

**Scheme for entanglement generation in an atom-cavity system via dissipation**

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We propose a dissipative scheme to generate a maximally entangled state for two  $\Lambda$ -type atoms trapped in an optical cavity. Different from the unitary-dynamics-based scheme, our work shows that both atomic spontaneous emission and cavity decay are no longer detrimental, but necessary for entanglement generation. The scheme is independent of initial states and does not require precise time control. A final numerical simulation with two groups of experimental parameters indicates that the present scheme is feasible and the performance could be better than the unitary-dynamics-based scheme.

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Dissipation induced by the interaction between the system and the environment is inevitable and has been traditionally considered as a detrimental factor in quantum-information processing. However, recent theories and experiments show an interesting fact that the dissipation can be used as resources for quantum computation and entanglement generation [1–26]. Compared to the unitary-dynamics-based schemes, these dissipative ones have some common merits, such as robust against parameter fluctuations, do not require specifying the initial state and controlling the evolution time accurately. In particular, Kastoryano, Reitor, and Sørensen considered a dissipative scheme of state generation [1] in one optical cavity, whose results are better than that based on the unitary dynamics. Shen *et al.* generalized the idea of the authors of Ref. [1] to the coupled cavity system [16] and atom-cavity-fiber system [17]. In addition, Reitor *et al.* presented a scheme for the dissipative preparation of an entangled steady state of two superconducting qubits in a circuit quantum electrodynamics setup [24]. Moreover, Zheng *et al.* also proposed a scheme to generate maximal entanglement between two atomic qubits coupled to a decaying resonator [26]. In these schemes, cavity decay is no longer undesirable but exerts positive effects on entanglement generation. Nevertheless, the imperfections of these schemes would be imposed by atomic spontaneous emission. Differently, Shao *et al.* proposed a scheme to generate high-dimensional entanglement, in which atomic spontaneous emission plays a positive role [22].

As is well known, atomic spontaneous emission and cavity decay are two typical dissipative factors in atom-cavity systems. Thus, dissipative schemes utilize both atomic spontaneous emission and cavity decay simultaneously as

resources have unique advantages compared to that only utilize one out of the two factors. In Ref. [3], Busch *et al.* designed a dissipative scheme to generate the maximally entangled atomic ground state, in which both of the dissipative factors were used as resources. The main idea in their scheme is to use classical fields to resonantly drive the undesired states in zero excitation subspace to the states in single excitation subspace, which could decay to the desired state probably through the dissipative process. Since there are four eigenstates in the zero excitation subspace and only one of them is the desired state, three classical fields are required. In Ref. [25], based on the effective operator formalism [27], Sweke *et al.* showed that both dissipative factors can be utilized to generate a large  $W$  state. Since the desired state of Ref. [25] is the multiqubit entanglement, the requirement of the cavity parameters is higher than that in Refs. [1–3].

In this Brief Report, we get inspiration from Refs. [1–3] and design a dissipative scheme to generate the maximal entanglement of two  $\Lambda$ -type atoms in an optical cavity. After employing a microwave field that shuffles the undesired states in the zero excitation subspace, only one classical field is required to resonantly drive the undesired states to the well-defined state in the single excitation subspace, which would decay to the desired state in the zero excitation subspace via dissipation. This scheme actively exploits the atomic spontaneous emission and cavity decay as resources to generate stationary state with higher fidelity.

The present work has the following features. (i) It performs well without specifying the initial state and controlling evolution time accurately. In addition, it is robustness on parameter fluctuations. (ii) Different from the schemes shown in Refs. [1,16,17,22,24,26], the present one uses two dissipative factors as resources rather than one. (iii) Compared to the scheme proposed by the authors of Ref. [3], the present one is more simple and economic since it takes advantage of one

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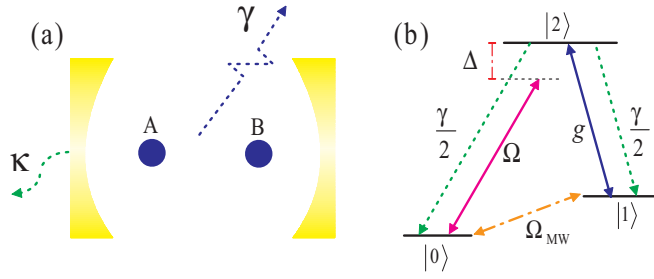


