Fragmentation of a spin-1 mixture in a magnetic field

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We study ground-state quantum fragmentation in a mixture of a polar condensate and a ferromagnetic condensate when subjected to an external magnetic field. We pay more attention to the polar condensate, due to the fact that fragmentation of a polar condensate, which typically occurs only in a very weak magnetic field, can occur in a mixture at higher magnetic fields, where both atom numbers and number fluctuations will remain of a macroscopic magnitude, of the order of N. The role of the ferromagnetic condensate is to provide a uniform and stable background which can delay the rapid shrinkage of the zero-component population and make it possible to capture "superfragmentation." Our method has potential applications in measurement of the interspecies spin-coupling interaction by adjusting the magnetic field.

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I. INTRODUCTION

Recent experimental breakthroughs in spinor Bose-Einstein condensates (BECs), such as sub-Poissonian spin correlations generated by atomic four-wave spin mixing [1], atomic squeezed states realized in spin-1 ultracold atomic ensembles [2], and antiferromagnetic spatial ordering observed in a quenched one-dimensional spin-1 gas [3], are all in connection with vacuum fluctuations and recall attention to the finite-particle-number effect beyond the mean-field treatment. Vacuum fluctuations become a significant subject in more and more experimental topics, e.g., atomic quantum matter-wave optics, atomic spin squeezing, and quantum information. With the basic interaction form $V(\mathbf{r}) = (\alpha + \beta \mathbf{F}\cdot\mathbf{F})\delta(\mathbf{r})$, the properties of such a three-component spinor condensate [4] have been demonstrated experimentally [5], and two phases, reflecting fundamental properties of spin correlation, identified: the so-called polar and ferromagnetic states for $\beta > 0$ (^{23}Na) and $\beta < 0$ (^{87}Rb) atomic condensates, respectively. Mixtures of two spinor condensates with ferromagnetic and polar atoms, respectively, show more attractive quantum effects [6–15]. With the help of sympathetic cooling, BEC mixtures of Na and Rb have been realized and it is interesting to observe the interspecies-interaction-induced immiscibility between the two condensates [15].

The ground state of the condensate with $\beta > 0$ has been predicted to be either polar ($n_0 = N$) or antiferromagnetic ($n_1 = n_{-1} = N/2$) within the mean-field treatment, where the condensate is usually described by a coherent state. However, the many-body theory of Law, Pu, and Bigelow [16] points out that the ground state of $\beta > 0$ atoms is a spin singlet with properties ($n_1 = n_0 = n_{-1} = N/3$) drastically different from the results predicted by the mean-field theory. Soon, Ho, and Yip [17] show that this spin-singlet state is a fragmented condensate with anomalously large number fluctuations and thus has a fragile stability. The remarkable nature of this superfragmentation is that the single-particle reduced density matrix gives three macroscopic eigenvalues ($N/3$) with large number fluctuations, $\Delta n_{1,0,-1} \sim N$. Similar considerations were also addressed by Koashi and Ueda [18–20]. The signature of fragmentation is then referred to the anomalously large fluctuations of the populations in the Zeeman levels. This is a super-Poissonian correlation character, and the large number fluctuations shrink rapidly when experimentally adventitious perturbations exist, such as a magnetic field or field gradient.

In this paper we report the influence of an external magnetic field on a spinor condensate with $\beta > 0$, but on the premise of doping many ferromagnetic atoms in it. An interspecies spin-coupling interaction arises and we propose a valid procedure to observe and control the fragmented states. If the ferromagnetic atoms in the mixture are condensed, the ground state favors all atoms being aligned along the same direction and provides a uniform and stable background which can delay the rapid shrinking of the number fluctuations when the interspecies coupling interaction is adjusted. The back action from polar atoms onto the more stable ferromagnetic atoms is negligible. Doping ferromagnetic atoms into spin-1 polar condensates can effectively influence the vacuum fluctuations and will have potential applications for quantum information and quantum-enhanced magnetometry.

II. HAMILTONIAN OF THE MIXTURE

We consider a mixture of two spinor condensates of N_1 ferromagnetic and N_2 polar atoms, respectively. The intracondensate atomic spin-1 interaction takes the standard interaction form, $V_k(\mathbf{r}) = (\alpha_k + \beta_k \mathbf{F}_k\cdot\mathbf{F}_k)\delta(\mathbf{r})$, with $k = 1,2$. The intercondensate interaction between ferromagnetic and polar atoms is $V_{12}(\mathbf{r}) = \frac{1}{2}(\alpha + \beta \mathbf{F}_1\cdot\mathbf{F}_2 + \gamma P_0)\delta(\mathbf{r})$, which is more complicated because collisions can occur in the total spin $F_{\text{tot}} = 1$ channel between different atoms [6,7]. The parameters α, β, and γ are related to the s-wave scattering lengths in the three total spin channels and the reduced mass μ for atoms of different species, and P_0 projects an interspecies pair into a spin-singlet state. Within the single spatial-mode approximation [16,21,22] for each of the two

