High-energy e^+e^- photoproduction in the field of a heavy atom accompanied by bremsstrahlung

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Helicity amplitudes and the differential cross section of high-energy e^+e^- photoproduction accompanied by bremsstrahlung in the electric field of a heavy atom (i.e., the amplitudes of the process $\gamma_1 Z \to e^+e^-\gamma_2 Z$) are derived. The results are exact in the nuclear charge number and are obtained in the leading quasiclassical approximation. They correspond to the leading high-energy small-angle asymptotics of the amplitude. It is shown that, in general, accounting for the Coulomb corrections essentially modify the differential cross section, which is different from the Born result. When the initial photon is circularly polarized, the Coulomb corrections lead to the asymmetry in the distribution over the azimuth angles φ_i of produced particles with respect to the replacement $\varphi_i \to -\varphi_i$.

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I. INTRODUCTION

QED processes at high energy in the field of a heavy nucleus or atom are the classical examples of the processes in a strong field. They show up in many experimental setups, including those designed for completely different purposes, not connected with the observation of these processes. Therefore, their investigation clearly has a practical value. From the theoretical point of view, these processes are interesting because they provide an important insight into the structure of the higher-order effects of the perturbation theory.

The general approach to the strong-field calculations is the use of the Furry representation. In this approach the wave functions and propagators of particles are replaced by the exact solutions and Green's functions of the wave equations in the external field. However, even for the pure Coulomb field, these objects are very complicated, and their use for practical calculations is limited. Fortunately, at high energies of initial particles the final particle momenta usually have small angles with respect to the incident direction. In this case typical angular momenta, which provide the main contribution to the amplitude, are large ($l \sim E/\Delta \gg 1$, where E is energy and Δ is the momentum transfer). This is where the quasiclassical approximation, based on the account of large angular momenta contributions, comes into play. In this approximation, the wave functions and propagators acquire remarkably simple forms which allow for their effective use in specific calculations. The quasiclassical Green's function of the Dirac equation in the external field has been derived for a number of field configurations; see Ref. [1] for the case of a pure Coulomb field, Ref. [2] for an arbitrary spherically symmetric field, Ref. [3] for a localized field which generally possesses no spherical symmetry, and Ref. [4] for combined strong laser and atomic fields. Even more surprising is the fact that within this approximation it appears to be possible to derive not only the results in the leading order but also a first quasiclassical correction to them.

Basic processes in the field of heavy atoms are the electron-positron pair photoproduction (PP) and electron bremsstrahlung (BS). They both have a long history of investigation; for the former process see reviews in Refs. [5,6]. For the total cross section of electron-positron pair photoproduction there is also a formal expression [7], exact in the parameter $\eta = Z\alpha$ and the photon energy ω (here Z is the atomic charge number, α is the fine-structure constant, $\hbar = c = 1$). It has the form of multiple slowly converging sums containing the hypergeometric function of two arguments F_2. Due to these complications, the computation based on this expression rapidly becomes intractable with the growth of ω, and the numerical results have been obtained so far only for $\omega < 12.5$ MeV [8]. At high energy the quasiclassical approximation is applicable, and the leading quasiclassical term for both pair production and bremsstrahlung has been obtained in [9–13]. The first quasiclassical corrections to the spectra of both processes as well as to the total cross section of pair production have been obtained in Refs. [14,15]. It is remarkable that the quasiclassical correction to the total cross section of pair production cannot be obtained by simply integrating the quasiclassical correction to the spectrum. This is because of the contribution of the tip regions of the spectrum, where only one particle can be considered quasiclassically. A detailed investigation of this region was made in Ref. [16]. The corresponding angular distribution was derived in Ref. [17]. Recently, the first quasiclassical correction to the fully differential cross section was obtained in Ref. [18] for e^+e^- pair photoproduction and in Ref. [19] for $\mu^+\mu^-$ pair photoproduction. As a result, charge asymmetry in these processes was predicted.

In the present paper we apply the quasiclassical approach to the investigation of e^+e^- photoproduction in the field of a heavy atom accompanied by bremsstrahlung. The cross section of this process is a significant part of the radiative corrections to e^+e^- photoproduction as well as noticeable background to such processes as Delbrück scattering [20]. This process should be taken into account when considering electromagnetic showers. In spite of its importance, there are only few theoretical results related to this process [21,22]. In those papers the Born approximation was used, while there are no theoretical results exact in the parameter η. The

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goal of the present paper is twofold. First, we would like to fill the gap in the theoretical description of the process and, in particular, determine the magnitude of the Coulomb corrections for various kinematic regions. We show that, apart from the region of very small momentum transfer, accounting for the Coulomb corrections for heavy atoms drastically change the result. Second, we would like to demonstrate how smoothly the quasiclassical approach works for this complicated case. We consider in detail the case of a pure Coulomb field and then present the modification due to screening by atomic electrons.

II. GENERAL DISCUSSION

The main contribution to the cross section of the process $\gamma_1 Z \to e^+e^-\gamma_2 Z$ is given by the region of small angles between the momenta of the incoming and outgoing particles. In this region

$$d\sigma = \alpha^2 |M|^2 \frac{d\boldsymbol{p}_\perp\, d\boldsymbol{q}_\perp\, d\boldsymbol{k}_{2\perp} d\varepsilon_p d\varepsilon_q}{(2\pi)^6 \omega_1\omega_2}\,, \tag{1}$$

where $\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{p}$, and $\boldsymbol{q}$ are the momenta of initial photon, final photon, electron, and positron, respectively, $\varepsilon_p = \sqrt{\boldsymbol{p}^2+m^2}$, $\varepsilon_q = \sqrt{\boldsymbol{q}^2+m^2}$, and $\omega_2 = \omega_1 - \varepsilon_p - \varepsilon_q$. We fix the coordinate system so that $\boldsymbol{\nu} = \boldsymbol{k}_1/\omega_1$ is directed along the z axis and $\boldsymbol{k}_2$ lies in the xz plane with $k_{2x} > 0$; the notation $\boldsymbol{X}_\perp = \boldsymbol{X} - (\boldsymbol{X}\cdot\boldsymbol{\nu})\boldsymbol{\nu}$ for any vector $\boldsymbol{X}$ is used.

