Single-photon modulation by the collective emission of an atomic chain

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We study the collective spontaneous emission of a linear atomic chain excited by a single photon. The interaction between the atoms and the common vacuum field can significantly change the eigenenergy and the spontaneous emission rate of the system. Due to the dipole-dipole interactions, the system prepared in a single-photon timed Dicke state is the superposition of superradiant and subradiant eigenstates that can have a nonexponential decay dynamics. We can tune the frequency and linewidth of the superradiant and subradiant emission from a timed Dicke state by changing the direction of the atomic dipole moment or the atomic separation. In addition, the emission direction of the superradiant and subradiant photons also depends on the polarization of the atoms.

I. INTRODUCTION

Since the pioneering work of Dicke [1], the collective spontaneous emission by an ensemble of identical atoms has been extensively studied [2–5]. The Dicke superradiance provides a valuable example for studying the many-body physics of photons and atoms [6]. The Dicke symmetric state of maximum cooperation has decay rate N times the single-atom decay rate [1]. In this model the atoms are assumed to be continuously distributed in the space. On the other hand, optically active materials with regular spatial distribution are also very important, mainly for their application in quantum information processing or quantum simulations for the condensed-matter many-body effects [7,8]. The general properties of the collective emission of an atomic chain have been widely studied and were found to show superradiant and subradiant emission and quantum state generation [9–11].

In addition to an initially fully inverted system, an atomic ensemble with single-photon excitation has also been widely studied [12–15]. These systems play an important role in quantum memory and quantum information science [16–18]. The collective light emission of a regular atomic chain with one-photon excitation has also been studied [19–22]. It is shown that the collective damping rate of the symmetric state depends on the chain length and the lattice constant. In addition, the dipole-dipole interaction between the atoms can significantly change the eigenstates of the system. The directionality of the emission pattern from each eigenstate with one-photon excitation has been studied and it was found that the collective dipole-dipole interaction can modify the emission direction [19]. However, the question about how the emission spectrum and the emission direction of an atomic chain change by altering the atomic transition dipole moment and the atomic separation has not been addressed so far.

In this paper we study the collective emission properties of a linear atomic chain including the effects of the dipole-dipole interaction. We first show that the spontaneous decay of an atomic chain prepared in a timed Dicke state can be nonexponential due to the coexistence of the superradiant and the subradiant modes [23–30]. The emission spectrum of a timed Dicke state shows that we can tune the frequency and the linewidth of the emitted superradiant and

subradiant photon by controlling the polarization of the atoms and the atomic separation. The emission direction of the superradiant and subradiant states also depends on the polarization of the atoms. The controllability may have important applications for the quantum information and precision measurement [31–33].

This paper is organized as follows. In Sec. II we introduce our model system and calculate theoretically the collective emission rate and emission spectrum of a timed Dicke state including the dipole-dipole interaction effects. In Sec. III we consider the example of five atoms to show how we can tune the frequency, linewidth, and direction of an emitted photon from a timed Dicke state. In Sec. IV we summarize our results.

II. THEORY

We consider a one-dimensional atomic chain with atomic positions $\vec{r}_n = na\vec{e}_z$ ($n = 1, \ldots, N$) in which n denotes the nth atom, a is the atomic separation, $\vec{e}_z$ is the direction along the atomic chain, and N is the total number of atoms (Fig. 1). A linearly polarized photon with the wave vector $\vec{k}$ is incident on the atomic chain. We assume that the frequency of the incident photon is resonant with the two atomic levels, i.e., $|\vec{k}| = k_a = \omega_a/c$, where ω_a is the transition frequency and c is the speed of light. By absorbing the single photon, the system can be prepared in the one-photon timed Dicke state, which is given by [34,35]

$$|\psi_0\rangle = \frac{1}{\sqrt{N}} \sum_{n=1}^{N} e^{i\vec{k}\cdot\vec{r}_n} |g_1 \cdots e_n \cdots g_N\rangle, \qquad (1)$$