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spinor condensates, the spin-dependent Hamiltonian for the mixture finally reads

$$\hat{H} = c_1\beta_1\hat{\mathbf{F}}_1^2 + c_2\beta_2\hat{\mathbf{F}}_2^2 + c_{12}\beta\hat{\mathbf{F}}_1\cdot\hat{\mathbf{F}}_2 + \frac{c_{12}\gamma}{3}\hat{\Theta}_{12}^\dagger\hat{\Theta}_{12}, \quad (1)$$

where $\hat{\mathbf{F}}_1 = \hat{a}_i^\dagger\mathbf{F}_{1ij}\hat{a}_j$ ($\hat{\mathbf{F}}_2 = \hat{b}_i^\dagger\mathbf{F}_{2ij}\hat{b}_j$) are defined in terms of 3×3 spin-1 matrices with $i(j) = 1,0,-1$, and $\hat{a}_i^\dagger$ ($\hat{b}_i^\dagger$) creates a ferromagnetic (polar) atom in the hyperfine state i. The operator

$$\hat{\Theta}_{12}^\dagger = \hat{a}_0^\dagger\hat{b}_0^\dagger - \hat{a}_1^\dagger\hat{b}_{-1}^\dagger - \hat{a}_{-1}^\dagger\hat{b}_1^\dagger \quad (2)$$

creates a singlet pair with one atom each from the two species, similar to

$$\hat{\Theta}_2^\dagger = (\hat{b}_0^\dagger)^2 - 2\hat{b}_1^\dagger\hat{b}_{-1}^\dagger, \quad (3)$$

for an intraspecies spin-singlet pair [17,18] when $\beta_2 > 0$. The interaction parameters are $c_1 = \frac{1}{2}\int d\mathbf{r}|\Psi(r)|^4$, $c_2 = \frac{1}{2}\int d\mathbf{r}|\Phi(r)|^4$, and $c_{12} = \int d\mathbf{r}|\Psi(r)|^2|\Phi(r)|^2$, which can be tuned through control of the frequency of the trapping potential [7].

III. FRAGMENTATION IN A MAGNETIC FIELD

A. Number distributions in a magnetic field

When the interspecies scattering parameters are calculated in the degenerate internal-state approximation [23–26], the low-energy atomic interactions can be mostly attributed to the ground-state configurations of the two valence electrons, and the noncommutative term $\hat{\Theta}_{12}^\dagger\hat{\Theta}_{12}$ can be neglected [6,7,9]. The ground states are classified into four distinct phases—FF, MM_-,MM_+, and AA—by three critical values: $c_{12}\beta = -\frac{(2N-1)c_2\beta_2}{N}$, 0, and $\frac{(2N-1)c_2\beta_2}{N+1}$ [9].

In this paper we discuss the atom number distribution and fluctuation in an external magnetic field. The spin-dependent Hamiltonian in the magnetic field reads

$$\hat{H} = c_1\beta_1\hat{\mathbf{F}}_1^2 + c_2\beta_2\hat{\mathbf{F}}_2^2 + c_{12}\beta\hat{\mathbf{F}}_1\cdot\hat{\mathbf{F}}_2 - c_1p_1\hat{F}_{1z} - c_2p_2\hat{F}_{2z}, \quad (4)$$

where only the linear Zeeman terms are considered. As the SU(2) symmetry is broken in a spinor mixture, one cannot eliminate the linear Zeeman effect through a spin rotation [27]. Meanwhile, the quadratic Zeeman energy, typically two orders of magnitude weaker than the linear terms, is negligible in the calculation of number distributions. For alkali atoms such as ^{23}Na and ^{87}Rb, in their subspace of hyperfine spin F = 1, both the nuclear spins and the valence electron spins are the same for the two species, and the linear Zeeman shifts are thus almost equal. In the following discussion, we take $p = c_1p_1 = c_1p_2$ for simplicity.

We consider the direct product of the Fock states of the two species $|n_1,n_0,n_{-1}\rangle_1 \otimes |n_1,n_0,n_{-1}\rangle_2$ and do not restrict the model in the subspace with zero total magnetization [9,10]. Instead, we consider the full space including all possible system magnetization $m = m_1 + m_2$. Using the full quantum approach of exact diagonalization, we can get the ground state of the system and study the response of the two species to the external magnetic field p for $N_1 = N_2 = 40$. The three critical points for the phase boundaries are approximately $c_{12}\beta = -4$, 0, and 4.