The matrix element M has the form

$$\begin{aligned} M = M^{(1)} + M^{(2)} = &-\int d\boldsymbol{r}_1\, d\boldsymbol{r}_2\, \bar{u}_{\boldsymbol{p}}^{(-)}(\boldsymbol{r}_1)\\ &\times \{(\boldsymbol{\gamma}\cdot\boldsymbol{e}_2^*)e^{-i\boldsymbol{k}_2\cdot\boldsymbol{r}_1} G(\boldsymbol{r}_1,\boldsymbol{r}_2|\varepsilon_p+\omega_2)e^{i\boldsymbol{k}_1\cdot\boldsymbol{r}_2}(\boldsymbol{\gamma}\cdot\boldsymbol{e}_1)\\ &+(\boldsymbol{\gamma}\cdot\boldsymbol{e}_1)e^{i\boldsymbol{k}_1\cdot\boldsymbol{r}_1} G(\boldsymbol{r}_1,\boldsymbol{r}_2|-\varepsilon_q-\omega_2)e^{-i\boldsymbol{k}_2\cdot\boldsymbol{r}_2}(\boldsymbol{\gamma}\cdot\boldsymbol{e}_2^*)\}\\ &\times v_{\boldsymbol{q}}^{(+)}(\boldsymbol{r}_2)\,. \end{aligned} \tag{2}$$

Here $u_{\boldsymbol{p}}^{(-)}(\boldsymbol{r})$ and $v_{\boldsymbol{q}}^{(+)}(\boldsymbol{r})$ are the positive- and negative-energy solutions of the Dirac equation in the external field, $\boldsymbol{e}_1$ and $\boldsymbol{e}_2$ are the polarization vectors of the initial and final photons, respectively, γ^μ are the Dirac matrices, and $G(\boldsymbol{r}_1,\boldsymbol{r}_2|\varepsilon)$ is the Green's function of the Dirac equation in the external field. The superscripts $(-)$ and $(+)$ remind us that the asymptotic forms of $u_{\boldsymbol{p}}^{(-)}(\boldsymbol{r})$ and $v_{\boldsymbol{q}}^{(+)}(\boldsymbol{r})$ at large $\boldsymbol{r}$ contain, in addition to the plane wave, the spherical convergent and divergent waves, respectively. The first term in Eq. (2), $M^{(1)}$, corresponds to radiation from the electron line, and the second term, $M^{(2)}$, corresponds to that from the positron line; see Figs. 1(a) and 1(b), respectively. It is convenient to write Eq. (2) in terms of the Green's function $D(\boldsymbol{r}_1,\boldsymbol{r}_2|\varepsilon)$ of the "squared" Dirac

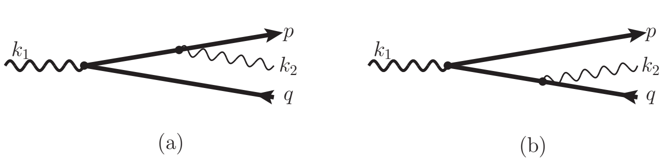

FIG. 1. Diagrams of the process $\gamma_1 Z \to e^+e^-\gamma_2 Z$. Thick solid lines denote exact propagators in the nuclear field.

equation,

$$\begin{aligned} G(\boldsymbol{r}_1,\boldsymbol{r}_2|\varepsilon) &= (\hat{\mathcal{P}}+m)D(\boldsymbol{r}_1,\boldsymbol{r}_2|\varepsilon),\\ D(\boldsymbol{r}_1,\boldsymbol{r}_2|\varepsilon) &= \langle\boldsymbol{r}_1|\frac{1}{\hat{\mathcal{P}}^2-m^2+i0}|\boldsymbol{r}_2\rangle, \end{aligned} \tag{3}$$

where $\hat{\mathcal{P}} = \gamma^\mu\mathcal{P}_\mu$, $\mathcal{P}_\mu = (\varepsilon - V(r), i\boldsymbol{\nabla})$, and $V(r)$ is the atomic potential. Substituting Eq. (3) in Eq. (2), performing integration by parts, and using the Dirac equation, we obtain

$$\begin{aligned} M = &-\int d\boldsymbol{r}_1\, d\boldsymbol{r}_2\, \bar{u}_{\boldsymbol{p}}^{(-)}(\boldsymbol{r}_1)\{e^{-i\boldsymbol{k}_2\cdot\boldsymbol{r}_1}[(\boldsymbol{\gamma}\cdot\boldsymbol{e}_2^*)\hat{k}_2 + 2(\boldsymbol{e}_2^*\cdot\boldsymbol{p}_1)]\\ &\times D(\boldsymbol{r}_1,\boldsymbol{r}_2|\varepsilon_p+\omega_2)e^{i\boldsymbol{k}_1\cdot\boldsymbol{r}_2}(\boldsymbol{\gamma}\cdot\boldsymbol{e}_1) + (\boldsymbol{\gamma}\cdot\boldsymbol{e}_1)e^{i\boldsymbol{k}_1\cdot\boldsymbol{r}_1}\\ &\times D(\boldsymbol{r}_1,\boldsymbol{r}_2|-\varepsilon_q-\omega_2)e^{-i\boldsymbol{k}_2\cdot\boldsymbol{r}_2}[(\boldsymbol{\gamma}\cdot\boldsymbol{e}_2^*)\hat{k}_2 + 2(\boldsymbol{e}_2^*\cdot\boldsymbol{p}_2)]\}\\ &\times v_{\boldsymbol{q}}^{(+)}(\boldsymbol{r}_2)\,. \end{aligned} \tag{4}$$

Here $\boldsymbol{p}_1 = -i\partial/\partial\boldsymbol{r}_1$, and $\boldsymbol{p}_2 = -i\partial/\partial\boldsymbol{r}_2$. We first calculate the term $M^{(1)}$ and then find $M^{(2)}$ by means of the C-parity transformation.

As shown in Ref. [19], the wave functions and the Green's function can be represented in the form

$$\begin{aligned} \bar{u}_{\boldsymbol{p}}^{(-)}(\boldsymbol{r}_1) &= \bar{u}_{\boldsymbol{p}}[f_0(\boldsymbol{r}_1,\boldsymbol{p}) - \boldsymbol{\alpha}\cdot\boldsymbol{f}_1(\boldsymbol{r}_1,\boldsymbol{p}) - \boldsymbol{\Sigma}\cdot\boldsymbol{f}_2(\boldsymbol{r}_1,\boldsymbol{p})]\,,\\ v_{\boldsymbol{q}}^{(+)}(\boldsymbol{r}_2) &= [g_0(\boldsymbol{r}_2,\boldsymbol{q}) + \boldsymbol{\alpha}\cdot\boldsymbol{g}_1(\boldsymbol{r}_2,\boldsymbol{q}) + \boldsymbol{\Sigma}\cdot\boldsymbol{g}_2(\boldsymbol{r}_1,\boldsymbol{q})]v_{\boldsymbol{q}}\,, \end{aligned} \tag{5}$$

$$D(\boldsymbol{r}_1,\boldsymbol{r}_2|\varepsilon) = [d_0(\boldsymbol{r}_1,\boldsymbol{r}_2) + \boldsymbol{\alpha}\cdot\boldsymbol{d}_1(\boldsymbol{r}_1,\boldsymbol{r}_2) + \boldsymbol{\Sigma}\cdot\boldsymbol{d}_2(\boldsymbol{r}_1,\boldsymbol{r}_2)], \tag{6}$$

where

$$\begin{aligned} u_{\boldsymbol{p}} &= \sqrt{\frac{\varepsilon_p+m}{2\varepsilon_p}}\begin{pmatrix}\phi\\ \dfrac{\boldsymbol{\sigma}\cdot\boldsymbol{p}}{\varepsilon_p+m}\phi\end{pmatrix},\\ v_{\boldsymbol{q}} &= \sqrt{\frac{\varepsilon_q+m}{2\varepsilon_q}}\begin{pmatrix}\dfrac{\boldsymbol{\sigma}\cdot\boldsymbol{q}}{\varepsilon_q+m}\chi\\ \chi\end{pmatrix}, \end{aligned} \tag{7}$$

