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Generation of rotary beams by interaction of moving solitons in nonlocal media

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We report that nonlocal nonlinear media allow a controlled interaction among coherent solitons with a relative tilt previously imposed, resulting in the generation of rotary self-trapped beams. We demonstrate that two initially separated fundamental solitons can interact, generating stable rotating dipoles for a continuum interval of relative tilt values. Surprisingly, we find that for a higher number of initial solitons launched and after some emission of radiation waves, the initial self-trapped structures can decay into rotating dipole solitons. The normalized orbital angular momentum of these rotating dipoles can be controlled by adjusting the initial tilt.

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I. INTRODUCTION

A balance between diffraction and nonlinearity allows the generation of self-trapped beams or optical spatial solitons [1,2]. A fundamental feature of solitons is their particle behavior. Interactions among these self-trapped nonlinear beams can result in the exchange of momentum [1], which produces propagation dynamics in which it is possible to observe phenomena such as attraction, repulsion, fusion, fission, and even spiraling interactions [3,4]. In particular, certain self-trapped structures such as propeller solitons [5], necklace beams [6], rotating soliton clusters [7], azimuthons [8], ellipticons [9], and rotating dipoles [10], among others, can experience an intensity rotation under propagation. These structures carry orbital angular momentum, which opens the possibility of using them in applications such as the rotation of microparticles in optical trapping [11], dynamic manipulation of Bose-Einstein condensates [12], and high-bandwidth information encoding in optical communication systems [13]. In general, a standard variational method [14] and specialized numerical relaxation algorithms [15] are common procedures used to theoretically study these rotary structures. In this paper, we show that it is also possible to generate stable rotary self-trapped structures by using only fundamental solitons with an initial relative tilt imposed and that in several cases a rotating dipole is formed, even in the case of multiple-soliton interaction.

In local nonlinear media, it is known that two fundamental solitons experience attraction or repulsion if they are in phase or out of phase, respectively, although there are more complex dynamics in any other case of phase difference. Thus, to generate a rotating structure starting from two far apart and copropagating solitons, it is necessary that, in principle, both solitons have the same relative phase, which allows us to cancel out the natural escaping dynamics. However, a very specific tilt value is necessary to neutralize either the merging or the escaping scenario, resulting, in any case, in unstable dynamics, as shown for the case of spiraling spatial solitons in saturable media [16]. In order to overcome this issue, Segev's group proposed and corroborated the use of incoherent solitons to generate spiraling in interacting spatial solitons [17]. Here we report that it is also possible to relax the very critical tilt condition by using coherent solitons in a nonlocal nonlinear medium. It has been demonstrated that nonlocality always provides an attraction mechanism among solitons in spite of their relative phase [1,18]. Therefore, we propose the use of nonlocal media to trap, merge, and stabilize the interaction among moving solitons. We start by demonstrating that using only two solitons with opposite phase and a relative initial tilt, it is possible to produce rotating dipole beams. In this scenario, as the solitons try to merge as a result of the trapping nonlocal potential, the phase opposite that of the solitons can prevent fusion dynamics. In a similar way, as the solitons try to escape, the nonlocality can keep them together. This scenario results in the existence of continuum intervals of tilt values where it is possible to generate rotary beams. Surprisingly, we find that for the case of multiple-soliton interaction and after some emission of radiation waves, in several cases a rotating dipole soliton is generated. The rest of the paper is organized as follows. The nonlocal model is discussed in Sec. II. The simplest generation of rotary beams, with only two fundamental solitons, is reported in Sec. III, while the generation of rotary beams using multiple fundamental solitons is reported in Sec. IV, demonstrating the recurrent morphing from initial multiple fundamental solitons into rotating dipole solitons. In Sec. V, we compare the generation of rotating dipoles for a Gaussian nonlocal response with an exponential response to show that this morphing phenomenon does not happen only for the Gaussian nonlocal response. Finally, the paper is concluded in Sec. VI.