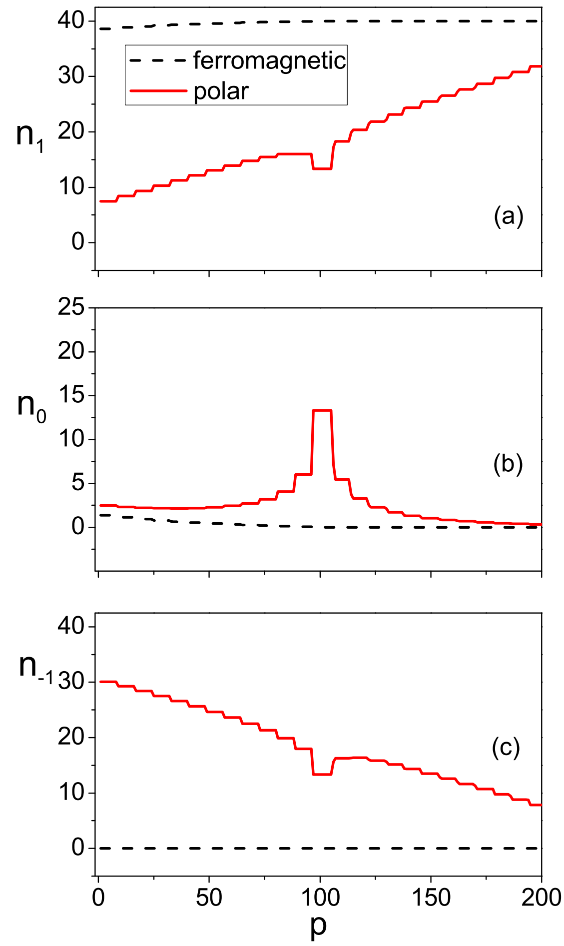

FIG. 1. (Color online) Dependence of atom numbers on p, at fixed values of $c_1\beta_1 = -1$, $c_2\beta_2 = 2$, and $c_{12}\beta = 2.5$. The total numbers of the two species are $N_1 = N_2 = 40$, and we consider the full space with total magnetization m a variable. Dashed black and solid (red) lines denote the number distributions in the ferromagnetic and polar condensate, respectively. All interaction parameters are in units of $|c_1\beta_1|$.

The field dependence of the population is shown in Fig. 1 for the MM_+ phase at $c_{12}\beta = 2.5$, where polar atoms are partly polarized in the direction opposite that for ferromagnetic atoms [9]. We note that the ferromagnetic atoms (dashed black lines) are very sensitive to the magnetic field, i.e., atoms quickly redistribute in the n_1 component and the magnetization of the ferromagnetic condensate $m_1 = n_1 - n_{-1}$ saturates immediately. The ferromagnetic atoms actually form a stable condensate and provide a uniform magnetic background in the mixture. The polar atoms present a stepwise increase (decrease) in the atom number distribution $n_1(n_{-1})$ when the field increases. For small p and positive $c_{12}\beta$, the system favors a negative magnetization ($m_2 = n_1 - n_{-1}$) of the polar condensate, and m_2 will reverse and tend to saturate for a large magnetic field. We note that a special number distribution, with $n_1 = n_0 = n_{-1} = \frac{N}{3}$, remarkably arises around the value of $p = 100$.

The situation becomes simples if the parameter $c_{12}\beta$ is negative, that is, in the FF phase (or MM_- phase), where polar atoms are fully (partly) polarized in the same direction as ferromagnetic atoms. The enhanced ferromagnetic effect and the external magnetic field jointly suppress the atom number distribution n_0 and n_{-1} of the polar condensate to 0 and, at the same time, saturate n_1 and the magnetization m_2 without magnetization reversal.

B. Retrieving the superfragmented state

According to the spin-space rotational-invariant Hamiltonian [17–20],

$$\hat{H}_0 = c_2\beta_2\hat{\mathbf{F}}^2 = c_2\beta_2\big[\hat{N}_2^2 - \hat{\Theta}_2^\dagger\hat{\Theta}_2\big],$$

the superfragmented state is named in [17] for the ground state of a pure spin-1 condensate with $c_2\beta_2 > 0$. This ground state is described by a many-body spin singlet with the form

$$|\phi_{\rm sup}\rangle \propto (\hat{\Theta}_2^\dagger)^{N_2/2}|0\rangle,$$

where $\hat{\Theta}_2^\dagger$ creates a singlet pair formed by two identical spin-1 bonsons. For spin-1 particles with three hyperfine spin states $|f,f_m\rangle = \hat{b}_m^\dagger|0\rangle$, the simplest spin singlet is formed by two spin-1 particles and described as

$$|F_{\rm tot}=0,F_m=0\rangle = \sum C|f_1,f_{m1}\rangle|f_2,f_{m2}\rangle, \qquad (5)$$

under the condition $f_m = f_{m1} + f_{m2} = 0$, and with the remainder corresponding to the Clebsch-Gordon coefficient C, one can get

$$|F_{\rm tot}=0,F_m=0\rangle = \frac{1}{\sqrt{3}}\big(\hat{b}_0^{\dagger 2} - 2\hat{b}_1^\dagger\hat{b}_{-1}^\dagger\big)|0\rangle. \qquad (6)$$

A many-body spin singlet is constructed by applying $\hat{\Theta}_2^\dagger$ as many times as needed to get the desired number of particles [28]. The particle density matrix will be $(\hat{\rho})_{mn} = \langle\hat{b}_m^\dagger\hat{b}_n\rangle = \frac{N}{3}\delta_{mn}$, which fulfills the condition [29] that the ground state can contain several condensates. The number fluctuations, as the signature of fragmentation, can be calculated algebraically [17,20] and satisfy $2\Delta n_1 = \Delta n_0 = 2\Delta n_{-1}$, with

$$\Delta n_0 = \frac{2\sqrt{N^2+3N}}{3\sqrt{5}}. \qquad (7)$$

Such a state with a polar interaction was not likely realized in typical experiments due to its fragility towards any perturbation-breaking spin rotational symmetry. For example, if subjected to an external magnetic field, the ground state of the system [17–20] will be

