Increase of entanglement by local $\mathcal{PT}$-symmetric operations

Shin-Liang Chen,[1] Guang-Yin Chen,[2] and Yueh-Nan Chen[1,*]

[1] *Department of Physics and National Center for Theoretical Sciences, National Cheng-Kung University, Tainan 701, Taiwan*

[2] *Department of Physics, National Chung Hsing University, Taichung 402, Taiwan*

(Received 30 May 2014; revised manuscript received 5 September 2014; published 10 November 2014)

Entanglement plays a central role in the field of quantum information science. It is well known that the degree of entanglement cannot be increased under local operations. Here, we show that the concurrence of a bipartite entangled state can be increased under the local $\mathcal{PT}$-symmetric operation. This violates the property of entanglement monotonicity. We also use the Bell–Clauser-Horne-Shimony-Holt and steering inequalities to explore this phenomenon.

DOI: 10.1103/PhysRevA.90.054301 PACS number(s): 03.67.Bg, 03.65.Ca, 03.67.Ac, 03.67.Hk

I. INTRODUCTION

In conventional quantum mechanics, one of the axioms is that the Hamiltonian of a closed system has to be Hermitian, leading to the following properties: (1) The eigenvalues of the Hamiltonian are real, and (2) the time evolution of the system is unitary. In 1998, Bender and Boettcher [1] found that the parity-time ($\mathcal{PT}$)-symmetric Hamiltonian, which is non-Hermitian, can still have real energy spectra under some conditions. Later, they reconstructed the mathematical form of the inner product by introducing C symmetry, such that the evolution of the $\mathcal{PT}$-symmetric system becomes unitary [2]. In the Schrödinger equation, the necessary but not sufficient condition for a Hamiltonian to be $\mathcal{PT}$ symmetric is $V(x) = V^*(-x)$ [3]. Recently, experimental realizations of the $\mathcal{PT}$-symmetric Hamiltonian in classical optical systems have been proposed and realized by using the spatially balanced gain and loss of energy [4–13]. However, even with the experimental success in classical optical systems, there are still controversial results in some $\mathcal{PT}$-symmetric quantum systems. For example, Bender *et al.* [14] found that the evolution time between two quantum states under the $\mathcal{PT}$-symmetry operation can be arbitrary small. Lee *et al.* [15] found that the no-signalling principle can be violated when applying the local $\mathcal{PT}$-symmetric operation on one of the entangled particles.

Quantum entanglement [16] is one of the most intriguing phenomena in quantum physics. Its history can be traced back to the challenge by Einstein, Podolsky, and Rosen (EPR) [17]. In 1964, Bell [18] proposed the famous "Bell's inequality" based on the local hidden variable (LHV) model. Subsequent experiments [19] have successfully demonstrated violations of Bell's inequality, meaning that quantum mechanics and the LHV theory are incompatible. In response to the EPR paradox, Schrödinger introduced a concept called "quantum steering." Steering was recently formalized as a quantum information task by Wiseman *et al.* [20]. The steering inequality was further introduced [21] to delineate the quantum steering from other nonlocal properties. For many years, Bell's inequality has been used as an experimental tool [22] to examine the nonlocality. Its relation with the steering inequality and entanglement has also attracted great attention very recently [20,23].

Motivated by these works, in this paper, we consider a bipartite system in which one of the particles undergoes a local $\mathcal{PT}$-symmetric operation. We examine the behavior of the bipartite entanglement through the concurrence, Bell's inequality, and the steering inequality. Not only is the behavior of entanglement restoration observed, but it is also found that its value can exceed the initial one. This violates the property of entanglement monotonicity [24,25] and is beyond the description of non-Markovian dynamics. We also show that the increase of entanglement is not a unique property of the $\mathcal{PT}$-symmetric system by considering the non-Hermitian Hamiltonian without $\mathcal{PT}$ symmetry.