II. DEFINITION OF THE NONLOCAL NONLINEAR MODEL

The propagation of paraxial optical beams in a nonlocal nonlinear medium can be described by a nonlinear Schrödinger equation in dimensionless units [1],

$$i\frac{\partial \Psi}{\partial z} + \nabla_{\perp}^2 \Psi + \int \mathcal{N}(|\mathbf{r}-\rho|)|\Psi(\rho)|^2 d\rho \Psi = 0, \quad (1)$$

where Ψ stands for the scalar field envelope, z and $\mathbf{r} = (x,y)$ are the propagation and transverse coordinates, respectively, and $\nabla_{\perp}^2$ stands for the transverse Laplacian. The response function $\mathcal{N}$, which satisfies the normalization condition $\int \mathcal{N}(\mathbf{r}) d\mathbf{r} = \mathbf{1}$, is defined by the physical process that generates the medium nonlinearity, and it can usually be modeled by a diffusion equation, which is the case for nematic liquid crystals [19,20]. Here we consider a nonlocal response

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function of the form

$$\mathcal{N}(r) = (1/\pi\sigma^2)\exp(-r^2/\sigma^2), \tag{2}$$

where σ is the degree of nonlocality of the medium. When $\sigma \to 0$, we recover the pure local Kerr medium, whereas when $\sigma \to \infty$, we have the highly nonlocal nonlinear limit proposed by Snyder and Mitchell [21], a scenario in which it is possible to find exact analytical solutions to Eq. (1), such as the Hermite-Bessel solitons [22] or the ellipticons [9], to name just a few examples.

Although there is not a real physical system truly associated with this Gaussian response, Eq. (2) has been broadly used as a theoretical model to investigate nonlocal nonlinearities [10,23,24]. It has been demonstrated that if the response function is real, positive definite, symmetric, and monotonically decaying, then the physical properties do not depend strongly on its shape [23]. In fact, some phenomena were predicted using a Gaussian response [10] and then were corroborated in experimental works, which is the case for self-induced mode transformations [25].

Next, we introduce the scaled variables $\Psi = \Psi'/\sigma$, $r = r'\sigma$, and $z = z'\sigma^2$ and omit primes in order to demonstrate that the physically important beam power is independent of σ. Using this scaling, we can quantify the degree of nonlocality by using the soliton power, where a higher power beam means a higher degree of nonlocality.

III. GENERATING ROTARY BEAMS WITH TWO FUNDAMENTAL SOLITONS

We look for N identical fundamental soliton solutions of Eq. (1) in the form $\Psi(\mathbf{r_n},z) = \sum_{n=1}^{N} U_n(\mathbf{r}_n)\exp(i\lambda z)$, where $U_n(\mathbf{r_n})$ is a purely real function obtained using a standard Petviashvili relaxation method [26] starting from an initial Gaussian ansatz centered at $\mathbf{r_n} = (x_n, y_n)$ and λ is the soliton propagation constant. In particular, when $N = 2$, we use as a field condition at $z = 0$

$$\Psi(x,y) = \Psi_1(x_o,y_o)\exp(-i\alpha x) + \exp(i\chi)\Psi_2(-x_o,-y_o) \times \exp(i\beta x), \tag{3}$$

where x_o and y_o must be chosen to be large enough to neglect an initial field interaction of the theoretical infinite tails of the solitons, and thus, the total initial power is $P = \int |\sum_{n=1}^{N} \Psi_n(\mathbf{r},0)|^2 d\mathbf{r} \simeq N \int |\Psi_n(\mathbf{r},0)|^2 d\mathbf{r}$. Here we report the most basic scenario that occurs when we set the symmetric condition $\alpha = \beta = \mu$. In this case the relative phase needed to achieve rotary structures is simply $\chi = \pi$. To study the corresponding dynamics of the beam given by Eq. (3), we use Eq. (3) as an initial condition in Eq. (1), and then we solve numerically using a split-step Fourier method. In Fig. 1, we show the propagation for different λ values. We find that rotating dipoles are generated when $\mu \in [\mu_{\min},\mu_{\max}]$, where $\mu_{\min}$ is the minimum tilt necessary to overcome the "local" repulsion effect produced by the phase χ and $\mu_{\max}$ is the maximum tilt that can be trapped by the nonlocal potential.

