Electron vortex beams subject to static magnetic fields

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The properties of electron vortex beams are examined when subject to static magnetic fields. The fields are assumed to be applied after the electron vortex beam carrying a well-defined orbital angular momentum has been created as a result of using a holographic mask. The shifts in the electron vortex beam energy momentum as well as its angular momentum due to the presence of an axial uniform magnetic field are evaluated. Order-of-magnitude estimates of the shifts are given with reference to typical electron vortex beams subject to moderate magnetic fields.

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Considerable emphasis has been laid in recent years on the physics and potential applications of electron vortex beams [1–3]. Advances in this area swiftly followed their prediction and realization in the laboratory and occurred after much work was carried out on optical vortex beams, leading to a great deal of fundamental physics as well as notable applications [4–7]. Electron vortex beams are a special case of particle vortices first discussed by Bilalyncki-Birula and coworkers [8–10] and the concept has recently been extended to neutral atom vortex beams [11,12].

From a quantum-mechanical point of view, an electron vortex beam consists of an axial flow of twisting probability density and probability current density distributions which implies that the beam is endowed with its own charge and current as well as intrinsic spin sources and, like other vortex beams, it carries orbital angular momentum $l\hbar$ per electron about the beam axis where l is the winding number with $|l| \geqslant 0$. In addition to generating electric and magnetic fields associated with its own sources, the electron vortex beam must respond to the application of externally applied magnetic fields. Recent studies involving electrons in magnetic fields have dealt with Landau levels and Aharonov–Bohm states [13], the vacuum Faraday effect for electrons [14], and the propagation of electron wave functions in a magnetic field [15].

Here we base our treatment on the realization that, for electron vortex beams of relatively low winding numbers l subject to moderate magnetic fields, the formation of Landau states is a weak feature and the physics is dominated by the electron vortex state. Our main task is to quantify the modifications to the properties of the electron vortex beam arising from the application of the magnetic field.

In the absence of the magnetic field the Hamiltonian for a vortex beam with a well-defined axis along z is given by

$$\begin{aligned}\mathcal{H}_{EV} &= \frac{P^2}{2m}\\ &= \frac{P_z^2}{2m} + \frac{P_\perp^2}{2m}\\ &\equiv \mathcal{H}_z + \mathcal{H}_\perp,\end{aligned} \tag{1}$$

where the subscripts z and $\perp$ describe axial and in-plane motions, respectively. A specific eigenfunction of $\mathcal{H}_{EV}$ is that for an electron vortex beam of winding number l in the Bessel form expressed conveniently in cylindrical coordinates $\mathbf{r} = (\rho,\phi,z)$:

$$\psi_l(\mathbf{r},t) = N_l J_l(k_\perp \rho) e^{ik_z z} e^{il\phi} e^{-i\mathcal{W}t/\hbar}, \tag{2}$$

where N_l is a normalization constant to be specified later, $k_\perp$ and k_z are the transverse and axial components of the wave vector of the vortex beam, respectively, such that $k_\perp^2 + k_z^2 = k^2 = \frac{2m\mathcal{W}}{\hbar^2}$, with $\mathcal{W}$ being the energy eigenvalue of $\mathcal{H}_{EV}$ corresponding to ψ_l, and $J_l(k_\perp\rho)$ is the lth-order Bessel function. A typical value in the case of an electron vortex beam created inside an electron microscope is $\mathcal{W} = 200$ keV. The transverse-wave-vector component is an adjustable parameter, which enters the mask design. It indicates the extent of the cross section of the electron vortex beam. Due to certain practical constraints that are imposed by the fabrication process involved in the creation of the holographic mask the transverse-wave-vector component is typically in the range $k_\perp \approx 10^9$ to 10^{10} m^{-1}.