where $|g\rangle$ ($|e\rangle$) is the ground (excited) state, $\vec{k}\cdot\vec{r}_n = nka\cos\theta$, and θ is the angle between the incident direction and the direction of the atomic chain. From Eq. (1) we see that the system can be prepared in different initial states by changing the angle θ or the atomic distance a. Let us take the two-atom system as an example. If $\vec{k}\cdot\vec{r}_{12} = 0$, the prepared initial state is $(|ge\rangle + |eg\rangle)/\sqrt{2}$, which is a symmetric state. If $\vec{k}\cdot\vec{r}_{12} = \pi$, the prepared initial state is $(|ge\rangle - |eg\rangle)/\sqrt{2}$, which is an antisymmetric state.
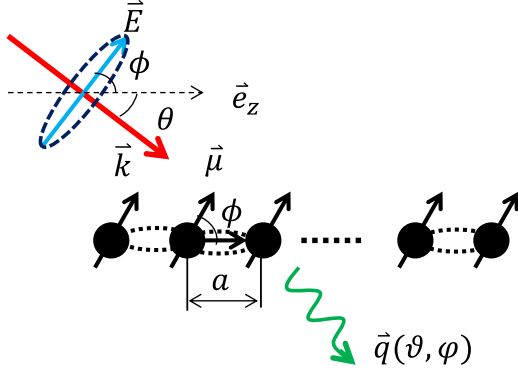
FIG. 1. (Color online) Collective emission of a linear atomic chain. A single photon is incident with angle θ and polarization angle ϕ with respect to the direction of the atomic chain $\vec{e}_z$. The direction of the transition dipole moment is aligned to the polarization of the incident photon. Here a is the nearest-neighbor atomic separation. A photon is emitted in a direction described by $\vec{q}(\vartheta,\varphi)$.

After preparing the system in the single-excitation state, the system undergoes collective spontaneous decay. The Hamiltonian of the atomic system is given by

$$H = \hbar\omega_a \sum_n S_n^+ S_n^- + \hbar\Omega_{dd} \sum_{\langle m,n\rangle} S_m^+ S_n^-, \quad (2)$$

where the first term is the energy of the atoms with ω_a being the angular frequency of the two atomic levels and S_n^+ (S_n^-) is the raising (lowering) operator of the nth atom and the second term is the nearest-neighbor dipole-dipole interaction energy. Here $\langle m,n\rangle$ denotes the nearest-neighbor pairs and Ω_{dd} is the dipole-dipole interaction energy, which is given by [24]

$$\Omega_{dd}(a,\phi) = \frac{3\gamma}{4}\left\{(1-3\cos^2\phi)\left[\frac{\sin(ka)}{(ka)^2}+\frac{\cos(ka)}{(ka)^3}\right]\right.$$
$$\left. -(1-\cos^2\phi)\frac{\cos(ka)}{ka}\right\}, \quad (3)$$

where $\gamma = \omega_a^3\mu^2/3\pi\epsilon_0\hbar c^3$ is the single-atom spontaneous decay rate (μ is the transition dipole moment and ϵ_0 is vacuum permittivity) and ϕ is the angle between the transition dipole moment and the direction of the atomic chain. The dipole-dipole interaction depends on the atomic separation and the direction of the transition dipole moment. We assume that the atoms do not have a permanent dipole moment, which is the usual case. We can control the direction of the transition dipole moment of the atomic chain by either changing the polarization of the pumping laser or applying an external linearly polarized electric field. Changing the direction of the transition dipole moment can change the dipole-dipole interaction and then modify the emission properties of the atomic chain. Here the nearest-neighbor approximation is applied since $\Omega_{dd} \propto 1/r^3$, which decays very rapidly as the distance increases.