II. RESTORATION OF ENTANGLEMENT BY LOCAL $\mathcal{PT}$-SYMMETRIC OPERATION

As shown in Fig. 1, we consider a composite system consisting of two identical qubits. Let qubit 1 undergo a coherent Rabi oscillation governed by $H_{\mathrm{Rabi},1} = \hbar g\,(\sigma_{1,+} + \sigma_{1,-})$, where $\sigma_{1,+}$ ($\sigma_{1,-}$) is the raising (lowering) operator and $\hbar g$ is the coupling strength. The evolution of the entire system can be obtained by solving the following equation:

$$\dot{\rho} = \frac{1}{i\hbar}[H,\rho(t)], \tag{1}$$

where $H = H_{\mathrm{Rabi},1} \otimes I_2$ is the total Hamiltonian of the composite system with I_2 denoting the identity operator of qubit 2.

Let us also consider a different scenario by replacing the coherent Rabi process with a local $\mathcal{PT}$-symmetric operation on qubit 1. The total Hamiltonian $H_{\mathcal{PT}}$ can then be written as [26]

$$H_{\mathcal{PT}} = H_{\mathcal{PT},1} \otimes I_2 = s\begin{pmatrix} i\sin\alpha & 1 \\ 1 & -i\sin\alpha \end{pmatrix} \otimes I_2, \tag{2}$$

where $H_{\mathcal{PT},1}$ is the Hamiltonian of qubit 1. The real number s is a scaling constant, and the real number α is the non-Hermiticity of $H_{\mathcal{PT},1}$. The condition $|\alpha| < \pi/2$ keeps the eigenvalues of $H_{\mathcal{PT},1}$ real, i.e., $\mathcal{PT}$ symmetric. The non-Hermitian Hamiltonian $H_{\mathcal{PT},1}$ can be decomposed into a Hermitian part (H_+) and an anti-Hermitian part (H_-):

$$H_{\mathcal{PT}} = s\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \otimes I_2 + s\begin{pmatrix} i\sin\alpha & 0 \\ 0 & -i\sin\alpha \end{pmatrix} \otimes I_2 = H_+ + H_-. \tag{3}$$

*yuehnan@mail.ncku.edu.tw

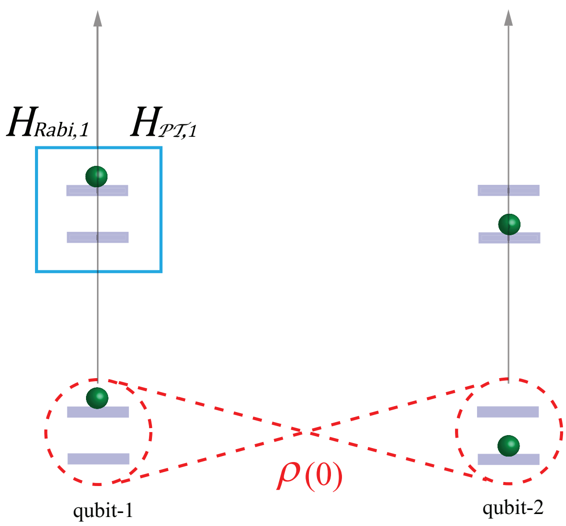

FIG. 1. (Color online) The schematic diagram of the dynamics of the two qubits. Qubit 1 undergoes either the Rabi process or a local $\mathcal{PT}$-symmetric operation, while qubit 2 remains isolated. The initial condition is the maximally entangled state: $(|00\rangle + |11\rangle)/\sqrt{2}$.

To obtain the evolution of the system, Eq. (1) has to be modified as [27,28]

$$\dot{\rho} = \frac{1}{i\hbar}[H_+,\rho(t)] + \frac{1}{i\hbar}\{H_-,\rho(t)\}. \tag{4}$$

The solution can also be obtained by introducing a time-evolving operator [14,15]:

$$U_1(t) = e^{-iH_{\mathcal{PT},1}t} = \frac{1}{\cos\alpha}\begin{pmatrix}\cos(t'-\alpha) & -i\sin t' \\ -i\sin t' & \cos(t'+\alpha)\end{pmatrix}, \tag{5}$$

where $t' = \Delta E t$, with $\Delta E \equiv \frac{E_+ - E_-}{2}$. Here, $E_\pm = \pm s\cos\alpha$ are the eigenvalues of $H_{\mathcal{PT},1}$.