The intrinsic properties of such an electron vortex beam in the absence of external fields have been explored recently [16–20]. In particular, it has been shown by explicit analysis that the beam is endowed with a linear momentum vector of magnitude $\hbar k_z$ parallel to the beam axis and also an axial orbital angular momentum vector of magnitude $\hbar l$ and all other vector components vanish [18]. The vortex beam also carries electric charge and current sources associated with its wave function and hence it possesses its own electric and magnetic fields due to these sources [19]. The coupling of the electron vortex beam to atomic matter has been explored theoretically and in relation to experiment [16,20].

It is interesting to explore how the transverse energy of the electron vortex beam is shared between the radial motion and the rotational motion. This can be seen by working out the expectation value of $\mathcal{H}_\perp$ in cylindrical polar coordinates. We find after some algebra

$$\langle \Psi_l|\mathcal{H}_\perp|\Psi_l\rangle = \frac{\hbar^2 k_\perp^2}{2m}\{\alpha_l + l^2\beta_l\}, \tag{3}$$

where α_l and β_l are definite integrals which depend on the winding number l. The first term between the brackets represents the fraction of energy associated with the radial

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motion, while the second term is the fraction associated with the rotational motion. It is straightforward to check that the sum between the brackets is unity:

$$\alpha_l + l^2\beta_l = 1, \tag{4}$$

as it should be.

In the presence of an external magnetic field $\mathbf{B}_0 = \nabla\times\mathbf{A}$ the vortex Hamiltonian becomes

$$\mathcal{H} = \frac{(\mathbf{P} - e\mathbf{A})^2}{2m} - \boldsymbol{\sigma}\cdot\mathbf{B}_0, \tag{5}$$

where $\boldsymbol{\sigma}$ is the spin magnetic moment. If the magnetic field is constant and uniform, the vector potential can be expressed as follows:

$$\mathbf{A} = \frac{1}{2}\mathbf{B}_0\times\mathbf{r}, \tag{6}$$

and the Hamiltonian can be shown to reduce to

$$\mathcal{H} = \mathcal{H}_{EV} - \boldsymbol{\mathcal{M}}\cdot\mathbf{B}_0 + \frac{e^2}{8m}\{|\mathbf{B}_0|^2r^2 - (\mathbf{B}_0\cdot\mathbf{r})^2\}, \tag{7}$$

where $\mathcal{H}_{EV}$ is given by Eq. (1) and $\boldsymbol{\mathcal{M}}$ is the total magnetic moment

$$\boldsymbol{\mathcal{M}} = \boldsymbol{\mathcal{M}}_L + \boldsymbol{\mathcal{M}}_S = -\frac{\mu_B}{\hbar}(\mathbf{L} + 2\mathbf{S}), \tag{8}$$

where $\mathbf{L}$ and $\mathbf{S}$ are the orbital and spin angular-momentum vector operators. Note that $\mu_B|\mathbf{B}_0|/\hbar = \omega_L$ is the Larmor angular frequency with $\mu_B = \hbar|e|/(2m)$ being the Bohr magneton.

As will be explained below for vortex beams of not too large winding numbers l in static magnetic fields of up to 10 T, all the Hamiltonian terms apart from the first term on the right-hand side of Eq. (7) are small and can be treated as perturbations. The dominant feature in this case is the electron vortex, and the effects of magnetic field are too small relative to the vortex for consideration of the formation of Landau states. Thus we write

$$\mathcal{H} = \mathcal{H}_{EV} + \mathcal{H}_{\rm int}, \tag{9}$$

where $\mathcal{H}_{EV}$ is the zero-order Hamiltonian given by Eq. (1) and $\mathcal{H}_{\rm int}$ is the interaction Hamiltonian

$$\mathcal{H}_{\rm int} = -\boldsymbol{\mathcal{M}}\cdot\mathbf{B}_0 + \frac{e^2}{8m}[|\mathbf{B}_0|^2r^2 - (\mathbf{B}_0\cdot\mathbf{r})^2]. \tag{10}$$