For a low degree of nonlocality, the two solitons can interact for a certain distance, and then the initial dipole configuration is destroyed [Fig. 1(a)]. However, if a certain degree of power is reached, the two solitons can orbit around each other for longer distances. Because of the acceleration experienced by the individual solitons, they radiate energy, decaying into more asymmetric structures but remaining self-trapped by the nonlocality [Fig. 1(b)]. We also find that for a high enough power, the solitons finally develop a more symmetrical spiraling structure. This occurs because the nonlocality tends to smooth out the spatial variations produced by the solitons' radiation. Furthermore, we find that for certain μ values, the resulting beam can achieve a rotating dipole or azimuthon-like structure [Fig. 1(c)]. Perturbing the initial solitons with 10% noise produces more complex dynamics, in which the remaining self-trapped beams can also show a translation movement [Fig. 1(d)], but the analysis of the dynamics in the presence of noise is beyond the scope of this work. Figure 2(a) shows the existence domain of the intervals whose initial tilts allow the generation of rotating dipole solitons. Note that in the case of higher λ values, i.e., higher power, it is possible to impose higher initial tilts to generate rotating dipole solitons. Due to the nonintegrability of Eq. (1), the interaction between the solitons produces radiation waves. In order to consider a more steady state after the initial power decay, we calculate the power beam $\overline{P}$ averaged from $z = 50$

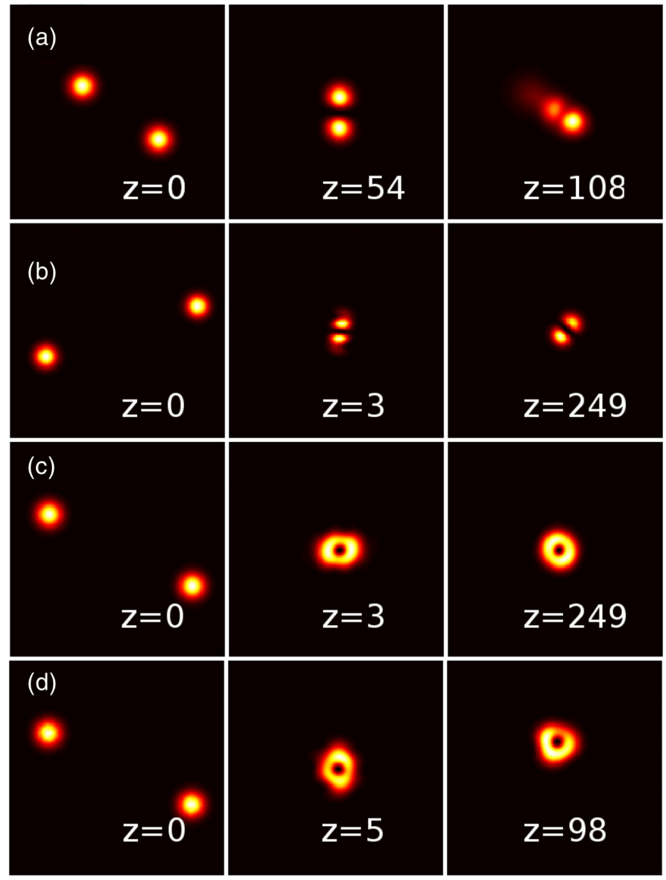

FIG. 1. (Color online) Propagation dynamics of two solitons with the following parameters: (a) $\lambda = 10$, $\mu = 0.06$, $(x_o, y_o) = (-2, 1.4)$, and $L = 15$, (b) $\lambda = 50$, $\mu = 0.5$, $(x_o, y_o) = (-3, 1)$, and $L = 17$, and (c) and (d) $\lambda = 100$, $\mu = 3$, $(x_o, y_o) = (-2, 1)$, and $L = 12$. In (d), there is an input noise of 10%. The profiles are shown in an x-y box of $L \times L$. (See the Supplemental Material [27] to observe propagation.)