$\boldsymbol{\alpha} = \gamma^0\boldsymbol{\gamma}$; $\boldsymbol{\Sigma} = \gamma^0\gamma^5\boldsymbol{\gamma}$; $\gamma^5 = -i\gamma^0\gamma^1\gamma^2\gamma^3$; f_0, $\boldsymbol{f}_{1,2}$, g_0, $\boldsymbol{g}_{1,2}$, d_0, and $\boldsymbol{d}_{1,2}$ are some functions; and ϕ and χ are spinors. In the quasiclassical approximation the relative magnitude of these functions is different, so that

$$f_0 \sim l_c f_1 \sim l_c^2 f_2\,,\quad g_0 \sim l_c g_1 \sim l_c^2 g_2\,,\quad d_0 \sim l_c d_1 \sim l_c^2 d_2\,, \tag{8}$$

where $l_c \sim \omega/\Delta \gg 1$ is the characteristic value of the angular momentum in the process and $\boldsymbol{\Delta} = \boldsymbol{p}+\boldsymbol{q}+\boldsymbol{k}_2-\boldsymbol{k}_1$ is the momentum transfer. Nevertheless, it appears that, due to cancellations in the matrix element M, it is necessary to keep not only the leading terms f_0, g_0, d_0 but also the subleading terms $\boldsymbol{f}_1$, $\boldsymbol{g}_1$, $\boldsymbol{d}_1$, while the terms $\boldsymbol{f}_2$, $\boldsymbol{g}_2$, $\boldsymbol{d}_2$ can be safely omitted in the leading approximation. Thus, we can write the term $M^{(1)}$ as follows:

$$\begin{aligned} M^{(1)} = &-\int d\boldsymbol{r}_1\, d\boldsymbol{r}_2\, \mathrm{Sp}\{(f_0 - \boldsymbol{\alpha}\cdot\boldsymbol{f}_1)\\ &\times[(\boldsymbol{\gamma}\cdot\boldsymbol{e}_2^*)\hat{k}_2 + 2(\boldsymbol{e}_2^*\cdot\boldsymbol{p}_1)]e^{-i\boldsymbol{k}_2\cdot\boldsymbol{r}_1}\\ &\times(d_0 + \boldsymbol{\alpha}\cdot\boldsymbol{d}_1)e^{i\boldsymbol{k}_1\cdot\boldsymbol{r}_2}(\boldsymbol{\gamma}\cdot\boldsymbol{e}_1)(g_0 + \boldsymbol{\alpha}\cdot\boldsymbol{g}_1)v_{\boldsymbol{q}}\bar{u}_{\boldsymbol{p}}\}\,. \end{aligned} \tag{9}$$

In what follows we calculate the matrix element for definite helicities of the particles. Let λ_1 , λ_2 , μ_p, and μ_q be the signs of the helicities of the initial photon, final photon, electron, and positron, respectively. Denoting helicities by the subscripts, we have

$$v_{\boldsymbol{q}\mu_q}\bar{u}_{\boldsymbol{p}\mu_p} = \frac{1}{8}\big(a_{\mu_p\mu_q} + \boldsymbol{\Sigma}\cdot\boldsymbol{b}_{\mu_p\mu_q}\big)[\gamma^0(Q+P) + \gamma^0\gamma^5(1+PQ) - (P-Q) - \gamma^5(1-PQ)],$$
$$P = \frac{\mu_p p}{\varepsilon_p + m}, \quad Q = -\frac{\mu_q q}{\varepsilon_q + m}, \tag{10}$$

where $a_{\mu_p\mu_q}$ and $\boldsymbol{b}_{\mu_p\mu_q}$ are defined from

$$\chi_{\mu_q}\phi^{\dagger}_{\mu_p} = \tfrac{1}{2}\big(a_{\mu_p\mu_q} + \boldsymbol{\sigma}\cdot\boldsymbol{b}_{\mu_p\mu_q}\big)\,. \tag{11}$$

Note that only the terms with $(P+Q)$ and $(1+PQ)$ in Eq. (10) contribute to the matrix element (9) because it contains the odd number of the γ matrices.

Let us fix the overall phase of the helicity amplitudes by choosing

$$\phi_{\mu_p} = \frac{1+\mu_p\boldsymbol{\sigma}\cdot\boldsymbol{n}_p}{4\cos(\theta_p/2)}\begin{pmatrix}1+\mu_p\\1-\mu_p\end{pmatrix} \approx \frac{1}{4}\left(1+\frac{\theta_p^2}{8}\right)(1+\mu_p\boldsymbol{\sigma}\cdot\boldsymbol{n}_p)\begin{pmatrix}1+\mu_p\\1-\mu_p\end{pmatrix},$$
$$\chi_{\mu_q} = -\frac{1-\mu_q\boldsymbol{\sigma}\cdot\boldsymbol{n}_q}{4\cos(\theta_q/2)}\begin{pmatrix}\mu_q-1\\\mu_q+1\end{pmatrix} \approx -\frac{1}{4}\left(1+\frac{\theta_q^2}{8}\right)(1-\mu_q\boldsymbol{\sigma}\cdot\boldsymbol{n}_q)\begin{pmatrix}\mu_q-1\\\mu_q+1\end{pmatrix}, \tag{12}$$
$$\boldsymbol{e}_{1\lambda_1} = \boldsymbol{e}_{\lambda_1} = \frac{1}{\sqrt{2}}(\boldsymbol{e}_x + i\lambda_1\boldsymbol{e}_y)\,, \quad \boldsymbol{e}_{2\lambda_2} = \frac{1}{\sqrt{2}}(\boldsymbol{e}'_x + i\lambda_2\boldsymbol{e}_y) \approx \frac{1}{\sqrt{2}}\big(\boldsymbol{e}_x + i\lambda_2\boldsymbol{e}_y - \theta_{k_2}\boldsymbol{\nu}\big)\,,$$

where θ_p, θ_q, and θ_{k_2} are the polar angles of the vectors $\boldsymbol{p}$, $\boldsymbol{q}$, and $\boldsymbol{k}_2$. Within our approximation it is convenient to introduce the vectors $\boldsymbol{\theta}_p = \boldsymbol{p}_\perp/p$ an so on. We recall that the orts $\boldsymbol{e}_x$ and $\boldsymbol{e}_y$ are directed along $\boldsymbol{k}_{2\perp}$ and $\boldsymbol{k}_1\times\boldsymbol{k}_2$, respectively.