FIG. 1. (Color online) (a) The schematic of two  $\Lambda$ -type atoms A and B trapped into an optical cavity.  $\kappa$  and  $\gamma$  stand for the cavity decay rate and atomic spontaneous emission rate, respectively. (b) Energy level structure of the  $\Lambda$ -type atom. Here we have assumed the excited state  $|2\rangle$  spontaneously decays into two ground states with branching rate  $\gamma/2$ .

classical laser field as well as one microwave field rather than three classical laser fields. Besides, the roles of the dissipative process are more concise. Moreover, the Raman transition between any two classical laser fields could be avoided since there is only one classical laser field. (iv) Compared to the other atom-cavity-system-based schemes [28–31], the one proposed here does not require the detection of photons or quantum feedback operations.

Consider the setup described in Fig. 1, where two identical  $\Lambda$ -type atoms are trapped into a resonant optical cavity. Each atom has two ground states  $|0\rangle$  and  $|1\rangle$  and one excited state  $|2\rangle$  with the corresponding energies  $\omega_0$ ,  $\omega_1$ , and  $\omega_2$ , respectively. The cavity mode with frequency  $\omega_c$  is coupled to the transition  $|1\rangle \leftrightarrow |2\rangle$  resonantly with the coupling constant  $g$ . In addition, an off-resonance classical optical laser with detuning  $\Delta$ , Rabi frequency  $\Omega$  is applied to drive the transition  $|0\rangle \leftrightarrow |2\rangle$ . The two ground states  $|0\rangle$  and  $|1\rangle$  are coupled resonantly by means of a microwave field with Rabi frequency  $\Omega_{\text{MW}}$ . Then, the total system Hamiltonian is (assuming  $\hbar = 1$ )  $H = H_{\text{WL}} + H_{\text{CL}} + H_{\text{MW}}$ ,

$$H_{\text{WL}} = \omega_c c^\dagger c + \sum_{i=A,B} \left( g|2\rangle_{ii} \langle 1|_i c + \text{H.c.} + \sum_{j=0,1,2} \omega_j |j\rangle_{ii} \langle j| \right), \quad (1)$$

$$H_{\text{CL}} = \frac{\Omega}{2} \sum_{i=A,B} e^{i\omega t} |0\rangle_{ii} \langle 2| + \text{H.c.}, \quad (2)$$

$$H_{\text{MW}} = \frac{\Omega_{\text{MW}}}{2} e^{i\omega_{\text{MW}} t} (|0\rangle_{\text{AA}} \langle 1| + e^{i\phi} |0\rangle_{\text{BB}} \langle 1|) + \text{H.c.}, \quad (3)$$

in which  $c$  and  $c^\dagger$  denote the annihilation and creation operators for the optical cavity mode, respectively.  $\omega$  and  $\omega_{\text{MW}}$  in the exponentials denote the frequencies of the classical optical laser and microwave field, respectively.  $\phi$  means the phase difference of the microwave field for the two atoms.

Note that the excitation number operator of the total system  $\sum_{i=A,B} |2\rangle_{ii} \langle 2| + c^\dagger c$  commutes with Hamiltonian  $H_{\text{WL}}$  and  $H_{\text{MW}}$ , the excitation number is thus preserved under the control of these two Hamiltonians. Nevertheless, Hamiltonian  $H_{\text{CL}}$  would change the excitation number since it does not commute with the excitation number operator. When the parameters

TABLE I. The eigenstate and corresponding eigenenergy of the Hamiltonian  $H_{\text{WL}}$  in the zero excitation subspace. Here,  $|xy\rangle|z\rangle$  represents that atom A (B) is in the state  $|x\rangle$  ( $|y\rangle$ ) and cavity mode is in the state  $|z\rangle$ . For simplicity, we set  $\omega_0 = 0$  throughout this paper.

Eigenstate	Eigenenergy
$ 00\rangle 0\rangle$	0
$ S\rangle 0\rangle$	$\omega_1$
$ T\rangle 0\rangle$	$\omega_1$
$ 11\rangle 0\rangle$	$2\omega_1$

satisfy  $\Omega \ll \{g, \Delta\}$  and the initial state is in the zero excitation subspace, the probability that the system is excited to the subspace more than a single excitation can be neglected.