$$|\phi_{\rm mag}\rangle \propto (\hat{b}_1^\dagger)^{m_2}(\hat{\Theta}_2^\dagger)^{(N_2-m_2)/2}|0\rangle; \qquad (8)$$

one can see a rapid decrease in the spin-0 component distribution n_0 and the fluctuations $\Delta n_{1,0,-1}$ when m_2 is increased. The superfragmented state then reduces to a much more generic fragmented state: a two-component number state with essentially zero fluctuations,

$$|\phi_{\rm num}\rangle \propto (\hat{b}_1^\dagger)^{(N_2+m_2)/2}(\hat{b}_{-1}^\dagger)^{(N_2-m_2)/2}|0\rangle. \qquad (9)$$

For a spin-1 polar condensate doped with many ferromagnetic atoms, we can retrieve this superfragmented state in the presence of an external field. For some special values of the magnetic field, both the spin-0 component population and the number fluctuations would not decrease but recover to macroscopic orders of N_2. In Fig. 2, we illustrate the recovery points for three interspecies coupling parameters $c_{12}\beta$ in the MM_+ phase ($0 < c_{12}\beta < 4$). These recovery points are found to move towards largers value of p as $c_{12}\beta$ increases.

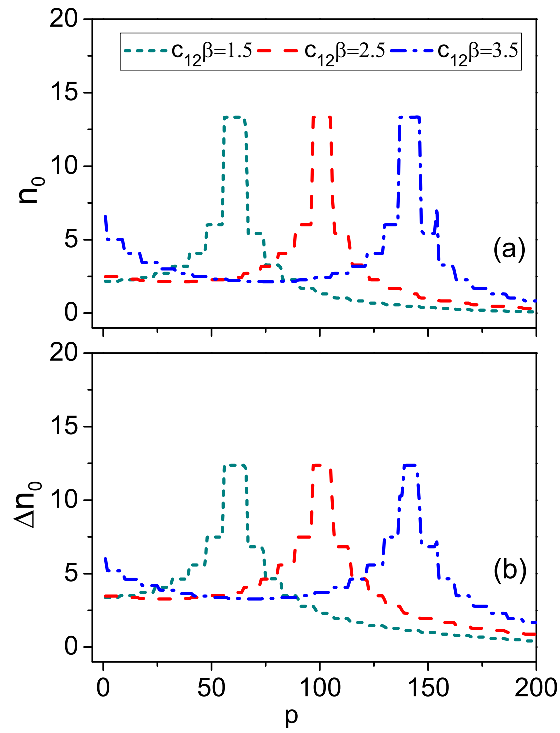

FIG. 2. (Color online) Dependence of atom numbers and fluctuations on $c_{12}\beta$ and p at fixed values of $c_1\beta_1 = -1$ and $c_2\beta_2 = 2$. Only the results of polar atoms with n_0 and Δn_0 are shown. When the extra magnetic field parameter p (in units of $|c_1\beta_1|$) increases, there are serval critical points associated with $c_{12}\beta$. All interaction parameters are in units of $|c_1\beta_1|$.

As learned from previous studies [22], the mean-field treatment is efficient for atomic interactions of the ferromagnetic type. The much more stable ferromagnetic condensate in the mixture can be formulated in the mean-field treatment as a boson-enhanced effective magnetic field. This simplifies the Hamiltonian, (4), to

$$\begin{aligned}\hat{H} &= c_1\beta_1\langle\hat{\mathbf{F}}_1^2\rangle + c_2\beta_2\hat{\mathbf{F}}_2^2 + c_{12}\beta\langle\hat{\mathbf{F}}_1\rangle\cdot\hat{\mathbf{F}}_2 \\ &\quad - c_1p_1\langle\hat{F}_{1z}\rangle - c_2p_2\hat{F}_{2z} \\ &= c_2\beta_2\hat{\mathbf{F}}_2^2 + A\hat{F}_{2z} + C,\end{aligned} \qquad (10)$$

where $\langle\hat{\mathbf{F}}_1\rangle = \langle\hat{F}_{1z}\rangle = N_1$, $A = c_{12}\beta N_1 - c_2p_2$, and $C = c_1\beta_1N_1(N_1+1) - c_1p_1N_1$. The criterion for the emergence of the superfragmented state is $p = c_{12}\beta N_1$, where the magnetic field (p), the optical trapping frequency (c_{12}), and the number of doped ferromagnetic atoms (N_1) are all adjustable. When the magnetic field matches the condition that $c_{12}\beta N_1$ and c_2p_2 cancel each other, we may achieve the superfragmented state in a magnetic field. The three critical points in Fig. 2 are found to agree with the numerical results exactly. The special value such as $p = 100$ in Fig. 1 can be predicted exactly here with $p = c_{12}\beta N_1 = 2.5\times 40 = 100$.