Perturbation theory is justifiable in this situation rather than consideration of Landau levels since the typical energy arising from the magnetic field is of the order $\hbar\omega_L$ and this is of the order of an meV for a magnetic field of a few T. This is rather small when compared with the zero-order vortex beam energy of the order of keV. However, the relevant energy to compare with the magnetic-field energy is the transverse energy of the electron vortex beam, given by $W_\perp = \hbar^2k_\perp^2/(2m)$ which is approximately equal to 31 meV for $k_\perp = 10^9$ m^{-1} and is approximately 3.1 eV for $k_\perp = 10^{10}$ m^{-1}

We take the magnetic-field vector pointing along the axis so that $\mathbf{B}_0 = |\mathbf{B}_0|\hat{z}$ and the leading interaction term is the $-\boldsymbol{\mathcal{M}}\cdot\mathbf{B}_0$ term where the magnetic moment is given by Eq. (8). The last terms in $\mathcal{H}_{\rm int}$ represent the diamagnetic energy and are much smaller that the $-\boldsymbol{\mathcal{M}}\cdot\mathbf{B}_0$ term.

We now show that the system displays dichroism. The zero-order state is an eigenfunction of $\mathcal{H}_0$ and is identified as the electron vortex wave function given by Eq. (2). The spin state can, as commonly done, be tagged to the space wave function. We write

$$|l;s_z\rangle = |\psi_l(\mathbf{r},t);\chi_s\rangle\,, \tag{11}$$

where $|\chi_s\rangle$ is the spin state and is either spin up or spin down such that $S_z\chi_{\pm1/2} = \pm\frac{\hbar}{2}\chi_{\pm1/2}$. The expectation value of $-\boldsymbol{\mathcal{M}}\cdot\mathbf{B}_0$ in the state $|l;s_z\rangle$ represents a shift in the vortex beam energy. For $l > 0$ and $s_z = \pm1/2$ we have

$$\Delta\mathcal{W}_{l,\pm1/2} = \mu_B|\mathbf{B}_0|\,(|l| \pm 1) = \hbar\omega_L\,(|l| \pm 1)\,. \tag{12}$$

On the other hand for $l = -|l|$ and $s_z = \pm1/2$ we have the shift in energy

$$\Delta\mathcal{W}_{-|l|,\pm1/2} = \mu_B|\mathbf{B}_0|\,(-|l| \pm 1) = \hbar\omega_L\,(-|l| \pm 1)\,. \tag{13}$$

Thus the two vortex beams with opposite but equal signs of l experience different energy shifts when subject to a static magnetic field, indicating dichroism. If the energy distributions of the beam first in the field-free region and second when the beam is subject to the influence of the magnetic field can both be measured, they should, in principle, show the differences in the effects of the different signs of the winding number l of the vortex. Note that the magnitudes of the energy shifts increase with increasing $|l|$.

In the presence of the magnetic field there exists an additional linear momentum for which the volume density is in the form

$$\boldsymbol{\pi} = \epsilon_0\mathbf{E}_v\times(\mathbf{B}_0 + \mathbf{B}_v), \tag{14}$$

where $\mathbf{E}_v$ and $\mathbf{B}_v$ are the electric and magnetic fields associated with the electron vortex by virtue of its charge and current density quantum distributions. These have been evaluated in detail by Lloyd *et al.* [18] who also found that the vortex fields are small for electron vortex beams created in a typical electron microscope. The leading momentum-density term arising from the presence of the external magnetic field is [21]

$$\boldsymbol{\pi} = \epsilon_0\mathbf{E}_v\times\mathbf{B}_0, \tag{15}$$

and this leading linear momentum density is also responsible for a leading shift in $\boldsymbol{\Delta L}$ in the angular momentum vector due to the presence of the external magnetic field

$$\boldsymbol{\Delta L} = \int d^3\mathbf{r}\mathbf{r}\times\boldsymbol{\pi}. \tag{16}$$

The total changes in the linear and angular momenta imparted on the vortex beam due to the external magnetic field are found by integration over space.

