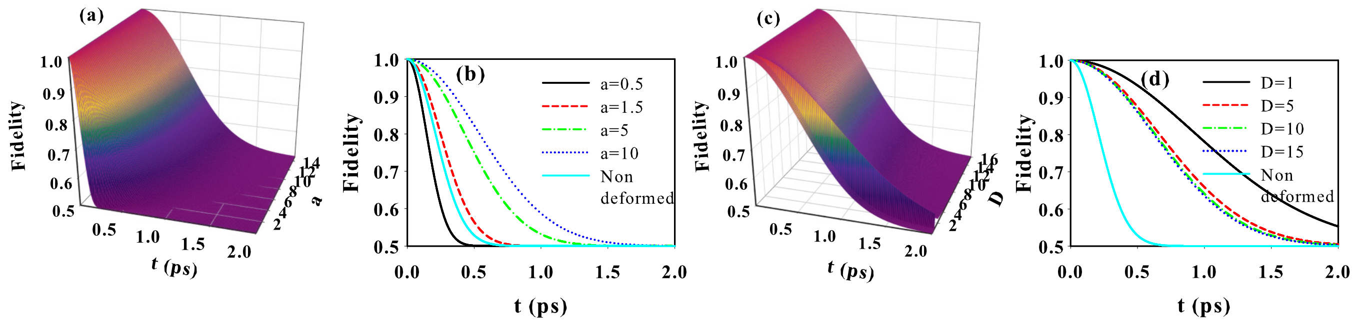

FIG. 1. (Color online) Variations of the relative purity of a spin system as a function of t and the dimensionless parameter a, which describes the deformed harmonic oscillator in the infinite well as a nonlinear environment, are shown in panel (a) and those for the dimensionless parameter of a well with a finite depth D are shown in panel (c). The figures show the evolution of an environment consisting of $N = 20$ harmonic oscillators. Also, the coupling g_i and frequencies ω_i were chosen randomly from the interval [0,1]. In addition, variations of relative purity as a function of t are plotted in panels (b) and (d), respectively, for different values of a and D.

We now turn to a more general case in which the environment is in a thermal state at the initial time, i.e.,

$$\hat{\rho}_{\varepsilon_i} = \frac{1}{Z_i} \exp\left(-\beta \hat{H}_{\varepsilon_i}\right) = \frac{1}{Z_i} \exp[-\beta\omega_i(\hat{\mathbb{A}}_i\hat{\mathbb{A}}_i^{\dagger} + \hat{\mathbb{A}}_i^{\dagger}\hat{\mathbb{A}}_i)], \tag{24}$$

where the partition function for the ith mode is defined by $Z_i = (\sum_{n=0}^{M} e^{-\beta\omega_i E(n_i)})^{-1}$, and $E(n_i) = \frac{1}{2}[(n+1)f(n+1) + nf(n)]$ are the eigenvalues of the ith mode which is described by a deformed oscillator. Thus, the initial state of the composite system is given by

$$\hat{\rho}(0) = \hat{\rho}_s(0) \otimes \prod_i \frac{1}{Z_i} \exp\left(-\beta \hat{H}_{\varepsilon_i}\right). \tag{25}$$

The time evolution of the reduced density matrix of the system, $\hat{\rho}_s(t)$, is obtained in the usual manner via

$$\hat{\rho}_s(t) = \mathrm{Tr}_{\varepsilon}[\hat{U}(t)\hat{\rho}(0)\hat{U}^{-1}(t)], \tag{26}$$

where $\hat{U}(t)$ is the time evolution operator given by Eq. (7). In this equation, $\mathrm{Tr}_{\varepsilon}$ designates the partial trace over the environmental degrees of freedom. It is clear that the diagonal elements of the density matrix $\rho_s^{ii}(t) = \langle i|\hat{\rho}_s(t)|i\rangle$, where $i = +,-$, are constant in time. Also, the off-diagonal matrix elements of $\hat{\rho}_s(t)$ are obtained by

$$\hat{\rho}_{s,-+}(t) = \hat{\rho}_{s,+-}^{*}(t) = \langle -|\mathrm{Tr}_{\varepsilon}[\hat{U}(t)\hat{\rho}(0)\hat{U}^{-1}(t)]|+\rangle. \tag{27}$$