In general, the evolution of a system with a non-Hermitian Hamiltonian is not trace preserving:

$$\frac{\partial}{\partial t}\mathrm{tr}(\rho) = \frac{2}{i\hbar}\mathrm{tr}(\rho H_-) \neq 0. \tag{6}$$

Thus, irrespective of whether one uses Eq. (4) or Eq. (5), to obtain a solution, $\rho(t)$ has to be renormalized,

$$\tilde{\rho}(t) = \frac{\rho(t)}{\mathrm{tr}[\rho(t)]} \tag{7}$$

or

$$\tilde{\rho}(t) = \frac{(U_1(t)\otimes I_2)\rho(0)(U_1(t)\otimes I_2)^\dagger}{\mathrm{Tr}[(U_1(t)\otimes I_2)\rho(0)(U_1(t)\otimes I_2)^\dagger]}, \tag{8}$$

because the observers live in the conventional quantum world [15,27,28]. The quantum average of an observable $\mathcal{A}$ can then be calculated as

$$\langle\mathcal{A}\rangle \equiv \mathrm{tr}(\mathcal{A}\tilde{\rho}(t)) = \frac{\mathrm{tr}[\mathcal{A}\rho(t)]}{\mathrm{tr}[\rho(t)]}. \tag{9}$$

In standard quantum mechanics, $\mathrm{tr}[\rho(t)] = 1$, so Eq. (9) coincides with the standard Born's rule.

To evaluate the degree of the entanglement between the two qubits, we use the concurrence [29]:

$$\mathcal{C}(\rho) = \mathrm{Max}\{0, \sqrt{\lambda_1} - \sqrt{\lambda_2} - \sqrt{\lambda_3} - \sqrt{\lambda_4}\}, \tag{10}$$

where $\{\lambda_i\}$, in decreasing order, are the eigenvalues of $\rho(\sigma_y\otimes\sigma_y)\rho^*(\sigma_y\otimes\sigma_y)$. Here, σ_y is the Pauli-y matrix, and ρ^* is the complex conjugate of ρ. To confirm the existence of the entanglement experimentally, the Bell–Clauser-Horne-Shimony-Holt (CHSH) inequality [18,22] and steering inequality [21,22] are commonly used. Therefore, it is useful to calculate the maximal mean value of the Bell kernel $\langle\mathcal{B}_{\max}\rangle$ [30]:

$$\langle\mathcal{B}_{\max}\rangle = 2\sqrt{M(\rho)} = 2\sqrt{u_1 + u_2}, \tag{11}$$

where u_1 and u_2 are the two largest eigenvalues of $T_\rho^T T_\rho$, and T_ρ^T is the transpose of T_ρ. The correlation tensor T_ρ is given by $t_{ij} = \mathrm{Tr}[\rho(\sigma_i\otimes\sigma_j)]$ for $i,j = 1,2,3$, where σ_i and σ_j are the Pauli matrices. If the correlation between the two qubits can be described by the LHV model, the Bell-CHSH inequality holds [30]:

$$\langle\mathcal{B}_{\max}\rangle \leqslant 2. \tag{12}$$

The violation of the Bell-CHSH inequality indicates the failure of the LHV model and can be viewed as a certification of quantum entanglement [20].

If the correlation between two qubits can be described by the LHS model, the steering inequality holds [22]:

$$S_N \equiv \sum_{i=1}^{N} E\big[\langle\hat{B}_i\rangle_{A_i}^2\big] \leqslant 1, \tag{13}$$

where $N(=2$ or $3)$ is the number of mutually unbiased measurement [31] (for example, the Pauli $\hat{X}, \hat{Y}$, and $\hat{Z}$ matrices) performed on qubit 2, and

$$E\big[\langle\hat{B}_i\rangle_{A_i}^2\big] \equiv \sum_{a=\pm1} P(A_i = a)\langle\hat{B}_i\rangle_{A_i=a}^2 \tag{14}$$

is the average expectation of qubit 2. Here, $P(A_i = a)$ is the probability of the measurement result of qubit 1, and

$$\langle\hat{B}\rangle_{A_i=a} \equiv \sum_{b=\pm1} bP(B_i = b|A_i = a) \tag{15}$$

is the expectation value of qubit 2 conditioned on the outcome of qubit 1. The violation of the steering inequality indicates the failure of the LHS model and can also serve as an entanglement certification [20].