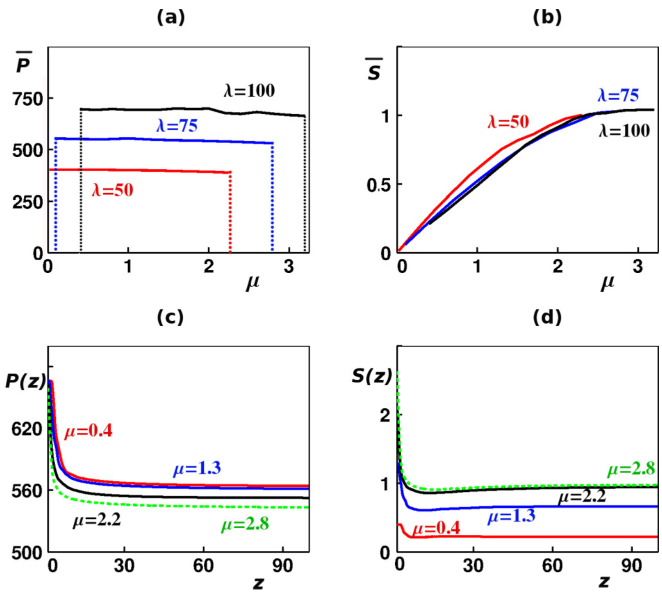

FIG. 2. (Color online) (a) Existence domain of the dipole solitons showing the averaged power beam $\overline{P}$. (b) Averaged fractional spin beam $\overline{S}$ of the dipole solitons. (c) Evolution of the power beam P in propagation for $\lambda = 75$. (d) Evolution of the fractional spin beam S for $\lambda = 75$.

to $z = 100$ over a fixed circular area centered at the origin that has a radius of five times the width of a single soliton. We find that the remaining power of the rotary self-trapped beams becomes practically constant, and it is almost independent of the μ previously imposed. Note that even though fundamental solitons have zero angular momentum, the final generated self-trapped beams do not have a trivial phase structure and hence carry a nonzero beam orbital angular momentum $M = \mathrm{Im} \int \Psi^* \partial_\phi \Psi d\mathbf{r}$. Next, we normalize M to calculate the fractional beam spin $S = M/P$, and we show in Fig. 2(b) the averaged $\overline{S}$ from $z = 50$ to $z = 100$. Note that the fractional beam spin is a monotonic increasing function of μ, and as $\mu \to \mu_{\max}$, $S \to 1$, resembling the case of a single-charged vortex soliton. In [28], Assanto's group demonstrated that the angular momentum depends linearly on the soliton mass of a two-soliton cluster when the angular momentum is conserved. Here we report an example in which even though both P and S suffer a considerable decay from their initial values due to radiation losses, it is still possible to achieve a steady state where either P or S can be stabilized, as shown in Figs. 2(c) and 2(d), respectively, allowing the generation and propagation of stable rotating dipole solitons in nonlocal media by using just fundamental solitons. Moreover, the spin beam achieved at steady state can be controlled by adjusting just the initial relative tilt previously imposed on the fundamental solitons.

IV. INTERACTIONS AMONG SEVERAL SOLITONS: DECAYING INTO ROTATING DIPOLE SOLITONS

For $N = 2$, we have only two forces between solitons, while for a more general case, the number of forces among solitons becomes $N^2 - N$. Thus, the number of initial configurations that can produce rotary structures also increases. Then for $N > 2$, for simplicity, we only report the propagation dynamics for the initial transverse field

$$\Psi(x,y) = \sum_{n=1}^{N} \Psi_n(x_n,y_n) \exp(-i\zeta_n x - i\vartheta_n y) \exp(i\Phi_n), \tag{4}$$

where each soliton is initially located at $x_n = R\cos(\Phi_n)$ and $y_n = R\sin(\Phi_n)$, where R is a radius length, $\Phi_n = 2\pi n m/N$ is the phase distribution imposed to mimic the corresponding phase distribution of a vortex soliton with a topological charge m, and $\sqrt{\zeta_n^2 + \vartheta_n^2} = \mu$ gives the magnitude of an initial tilt imposed on each soliton in the direction $-\widehat{r}$. Similar configurations were studied for the case of soliton clusters [7] and optical necklaces [6], but here we focus on the dynamics that occur when a tilt other than the initial tilt μ is imposed.