The Hamiltonian in Eq. (2) can be diagonalized by applying the Jordan-Wigner transformation. The eigenenergy and eigenfunction in the one-photon excitation subspace are given by [4]

$$E_j = \hbar\omega_0 + 2\hbar\Omega_{dd}\cos\left(\frac{j\pi}{N+1}\right), \quad (4)$$

$$|\psi_j\rangle = \sqrt{\frac{2}{N+1}}\sum_{n=1}^{N}\sin\left(\frac{jn\pi}{N+1}\right)|g_1\cdots e_n\cdots g_N\rangle, \quad (5)$$

where $j = 1,2,\ldots,N$. If the atomic distance is comparable to or larger than the atomic transition wavelength, the dipole-dipole interaction energy is negligible and the one-photon excitation sublevels are degenerate. However, if the atomic distance is much smaller than the atomic transition wavelength, the dipole-dipole interaction energy is significant and the one-photon excitation sublevels split into N sublevels.

The initial timed Dicke state can be decomposed as

$$|\psi_0\rangle = \sum_{j=1}^{N}C_j(a,\theta)|\psi_j\rangle, \quad (6)$$

where $C_j(a,\theta)$ is given by

$$C_j(a,\theta) = \sqrt{\frac{2}{N(N+1)}}\sum_{n=1}^{N}e^{ikna\cos\theta}\sin\left(\frac{jn\pi}{N+1}\right). \quad (7)$$

The probability for the system being in the eigenstate $|\psi_j\rangle$ is given by

$$P_j(a,\theta) = |C_j(a,\theta)|^2 = \frac{1}{N} + \frac{4}{N(N+1)}\sum_{m<n=1}^{N}\sin\left(\frac{jm\pi}{N+1}\right)$$
$$\times \sin\left(\frac{jn\pi}{N+1}\right)\cos[(m-n)ka\cos\theta], \quad (8)$$

where we can see that the probability in each eigenstate depends on the atomic separation a and the incident angle θ.

The interaction between the atom and the vacuum field causes the system to decay and the interaction Hamiltonian is given by [36]

$$H_I = \sum_{\vec{q}\lambda}V(\vec{\mu},\vec{q},\lambda), \quad (9)$$

with

$$\hat{V}(\vec{\mu},\vec{q},\lambda) = -i\sqrt{\frac{\hbar cq}{2\epsilon_0 V}}(\vec{u}\cdot\vec{e}_{\vec{q}\lambda})$$
$$\times \sum_{n=1}^{N}(a_{\vec{q}\lambda}S_n^+e^{i\vec{q}\cdot\vec{r}_n}-a_{\vec{q}\lambda}^\dagger S_n^-e^{-i\vec{q}\cdot\vec{r}_n}) \quad (10)$$

being the interaction between the atomic chain and a vacuum photon with wave vector $\vec{q}$ and polarization $\vec{e}_{\vec{q}\lambda}$. Here $a_{q\lambda}^\dagger$ ($a_{q\lambda}$) is the creation (annihilation) operator of the vacuum photons. The spontaneous emission rate for each eigenstate can be calculated by the Fermi golden rule, which reads [19,20]

$$\Gamma_j = \frac{2\pi}{\hbar}\sum_{\vec{q},\lambda}|\langle f|\hat{V}(\vec{\mu},\vec{q},\lambda)|\psi_j\rangle|^2\delta(E_a-E_{ph}), \quad (11)$$

where $|f\rangle = |g\cdots g,1_{\vec{q},\lambda}\rangle$ is the state when all the atoms are in the ground state and one photon is emitted, E_a is the atomic

transition energy, and E_{ph} is the photon energy. The emission rate per unit solid angle for one specific direction is given by [19]

$$\frac{d\Gamma_j(\hat{q})}{d\Omega} = \frac{2\pi}{\hbar}\frac{V}{(2\pi)^3}\int_0^\infty q^2 dq |\langle f|\hat{V}|\psi_j\rangle|^2 \delta(E_a - E_{ph})$$

$$= \frac{V\omega^2}{4\pi^2\hbar^2 c^3}|\langle f|\hat{V}|\psi_j\rangle|^2|_{|\vec{q}|=k_a}. \qquad (12)$$