In Fig. 2, we plot the dynamics of $\mathcal{C}$, S_3, and $\langle\mathcal{B}_{\max}\rangle$ for both the coherent Rabi process and the local $\mathcal{PT}$-symmetric operation. We set the initial state $|\psi_{AB}\rangle$ to be one of the Bell states: $|\psi_{AB}\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$. In Figs. 2(a) and 2(b), we can see that the values $\mathcal{C}$ and $\langle\mathcal{B}_{\max}\rangle$ remain unchanged when qubit 1 undergoes the coherent Rabi process. On the other hand, $\mathcal{C}$ and $\langle\mathcal{B}_{\max}\rangle$ oscillate with time when performing the local $\mathcal{PT}$ -symmetric operation. Thus, the entanglement between two parties can be restored when one of them undergoes the $\mathcal{PT}$-symmetric operation.

III. INCREASE OF ENTANGLEMENT BY LOCAL $\mathcal{PT}$-SYMMETRIC OPERATION

Let us consider qubit 1 embedded in an environment, while qubit 2 is still isolated. From the angle of non-Markovian dynamics [32], one can observe the behavior of *entanglement restoration* if there exists some memory effect; i.e., quantum

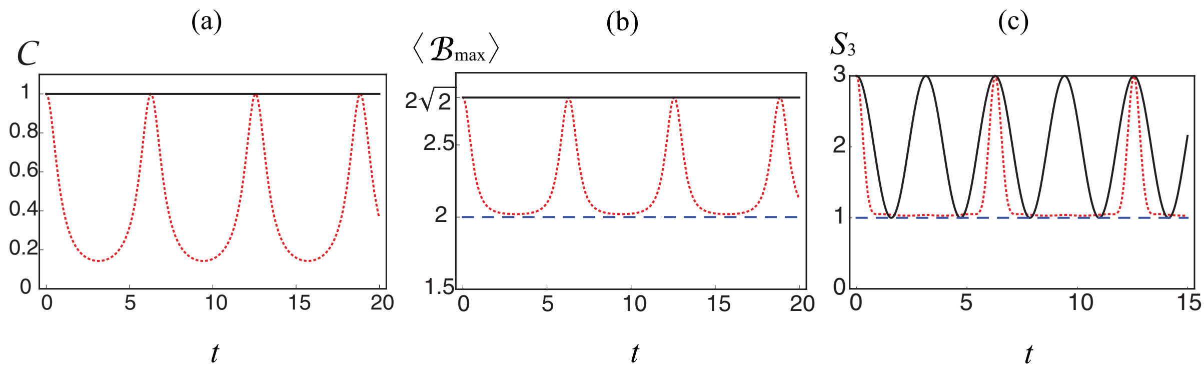

FIG. 2. (Color online) The dynamics of (a) the concurrence $\mathcal{C}$, (b) the maximal mean value of the Bell kernel $\langle\mathcal{B}_{\max}\rangle$, and (c) the steering parameter S_3. The black solid curve in each figure represents that qubit 1 undergoes the coherent Rabi oscillation, while the red dotted curve represents that the local $\mathcal{PT}$-symmetric operation is performed on qubit 1. The blue dashed lines in (b) and (c) are the classical bounds [i.e., upper bounds in Eqs. (12) and (13)] of $\langle\mathcal{B}_{\max}\rangle$ and S_3, respectively. In plotting the figure, the time t is in units of $1/\Delta E$, and the initial condition is $(|00\rangle+|11\rangle)/\sqrt{2}$.

coherence is built between qubit 1 and the environment during the evolution. So, one may speculate that the entanglement restoration in Fig. 2 is similar to that derived from the non-Markovian effect. However, we should note that the entanglement restored from the environment cannot exceed the initial one for any non-Markovian process [32]. Otherwise, the property of entanglement monotonicity is violated; i.e., entanglement cannot be created (or increased) by performing any local operation [24,25]. In this section, we will show that the degree of the entanglement can exceed the initial value for the local $\mathcal{PT}$-symmetric operation.