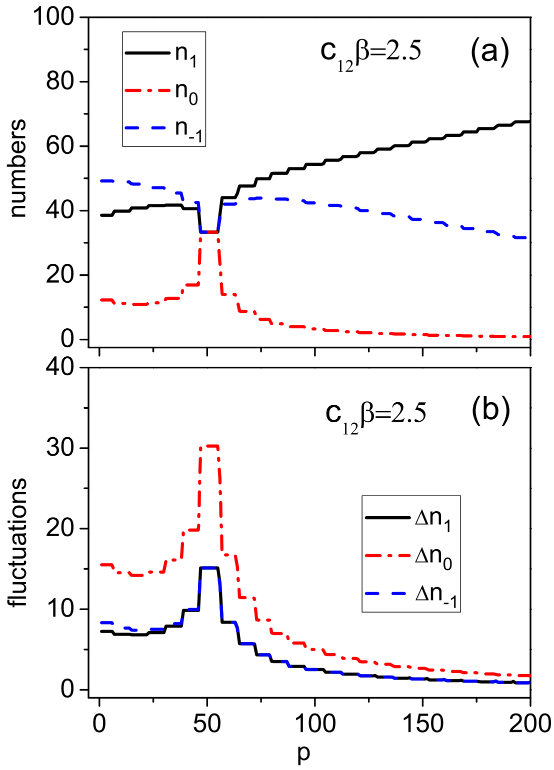

FIG. 3. (Color online) Dependence of atom number distributions and number fluctuations in the polar condensate on the magnetic coefficient p at fixed values of $c_1\beta_1 = -1$, $c_{12}\beta = 2.5$, and $c_2\beta_2 = 2$. Total numbers of the two species are $N_1 = 20$ and $N_2 = 100$. Solid black, dash-dotted (red), and dashed (blue) lines denote atom numbers and fluctuations on the 1, 0, and -1 sublevels, respectively. All interaction parameters are in units of $|c_1\beta_1|$.

Next, we turn to the situation with population imbalance in the two species. Figure 3 illustrates the location of the critical point when the interspecies coupling parameter $c_{12}\beta$ is fixed at 2.5 and the atom numbers for the two species are $N_1 = 20$ and $N_2 = 100$. As the mean-field picture works well for ferromagnetic atoms, we still get the crucial point $p = 2.5 \times N_1 = 50$ in Fig. 3. When equal populations $n_1 = n_0 = n_{-1} = N/3$ occur for the polar condensate, the number fluctuations also instantaneously reach macroscopic levels. Our numerical results for the fluctuation relation ($\Delta n_0 = 2\Delta n_{\pm 1}$) agree exactly with the algebraic results in [17] for a pure polar condensate. With the emergence of equal populations $N/3$ regarded as a sign of antiferromagnetic spin interaction, the interspecies spin-coupling interaction $c_{12}\beta$ can be estimated by the location of the critical magnetic field.

C. AA phase in a magnetic field

When the interaction parameter $c_{12}\beta > \frac{(2N-1)c_2\beta_2}{N+1}$, the system spontaneously breaks into a high-symmetry state called the AA phase. The AA phase is another superfragmented state which has been predicted in the absence of magnetic fields [9]. It is also a many-body spin singlet, which requires exactly the same atom number for the two species ($N = N_1 = N_2$) and total spins of different species that are polarized in opposite directions. In the notation of the angular momentum representation,

$$|F_1, F_2, F, m\rangle = \sum C^{F,m}_{F_1,m_1;F_2,m_2} |F_1, m_1\rangle |F_2, m_2\rangle, \qquad (11)$$

where the AA phase is denoted $|\phi_{AA}\rangle = |N, N, 0, 0\rangle$, with F_1, F_2, and F the total spin quantum numbers of ferromagnetic atoms, polar atoms, and the mixture and m_1, m_2, and m the corresponding z components. The intraspecies angular momentum states involved in the AA phase, $|N, m_1\rangle$ and $|N, m_2\rangle$, should obey the constraint $m_1 + m_2 = 0$. The interesting feature of the AA phase is the equal distribution of atoms in the six components ($N/3$) and the large number fluctuations. To calculate the number distribution and the number fluctuation, one has to expand the two species states $|N, m_1\rangle$ and $|N, m_2\rangle$ into Fock states [9,30], and the number fluctuations without a magnetic field are calculated to be

$$\Delta n_0^{(1,2)} = \frac{\sqrt{N^2 + 9N}}{3\sqrt{5}}, \quad \Delta n_{\pm 1}^{(1,2)} = \frac{2\sqrt{N^2 + 3N/2}}{3\sqrt{5}}. \qquad (12)$$

However, unlike the superfragmented state, we cannot give the perfect creation operator description of the AA phase, due to the more complicated symmetry originating from the collision occurring in the total spin $F_{\text{tot}} = 1$ channel.