On substituting the eigenstate in Eq. (5) into Eq. (12) we obtain

$$\frac{d\Gamma_j(\hat{q})}{d\Omega} = \frac{\omega^3\mu^2}{8\pi^2 c^3\hbar\epsilon_0}[1 - (\hat{q}\cdot\hat{\mu})]G(\vec{q},j,N), \qquad (13)$$

where $\hat{q} = (\sin\vartheta\cos\varphi, \sin\vartheta\sin\varphi, \cos\vartheta)$ is the direction of the emitted photon and $\hat{\mu} = (\sin\phi, 0, \cos\phi)$ is the direction of the transition dipole moment. The term in the square brackets is the directional distribution of a single-atom emission and $G(\hat{q},j,N)$ is the collective effect on the emission direction, which is given by [19]

$$G(\vec{q},j,N) = \frac{1 - (-1)^j\cos[(N+1)\vec{q}\cdot\vec{a}]}{\left\{\cos(\vec{q}\cdot\vec{a}) - \cos\left[\frac{j\pi}{N+1}\right]\right\}^2}\cdot\frac{\sin^2\left[\frac{j\pi}{N+1}\right]}{N+1}. \qquad (14)$$

Summing over all directions, the total decay rate is given by [19]

$$\frac{\Gamma_j}{\gamma} = 1 + \frac{4}{N+1}\sum_{n<m=1}^N \sin\left(\frac{mj\pi}{N+1}\right)$$

$$\times \sin\left(\frac{nj\pi}{N+1}\right)F[(m-n)ka,\phi], \qquad (15)$$

where

$$F(x,\phi) = \frac{3}{2}\left\{\left(\frac{\cos x}{x^2} - \frac{\sin x}{x^3}\right)(1 - 3\cos^2\phi)\right.$$

$$\left. + \frac{\sin x}{x}(1 - \cos^2\phi)\right\} \qquad (16)$$

describes the collective damping. We can see that the decay rate for each eigenstate depends on the polarization angle ϕ and the atomic separation a. We can therefore control the emission pattern by tuning the direction of the transition dipole moment and the atomic separations.

The probability in the excited state decays with time as

$$P_e(t) = \sum_{j=1}^N P_j(a,\theta)e^{-\Gamma_j t}, \qquad (17)$$

where P_j is given by Eq. (8) if the system is initially prepared in a timed Dicke state. The collective emission spectrum of the atomic chain can be evaluated by summing over the emission from all the eigenstates and is given by

$$S(\omega) \propto \sum_{j=1}^N P_j(a,\theta)\frac{\Gamma_j/2}{(\omega - \omega_j)^2 + (\Gamma_j/2)^2}. \qquad (18)$$

III. NUMERICAL EXAMPLE

As an example we consider the case when $N = 5$. The energy splitting as a function of the polarization angle ϕ of

the incident photon is shown in Fig. 2(a). Here the separation between the atoms is assumed to be $\lambda/20$. We can see that the one-photon excitation eigenenergy splits into five levels and the splitting can be tuned by changing the angle ϕ. This can be used to tune the frequency of the emitted photon. The energy sublevels can merge at two special polarization angles where the dipole-dipole interaction is eliminated.

The population in each eigenstate as a function of incident angle θ with $a = \lambda/20$ and $\phi = \pi/2$ is plotted in Fig. 2(b). The population distribution among different eigenstates can be modulated by the incident angle θ. When $\theta = \pi/2$, only two eigenstates $j = 1$ and 3 are excited. However, when $\theta = 0$ all eigenstates have a non-negligible population.

The decay rate for each eigenstate as a function of ka with $\theta = \pi/2$ and $\phi = \pi/2$ is shown in Fig. 2(c). We see that different eigenstates have different decay rates and they change as the atomic distance changes. In the limit of zero atomic distance, there is one superradiant eigenstate ($j = 1$) whose decay rate is about 4.65γ. The $j = 3$ eigenstate is subradiant with a decay rate of about $\gamma/3$. The other three eigenstates have a negligible decay rate. As the atomic distance increases, the superradiant state may become subradiant and the subradiant states may become superradiant.