The first step is to prepare the quantum state which is not maximally entangled. To accomplish this, let us start from the maximally entangled state, $\frac{1}{\sqrt{2}}(|00\rangle+|11\rangle)$, and subject qubit 1 to a Markovian amplitude damping (with rate γ) for a time t_c, as shown in Fig. 3. By solving the following Lindblad-form [33,34] master equation,

$$\dot{\rho}_c=\frac{\gamma}{2}(2\sigma_1^-\rho_c\sigma_1^+-\sigma_1^+\sigma_1^-\rho_c-\rho_c\sigma_1^+\sigma_1^-), \quad (16)$$

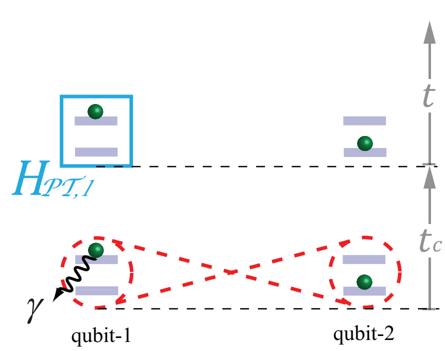

FIG. 3. (Color online) The schematic diagram of the dynamics of the two qubits. Qubit 1 is subjected to an amplitude damping for a time t_c. At the time t_c, the damping is turned off, and the local $\mathcal{PT}$-symmetric operation is then performed on qubit 1. The initial state of the system is $(|00\rangle+|11\rangle)/\sqrt{2}$.

one can obtain the state $\rho_c(t_c)$ at the cutoff time t_c:

$$\rho_c(t_c)=\frac{1}{2}\begin{pmatrix} e^{-\gamma t_c} & 0 & 0 & e^{-\gamma t_c/2}\\ 0 & 0 & 0 & 0\\ 0 & 0 & 1-e^{-\gamma t_c} & 0\\ e^{-\gamma t_c/2} & 0 & 0 & 1\end{pmatrix}, \quad (17)$$

with the concurrence $\mathcal{C}=e^{-\gamma t_c/2}$, which is a monotonically decreasing function of t_c. For comparisons, we choose $t_c=0.5$, 1, and 1.6 (in units of $1/\gamma$) to have three different initial states:

$$\rho_c(t_c=0.5)=\begin{pmatrix} 0.3033 & 0 & 0 & 0.3894\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0.1967 & 0\\ 0.3894 & 0 & 0 & 0.5\end{pmatrix}, \quad (18)$$

$$\rho_c(t_c=1)=\begin{pmatrix} 0.1839 & 0 & 0 & 0.3033\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0.3161 & 0\\ 0.3033 & 0 & 0 & 0.5\end{pmatrix}, \quad (19)$$

and

$$\rho_c(t_c=1.6)=\begin{pmatrix} 0.1009 & 0 & 0 & 0.2247\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0.3991 & 0\\ 0.2247 & 0 & 0 & 0.5\end{pmatrix}. \quad (20)$$

With these three initial states, we let the system undergo the local $\mathcal{PT}$-symmetric operation [Eq. (2)] to obtain the entanglement dynamics. In Fig. 4, we plot $\mathcal{C}$, S_3, and $\langle\mathcal{B}_{\max}\rangle$ under the local $\mathcal{PT}$-symmetric operation. From $\langle\mathcal{B}_{\max}\rangle$ (or S_3), we can see that it is possible to certify the entanglement (values above the classical bound shown by the horizontal green line) at a later time, even if initially the entanglement is not certified (values below the classical bound). We can also see that the degree of the entanglement $\mathcal{C}$ can *exceed* the initial value under the local $\mathcal{PT}$-symmetric operation. This violates the property of entanglement monotonicity; i.e., entanglement cannot be created (or increased) by performing any local operation [24,25].