In this section, we numerically discuss the AA phase ($c_{12}\beta > 4$) subject to external magnetic fields using the full quantum approach of exact diagonalization and compare the results with the superfragmented state in the pure condensate [17]. The features of these two typical fragmented ground states, which belong to two special phases characterized by typical values of the interaction parameter $c_{12}\beta = 4.5$ and $c_{12}\beta = 0$, are illustrated in Fig. 4. First, we note that the numerical results for the number distributions and fluctuations

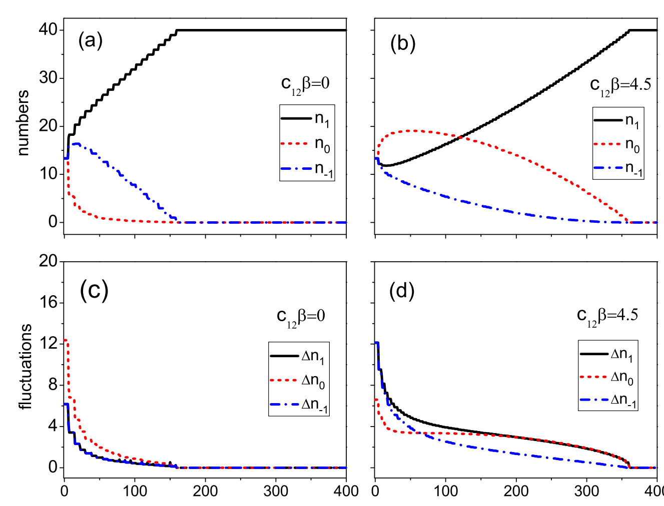

FIG. 4. (Color online) Dependence of atom number distributions $n_{1,0,-1}$ and number fluctuations $\Delta n_{1,0,-1}$ of the polar condensate on both $c_{12}\beta$ and magnetic field p at fixed values of $c_1\beta_1 = -1$ and $c_2\beta_2 = 2$. Total numbers of the two species are $N_1 = N_2 = 40$. Solid black, dotted (red), and dash-dotted (blue) lines denote the numbers (fluctuations) on the 1, 0, and -1 sublevels, respectively. All interaction parameters are in units of $|c_1\beta_1|$.

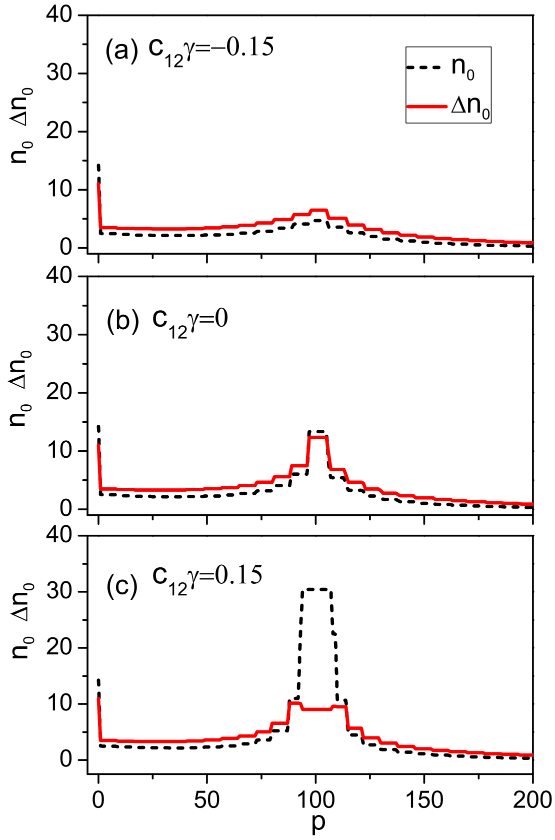

FIG. 5. (Color online) Dependence of the atom number distribution n_0 and Δn_0 in the polar condensate on the magnetic coefficient p and $c_{12}\gamma$ at fixed values of $c_1\beta_1 = -1$, $c_{12}\beta = 2.5$, and $c_2\beta_2 = 2$. Total numbers of the two species are $N_1 = N_2 = 40$. Dashed black lines and solid (red) lines denote the value of n_0 and Δn_0, respectively. All interaction parameters are in units of $|c_1\beta_1|$.

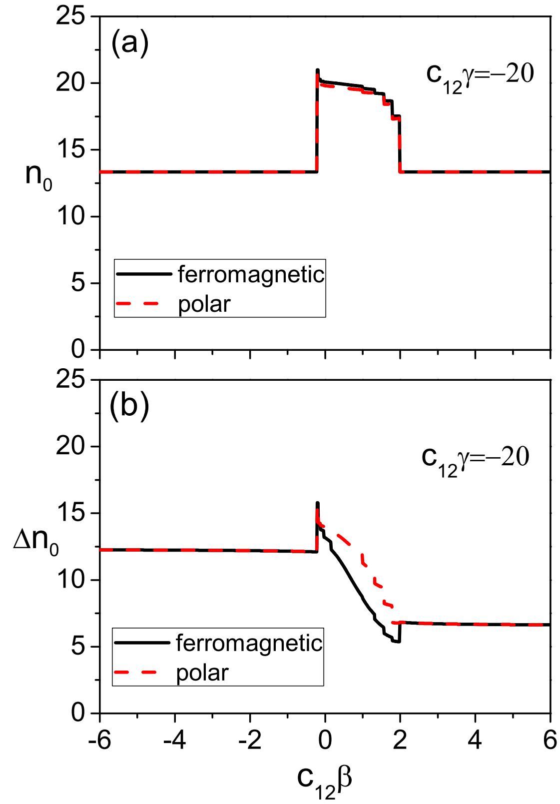

FIG. 6. (Color online) Dependence of atom numbers and fluctuations on $c_{12}\beta, c_{12}\gamma$ at fixed values of $p = 0, c_1\beta_1 = -1$, and $c_2\beta_2 = 2$. This graph shows only the results for n_0 and Δn_0, when the interaction parameter $c_{12}\gamma$ equals -20. Total numbers of the two species are $N_1 = N_2 = 40$ and we restrict the problem in full space without an external magnetic field. Solid black lines and dashed (red) lines denote the ferromagnetic and polar condensate, respectively. All interaction parameters are in units of $|c_1\beta_1|$.