Using some approximations and neglecting the second-order terms in $|g_i|$ in exponentials, we obtain the following off-diagonal term

$$\hat{\rho}_{s,+-}(t) \approx \hat{\rho}_{s,+-}(0) \prod_j \frac{1}{Z_j} \left(\sum_{n_j=0}^{M} n_j![|\mathbb{F}(n_j)|!]^2 e^{-\beta\omega_j E(n_j)} \right) \times \left(\sum_{p=0}^{n_j} \frac{|g_j/2|^{2p}(-1)^p}{(p\,!)^2(n_j-p)!\,|[\mathbb{F}(n_j-p)]!|^2} \right). \tag{28}$$

Therefore, the decoherence factor $r(t)$ is given by [43]

$$r(t) = \prod_j \frac{1}{Z_j} \left(\sum_{n_j=0}^{M} n_j![|\mathbb{F}(n_j)|!]^2 e^{-\beta\omega_j E(n_j)} \right) \times \left(\sum_{p=0}^{n_j} \frac{|g_j/2|^{2p}(-1)^p}{(p\,!)^2(n_j-p)!\,|[\mathbb{F}(n-p)]!|^2} \right). \tag{29}$$

In the limit of the nondeformed case, $f \to 1$, one can show that the decoherence rate will reduce to

$$r(t)_{f\to 1} = \prod_i \exp\left[\frac{-|\lambda_i|^2}{2} \coth\left(\frac{\beta\omega_i}{2}\right)\right]. \tag{30}$$

In Fig. 2, we consider the relation between the dynamics of the fidelity of the two-state system with the width of the infinite well a and the depth of the finite well D, when the environment

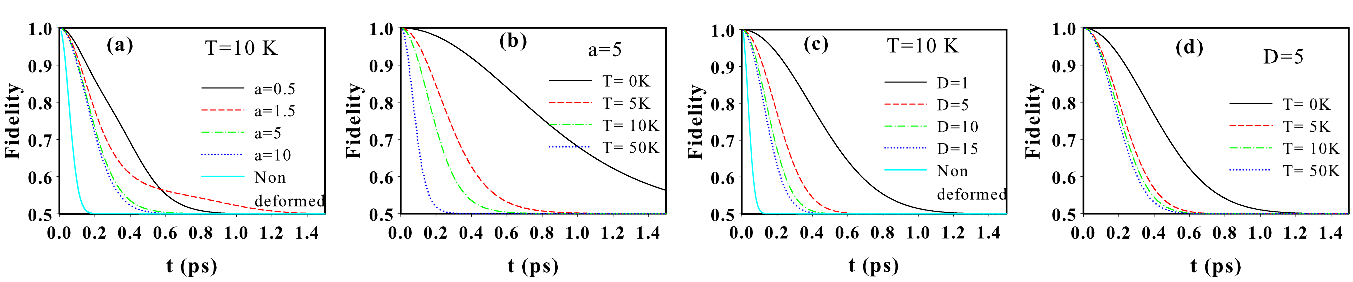

FIG. 2. (Color online) (a) The variation of the relative purity of a spin system as a function of t, for definite values of a and a constant temperature. (b) Variation of the relative purity of a spin system as a function of T, with a constant value of a. Panels (c) and (d) indicate the variation of relative purity as function of D with a constant temperature and as a function of T with a constant D, respectively.

is assumed to be in the thermal equilibrium. Figures 2(a) and 2(c) indicate that, in the case of the deformed environment, the system is more resistant against decoherence. Moreover, the decoherence rate will increase with increasing values of a and D. In addition, Figs. 2(b) and 2(d) show the fidelity evolution, for different temperatures, with known values of width a and depth D, respectively. It may be understood from these plots that, the more the temperature increases, the faster the decoherence rate will become. However, given the role of temperature in decoherence and dissipation, one expects this result. In addition, a comparison of Figs. 2(b) and 2(d) reveals that any increase in temperature in the confined environment with an infinite depth leads to only a smaller robustness against decoherence. In other words, modeling the environment with truncated harmonic oscillators causes the system to be more resistant against decoherence.