Here, we identify the relevant energy eigenstates of Hamiltonian  $H_{\text{WL}}$  and use it as dressed states to see clearly the roles of  $H_{\text{MW}}$  and  $H_{\text{CL}}$  after choosing appropriate laser driving and detuning. In Tables I and II, we show the eigenstates and the corresponding eigenvalues of zero and single excitation subspace with the notation

$$|S\rangle \equiv (|01\rangle - |10\rangle)/\sqrt{2},$$

$$|T\rangle \equiv (|01\rangle + |10\rangle)/\sqrt{2},$$

$$|S'_1\rangle \equiv (|21\rangle|0\rangle - |12\rangle|0\rangle)/\sqrt{2},$$

$$|T'_1{}^\pm\rangle \equiv (|21\rangle|0\rangle + |12\rangle|0\rangle \pm \sqrt{2}|11\rangle|1\rangle)/2,$$

$$|T_1{}^\pm\rangle \equiv (\pm|01\rangle|1\rangle \pm |10\rangle|1\rangle + |02\rangle|0\rangle + |20\rangle|0\rangle)/2,$$

$$|S_1{}^\pm\rangle \equiv (\mp|01\rangle|1\rangle \pm |10\rangle|1\rangle - |02\rangle|0\rangle + |20\rangle|0\rangle)/2, \quad (4)$$

in which  $|T\rangle$  is the desired state of our scheme. Since  $|T\rangle|0\rangle$  is a Kronecker product state of the desired state and the cavity mode vacuum state, and could be transformed to  $|T\rangle$  after performing a partial trace over the cavity mode degrees of freedom, the scheme would be considered successful if  $|T\rangle|0\rangle$  is generated. Hamiltonian  $H_{\text{MW}}$  can be rewritten as

$$\begin{aligned} H_{\text{MW}} &= \frac{1 + e^{i\phi}}{2\sqrt{2}} \Omega_{\text{MW}} e^{i\omega_{\text{MW}} t} (|T\rangle|0\rangle \langle 0| \langle 11| + |00\rangle|0\rangle \langle 0| \langle T|) \\ &+ \frac{1 - e^{i\phi}}{2\sqrt{2}} \Omega_{\text{MW}} e^{i\omega_{\text{MW}} t} (|S\rangle|0\rangle \langle 0| \langle 11| - |00\rangle|0\rangle \langle 0| \langle S|) \\ &+ \text{H.c.} \end{aligned} \quad (5)$$

TABLE II. The eigenstate and corresponding eigenenergy of the Hamiltonian  $H_{\text{WL}}$  in the single excitation subspace.

Eigenstate	Eigenenergy
$ 00\rangle 1\rangle$	$\omega_2 - \omega_1$
$ T_1^-\rangle,  S_1^-\rangle$	$\omega_2 - g$
$ T_1^+\rangle,  S_1^+\rangle$	$\omega_2 + g$
$ S'_1\rangle$	$\omega_2 + \omega_1$
$ T'_1{}^\pm\rangle$	$\omega_2 + \omega_1 \pm \sqrt{2}g$

under the dressed states. Setting  $\phi = \pi$  and moving to the interaction picture with respect to the Hamiltonian  $H_{WL}$ , Equation (5) can be re-expressed as

$$H_{MW} = \frac{1}{\sqrt{2}} \Omega_{MW} (|S\rangle|0\rangle\langle 0|\langle 11| - |00\rangle|0\rangle\langle 0|\langle S|) + \text{H.c.} \quad (6)$$

Similar to the above process,  $H_{CL}$  can be rewritten as

$$\begin{aligned} H_{CL} = & \frac{1}{2\sqrt{2}} \Omega e^{i(\omega - \omega_2 - \sqrt{2}g)t} |T\rangle|0\rangle\langle T_1^+| \\ & + \frac{1}{2\sqrt{2}} \Omega e^{i(\omega - \omega_2 + \sqrt{2}g)t} |T\rangle|0\rangle\langle T_1^-| \\ & + \frac{1}{2} \Omega e^{i(\omega - \omega_2)t} |S\rangle|0\rangle\langle S'_1| \\ & + \frac{1}{2} \Omega e^{i(\omega - \omega_2 + g)t} |00\rangle|0\rangle\langle T_1^-| \\ & + \frac{1}{2} \Omega e^{i(\omega - \omega_2 - g)t} |00\rangle|0\rangle\langle T_1^+| + \text{H.c.} \end{aligned} \quad (7)$$

That is, we can choose different values of  $\omega_2 - \omega$ , i.e.,  $\Delta$ , to realize resonant or nonresonant couplings between states in zero excitation subspace and single excitation subspace according to our requirements. For instance, if we set  $\Delta = -g$ ,  $|00\rangle|0\rangle$  would couple resonantly to  $|T_1^+\rangle$  while the other terms in Eq. (7) undergo nonresonant transitions.