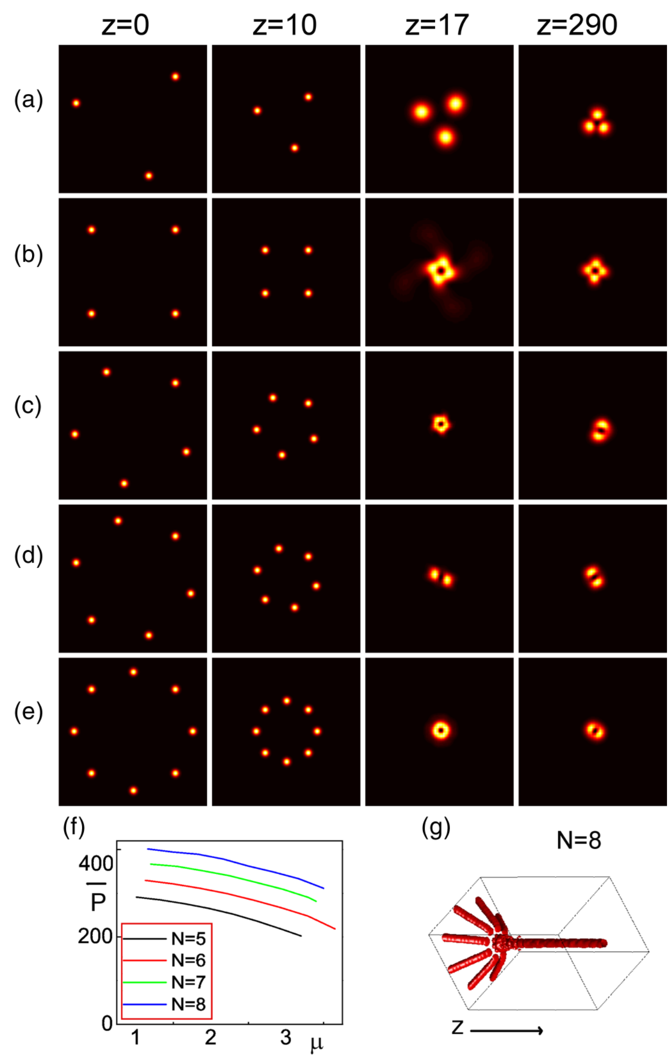

FIG. 3. (Color online) Propagation dynamics of multiple solitons with the parameters $\lambda = 15, \mu = 1$, and $R = 10$ for (a) $N = 3$, (b) $N = 4$, (c) $N = 5$, (d) $N = 6$, and (e) $N = 8$. The profiles are shown in a x-y box of $L \times L$, where $L = 25$ for the first two columns and $L = 10$ for the last two columns. (f) Existence domain for the generation of rotating dipoles from $N = 5$ (lowest power beam $\overline{P}$) to $N = 8$ (highest power beam $\overline{P}$). (g) Three-dimensional representation for propagation dynamics when $N = 8$. (See Supplemental Material [27] to observe propagation.)

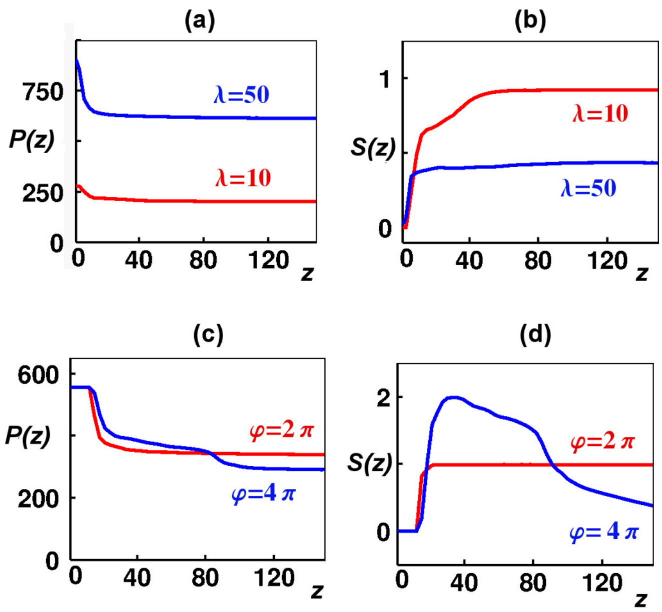

FIG. 4. (Color online) Evolution of (a) power beam P and (b) spin beam S when $N = 4$. For $\lambda = 10$, $\mu = 0.25$ and $R = 4$, and for $\lambda = 50$, $\mu = 0.1$ and $R = 3$. Variation of (c) power beam P and (d) spin beam S when $N = 8$. In both cases $\lambda = 10$, $\mu = 1$, and $R = 9$.