To understand this thoroughly, let us examine the reduced density state of qubit 2. In conventional quantum mechanics,

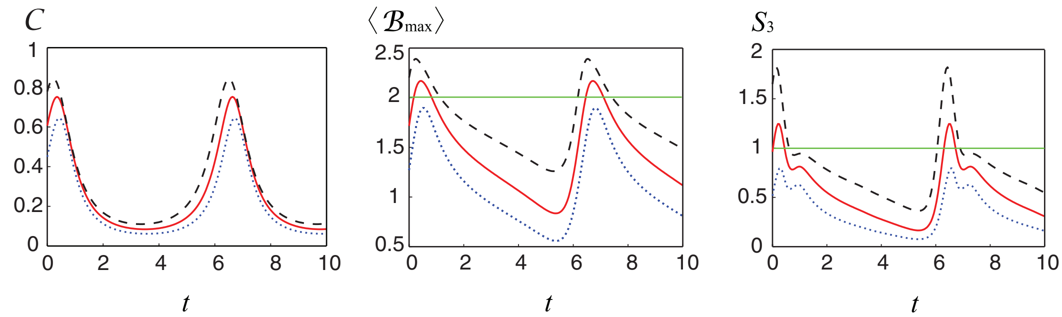

FIG. 4. (Color online) The dynamics of (a) the concurrence $\mathcal{C}$, (b) the maximal mean value of the Bell kernel $\langle\mathcal{B}_{\max}\rangle$, and (c) the steering parameter S_3 when the local $\mathcal{PT}$-symmetric operation is performed on qubit 1. The black dashed, red solid, and blue dotted curves represent the results of different initial states of the $\mathcal{PT}$-symmetric evolution given by Eqs. (18), (19), and (20), respectively. The green horizontal lines in (b) and (c) are the classical bounds [i.e., upper bounds in Eqs. (12) and (13)] of $\langle\mathcal{B}_{\max}\rangle$ and S_3, respectively. In plotting the figure, the time t of the $\mathcal{PT}$-symmetric evolution is in units of $1/\Delta E$.

a local operation on qubit 1 cannot alter the reduced state of qubit 2. Under the local $\mathcal{PT}$-symmetric operation, however, the reduced density states of qubit 2 at $t' = 0$ and $\pi/2$ are

$$\rho_B(0) = \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix} \tag{21}$$

and

$$\rho_B(\frac{\pi}{2}) = \begin{pmatrix} \frac{1}{2} & \frac{i\sin\alpha}{(1+\sin^2\alpha)} \\ \frac{-i\sin\alpha}{(1+\sin^2\alpha)} & \frac{1}{2} \end{pmatrix}, \tag{22}$$

respectively, while the initial state is $\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$. This means that the reduced density state of qubit 2 is changed under the local $\mathcal{PT}$-symmetric operation on qubit 1. From this viewpoint, we know that the local $\mathcal{PT}$-symmetric operation is not a genuine local operation in the conventional quantum world. More importantly, when we deal with a $\mathcal{PT}$-symmetric Hamiltonian, renormalization [Eqs. (7) and (8)] is required to ensure trace preserving. Such an action directly affects the quantum state of the two qubits and results in the violation of entanglement monotonicity [35].

To check whether the increase of entanglement is a unique property only for the $\mathcal{PT}$-symmetric Hamiltonian, let us consider the following Hamiltonian:

$$H_1 = s\begin{pmatrix} i\sin\alpha & 1 \\ 1 & -i\sin\alpha \end{pmatrix} + \epsilon\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \tag{23}$$

where ϵ is a real number. If ϵ is not equal to zero, H_1 is no longer a $\mathcal{PT}$-symmetric Hamiltonian but a normal non-Hermitian Hamiltonian. As shown in Fig. 5, the entanglement can still be increased with the same initial conditions given in Eqs. (18)–(20). The reason is the same as that of the $\mathcal{PT}$-symmetric Hamiltonian. When we deal with the non-Hermitian Hamiltonian, the renormalization procedure induces a nonlocal effect, and the entanglement monotonicity is violated.