Affected jointly by the dissipative factors mentioned above, states in single excitation subspace would undoubtedly decay to zero excitation subspace. Interestingly, it is easy to find that the first two terms of  $|T_1^+\rangle$  would be converted to  $|T\rangle|0\rangle$  through cavity decay. Meanwhile, the last two terms of  $|T_1^+\rangle$  would be translated to  $|T\rangle|0\rangle$  through atomic spontaneous emission  $|2\rangle \rightarrow |1\rangle$ . Therefore, to generate the desired state  $|T\rangle$  via dissipation, the other three states in the zero excitation subspace should be coupled to  $|T_1^+\rangle$  directly or indirectly.

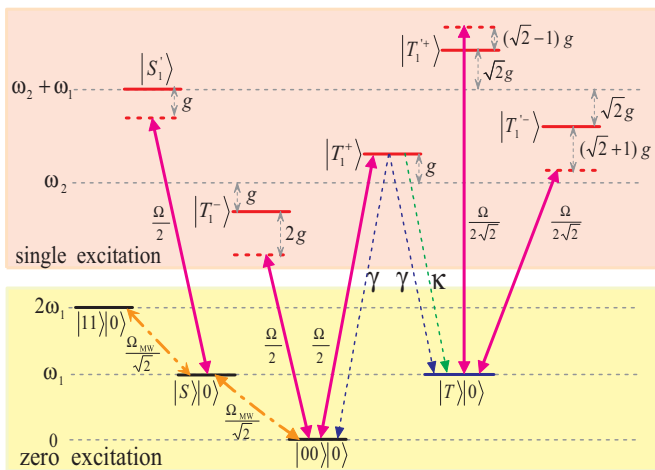


FIG. 2. (Color online) Level configuration and transitions in the dressed state picture. After choosing the frequency suitably, microwave field causes resonant transitions among states  $|00\rangle|0\rangle$ ,  $|S\rangle|0\rangle$ , and  $|11\rangle|0\rangle$ . And classical laser field causes resonant transition from state  $|00\rangle|0\rangle$  to state  $|T_1^+\rangle$ , but causes nonresonant transitions from  $|T\rangle|0\rangle$  to the states in the single excitation subspace.

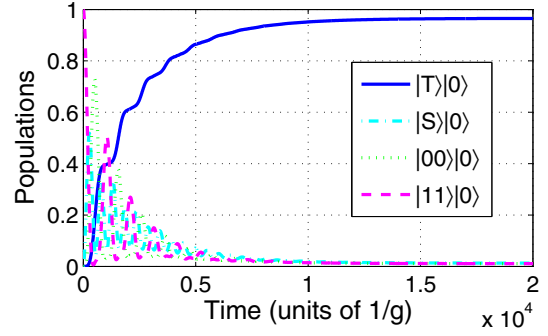


FIG. 3. (Color online) Populations of  $|T\rangle|0\rangle$ ,  $|S\rangle|0\rangle$ ,  $|00\rangle|0\rangle$ , and  $|11\rangle|0\rangle$  versus time in units of  $g^{-1}$  with the initial state  $|11\rangle|0\rangle$ . The parameters are chosen as  $\Omega = 0.015g$ ,  $\Omega_{MW} = 0.4\Omega$ ,  $\Delta = -g$ , and  $\kappa = \gamma = 0.1g$ .

Nevertheless, atomic spontaneous emission has another passage,  $|2\rangle \rightarrow |0\rangle$ , which would translate the last two terms of  $|T_1^+\rangle$  to  $|00\rangle|0\rangle$ . When it happens, the classical laser field would redrive the state  $|00\rangle|0\rangle$  to the state  $|T_1^+\rangle$  resonantly if we choose  $\Delta = -g$ .