agree exactly with the algebraic results for the special point $p = 0$. The AA phase is as fragile as the pure polar singlet, with the number fluctuations dropping rapidly [Fig. 4(d)] and the number distributions finally decreasing to a generic number state, $(b_1^\dagger)^{N_2}|0\rangle$. It is interesting that the responses of the n_0 component to the magnetic field are quite different. For a pure polar condensate [Fig. 4(a)], as p is increased, the zero-component distribution n_0 [dashed (red) line] decreases rapidly, which agrees with the algebraic results in [17]. For the AA phase [Fig. 4(b)], we note that n_0 does not decrease rapidly in the beginning; instead, it increases first and remains at a high value for a certain range of p. The applied external magnetic field can be used to characterize these two spin singlets by tracing the atom numbers of the n_0 component of the polar atoms.

D. The interspecies P_0 effect

If we refer to a more general case beyond the degenerate internal-state approximation, the γ term of Hamiltonian (1) should be considered. We note that

$$[\hat{\mathbf{F}}^2_{1,2}, \hat{\Theta}^\dagger_{12}\hat{\Theta}_{12}] \neq 0, \quad [\hat{\mathbf{F}}^2, \hat{\Theta}^\dagger_{12}\hat{\Theta}_{12}] = 0, \qquad (13)$$

which means, in general, that they do not belong to a set of commutative operators. However, we can numerically study the phase transition through the order parameter $\langle\hat{\Theta}^\dagger_{12}\hat{\Theta}_{12}\rangle$ [10]. To see more clearly the role played by the parameter $c_{12}\gamma$ in fragmentation, we numerically diagonalize Hamiltonian (1) with $N_1 = N_2 = 40$.

In Fig. 5, we illustrate the influence of a small $c_{12}\gamma \neq 0$ on the population n_0 and Δn_0 of a superfragmented state which has been retrieved in the MM_+ phase. We find that the crucial point is still located at $p = 2.5 \times N_1$, but a tiny $c_{12}\gamma = 0.15$ will increase the n_0 component to a dominant value and, meanwhile, suppress the n_1 and n_{-1} components to a lower level. A high occupation of n_0 components is evidence of the nematic order [2], and the signature of fragmentation disappears. For $c_{12}\gamma = -0.15$, on the contrary, the n_0 component is totally suppressed, with both n_0 and Δn_0 decreasing. Away from the critical point, the system is dominated by the magnetic field, with the magnetization $m_2 = n_1 - n_{-1}$ increasing linearly.

A negative γ term encourages pairing two different types of atoms in singlets [10]. In Fig. 6, the influences of a *negative* singlet pairing coefficient $c_{12}\gamma$ on the numbers and quantum fluctuations of the two species are illustrated. We note that the typical $N/3$ number distributions arise in both the $c_{12}\beta < 0$ and the $c_{12}\beta > 0$ regions when $c_{12}\gamma$ reaches -20. The number fluctuation $\triangle n_0$ gives two steady values, which represent two typical fragmented ground states: the interspecies entangled fragmented state for $c_{12}\beta > 2$ and the pure species independent fragmented state for $c_{12}\beta < 0$. The fluctuations for these two states,

$$\Delta n_0 = \frac{\sqrt{N^2 + 9N}}{3\sqrt{5}}, \quad c_{12}\beta > 2,$$

$$\Delta n_0 = \frac{2\sqrt{N^2 + 2N}}{3\sqrt{5}}, \quad c_{12}\beta < 0,$$

are found to match the numerical results in Fig. 6.

IV. CONCLUSION

To conclude, we have studied the ground-state properties of a binary mixture of ferromagnetic and polar spinor condensates in a magnetic field. Using the full quantum approach of exact diagonalization, we can study the competition between the magnetic linear Zeeman effect and the interspecies spin-coupling interaction $c_{12}\beta$. The large vacuum fluctuation of number distributions on the three Zeeman levels in the polar condensate is worthy of investigation. We point out that the fragmentation properties of the polar condensate can be adjusted through the magnetic field (p), trapping frequency (c_{12}), and number of doped ferromagnetic atoms (N_1). The ferromagnetic condensate is involved in providing a uniform and stable background, which can delay the rapid decrease in the large number fluctuations. We have illustrated the influences of the magnetic parameter p and identified two typical fragmented states with total spin $\langle \hat{\mathbf{F}}^2 \rangle = 0$. A positive interspecies spin-coupling interaction ($c_{12}\beta > 0$) can effectively entangle the different species, while for $c_{12}\beta < 0$ the different species on their $F = 1$ manifold are essentially independent. We propose a possible mechanism to effectively measure interspecies spin-coupling interactions by applying a magnetic field, as well as discriminate the two types of many-body spin singlets. Our work highlights the significant promise of experimental work on sodium and rubidium atomic condensate mixtures and provides some useful information for the study of photoassociation of heteronuclear molecules.