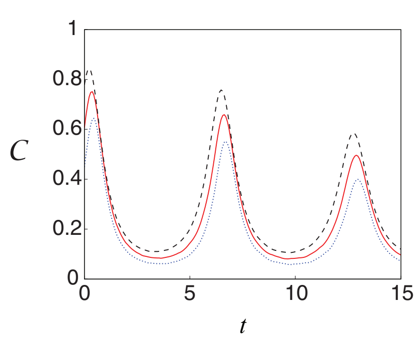

FIG. 5. (Color online) The concurrence as a function of time when qubit 1 is under the Hamiltonian of Eq. (23). The black dashed, red solid, and blue dotted curves represent the results of the different initial states given by Eqs. (18), (19), and (20) respectively. In plotting the figure, the time t is in units of $1/\Delta E$, and ϵ is set to 0.01.

IV. CONCLUSION

In summary, we find that the degree of entanglement between the two particles oscillates with time, when the local $\mathcal{PT}$-symmetric operation is performed on one of the qubits. To check whether this is similar to results of non-Markovian effects, we consider a maximally entangled state subjected to Markovian damping for some time t_c and then replace the damping with the local $\mathcal{PT}$-symmetric operation. It is found that the entanglement can be increased with the local $\mathcal{PT}$-symmetric operation. This contradicts the fact that entanglement cannot be increased by any local operation. We also consider the non-Hermitian Hamiltonian without $\mathcal{PT}$ symmetry and show that the increase of entanglement is not a unique property of the $\mathcal{PT}$-symmetric system.

ACKNOWLEDGMENTS

This work is supported partially by the National Center for Theoretical Sciences and Ministry of Science and Technology, Taiwan, MOST Grants No. 101-2628-M-006-003-MY3, No. 102-2112-M-005-009-MY3, and No. 103-2112-M-006-017-MY4.