The decay of the excited state as a function of time is shown in Fig. 2(d). The red dashed line is the single-atom decay $e^{-\gamma t}$ while the black solid line is the traditional superradiant decay $e^{-5\gamma t}$. The other two lines with symbols include the dipole-dipole interaction effects. We can see that the collective spontaneous decay of this system can be a nonexponential function, faster at the beginning and slower at the end. The reason is that the majority of the population in the excited states at the beginning is in the superradiant eigenstate, while at the end the majority of the population in the excited states is in the subradiant states. The incident angle θ and the polarization angle ϕ also affect the collective decay dynamics. The decay dynamics when $\theta = 0$ is slower than the case when $\theta = \pi/2$ because more subradiant states are excited when $\theta = 0$.

The collective emission spectrum of the atomic chain for different polarization angles when $a = \lambda/20$ and $\theta = \pi/2$ is shown in Fig. 3(a). Although the incident photon has a single frequency, the emitted photon can have different frequencies due to the dipole-dipole interaction. In this example we notice that the emitted photon can be either a superradiant photon with a superbroad linewidth or a subradiant photon with a supernarrow linewidth. When $\phi = 0$, the superradiant photon has a linewidth of about 4.5γ, while the subradiant photon has a linewidth of about $\gamma/50$. In addition, we can tune the polarization angle to shift the frequency of the superradiant and the subradiant photon. The frequency of the superradiant photon can be shifted from the red side of the resonant frequency to the blue side by just increasing the polarization angle ϕ. The frequency of the subradiant photon can also be shifted from the blue side of the resonant frequency to the red side. Since the superradiant photon and the subradiant photon have different frequencies, we can use a filter to separate them. From Eq. (3) we see that the dipole-dipole interaction depends on the angle ϕ. There is an angle when the superradiant and subradiant photons merge. The crossover angle is the angle when the dipole-dipole interaction vanishes.
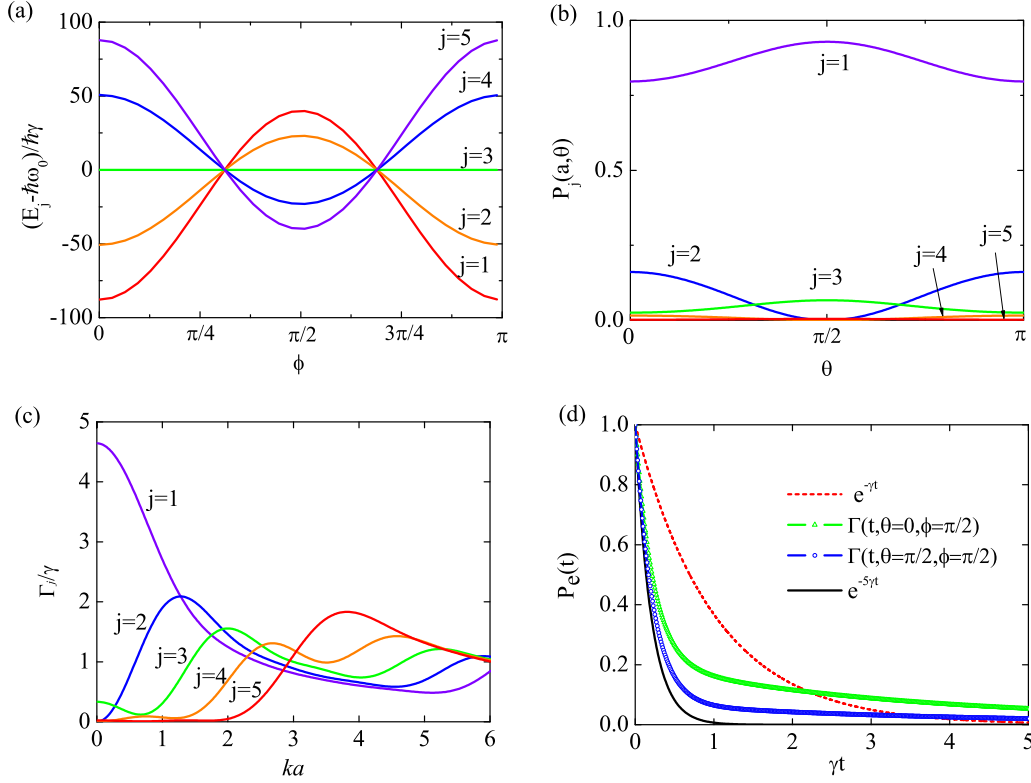
FIG. 2. (Color online) Plots of the $N = 5$ example: (a) one-photon excitation eigenenergy splitting as a function of the polarization angle ϕ with $\theta = \pi/2$ and $a = \lambda/20$, (b) population in each eigenstate, when the system is prepared in the timed Dicke state, as a function of the incident angle θ, (c) collective decay rate for each eigenstate as a function of atomic distance with $\theta = \pi/2$ and $\phi = \pi/2$, and (d) probability in the excited-state decays with time when $a = \lambda/20$.