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[1] E. M. Bookjans, C. D. Hamley, and M. S. Chapman, Phys. Rev. Lett. **107**, 210406 (2011).

[2] C. D. Hamley, C. S. Gerving, T. M. Hoang, E. M. Bookjans, and M. S. Chapman, Nat. Phys. **8**, 305 (2012); T. M. Hoang, C. S. Gerving, B. J. Land, M. Anquez, C. D. Hamley, and M. S. Chapman, Phys. Rev. Lett. **111**, 090403 (2013).

[3] A. Vinit, E. M. Bookjans, C. A. R. Sáde Melo, and C. Raman, Phys. Rev. Lett. **110**, 165301 (2013).

[4] T.-L. Ho, Phys. Rev. Lett. **81**, 742 (1998); T. Ohmi and K. Machida, J. Phys. Soc. Jpn. **67**, 1822 (1998).

[5] J. Stenger, S. Inouye, D. M. Stamper-Kurn, H.-J. Miesner, A. P. Chikkatur, and W. Ketterle, Nature (London) **396**, 345 (1998); L. E. Sadler, J. M. Higbie, S. R. Leslie, M. Vengalattore, and D. M. Stamper-Kurn, *ibid.* **443**, 312 (2006).

[6] M. Luo, Z. Li, and C. Bao, Phys. Rev. A **75**, 043609 (2007); M. Luo, C. Bao, and Z. Li, J. Phys. B: At. Mol. Opt. Phys. **41**, 125101 (2008).

[7] Z. F. Xu, Y. Zhang, and L. You, Phys. Rev. A **79**, 023613 (2009).

[8] Z. F. Xu, J. Zhang, Y. Zhang, and L. You, Phys. Rev. A **81**, 033603 (2010).

[9] J. Zhang, Z. F. Xu, L. You, and Y. Zhang, Phys. Rev. A **82**, 013625 (2010).

[10] J. Zhang, T. T. Li, and Y. Zhang, Phys. Rev. A **83**, 023614 (2011).

[11] Yu Shi, Phys. Rev. A **82**, 023603 (2010); Y. Shi and G. Li, *ibid.* **83**, 013616 (2011); Int. J. Mod. Phys. B **26**, 1250002 (2012); L. Ge, F. Zhou, and Y. Shi, Phys. Rev. A **89**, 043623 (2014).

[12] Z. F. Xu, J. W. Mei, R. Lu, and L. You, Phys. Rev. A **82**, 053626 (2010).

[13] Z. F. Xu, R. Lu, and L. You, Phys. Rev. A **84**, 063634 (2011).

[14] Z. F. Xu, D. J. Wang, and L. You, Phys. Rev. A **86**, 013632 (2012).

[15] F. D. Wang, D. Z. Xiong, X. K. Li, D. J. Wang, and E. Tiemann, Phys. Rev. A **87**, 050702(R) (2013).

[16] C. K. Law, H. Pu, and N. P. Bigelow, Phys. Rev. Lett. **81**, 5257 (1998).

[17] T.-L. Ho and S.-K. Yip, Phys. Rev. Lett. **84**, 4031 (2000).

[18] M. Koashi and M. Ueda, Phys. Rev. Lett. **84**, 1066 (2000).

[19] M. Ueda and M. Koashi, Phys. Rev. A **65**, 063602 (2002).

[20] E. J. Mueller, T.-L. Ho, M. Ueda, and G. Baym, Phys. Rev. A **74**, 033612 (2006).

[21] H. Pu, C. K. Law, S. Raghavan, J. H. Eberly, and N. P. Bigelow, Phys. Rev. A **60**, 1463 (1999).

[22] S. Yi, Ö. E. Müstecaplioglu, C. P. Sun, and L. You, Phys. Rev. A **66**, 011601(R) (2002).

[23] H. T. C. Stoof, J. M. V. A. Koelman, and B. J. Verhaar, Phys. Rev. B **38**, 4688 (1988).

[24] A. C. Maan, E. Tiesinga, H. T. C. Stoof, and B. J. Verhaar, Physica B **165-166**, 17 (1990).

[25] S. B. Weiss, M. Bhattacharya, and N. P. Bigelow, Phys. Rev. A **68**, 042708 (2003); **69**, 049903(E) (2004).

[26] A. Pashov, O. Docenko, M. Tamanis, R. Ferber, H. Knoeckel, and E. Tiemann, Phys. Rev. A **72**, 062505 (2005).
[27] W. X. Zhang, D. L. Zhou, M.-S. Chang, M. S. Chapman, and L. You, Phys. Rev. A **72**, 013602 (2005).
[28] A. Imambekov, M. Lukin, and E. Demler, Phys. Rev. A **68**, 063602 (2003).
[29] P. Nozières and D. Saint James, J. Phys. (Paris) **43**, 1133 (1982).
[30] Y. Wu, Phys. Rev. A **54**, 4534 (1996).