Using Eq. (12), we obtain

$$a_{+-} = 1 - \frac{\theta_{pq}^2}{8} + \frac{i}{4}\boldsymbol{\nu}\cdot[\boldsymbol{\theta}_p\times\boldsymbol{\theta}_q]\,, \quad a_{-+} = -1 + \frac{\theta_{pq}^2}{8} + \frac{i}{4}\boldsymbol{\nu}\cdot[\boldsymbol{\theta}_p\times\boldsymbol{\theta}_q]\,,$$
$$a_{++} = -\frac{1}{\sqrt{2}}\boldsymbol{e}_-\cdot\boldsymbol{\theta}_{pq}\,, \quad a_{--} = -\frac{1}{\sqrt{2}}\boldsymbol{e}_+\cdot\boldsymbol{\theta}_{pq}\,,$$
$$\boldsymbol{b}_{+-} = \left[1 - \frac{1}{8}(\boldsymbol{\theta}_p+\boldsymbol{\theta}_q)^2 - \frac{i}{4}\boldsymbol{\nu}\cdot[\boldsymbol{\theta}_p\times\boldsymbol{\theta}_q]\right]\boldsymbol{\nu} + \frac{1}{2}(\boldsymbol{\theta}_p+\boldsymbol{\theta}_q) + \frac{i}{2}[\boldsymbol{\nu}\times\boldsymbol{\theta}_{pq}]\,, \tag{13}$$
$$\boldsymbol{b}_{-+} = \left[1 - \frac{1}{8}(\boldsymbol{\theta}_p+\boldsymbol{\theta}_q)^2 + \frac{i}{4}\boldsymbol{\nu}\cdot[\boldsymbol{\theta}_p\times\boldsymbol{\theta}_q]\right]\boldsymbol{\nu} + \frac{1}{2}(\boldsymbol{\theta}_p+\boldsymbol{\theta}_q) - \frac{i}{2}[\boldsymbol{\nu}\times\boldsymbol{\theta}_{pq}]\,,$$
$$\boldsymbol{b}_{++} = \frac{1}{\sqrt{2}}(\boldsymbol{e}_-,\boldsymbol{\theta}_p+\boldsymbol{\theta}_q)\boldsymbol{\nu} - \sqrt{2}\boldsymbol{e}_-\,, \quad \boldsymbol{b}_{--} = -\frac{1}{\sqrt{2}}(\boldsymbol{e}_+,\boldsymbol{\theta}_p+\boldsymbol{\theta}_q)\boldsymbol{\nu} + \sqrt{2}\boldsymbol{e}_+\,,$$

where $\boldsymbol{\theta}_{pq} = \boldsymbol{\theta}_p - \boldsymbol{\theta}_q$.

The main contribution to the integrals in Eq. (9) is given by $r_{1,2}\sim\omega_1/m^2$ and by the impact parameters $r_{1\perp}\sim r_{2\perp}\sim 1/\Delta$. If $\Delta\gg m^2/\omega_1$, then the angle between vectors $-\boldsymbol{r}_2$ and $\boldsymbol{k}_1$ is small. The angle between vectors $\boldsymbol{r}_1$ and $\boldsymbol{k}_1$ may be either small or close to π, and we will call $M^{(1,1)}$ and $M^{(1,2)}$ the corresponding contributions to $M^{(1)} = M^{(1,1)} + M^{(1,2)}$.

For a small angle between vectors $\boldsymbol{r}_1$ and $\boldsymbol{k}_1$ one can use the quasiclassical form of the Green's function $D(\boldsymbol{r}_1,\boldsymbol{r}_2|\varepsilon_p+\omega_2)$ of the squared Dirac equation in the Coulomb field [14]:

$$D(\boldsymbol{r}_1,\boldsymbol{r}_2|\varepsilon) = \frac{i\kappa}{8\pi^2 r_1r_2}e^{i\kappa(r_1+r_2)}\int d\boldsymbol{s}\,\exp\left\{i\kappa\left[\frac{(r_1+r_2)}{2r_1r_2}s^2 + \boldsymbol{s}\cdot\boldsymbol{\theta}_{12}\right]\right\}\left(\frac{s^2}{4r_1r_2}\right)^{-i\eta}\left[1 - \frac{1}{2}\boldsymbol{\alpha}\cdot\left(\frac{r_1+r_2}{r_1r_2}\boldsymbol{s} + \boldsymbol{\theta}_{12}\right)\right], \tag{14}$$

where $\kappa = \sqrt{\varepsilon^2 - m^2}$, $\boldsymbol{s}$ is the two-dimensional vector in the plane perpendicular to $\boldsymbol{r}_1-\boldsymbol{r}_2$, and $\boldsymbol{\theta}_{12} = \boldsymbol{r}_1/r_1 + \boldsymbol{r}_2/r_2$. We can also use the quasiclassical form of the wave function $v^{(+)}_{\boldsymbol{q}}(\boldsymbol{r}_2)$ and the eikonal form of the wave function $\bar{u}^{(-)}_{\boldsymbol{p}}(\boldsymbol{r}_1)$:

$$v^{(+)}_{\boldsymbol{q}}(\boldsymbol{r}_2) = \frac{q}{2i\pi r_2}e^{iqr_2}\int d\boldsymbol{\tau}\,\exp\left[iq\left(\frac{\tau^2}{2r_2} + \boldsymbol{\tau}\cdot\boldsymbol{\theta}_{2q}\right)\right]\left(\frac{q\tau^2}{4r_2}\right)^{i\eta}\left[1 + \frac{1}{2}\boldsymbol{\alpha}\cdot\left(\frac{\boldsymbol{\tau}}{r_2} + \boldsymbol{\theta}_{2q}\right)\right]v_{\boldsymbol{q}},$$
$$\bar{u}^{(-)}_{\boldsymbol{p}}(\boldsymbol{r}_1) = \bar{u}_{\boldsymbol{p}}e^{-i\boldsymbol{p}\cdot\boldsymbol{r}_1}(pr_1)^{-i\eta}. \tag{15}$$

Here $\boldsymbol{\theta}_{2q} = -\boldsymbol{r}_2/r_2 - \boldsymbol{q}/q$, and $\boldsymbol{\tau}$ is the two-dimensional vector in the plane perpendicular to $\boldsymbol{q}$. For a small angle between vectors $-\boldsymbol{r}_1$ and $\boldsymbol{k}_1$ one can use the eikonal form of the Green's function $D(\boldsymbol{r}_1,\boldsymbol{r}_2|\varepsilon_p+\omega_2)$ and the quasiclassical form of the wave function $\bar{u}_{\boldsymbol{p}}^{(-)}(\boldsymbol{r}_1)$ [14]:

$$D(\boldsymbol{r}_1,\boldsymbol{r}_2|\varepsilon) = -\frac{1}{4\pi r_{12}}e^{i\kappa r_{12}}\left(\frac{r_2}{r_1}\right)^{i\eta}, \quad r_2 > r_1\,, \quad r_{12} = |\boldsymbol{r}_1 - \boldsymbol{r}_2|\,,$$
$$\bar{u}_{\boldsymbol{p}}^{(-)}(\boldsymbol{r}_1) = \frac{p}{2i\pi r_1}e^{ipr_1}\bar{u}_{\boldsymbol{p}}\int d\boldsymbol{s}\,\exp\left[ip\left(\frac{s^2}{2r_1}+\boldsymbol{s}\cdot\boldsymbol{\theta}_{1p}\right)\right]\left(\frac{ps^2}{4r_1}\right)^{-i\eta}\left[1-\frac{1}{2}\boldsymbol{\alpha}\cdot\left(\frac{\boldsymbol{s}}{r_1}+\boldsymbol{\theta}_{1p}\right)\right], \tag{16}$$

where $\boldsymbol{s}$ is the two-dimensional vector in the plane perpendicular to $\boldsymbol{p}$ and $\boldsymbol{\theta}_{1p} = -\boldsymbol{r}_1/r_1 - \boldsymbol{p}/p$. The quasiclassical wave functions in (15) and (16) are the integral representations of the Furry-Sommerfeld-Maue wave functions [23,24] (see also [25]). The simplest way to derive this integral representations is to use the relation between the wave functions and the Green's function $D(\boldsymbol{r}_1,\boldsymbol{r}_2|\varepsilon)$ (see [19]).