[1] C. M. Bender and S. Boettcher, Phys. Rev. Lett. **80**, 5243 (1998).
[2] C. M. Bender, D. C. Brody, and H. F. Jones, Phys. Rev. Lett. **89**, 270401 (2002).
[3] C. M. Bender, S. Boettcher, and P. N. Meisinger, J. Math. Phys. **40**, 2201 (1999).
[4] R. El-Ganainy, K. G. Makris, D. N. Christodoulides, and Z. H. Musslimani, Opt. Lett. **32**, 2632 (2007).
[5] K. G. Makris, R. El-Ganainy, D. N. Christodoulides, and Z. H. Musslimani, Phys. Rev. Lett. **100**, 103904 (2008).
[6] K. G. Makris, R. El-Ganainy, D. N. Christodoulides, and Z. H. Musslimani, Phys. Rev. A **81**, 063807 (2010).
[7] C. E. Rüter, K. G. Makris, R. El-Ganainy, D. N. Christodoulides, M. Segev, and D. Kip, Nat. Phys. **6**, 192 (2010).
[8] A. Guo, G. J. Salamo, D. Duchesne, R. Morandotti, M. Volatier-Ravat, V. Aimez, G. A. Siviloglou, and D. N. Christodoulides, Phys. Rev. Lett. **103**, 093902 (2009).
[9] S. Bittner, B. Dietz, U. Günther, H. L. Harney, M. Miski-Oglu, A. Richter, and F. Schäfer, Phys. Rev. Lett. **108**, 024101 (2012).
[10] J. Schindler, A. Li, M. C. Zheng, F. M. Ellis, and T. Kottos, Phys. Rev. A **84**, 040101(R) (2011).
[11] A. Regensburger, C. Bersch, M.-A. Miri, G. Onishchukov, D. N. Christodoulides, and U. Peschel, Nature (London) **488**, 167 (2012).
[12] B. Peng, Ş. K. Özdemir, F. Lei, F. Monifi, M. Gianfreda, G. L. Long, S. Fan, F. Nori, C. M. Bender, and L. Yang, Nat. Phys. **10**, 394 (2014).
[13] C. M. Bender, B. K. Berntson, D. Parker, and E. Samuel, Am. J. Phys. **81**, 173 (2013).
[14] C. M. Bender, D. C. Brody, H. F. Jones, and B. K. Meister, Phys. Rev. Lett. **98**, 040403 (2007).
[15] Y. C. Lee, M. H. Hsieh, S. T. Flammia, and R. K. Lee, Phys. Rev. Lett. **112**, 130404 (2014).
[16] R. Horodecki, P. Horodecki, M. Horodecki, and K. Horodecki, Rev. Mod. Phys. **81**, 865 (2009).
[17] A. Einstein, B. Podolsky, and N. Rosen, Phys. Rev. **47**, 777 (1935).
[18] J. S. Bell, Physics **1**, 195 (1964).
[19] A. Aspect, J. Dalibard, and G. Roger, Phys. Rev. Lett. **49**, 1804 (1982); W. Tittel, J. Brendel, H. Zbinden, and N. Gisin, *ibid.* **81**, 3563 (1998); G. Weihs, T. Jennewein, C. Simon, H. Weinfurter, and A. Zeilinger, *ibid.* **81**, 5039 (1998); M. A. Rowe, D. Kielpinski, V. Meyer, C. A. Sackett, W. M. Itano, C. Monroe, and D. J. Wineland, Nature (London) **409**, 791 (2001); D. N. Matsukevich, P. Maunz, D. L. Moehring, S. Olmschenk, and C. Monroe, *ibid.* **100**, 150404 (2008).
[20] H. M. Wiseman, S. J. Jones, and A. C. Doherty, Phys. Rev. Lett. **98**, 140402 (2007).
[21] E. G. Cavalcanti, S. J. Jones, H. M. Wiseman, and M. D. Reid, Phys. Rev. A **80**, 032112 (2009).
[22] J. F. Clauser, M. A. Horne, A. Shimony, and R. A. Holt, Phys. Rev. Lett. **23**, 880 (1969).
[23] G. Y. Chen, S. L. Chen, C. M. Li, and Y. N. Chen, Sci. Rep. **3**, 2514 (2013).
[24] C. H. Bennett, G. Brassard, S. Popescu, B. Schumacher, J. A. Smolin, and W. K. Wootters, Phys. Rev. Lett. **76**, 722 (1996).
[25] C. H. Bennett, H. J. Bernstein, S. Popescu, and B. Schumacher, Phys. Rev. A **53**, 2046 (1996).
[26] U. Günther and B. F. Samsonov, Phys. Rev. Lett. **101**, 230404 (2008).
[27] A. Sergi and K. G. Zloshchastiev, Int. J. Mod. Phys. B **27**, 1350163 (2013); A. K. Pati, arXiv:1404.6166 (2014).
[28] D. C. Brody and E.-M. Graefe, Phys. Rev. Lett. **109**, 230405 (2012).
[29] W. K. Wootters, Phys. Rev. Lett. **80**, 2245 (1998).
[30] R. Horodecki, P. Horodecki P, and M. Horodecki, Phys. Lett. A **200**, 340 (1995).
[31] W. K. Wootters and B. D. Fields, Ann. Phys. **191**, 363 (1989).
[32] A. Rivas, S. F. Huelga, and M. B. Plenio, Phys. Rev. Lett. **105**, 050403 (2010); A. Orieux, A. D'Arrigo, G. Ferranti, R. Lo Franco, G. Benenti, E. Paladino, G. Falci, F. Sciarrino, and P. Mataloni, arXiv:1410.3678 (2014).
[33] G. Lindblad, Commun. Math. Phys. **48**, 119 (1976).
[34] V. Gorini, A. Kossakowski, and E. C. G. Sudarshan, J. Math. Phys. **17**, 821 (1976).
[35] We should note that the reduced density matrix of qubit 2 is changed even without the renormalization. However, the change of the reduced state does not necessarily lead to the increase of the entanglement. It just shows that the $\mathcal{PT}$-symmetric operation is not a genuine local operation. For example, when qubit 1 suffers Markovian amplitude damping (with a coherent coupling between two qubits), the reduced state of qubit 2 is changed during the evolution. Depending on the initial conditions, the entanglement may or may not increase during the evolution.