Besides, the generation of the steady state is independent of the initial states. Consider the initial state  $|01\rangle|0\rangle$  or  $|10\rangle|0\rangle$ , which can be regarded as a superposition of the states  $|T\rangle|0\rangle$  and  $|S\rangle|0\rangle$ . As has been shown,  $|S\rangle|0\rangle$ ,  $|11\rangle|0\rangle$ , and  $|00\rangle|0\rangle$  can be translated to each other through microwave fields. And  $|00\rangle|0\rangle$  would be transformed to  $|T\rangle|0\rangle$  due to the combined effect of the unitary dynamics and the dissipative process. That is,  $|S\rangle|0\rangle$  is transformed to  $|T\rangle|0\rangle$ , and thus the initial state  $|01\rangle|0\rangle$  or  $|10\rangle|0\rangle$  would be converted to the desired state finally. Similarly, if the initial state is  $|11\rangle|0\rangle$ , it would be, no doubt, transformed to the state  $|T\rangle|0\rangle$ . The specific level configuration and transitions in the dressed picture is shown in Fig. 2.

The dynamics of the open dissipative system in Lindblad form could be described by the master equation

$$\dot{\rho} = -i[H, \rho] + \sum_j \left[ L^j \rho L^{j\dagger} - \frac{1}{2} (L^{j\dagger} L^j \rho + \rho L^{j\dagger} L^j) \right]. \quad (8)$$

In the present scheme, the Lindblad operators associated with the cavity decay and atomic spontaneous emission can be expressed as  $L^\kappa = \sqrt{\kappa}c$ ,  $L^{\gamma 1(2)} = \sqrt{\gamma/2}|0(1)\rangle_{AA}\langle 2|$ , and

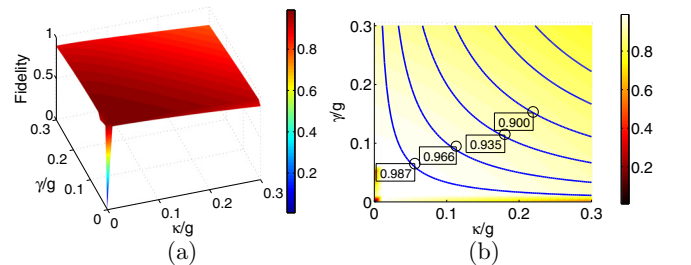


FIG. 4. (Color online) (a) Fidelity of  $|T\rangle|0\rangle$  versus  $\kappa$  and  $\gamma$  at the time  $4 \times 10^4/g$ . The parameters are chosen as  $\Omega = 0.015g$ ,  $\Omega_{MW} = 0.4\Omega$ , and  $\Delta = -g$ . (b) Fidelity of  $|T\rangle|0\rangle$  versus  $\kappa$  and  $\gamma$  at the time  $4 \times 10^5/g$ . Curves in the figure denotes  $C = 300, 100, 50, 30, 20$ , and 15 (from left to right), respectively.

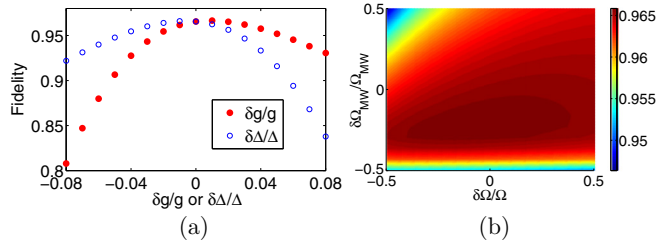


FIG. 5. (Color online) (a) Fidelity of  $|T\rangle|0\rangle$  at the time  $4 \times 10^5/g$  versus the relative fluctuations  $\delta g/g$  or  $\delta \Delta/\Delta$ . (b) Fidelity of  $|T\rangle|0\rangle$  versus the relative fluctuations  $\delta \Omega/\Omega$  and  $\delta \Omega_{MW}/\Omega_{MW}$ . The rest of the parameters are the same as in Fig. 3.