III. DECOHERENCE SPEED LIMIT

In this section, as we mentioned before, by using the approach of del Campo *et al.*, we study the decoherence speed limit [18]. Using Relation (18), we have

$$\frac{d\mathsf{F}}{dt} = \mathrm{tr}[\rho(0)\dot{\rho}(t)]. \tag{31}$$

According to the Cauchy-Schwarz inequality for operators, $|\mathrm{tr}(\hat{A}^\dagger\hat{B})|^2 \leqslant \mathrm{tr}(\hat{A}^\dagger\hat{A})\mathrm{tr}(\hat{B}^\dagger\hat{B})$, the rate of change of F can be bounded. In the case of a spin-deformed bosonic system, we may obtain the following relation:

$$\dot{\mathsf{F}}(t) \leqslant \sqrt{\mathrm{tr}\big[\rho_S^2(t)\big]}. \tag{32}$$

If we reparametrize $\mathsf{F} = \cos\zeta$ with $\zeta \in [0,\pi/2]$ and consider $\bar{X} = \tau_\zeta^{-1}\int_0^\zeta dt X$, we achieve

$$\tau_\zeta \geqslant \frac{|\cos\zeta - 1|}{\sqrt{\mathrm{tr}\big[\rho_S^2(t)\big]}} \geqslant \frac{4\zeta^2}{\pi^2\sqrt{\mathrm{tr}\big[\rho_S^2(t)\big]}}. \tag{33}$$

Therefore, the quantum speed limit of the spin system in the nonlinear environment is obtained as

$$\tau_\zeta \geqslant \frac{4\zeta^2}{\pi^2}\bigg[\cos^4\frac{\theta}{2}\langle\varepsilon_+(t)|\varepsilon_+(t)\rangle + \sin^4\frac{\theta}{2}\langle\varepsilon_-(t)|\varepsilon_-(t)\rangle + 2\cos^2\frac{\theta}{2}\sin^2\frac{\theta}{2}|r(t)|^2\bigg]^{-1/2}. \tag{34}$$

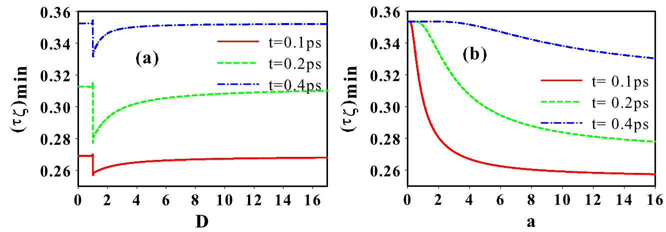

FIG. 3. (Color online) Variation of the minimum bound passage $(\tau_\zeta)_{\min}$ as a function of well depth D in panel (a) and as a function of well width a in panel (b).

Similar to the previous section, we consider the decoherence speed limit for two cases, an environment in the ground state and one in the thermal state.

First, we assume that each harmonic oscillator in the environment is initially in the ground state $|0\rangle$ (the environment in the vacuum state). Therefore, by substituting Eq. (21) into Relation (34), the quantum speed limit of a spin system interacting with the nonlinear environment is obtained. It is clear that in the nondeformed limitation, the quantum speed limit is achieved by

$$\tau_\zeta \geqslant \frac{4\zeta^2/\pi^2}{\sqrt{\cos^4\frac{\theta}{2} + \sin^4\frac{\theta}{2} + 2\cos^2\frac{\theta}{2}\sin^2\frac{\theta}{2}\prod_i e^{|\lambda_i(t)|^2}}}. \tag{35}$$

In Fig. 3, the minimum quantum passage time, $(\tau_\zeta)_{\min}$, is plotted as a function of the depth of the finite well D and the width range of the confined environment a, respectively. Figure 3(a) shows that increasing the well depth D causes the bound passage time $(\tau_\zeta)_{\min}$ to be slightly increased. Also, Fig. 3(b) indicates that the minimum bound passage time $(\tau_\zeta)_{\min}$ declines sharply with increasing well width a. Moreover, it is shown that the physical parameters a and D have different effects for different times. For longer times, the increments in a and D do not have any important effects. For short times, however, the physical parameters a and D exhibit more remarkable roles.

Also, we assume that each harmonic oscillator in the environment is initially in the thermal equilibrium. Therefore, by substituting Eq. (29) into Relation (34), the quantum speed limit of a spin system interacting with the nonlinear environment is obtained. It is evident that in the nondeformed limitation, the quantum speed limit is obtained by

$$\tau_\zeta \geqslant \frac{4\zeta^2/\pi^2}{\sqrt{\cos^4\frac{\theta}{2} + \sin^4\frac{\theta}{2} + 2\cos^2\frac{\theta}{2}\sin^2\frac{\theta}{2}\prod_i \exp\left(\frac{-1}{2}\lambda_i\coth\beta\omega_i\right)}}. \tag{36}$$