The external field is axial so that $\mathbf{B}_0 = |\mathbf{B}_0|\hat{z}$ and the vortex electric field is radial and depends only on the radial coordinate ρ, so that $\mathbf{E}_v = \mathcal{E}_l(\rho)\hat{\boldsymbol{\rho}}$. Thus the change in the linear momentum density is purely azimuthal and is given by

$$\begin{aligned}\boldsymbol{\pi} &= \epsilon_0\mathbf{E}_v\times\mathbf{B}_0\\ &= \epsilon_0|\mathbf{B}_0|\mathcal{E}_l(\rho)\,\hat{\boldsymbol{\phi}}.\end{aligned} \tag{17}$$

However, the volume integral of $\boldsymbol{\pi}$ is zero because the azimuthal component when integrated over all angles leads to a null result. Thus we have

$$\boldsymbol{\Delta P} = \int \boldsymbol{\pi} d^3\mathbf{r} = 0. \tag{18}$$

This means that the linear momentum of the electron vortex is conserved in the presence of the magnetic field and remains, as given in Ref. [18], equal to $\hbar k_z \hat{\mathbf{z}}$.

Consider next the change in the orbital angular momentum due to the presence of the external magnetic field, defined in Eq. (16). Substituting from Eq. (17) into Eq. (16) and using $\mathbf{r} = \rho\hat{\boldsymbol{\rho}} + z\hat{\mathbf{z}}$, we find

$$\begin{aligned}\boldsymbol{\Delta L}_l &= \epsilon_0|\mathbf{B}_0| \int_0^{2\pi} d\phi \int_{-D/2}^{D/2} dz \int_0^{\rho_m} \rho d\rho \\ &\quad \times (\rho\hat{\boldsymbol{\rho}} + z\hat{\mathbf{z}}) \times \hat{\boldsymbol{\phi}}\mathcal{E}_l(\rho) \\ &= 2\pi\epsilon_0|\mathbf{B}_0| D \int_0^{\rho_m} d\rho \mathcal{E}_l(\rho)\, \rho^2 d\rho \hat{\mathbf{z}},\end{aligned} \tag{19}$$

where only the term involving the cross product $\hat{\boldsymbol{\rho}} \times \hat{\boldsymbol{\phi}} = \hat{\mathbf{z}}$ survives after integration, so there is no in-plane vector component in $\boldsymbol{\Delta L}_l$. Here D is the beam length and the electric-field function $\boldsymbol{\mathcal{E}}_l(\rho)$ is given by [19]

$$\boldsymbol{\mathcal{E}}_l(\rho) = -\frac{e|N_l|^2}{2\epsilon_0}\rho \left\{ J_{|l|}^2(k_\perp\rho) - J_{|l|-1}(k_\perp\rho)\, J_{|l|+1}(k_\perp\rho) \right\}, \tag{20}$$

where $|N_l|$ is the vortex wave function normalization factor

$$|N_l|^2 = \frac{1}{2\pi D \int_0^{\rho_m} J_l^2(k_\perp\rho)\, \rho d\rho}. \tag{21}$$

Note that $\mathcal{E}(\rho)$ is invariant under the change of the sign of l. In the evaluation of the ρ integrals we follow the procedure by Lloyd *et al.* [19] where the upper integration limit is appropriately chosen to coincide with the first zero $x_m = \rho_m k_\perp$ of the Bessel function in question. Substituting in Eq. (19) we find

$$\boldsymbol{\Delta L}_l = \frac{e|\mathbf{B}_0|}{2k_\perp^2} Q_l \hat{\mathbf{z}}, \tag{22}$$

where Q_l is a dimensionless factor, which depends on l, and is given by

$$Q_l = \frac{\int_0^{x_m} x^3 \left[J_{|l|}^2(x) - J_{|l|-1}(x) J_{|l|+1}(x) \right] dx}{\int_0^{x_m} x J_{|l|}^2(x) dx}. \tag{23}$$