In addition, we can also control the emitted spectrum by controlling the separation between the atoms. An example is shown in Fig. 3(b). Here we change the atomic separation from 0.08λ to 0.04λ but keep $\theta = \pi/2$ and $\phi = \pi/2$. When we decrease the atomic distance, the frequency of the superradiant photon is blueshifted, while the frequency of the subradiant photon is redshifted. Meanwhile, the linewidth of the superradiant photon increases from 4γ to 4.5γ and the linewidth of the subradiant photon increases from $\gamma/100$ to $\gamma/50$.

The direction of the emitted photon can be calculated from Eqs. (13) and (14). Here we numerically show the direction distribution of the superradiant and subradiant photons for

two polarizations. The emission rate as a function of emission angle is shown in Fig. 4. The results when all the atomic dipole moments are along the atomic chain are shown in Fig. 4(a) with j being odd and in Fig. 4(b) with j being even. We see that when $j = 1$ the emission rate is strongest in the perpendicular direction, which is similar to the single-atom case. When $a \ll \lambda$, $j = 1$ corresponds to the superradiant state, so the superradiant photon emits in the direction perpendicular to the dipole moment with larger probability. However, when $j = 2$, which is the subradiant state when $a \ll \lambda$, the photon tends to emit in the direction about 45° away from the atomic chain. The probability that the photon emitted from the $j = 2$ state to the
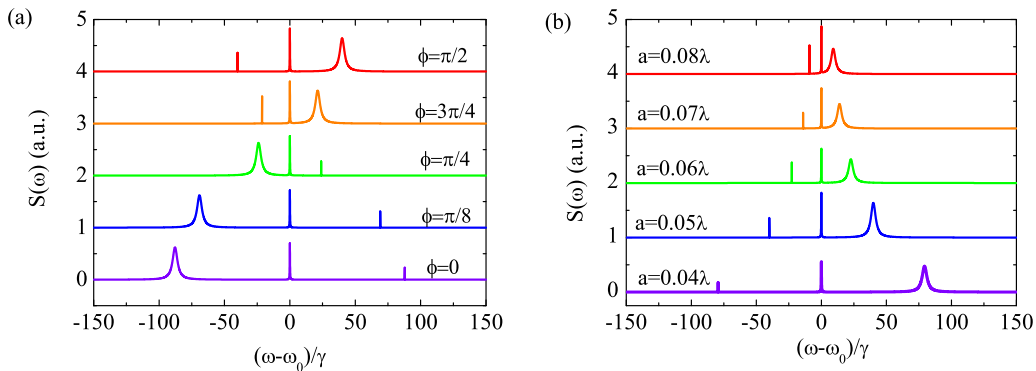