III. CALCULATION OF THE MATRIX ELEMENT

To calculate the matrix element (9) at $\Delta \gg \Delta_{\min} = m^2(\varepsilon_p+\varepsilon_q)/2\varepsilon_p\varepsilon_q$ we substitute the wave functions and the Green's function into Eq. (9), take the trace, perform the expansion of the integrand in the phase and in the preexponent with respect to small angles, taking into account the leading terms, and then take the integrals over $\boldsymbol{\theta}_{1p}$ and $\boldsymbol{\theta}_{2q}$. These integrals have the form $\int d\boldsymbol{\theta}\,\exp(ia\boldsymbol{\theta}^2 + i\boldsymbol{b}\cdot\boldsymbol{\theta})$, where a and $\boldsymbol{b}$ are some real constants and can be easily taken. Note that, within our accuracy, $\boldsymbol{s}$, $\boldsymbol{\tau}$, $\boldsymbol{\theta}_{12}$, $\boldsymbol{\theta}_{1p}$, and $\boldsymbol{\theta}_{2q}$ are perpendicular to $\boldsymbol{k}_1$. Then we pass from the variables $\boldsymbol{s}$ and $\boldsymbol{\tau}$ (in the integral representation of the quasiclassical Green's function and the quasiclassical wave functions or two quasiclassical wave functions) to the variables $\boldsymbol{T} = \boldsymbol{\tau}+\boldsymbol{s}$ and $\boldsymbol{\xi} = \boldsymbol{\tau}-\boldsymbol{s}$. After that both contributions $M_1^{(1,1)}$ and $M^{(1,2)}$ have the form

$$M^{(1,i)}_{\lambda_1\lambda_2\mu_p\mu_q} = \int_0^\infty dr_2\int_0^{L_i}dr_1\int d\boldsymbol{T}\int d\boldsymbol{\xi}\left(\frac{|\boldsymbol{T}+\boldsymbol{\xi}|}{|\boldsymbol{T}-\boldsymbol{\xi}|}\right)^{2i\eta}\exp\left[-\frac{i}{2}\boldsymbol{T}\cdot\boldsymbol{\Delta}_\perp\right]\mathcal{G}_i(r_1,r_2,\boldsymbol{\xi})\,, \tag{17}$$

where $\mathcal{G}_{1,2}(r_1,r_2,\boldsymbol{\xi})$ are some functions, $L_1=\infty$, and $L_2 = r_2$. To perform further integration we use the transformation [26]

$$\begin{aligned}&\int d\boldsymbol{T}\left(\frac{|\boldsymbol{T}+\boldsymbol{\xi}|}{|\boldsymbol{T}-\boldsymbol{\xi}|}\right)^{2i\eta}\exp\left[-\frac{i}{2}\boldsymbol{T}\cdot\boldsymbol{\Delta}_\perp\right]\\
&=\int d\boldsymbol{T}\left(\frac{|\boldsymbol{T}+\boldsymbol{\Delta}_\perp|}{|\boldsymbol{T}-\boldsymbol{\Delta}_\perp|}\right)^{2i\eta}\exp\left[-\frac{i}{2}\boldsymbol{T}\cdot\boldsymbol{\xi}\right]\frac{\xi^2}{\Delta_\perp^2} = -\frac{4}{\Delta_\perp^2}\int d\boldsymbol{T}\left(\frac{|\boldsymbol{T}+\boldsymbol{\Delta}_\perp|}{|\boldsymbol{T}-\boldsymbol{\Delta}_\perp|}\right)^{2i\eta}\nabla_{\boldsymbol{T}}^2\exp\left[-\frac{i}{2}\boldsymbol{T}\cdot\boldsymbol{\xi}\right]\\
&=\frac{8i\eta}{\Delta_\perp^2}\int d\boldsymbol{T}\left(\frac{|\boldsymbol{T}+\boldsymbol{\Delta}_\perp|}{|\boldsymbol{T}-\boldsymbol{\Delta}_\perp|}\right)^{2i\eta}\boldsymbol{\chi}\cdot\nabla_{\boldsymbol{T}}\exp\left[-\frac{i}{2}\boldsymbol{T}\cdot\boldsymbol{\xi}\right],\\
&\boldsymbol{\chi} = \frac{\boldsymbol{T}+\boldsymbol{\Delta}_\perp}{(\boldsymbol{T}+\boldsymbol{\Delta}_\perp)^2} - \frac{\boldsymbol{T}-\boldsymbol{\Delta}_\perp}{(\boldsymbol{T}-\boldsymbol{\Delta}_\perp)^2}\,.\end{aligned} \tag{18}$$

After this transformation the integrals over $\boldsymbol{\xi}$, r_1, and r_2 can be easily taken, and we obtain for the total amplitude $M_{\lambda_1\lambda_2\mu_p\mu_q} = M^{(1)}_{\lambda_1\lambda_2\mu_p\mu_q} + M^{(2)}_{\lambda_1\lambda_2\mu_p\mu_q}$:

$$M_{\lambda_1\lambda_2\mu_p\mu_q} = \frac{32\eta}{\omega_1\omega_2\Delta^2}\int d\boldsymbol{T}\left(\frac{|\boldsymbol{T}+\boldsymbol{\Delta}_\perp|}{|\boldsymbol{T}-\boldsymbol{\Delta}_\perp|}\right)^{2i\eta}\boldsymbol{\chi}\cdot\nabla_{\boldsymbol{T}}\mathcal{F}_{\lambda_1\lambda_2\mu_p\mu_q}(\boldsymbol{T})\,,$$
$$\mathcal{F}_{\lambda_1\lambda_2\mu_p\mu_q}(\boldsymbol{T}) = F_{\lambda_1\lambda_2\mu_p\mu_q}(\boldsymbol{p},\boldsymbol{q},\boldsymbol{T}) - F_{\lambda_1\lambda_2\mu_q\mu_p}(\boldsymbol{q},\boldsymbol{p},-\boldsymbol{T}), \tag{19}$$

where $\boldsymbol{\chi}$ is defined in (18) and the functions $F_{\lambda_1\lambda_2\mu_p\mu_q}(\boldsymbol{p},\boldsymbol{q},\boldsymbol{T})$ are