Equation (22) is the predicted orbital angular momentum gained by the electron vortex in the presence of the magnetic field. Note that it is axial in direction and therefore imparts an additional twisting action to the electron vortex. The orbital-angular-momentum change can be written alternatively in terms of an additional azimuthal speed v_l and a change in the magnitude of the radius a_l as follows:

$$\begin{aligned}\boldsymbol{\Delta L}_l &= \frac{m\omega_L}{k_\perp^2} Q_l \hat{\mathbf{z}} \\ &= \frac{m v_l}{k_\perp} \hat{\mathbf{z}} \\ &= m v_l \bar{\lambda} \hat{\mathbf{z}},\end{aligned} \tag{24}$$

TABLE I. Q_l versus $|l|$.

$\lvert l \rvert$	Q_l
1	9.77
2	15.58
3	21.80
4	28.38

where $\bar{\lambda} = k_\perp^{-1}$ is the reduced transverse wavelength. The corresponding change v_l in the rotational speed is given by

$$v_l = \omega_L Q_l \bar{\lambda} = \omega_L a_l, \tag{25}$$

where we have identified a_l as the change in the radius of the circular motion

$$a_l = Q_l \bar{\lambda}. \tag{26}$$

The l dependence appears only in the change in the rotational speed v_l and the change in the orbit radius a_l. The l dependence of the shift in the orbital angular momentum $\boldsymbol{\Delta L}_l$ enters through the constant Q_l, defined in Eq. (23). Table I lists the values of Q_l for the lowest winding numbers, evaluated along the lines of Ref. [18]. Note that Q_l increases only slowly with increasing $|l|$. We may now consider order-of-magnitude estimates leading to values of the shifts arising from the presence of the axial magnetic field.

In the first instance we explore the scenario of electron vortex beams generated in an electron microscope with the following parameters:

$$k_\perp \approx 10^{10}\ \mathrm{m}^{-1}, \quad \bar{\lambda} = 10^{-10}\ \mathrm{m}, \tag{27}$$

so that $\mathcal{W}_\perp \approx 3.1$ eV. The reduced transverse wavelength $\bar{\lambda}$ is in fact a measure of the spread of the electron vortex beam about its axis; this is the origin of referring to such electron vortex beams as Angstrom beams. In the context of an electron microscope the B fields are of the order of 1 T. The Larmor frequency is $\omega_L = e|\mathbf{B}_0|/(2m) \approx 10^{11}\ \mathrm{s}^{-1}$. The change in the radius for the lowest value of l with reference to Table I is $a_l = Q_1\bar{\lambda} \approx 10^{-9}$ m. The additional rotational velocity v_l is then $v_l = \omega_L a_l \approx 10^2$ m/s. The magnitude of the largest energy shift for a given l is

$$\Delta\mathcal{W}_l = \hbar\omega_L(|l| + 1) \approx 0.05\,(|l| + 1)\ \mathrm{meV}. \tag{28}$$

This energy shift increases with the magnitude of the winding number. For sufficiently large values of $|l|$, the energy shift could be measurable as the accuracy in the energy measurement is currently about 10 meV. The shift in the orbital angular momentum per electron in the beam can be estimated for $l = 1$ and turns out to be

$$|\boldsymbol{\Delta L}_1| = m v_l \bar{\lambda} = 10^{-4}\hbar. \tag{29}$$

Thus the magnitude of the shift per electron of the orbital angular momentum for vortex beams generated in an electron microscope with the parameters given in Eq. (27) interacting with a 1 T magnetic field turns out to be a small fraction of the unit $\hbar$.