FIG. 3. (Color online) (a) Collective emission spectrum of the atomic chain for different polarization angles ϕ when $a = \lambda/20$ and $\theta = \pi/2$. (b) Collective emission spectrum for different atomic separation a when $\theta = \phi = \pi/2$.
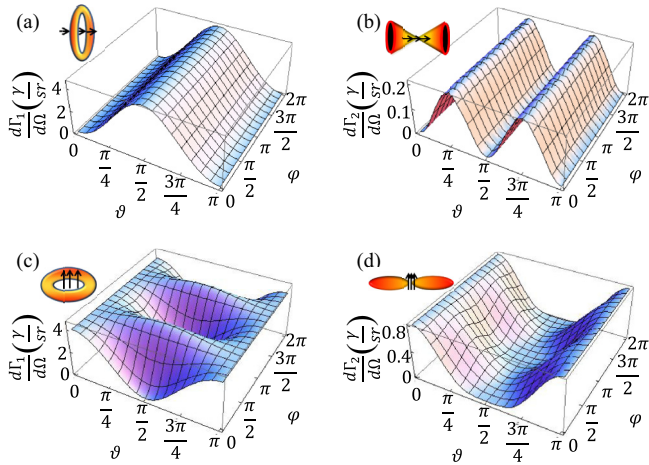
FIG. 4. (Color online) Emission rate per solid angle as a function of emission angles. (a) and (b) All the transition dipole moments are along the atomic chain with (a) $j = 1$ and (b) $j = 2$. (c) and (d) All the transition dipole moments are perpendicular to the atomic chain with (c) $j = 1$ and (d) $j = 2$.
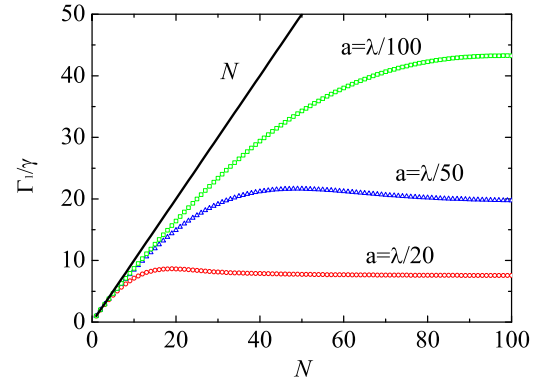

FIG. 5. (Color online) Superradiant decay rate as a function of the number of atoms for different atomic separations, with $\phi = \pi/2$ and $\theta = \pi/2$.

perpendicular direction is almost zero. Actually, the emission pattern from the odd j states is similar to the $j = 1$ case and the emission pattern from the even j states is similar to the $j = 2$ case. Therefore, the photon observed from the perpendicular direction must come from the odd j state. The results for when the atomic transition dipole moments are perpendicular to the atomic chain are shown in Figs. 4(c) and 4(d). Similar to the previous cases, the photon emitted from the odd j states tends to emit in the direction perpendicular to the dipole moment. However, the photon emitted from the even j states can only emit in the direction along the atomic chain.

The superradiant decay rate as a function of the number of atoms for different atomic separations is shown in Fig. 5. For all three cases, the decay rate increases as the number of atoms increase, but it saturates at some atomic number. For a fixed atomic separation, the decay rate deviates from the traditional superradiant decay rate $N\gamma$ significantly at large atomic number. If we want to increase the linewidth of the

superradiant photon, we may increase the number of atoms and while decreasing the atomic separation. In the limit $a \to 0$, the decay rate can approach $N\gamma$.

IV. SUMMARY

We studied the collective spontaneous decay of a linear atomic chain including the dipole-dipole interaction effects. We showed that the spontaneous decay of an atomic chain prepared in a timed Dicke state can be nonexponential due to the coexistence of the superradiant and the subradiant eigenmodes. We can generate either the superradiant photon with a linewidth much broader than that of the incident photon or the subradiant photon with a linewidth much narrower than that of the incident photon. The frequency, linewidth, and direction of the superradiant and subradiant photons can be simply tuned by changing the direction of the atomic transition dipole moment or the atomic separation. This tunability may have important applications in quantum information, quantum computation, and quantum sensing.

ACKNOWLEDGMENT

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