$$\begin{aligned}&F_{+++-} = -(\varepsilon_p+\omega_2)^2\boldsymbol{e}_+\cdot(\boldsymbol{T}-\boldsymbol{\delta}_q)(\boldsymbol{e}_-\cdot\boldsymbol{A}), \quad F_{+-+-} = -\varepsilon_p(\varepsilon_p+\omega_2)\boldsymbol{e}_+\cdot(\boldsymbol{T}-\boldsymbol{\delta}_q)(\boldsymbol{e}_+\cdot\boldsymbol{A})\,,\\
&F_{++-+} = \varepsilon_p\varepsilon_q\boldsymbol{e}_+\cdot(\boldsymbol{T}-\boldsymbol{\delta}_q)(\boldsymbol{e}_-\cdot\boldsymbol{A}) + 2m^2\omega_1\omega_2B - \frac{\varepsilon_q\varepsilon_p\omega_1\omega_2}{2(\varepsilon_p+\omega_2)D_2}\,,\\
&F_{+--+} = \varepsilon_q(\varepsilon_p+\omega_2)\boldsymbol{e}_+\cdot(\boldsymbol{T}-\boldsymbol{\delta}_q)(\boldsymbol{e}_+\cdot\boldsymbol{A})\,, \quad F_{++++} = \sqrt{2}m(\varepsilon_p+\omega_2)\omega_1(\boldsymbol{e}_-\cdot\boldsymbol{A})\,,\\
&F_{++--} = -\sqrt{2}m(\varepsilon_p+\omega_2)\omega_2\boldsymbol{e}_+\cdot(\boldsymbol{T}-\boldsymbol{\delta}_q)B\,,\\
&F_{+-++} = \sqrt{2}m\varepsilon_p\omega_1(\boldsymbol{e}_+\cdot\boldsymbol{A}) - \sqrt{2}m\varepsilon_q\omega_2\boldsymbol{e}_+\cdot(\boldsymbol{T}-\boldsymbol{\delta}_q)B, \quad F_{+---} = 0\,, \quad F_{\lambda_1\lambda_2\mu_p\mu_q} = -\mu_p\mu_q\left(F_{\bar{\lambda}_1\bar{\lambda}_2\bar{\mu}_p\bar{\mu}_q}\right)^*.\end{aligned} \tag{20}$$

Here $\overline{\mu}_{p,q} = -\mu_{p,q}$, $\overline{\lambda}_{1,2} = -\lambda_{1,2}$, and

$$A = \frac{1}{D_1}\left[\frac{\varepsilon_p \boldsymbol{\theta}_{pk_2}}{2\left(m^2+\varepsilon_p^2\boldsymbol{\theta}_{pk_2}^2\right)} + \frac{\omega_2\varepsilon_q(\boldsymbol{T}+\boldsymbol{\Delta}_\perp - 2\varepsilon_p\boldsymbol{\theta}_{pk_2})}{(\varepsilon_p+\omega_2)D_2}\right],$$
$$B = \frac{1}{D_1}\left[\frac{1}{4\left(m^2+\varepsilon_p^2\boldsymbol{\theta}_{pk_2}^2\right)} - \frac{\omega_2\varepsilon_q}{(\varepsilon_p+\omega_2)D_2}\right],$$
$$D_1 = 4m^2 + (\boldsymbol{T}-\boldsymbol{\delta}_q)^2\,,\quad \boldsymbol{\delta}_q = \boldsymbol{\Delta}_\perp - 2\varepsilon_q\boldsymbol{\theta}_q\,,\quad \boldsymbol{\theta}_{pk_2} = \boldsymbol{\theta}_p - \boldsymbol{\theta}_{k_2}, \tag{21}$$

$$D_2 = \frac{4\omega_1\omega_2\varepsilon_p\varepsilon_q}{\varepsilon_p+\varepsilon_q}\boldsymbol{\theta}_{k_2}^2 + (\varepsilon_p+\varepsilon_q)\left[\left(\boldsymbol{T} - \varepsilon_p\boldsymbol{\theta}_p + \varepsilon_q\boldsymbol{\theta}_q - \frac{\varepsilon_p-\varepsilon_q}{\varepsilon_p+\varepsilon_q}\omega_2\boldsymbol{\theta}_{k_2}\right)^2 + 4m^2\right]. \tag{22}$$

In Eq. (19) we have omitted for convenience the inessential factor $(q/p)^{i\eta}$ and have replaced $\Delta_\perp^2$ by Δ^2 in the coefficient of Eq. (19). After such replacement Eq. (19) can be used not only at $\Delta_\perp \gg \Delta_z \sim m^2/\omega_1$ but also at $\Delta_\perp \sim \Delta_z$ (see [26]). We recall that $d\boldsymbol{T} = dT_x\, dT_y$.

In Ref. [27] the impact-factor approach has been suggested. In this approach contributions of higher-order terms with respect to the external field are accumulated in the so-called impact factors. It is not quite clear what the accuracy and the region of applicability of this approach are. On the other hand, the quasiclassical approach, used in our paper, is much more established and systematic since it allows one to calculate not only the leading term but also the next-to-leading quasiclassical corrections. Therefore, it is interesting to check the validity of the impact-factor approach for the process under consideration. We have derived the amplitudes of the process within the impact-factor approach and have obtained the result, which is in agreement with our result (19).

For $\omega_2 \ll p, q$ expression (19) is essentially simplified,

$$M_{\lambda_1\lambda_2\mu_p\mu_q} = \frac{16\eta}{\omega_1\omega_2\Delta^2}\left[\frac{\varepsilon_p^2\left(\boldsymbol{e}_{\lambda_2}^*\cdot\boldsymbol{\theta}_p\right)}{m^2+\varepsilon_p^2\theta_p^2} - \frac{\varepsilon_q^2\left(\boldsymbol{e}_{\lambda_2}^*\cdot\boldsymbol{\theta}_q\right)}{m^2+\varepsilon_q^2\theta_q^2}\right]\int d\boldsymbol{T}\left(\frac{|\boldsymbol{T}+\boldsymbol{\Delta}_\perp|}{|\boldsymbol{T}-\boldsymbol{\Delta}_\perp|}\right)^{2i\eta}\boldsymbol{\chi}\cdot\nabla_{\boldsymbol{T}}\frac{1}{4m^2+(\boldsymbol{\delta}_0-\boldsymbol{T})^2}$$
$$\times\left[\delta_{\mu_p,-\mu_q}\left(\varepsilon_p\delta_{\mu_p,\lambda_1} - \varepsilon_q\delta_{\mu_q,\lambda_1}\right)\left(\boldsymbol{e}_{\lambda_1},\boldsymbol{\delta}_0-\boldsymbol{T}\right) + \sqrt{2}m\omega_1\lambda_1\delta_{\mu_q,\lambda_1}\delta_{\mu_p,\lambda_1}\right], \tag{23}$$

where $\boldsymbol{\delta}_0 = \varepsilon_p\boldsymbol{\theta}_p - \varepsilon_q\boldsymbol{\theta}_q$. This result can also be obtained directly within the soft-photon-emission approximation [25].