Figure 4 depicts the counter plot of the minimum quantum passage time, $(\tau_\zeta)_{\min}$, as a function of temperature and confinement size. Figures 4(a)–4(c) correspond to the finite range potential, whereas Figs. 4(d)–4(f) correspond to the infinite well. First of all, it seems that the control of the minimum bound passage time is easier at the initial time than when the time is increased. Second, increasing of well depth D causes the bound passage time $(\tau_\zeta)_{\min}$ to rise too, as is clear in Figs. 4(a)–4(c). Also, Fig. 4(a), which indicates the earlier time, shows that D has the central role in the variations of the bound passage time $(\tau_\zeta)_{\min}$. As time elapses, as plotted in Figs. 4(b) and 4(c), temperature plays the most

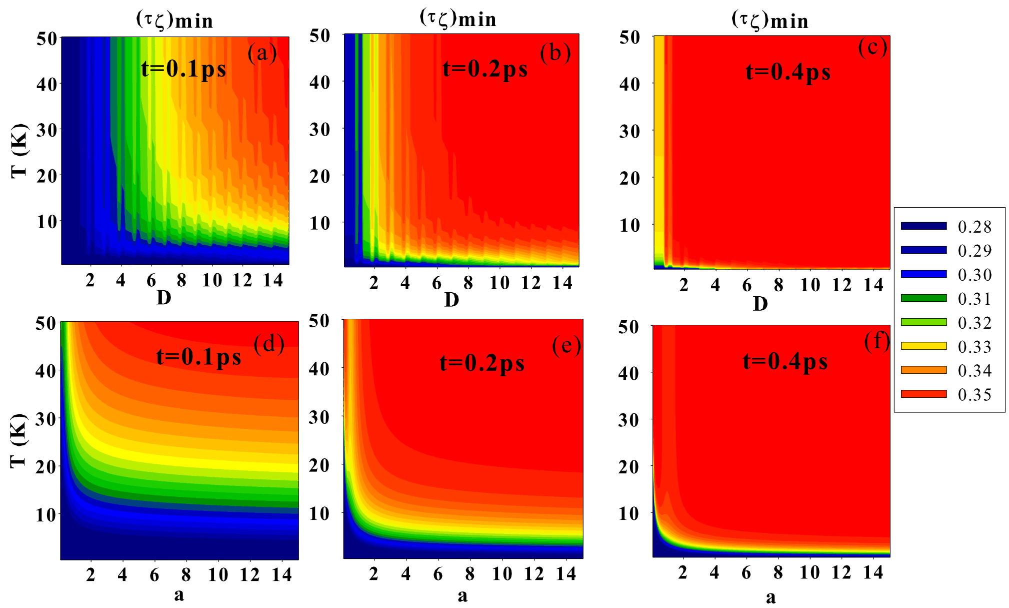

FIG. 4. (Color online) Counter plot of variation of the minimum bound passage $(\tau_\zeta)_{\min}$ as a function of temperature and well depth D in panels (a)–(c) and as functions of temperature and well width a in panels (d)–(f).

important role in the variations of $(\tau_\zeta)_{\min}$. The same is true for the minimum bound passage time with variations of the temperature T and well width a, as seen in Figs. 4(d)–4(f). Finally, comparison of plots 4(a)–4(c) and 4(d)–4(f) shows that, in the nonlinear environment with a deformation of an infinite well, temperature plays a more important role in the variation of the minimum bound passage time $(\tau_\zeta)_{\min}$ than the finite well does.

Inversely, in an environment consisting of a deformation of a finite well, well depth has a stronger impact in controlling the minimum bound passage time $(\tau_\zeta)_{\min}$.

IV. CONCLUSION AND REMARKS

In this paper, we studied the effects of temperature and nonlinearity of environment on the decoherence process of a superposition of the two-state system. For this case, we considered the relative purity as a measure of the decoherence rate and studied the quantum speed limit of open quantum systems. It was shown that nonlinearity causes the decoherence process to become more resistant. In addition, by choosing two different deformation functions, it was shown that, in the nonlinear environment, some environmental features exhibit more remarkable roles in the decoherence process.

As another contribution, it was demonstrated that there is a minimum bound passage time for the decoherence process which depends on the nonlinear features and the environment temperature. Assuming the environment to consist of deformed oscillators, we investigated two different deformed oscillators, i.e., the confined harmonic oscillator in an infinite well and a truncated harmonic oscillator. It was shown that at earlier times one can control the decoherence by physical features of the environment. On the other hand, it was shown that the temperature of the environment plays a remarkable role when enough time has elapsed. Finally, the study provided a method for investigating the role of environmental parameters on decoherence.

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