This initial configuration produces stronger interactions between adjacent solitons for larger N as they move towards the center, generating more complex propagation dynamics, as shown in Figs. 3(a)–3(e).

Using $N = 3$ and $N = 4$, it is possible to generate rotating tripoles and quadrupoles, respectively, as expected. However, quite remarkably, we find that for $N \geqslant 5$, after some emission of radiation waves, the fundamental moving soliton configuration given by Eq. (4) decays, generating rotating dipoles that can be stabilized when enough beam power is reached. In Fig. 3(f), we show the corresponding existence region for the generation of these rotating dipoles when $N \geqslant 5$. Thus, for initial configurations given by either Eq. (3) or (4), the final rotary beam more frequently observed in our numerical simulations was by far the rotating dipole, although it was possible to generate other rotary beams. Similar results were observed experimentally by Kivshar's group [29], but for the case of soliton vortices, which transformed into spiraling dipole azimuthons. Here we also report that it is possible to excite structures that resemble a single-charge vortex soliton due to symmetric initial conditions for the specific case of $N = 4$. Figures 4(a) and 4(b) show the corresponding power and S value, respectively. Note that both values tend to a stable value and that the S value is initially zero. We also observe the dynamics of N moving fundamental solitons but with phase distribution $\Phi_n = 4\pi n/N$. We report that in several cases a two-charge vortex soliton is generated, but eventually, it decays into a rotating dipole soliton.

The initially imposed relative phase structure Φ_n is crucial for stable propagation. For example, for $N = 8$, using either $\Phi_n = 2\pi n/N$ or $\Phi_n = 4\pi n/N$, although both configurations have the same initial P and S values, they produce very different values in steady stable state for the power and spin beams, as can be seen in Figs. 4(c) and 4(d), respectively. In both scenarios, either a single- or a two-charge vortexlike soliton is formed at the beginning of propagation. However, only for the single charge does the beam remain stable for longer propagations. Nevertheless, the single-charge vortexlike beam eventually decays again into a rotating dipole soliton.

V. COMPARISON BETWEEN THE GAUSSIAN AND EXPONENTIAL NONLOCAL RESPONSES

It has been demonstrated that while the response function $\mathcal{N}(r)$, defined by the physical process that generates the medium nonlinearity, remains real, positive definite, symmetric, and monotonically decaying, certain physical phenomena, such as the collapse arrest, do not depend strongly on the particular shape of $\mathcal{N}(r)$ [23]. However, other physical phenomena, such as the stability of higher-order nonlinear modes, do depend on the shape of the nonlocal response function [30]. Thus, it is natural and very important to ask if the generation of the rotary beams from moving fundamental solitons reported here can be achieved with another kind of nonlocality response. For comparison with the Gaussian nonlocal model, we report the generation of rotary beams for the case of an exponential response function [31],

$$\mathcal{N}(r) = (1/2\pi\sigma^2)\exp(-r/\sigma); \quad (5)$$

this nonlocal response function arises naturally in the one-dimensional case, where thermal nonlinearities generate this exponential nonlocal response, which allows us to find analytical solutions for several kinds of higher-order solitons [31]. For the two-dimensional case, we have used the Petviashvili relaxation method [26] to obtain the corresponding fundamental solitons, and then the solitons were propagated using a split-step Fourier method in a way similar to what was done for the case of the Gaussian nonlocal response. We choose the exponential nonlocal response as a second model to consider because this response has a noncontinuous first derivative at $r = 0$, which might lead to more instabilities during the soliton propagation. Future work will extend our analysis to cases with more complex nonlocal responses, such as media which possess singularities at $r = 0$ [32], media with asymmetric nonlocal responses [33], media where $\mathcal{N}(r)$ is described by a diffusion-type equation [34], or even media with competing nonlocal cubic and local quintic nonlinearity interactions, which have drawn considerable attention recently [32,35–40] because several popular nonlinear media, such as nematic liquid crystals and Bose-Einstein condensates, can exhibit this type of nonlocal response. However, in this paper, we focus on only the case in which $\mathcal{N}(r)$ is either Gaussian or exponential.