IV. BORN AMPLITUDES AND COULOMB CORRECTIONS

Let us represent the amplitude M as

$$M = M^B + M^C\,, \tag{24}$$

where M^B is linear in the η term (Born amplitude) and M^C is the contribution of the higher-order terms (Coulomb corrections). In order to find the Born term we omit the factor $(|\boldsymbol{T}+\boldsymbol{\Delta}_\perp|/|\boldsymbol{T}-\boldsymbol{\Delta}_\perp|)^{2i\eta}$ and perform the integration by parts using the relation

$$\nabla_{\boldsymbol{T}}\cdot\boldsymbol{\chi} = 2\pi[\delta(\boldsymbol{T}+\boldsymbol{\Delta}_\perp) - \delta(\boldsymbol{T}-\boldsymbol{\Delta}_\perp)]\,. \tag{25}$$

As a result we obtain

$$M^B_{\lambda_1\lambda_2\mu_p\mu_q} = \frac{64\pi\eta}{\omega_1\omega_2\Delta^2}\left[\mathcal{F}_{\lambda_1\lambda_2\mu_p\mu_q}(\boldsymbol{\Delta}_\perp) - \mathcal{F}_{\lambda_1\lambda_2\mu_p\mu_q}(-\boldsymbol{\Delta}_\perp)\right]. \tag{26}$$

In order to derive the explicit expression for the Coulomb corrections we write

$$\boldsymbol{\chi}\cdot\nabla_{\boldsymbol{T}}\mathcal{F}(\boldsymbol{T}) = \frac{(\boldsymbol{T}+\boldsymbol{\Delta}_\perp)}{(\boldsymbol{T}+\boldsymbol{\Delta}_\perp)^2}\cdot\nabla_{\boldsymbol{T}}[\mathcal{F}(\boldsymbol{T})-\mathcal{F}(-\boldsymbol{\Delta}_\perp)] - \frac{(\boldsymbol{T}-\boldsymbol{\Delta}_\perp)}{(\boldsymbol{T}-\boldsymbol{\Delta}_\perp)^2}\cdot\nabla_{\boldsymbol{T}}[\mathcal{F}(\boldsymbol{T})-\mathcal{F}(\boldsymbol{\Delta}_\perp)] \tag{27}$$

and perform integration by parts over $\boldsymbol{T}$ in Eq. (19). The surface term gives the Born amplitude (26), and the Coulomb corrections read

$$M^C_{\lambda_1\lambda_2\mu_p\mu_q} = -\frac{128i\eta^2}{\omega_1\omega_2\Delta^2}\int\frac{d\boldsymbol{T}}{(\boldsymbol{T}+\boldsymbol{\Delta}_\perp)^2(\boldsymbol{T}-\boldsymbol{\Delta}_\perp)^2}\left(\frac{|\boldsymbol{T}+\boldsymbol{\Delta}_\perp|}{|\boldsymbol{T}-\boldsymbol{\Delta}_\perp|}\right)^{2i\eta}\left\{(\boldsymbol{\Delta}_\perp^2+\boldsymbol{T}\cdot\boldsymbol{\Delta}_\perp)\left[\mathcal{F}_{\lambda_1\lambda_2\mu_p\mu_q}(\boldsymbol{T}) - \mathcal{F}_{\lambda_1\lambda_2\mu_p\mu_q}(\boldsymbol{\Delta}_\perp)\right]\right.$$
$$\left.+(\boldsymbol{\Delta}_\perp^2-\boldsymbol{T}\cdot\boldsymbol{\Delta}_\perp)\left[\mathcal{F}_{\lambda_1\lambda_2\mu_p\mu_q}(\boldsymbol{T}) - \mathcal{F}_{\lambda_1\lambda_2\mu_p\mu_q}(-\boldsymbol{\Delta}_\perp)\right]\right\}. \tag{28}$$

Note that it is possible to reduce expression (28) to a onefold integral using the trick from Ref. [28]. However, the resulting formulas are very cumbersome, and we do not present them here.

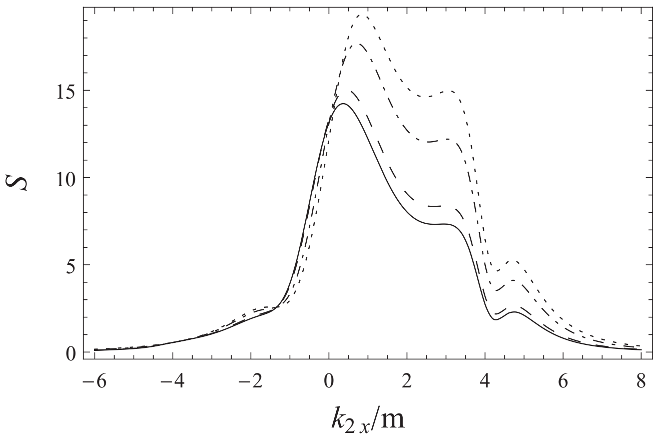

FIG. 2. The quantity S [the differential cross section in units of σ_0, averaged over the polarization of the initial photon and summed over polarizations of the final particles; see Eq. (29)] as a function of k_{2x}/m for $\varepsilon_p = 0.4\omega_1$, $\varepsilon_q = 0.25\omega_1$, $p_x = 4.7m$, $q_x = -0.8m$, $p_y = q_y = k_{2y} = 0$: Born result (dotted curve), $Z = 47$ (Ag, dash-dotted curve), $Z = 82$ (Pb, dashed curve), and $Z = 92$ (U, solid curve). The quantity S, calculated for the Coulomb field, is independent of ω_1. Accounting for screening, where the dependence on ω_1 still exists due to the atomic form factor, modifies S only in the narrow vicinity of the point $\Delta_\perp = 0$ ($k_{2x} = -3.9\,m$ here), where the Coulomb corrections are unimportant.

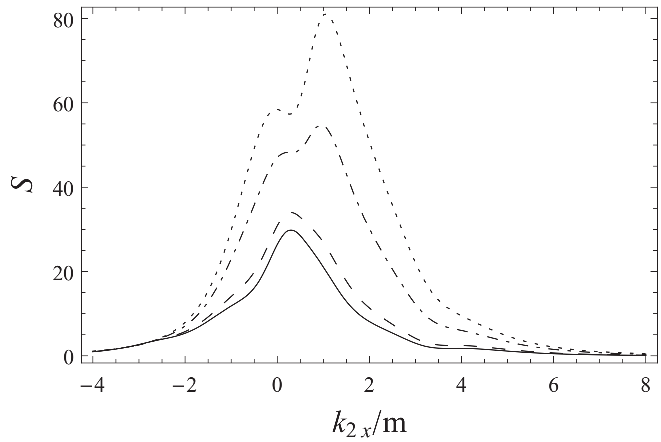

FIG. 3. Same as Fig. 2, but for $p_x = 0.7m$, $q_x = 2.33m$.

V. RESULTS AND DISCUSSION

Let us discuss the effect of screening. This effect is important only for small $\Delta \lesssim r_{\mathrm{scr}}^{-1} \ll m$, where $r_{\mathrm{scr}} \sim m\alpha Z^{1/3}$ is the screening radius. For such small Δ the amplitude (19) coincides with the Born amplitude at small Δ, where the effect of screening may be accounted for by multiplying the amplitude $M^B_{\lambda_1\lambda_2\mu_p\mu_q}$ by an atomic form factor $[1 - F_e(\Delta^2)]$. This form factor vanishes at $\Delta = 0$ and tends to unity at $\Delta \to \infty$. A simple parametrization of this form factor can be found in Ref. [29]. Thus, if we multiply the amplitude (19) for the case of a pure Coulomb field by the atomic form factor $[1 - F_e(\Delta^2)]$, we obtain a result which is valid in the atomic field for any Δ.