$L^{\gamma^{3(4)}} = \sqrt{\gamma/2}|0(1)\rangle_{BB}\langle 2|$ . We then solve the master equation in the zero and single excitation subspaces numerically, as plotted in Fig. 3. The results show that the populations of the desired state would be higher than 96.5% even when the cooperativity  $C \equiv g^2/\kappa\gamma$  is as low as 100. In Fig. 4(a), we plot the fidelity of  $|T\rangle|0\rangle$   $F = \langle 0|\langle T|\rho|T\rangle|0\rangle$  versus two dissipative factors, from which one can see that both  $\kappa$  and  $\gamma$  are utilized as resources since the fidelity would be zero when  $\kappa$  and  $\gamma$  are both set to zero. In Fig. 4(b), we plot the planform of the fidelity and add six curves which denote that  $C$  equals 300, 100, 50, 30, 20, and 15, respectively. Numerical simulations show that the fidelity can reach 98.7% ( $C = 300$ ), 96.6% ( $C = 100$ ), 93.5% ( $C = 50$ ), 89.9% ( $C = 30$ ), 86.0% ( $C = 20$ ), and 82.5% ( $C = 15$ ), respectively. Moreover, in Fig. 5, we plot the influence of different parameters' fluctuations. Figure 5(a) shows that the fidelity can still close to 81% and 84% when the fluctuations of  $g$  and  $\Delta$  are  $-8\%$  and  $8\%$ , respectively. And Fig. 5(b) shows that the fidelity keeps above 94.6% even if  $\Omega$  and  $\Omega_{MW}$  both have 50% fluctuations. Figure 6 illustrates the evolution of fidelity with the typical experimental parameters ( $g, \kappa, \gamma$ ) equal to  $2\pi \times (34, 4.1, 2.6)$  MHz [32–34] and  $2\pi \times (750, 2.62, 3.5)$  MHz [35], respectively. The other parameters are chosen as  $\Omega = 0.015g$ ,  $\Omega_{MW} = 0.4\Omega$ , and  $\Delta = -g$ .

The basic physical thought of the present scheme is to resonantly drive the undesired states in the zero excitation subspace to the state in the single excitation subspace directly or indirectly by choosing suitable frequencies of the microwave field and classical field. The corresponding state in the single excitation subspace has well-defined responsibilities since it can be transformed to the desired state through the dissipative process. Meanwhile, the desired state should undergo off-resonant driving. As a result, the population of the desired state accumulates for any initial state with time growing.

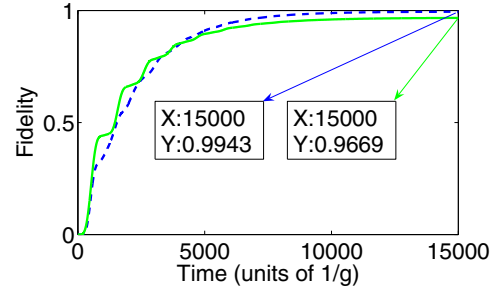


FIG. 6. (Color online) Fidelity of  $|T\rangle|0\rangle$  with two groups of experimental parameters versus time in units of  $g^{-1}$ . The green (solid) and blue (dashed) lines are plotted with the typical experimental parameters extracted from the Refs. [32–34] and Ref. [35], respectively.

The basic model used here is almost the same to Ref. [1]. The difference is that the detuned interaction between atomic state transition  $|2\rangle \leftrightarrow |1\rangle$  and the cavity mode in Ref. [1] is replaced by the resonant interaction. However, the method, the physical effect of the atomic spontaneous emission on the entanglement generation, the desired state, the Hamiltonians of classical laser field, and the microwave field are different. The method used by the authors of Ref. [1] is the effective operator method [27], while the one used in our scheme is the cooling method [3]. Spontaneous emission is an imperfect factor in Ref. [1], while it plays a positive role in our scheme. Another thing that should be noted here is that although spontaneous emission and cavity decay are critical for our scheme, it does not mean that the greater the dissipative factor the better. From Fig. 4, one can see that the fidelity decreases with the further increase of the dissipative factors. Like most of the dissipative schemes, the present one is based on the combined effect of the unitary dynamics and the dissipative process. On one hand, when values of the dissipative factors equal zero, the dissipative process would not work and the scheme would not succeed. On the other hand, further increase of the dissipative factors would influence the unitary dynamics and thus decrease the overall performance of the whole scheme.

In conclusion, we have proposed an alternative dissipative scheme for maximal entanglement generation between two  $\Lambda$ -type atoms in an optical cavity. The above analysis and numerical simulations show that the present scheme is feasible with current technology.

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