We find that an exponential nonlocal response also allows the generation of rotary beams from fundamental tilted solitons. Similar to the case of the Gaussian response, with the exponential response, it is possible to generate rotating dipoles with at least two fundamental solitons as well as other kinds of rotary self-trapped structures when $N \geqslant 2$. Remarkably, we also corroborate the frequent generation of rotating dipoles in the case in which multiple fundamental tilted solitons are present from the start for a range of initial tilt values. Thus, nonlocality can provide the physical mechanism necessary to stabilize the rotating dipole solitons generated, independent

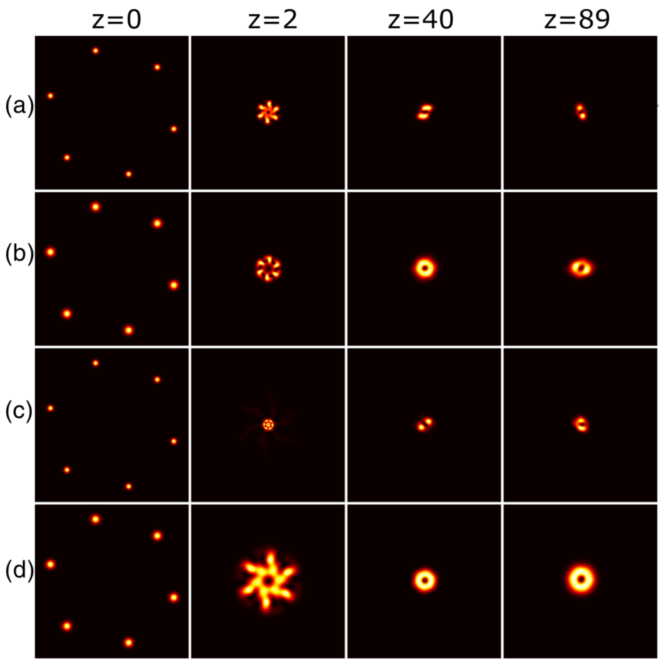

FIG. 5. (Color online) Evolution of six ($N = 6$) fundamental tilted solitons in nonlocal media with a Gaussian [(a) and (c)] and exponential [(b) and (d)] response function $\mathcal{N}(r)$. (a) $\mu = 2.3$, (b) $\mu = 2.3$, (c) $\mu = 2.7$, and (d) $\mu = 2.7$. In all cases $R = 10$ and $P = 220.69$ for each fundamental soliton. The profiles are shown in an x-y box of $L \times L$, where $L = 24$ for the first two columns and $L = 12$ for the last two columns.