It is clear that, to achieve a larger shift in the orbital angular momentum, we need to increase the Larmor frequency ω_L

(equivalent to increasing the external magnetic field) and the value of $\bar{\lambda}$, which is controlled by the choice of $k_\perp$ entering the mask function as a basic property of the vortex. We shall now assume that the magnetic-field strength can be increased to 10 T and we can arrange for $k_\perp$ to be one order of magnitude smaller than in Eq. (27). We have

$$|\mathbf{B}_0| = 10\ \mathrm{T}, \quad \bar{\lambda} = 10^{-9}\ \mathrm{m}. \tag{30}$$

This corresponds to a nanovortex rather than an Angstrom vortex. We then have $\omega_L = e|\mathbf{B}_0|/(2m) \approx 10^{12}\ \mathrm{s}^{-1}$, and $a_l = Q_1\bar{\lambda} \approx 10^{-8}$ m. These lead us to $v_l = 10^4$ m/s and we finally have for $l = 1$,

$$\mathbf{\Delta} L_1 = 0.1\hbar\hat{z}. \tag{31}$$

For a large value of $|l|$ there would be a larger shift of the beam orbital angular momentum. The largest energy shifts for this case are given by

$$\Delta\mathcal{W} \approx 0.5\,(|l| \pm 1)\ \mathrm{meV}. \tag{32}$$

This would be quite substantial for very large values of $|l|$. However, the transverse energy for this scenario is $\mathcal{W}_\perp \approx 31$ meV, so that for not too large values of $|l|$ this case is still within the regime in which the electron vortex is dominant. However, any sufficiently large increases in $\bar{\lambda}$ and/or the magnetic field, or in the magnitude of the winding number $|l|$, would result in the perturbative regime being no longer valid. This would be the scenario reported in a recent study where the electron vortex beam was created with a radius chosen to match the waist of the Landau states in the magnetic field [22].

In conclusion, we investigated the effects introduced by a constant axial magnetic field on the basic properties of an initially well-defined electron vortex beam. Our analysis and results demonstrate that accompanying the presence of the magnetic field are additional linear momentum and orbital angular momentum densities which influence the shape and propagation of the vortex by introducing additional azimuthal momentum flow and axial as well as radial orbital-angular-momentum flows. However, the volume integrals of the densities yield a null value for the shift in the linear momentum, but they give rise to a finite axial orbital-angular-momentum shift. The shift in the electron vortex orbital angular momentum depends on the vortex parameters and the value of the magnetic field. For a scenario in which a typical electron vortex beam is generated in an electron microscope subject to a magnetic field of 1 T, the shift in the orbital angular momentum due to the presence of the magnetic field turned out to be a relatively small fraction of the angular-momentum-unit $\hbar$ per electron. However, in a second scenario where the transverse reduced wavelength of the electron vortex and the magnetic field strength are each an order of magnitude larger, the shift of the orbital angular momentum turned out to be a sizable fraction of $\hbar$. This is despite the fact that, for small values of $|l|$, the energy change due to the increased magnetic field remains within the regime in which the electron vortex is dominant. It is interesting to explore whether such shifts in the properties of the electron vortex beam can be realized in the laboratory.

In an electron microscope the beam energy is currently measurable to an accuracy of about 10 meV, so the rotational energy shifts in Eqs. (28) and (32) could be detectable within an electron microscope for large values of $|l|$. The shift in the orbital angular momentum should, in principle, be measurable in terms of the corresponding change in the rotational velocity v_l and also in the change of the radius of the electron vortex beam due to the presence of the magnetic field. We estimated that the presence of the magnetic field leads to a change of the beam radius one order of magnitude larger than the vortex beam radius in the absence of the magnetic field.

The theory presented here assumes that the measurements are taken while the vortex is in the region of the magnetic field. A related problem which needs to be investigated concerns the case when the vortex enters the region of the magnetic field and measurement is carried out on exit in the field-free region. This is essentially a scattering problem and will require an entirely different treatment based on scattering theory, but we shall not pursue this matter any further here.

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