In order to demonstrate the importance of the Coulomb corrections in the process, we plot in Figs. 2 and 3 the quantity S (the differential cross section in units of σ_0, averaged over the polarization of the initial photon and summed over polarizations of the final particles),

$$S = \frac{1}{2}\sum_{\lambda_1\lambda_2\mu_p\mu_q} \frac{\sigma_0^{-1} d\sigma_{\lambda_1\lambda_2\mu_p\mu_q}}{d\boldsymbol{p}_\perp\, d\boldsymbol{q}_\perp\, d\boldsymbol{k}_{2\perp} d\varepsilon_p d\varepsilon_q},$$
$$\sigma_0 = \frac{\alpha^2\eta^2\Delta_\perp^2}{(2\pi)^6 m^6 \omega_1\omega_2\Delta^4}, \tag{29}$$

as a function of k_{2x} at fixed $\boldsymbol{p}_\perp$, $\boldsymbol{q}_\perp$, ε_p, ε_q, $k_{2y} = 0$ and different values of the atomic charge number Z. For numerical calculations we used the twofold integral representation (28). In the vicinity of the point $\Delta_\perp = 0$ ($k_{2x} = -3.9m$ in Fig. 2 and $k_{2x} = -3.03m$ in Fig. 3), the Born result dominates over the Coulomb corrections, as it should. However, it is seen that, in general, the Coulomb corrections significantly modify the cross section. Note that the quantity S for the Coulomb field is independent of ω_1 for given values of ε_p/ω_1, ε_q/ω_1, $\boldsymbol{p}_\perp/m$, $\boldsymbol{q}_\perp/m$, and $\boldsymbol{k}_{2\perp}/m$. For the atomic field, the dependence on ω_1 still exists via the dependence of the atomic form factor on Δ. However, this form factor is important only in the narrow vicinity of the point $\Delta_\perp = 0$, where the Coulomb corrections are unimportant.

There is an interesting question about the asymmetry A in the differential cross section for a circularly polarized initial photon,

$$A = \frac{d\sigma_+ - d\sigma_-}{d\sigma_+ + d\sigma_-}, \quad d\sigma_\pm = \sum_{\lambda_2\mu_p\mu_q} d\sigma_{\pm\lambda_2\mu_p\mu_q}. \tag{30}$$

In the Born approximation the asymmetry vanishes for any $\boldsymbol{p}$, $\boldsymbol{q}$, and $\boldsymbol{k}_2$. This fact follows from the relation

$$M^B_{\lambda_1\lambda_2\mu_p\mu_q} = -\mu_p\mu_q\big(M^B_{\bar\lambda_1\bar\lambda_2\bar\mu_p\bar\mu_q}\big)^*; \tag{31}$$

see Eqs. (20) and (26). However, for the Coulomb corrections this relation is not valid due to the complex factor $(\frac{|\boldsymbol{T}+\boldsymbol{\Delta}_\perp|}{|\boldsymbol{T}-\boldsymbol{\Delta}_\perp|})^{2i\eta}$ in the integrand in Eq. (28). In Figs. 4 and 5 the asymmetry

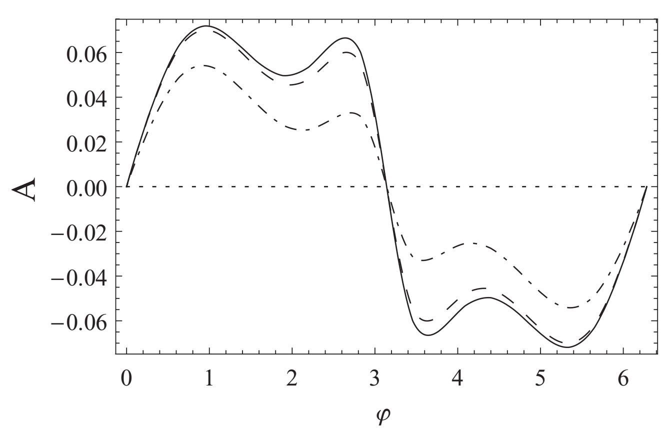

FIG. 4. Asymmetry A, Eq. (30), as a function of the angle φ between $\boldsymbol{k}_{2\perp}$ and $\boldsymbol{p}_\perp$ for $\varepsilon_p = 0.4\omega_1$, $\varepsilon_q = 0.25\omega_1$, $\boldsymbol{p}_\perp \parallel -\boldsymbol{q}_\perp$, $p_\perp = 4.7m$, $q_\perp = 0.8m$, $k_{2\perp} = m$: Born result (dotted curve), $Z = 47$ (Ag, dash-dotted curve), $Z = 82$ (Pb, dashed curve), and $Z = 92$ (U, solid curve).

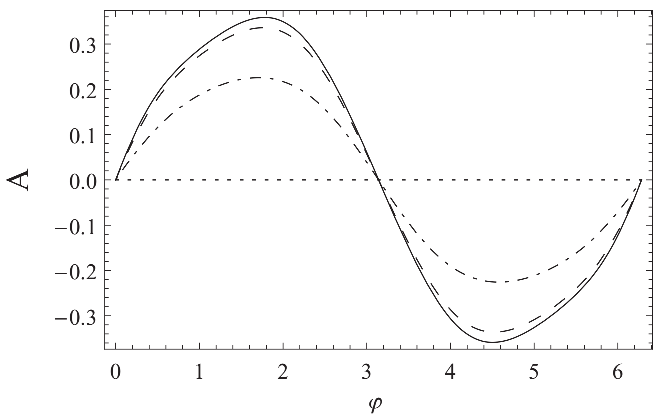

FIG. 5. Same as Fig. 4, but for $\boldsymbol{p}_\perp \parallel \boldsymbol{q}_\perp$, $p_\perp = 0.7m$, $q_\perp = 2.33m$.

is shown as a function of the angle φ between vectors $\boldsymbol{k}_{2\perp}$ and $\boldsymbol{p}_\perp$. As it should, the asymmetry vanishes when $\boldsymbol{k}_1$, $\boldsymbol{k}_2$, $\boldsymbol{p}$, and $\boldsymbol{q}$ lie in the same plane ($\varphi = 0,\pi$ in Figs. 4 and 5). It is seen that the asymmetry can reach tens of percent even for moderate values of Z.

VI. CONCLUSION

Using the quasiclassical approximation, we have derived exactly, in the parameter $\eta = Z\alpha$, the helicity amplitudes of e^+e^- photoproduction in the atomic field accompanied by bremsstrahlung. The results obtained, Eqs. (19), (26), and (28), have a compact form and are convenient for numerical calculations. They correspond to the leading high-energy small-angle asymptotics of the amplitude and have the relative uncertainty $\sim\max(\theta_p,\theta_q,\theta_{k_2},m/\omega_1)$. It is shown that, in general, accounting for the Coulomb corrections essentially modify the differential cross section, which is different from the Born result. Moreover, when the initial photon is circularly polarized, the Coulomb corrections lead to the asymmetry in the distribution over the azimuth angles φ_i of produced particles with respect to the replacement $\varphi_i \to -\varphi_i$, Eq. (30).

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