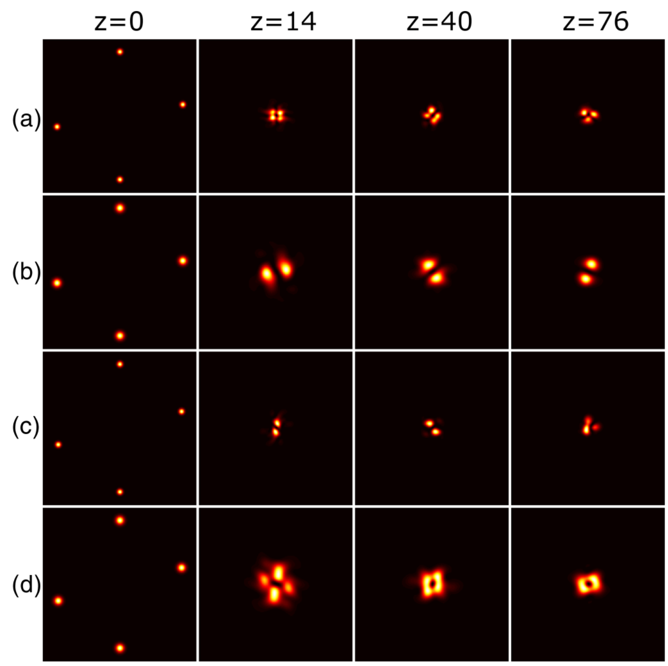

FIG. 6. (Color online) Evolution of four fundamental solitons ($N = 4$) in Gaussian [(a) and (c)] and exponential [(b) and (d)] nonlocal response, where two of the solitons have been initially moved by an angular displacement given by Φ_ϵ. (a) $\Phi_\epsilon = \pi/12$, (b) $\Phi_\epsilon = \pi/12$, (c) $\Phi_\epsilon = \pi/18$, and (d)$\Phi_\epsilon = \pi/18$. For each fundamental soliton, $P = 220.69$ and $\mu = 0.5$. In all cases $R = 10$. The profiles are shown in an x-y box of $L \times L$, where $L = 24$ for the first column and $L = 12$ for the other three columns.

of the particular response function $\mathcal{N}(r)$. However, we also find in our numerical simulations that identical initial physical conditions, i.e., an identical number of initial fundamental solitons N, power P, distribution of the solitons given by (x_n, y_n), relative phase, and the corresponding momentum imposed by (ζ_n, ϑ_n), can produce quite similar nonlinear dynamics with either a Gaussian [Fig. 5(a)] or an exponential [Fig. 5(b)] nonlocal response function. However, we also observed cases where identical initial conditions with either a Gaussian [Fig. 5(c)] or an exponential [Fig. 5(d)] nonlocal response function generate different rotary beams. Thus, we find that, in principle, predict the final rotary beam generated remains complex, mainly due to the strong decay of initial power by the emission of radiation waves, and we find that the final rotary beam generated does depend on the kind of $\mathcal{N}(r)$ used.

Finally, we also report that the stable rotary beams generated here depend strongly on the initial position of the tilted solitons. In the particular case of $N = 4$, using the initial configuration given by Eq. (4), we never observed the formation of stable rotating dipoles, but this situation changes drastically if we slightly modify the initial position of two fundamental solitons, for example, Φ_1 and Φ_3, by adding an angular displacement Φ_ϵ, as shown in Fig. 6. Under this new initial asymmetrical configuration, it is possible to generate rotating dipole solitons. The final rotary beam generated depends again on the particular choice of $\mathcal{N}(r)$. In our simulations, under this initial configuration, we only observed the generation of rotating dipoles for the case with an exponential nonlocal response [Figs. 6(b)–6(d)], even though the Gaussian nonlocal response is normally considered more appropriate for stabilizing self-trapped beams.

Thus, we find that the minimum critical number of initial solitons which is required to develop a stable rotary dipole structure is $N = 2$, but for an accurate prediction of the rotary beam generated, in addition to the physical variables of the fundamental solitons such as their power, initial momentum, and position and the corresponding distribution phase, the nonlocal response $\mathcal{N}(r)$ plays a very crucial and quite complex role in generating stable rotating dipole solitons. Even though we cannot claim an accurate stability analysis from our purely numerical simulations, we do observe the generation of rotating dipoles that remain stable during propagations up to $z = 500$ for either the Gaussian or the exponential nonlocal response.

VI. CONCLUSIONS

In summary, we study the morphing of fundamental moving solitons into basic rotary self-trapped structures. We found that nonlocality provides the physical mechanism that is crucial for the generation of stable rotating dipoles from fundamental moving solitons. The rotary self-trapped beams can be generated with at least two transverse counterpropagating solitons. Using a higher number of solitons can also produce several complex self-trapped structures, but in many cases, remarkably, a stable dipole soliton is finally generated. We demonstrate that this phenomenon is observed not only for the case of a Gaussian nonlocal response but also for an exponential nonlocal response. However, we also find that Gaussian and exponential nonlocal responses may produce very different soliton propagation dynamics even if they have the same initial conditions, demonstrating a certain complexity to predicting the generation of rotating dipoles or another kind of rotary beam, mainly due to radiation losses. We hope that this work helps develop new tools to produce rotary self-trapped nonlinear beams.

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