Controlling the magnetic-field sensitivity of atomic-clock states by microwave dressing

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We demonstrate control of the differential Zeeman shift between clock states of ultracold rubidium atoms by means of nonresonant microwave dressing. Using the dc field dependence of the microwave detuning, we suppress the first- and second-order differential Zeeman shift in magnetically trapped ⁸⁷Rb atoms. By dressing the state pair $5S_{1/2}$ F = 1, $m_F = -1$ and F = 2, $m_F = 1$, a residual frequency spread of <0.1 Hz in a range of 100 mG around a chosen magnetic offset field can be achieved. This is one order of magnitude smaller than the shift of the bare states at the magic field of the Breit-Rabi parabola. We further identify *double magic* points, around which the clock frequency is insensitive to fluctuations both in the magnetic field and in the dressing Rabi frequency. The technique is compatible with chip-based cold-atom systems and allows the creation of clock and qubit states with reduced sensitivity to magnetic-field noise.

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I. INTRODUCTION

The sensitivity of atomic transitions to external field perturbations represents a major limitation for the accuracy and stability of atomic clocks [1,2] and for the time of quantum-information storage in ultracold atoms and atomic gases [3–5]. Electromagnetic-field fluctuations and inhomogeneous trapping potentials give rise to temporal and spatial variations of atomic transition frequency. A common approach to reduce the frequency broadening observed in the preparation and readout of atomic superposition states is the use of "magic" magnetic fields [6,7] and wavelengths [8-10] for which the differential Zeeman and Stark shifts of a state pair are minimized, respectively. In addition, density-dependent collisional shifts [11,12] and the effect of identical spin rotation [13–15] have been shown to counteract inhomogeneous dephasing of superposition states in trapped atomic clouds. Possible realizations of cold-atom quantum memories on atom chips [15–19] and chip-based atomic clocks [12,20–22] have to face additional perturbations due to the proximate solid surface [23]. Controlling differential shifts between clock states is one of the key requirements for the realization of a coherent interface between cold atoms and solid-state quantum electronic circuits [24–32]. It was recently shown that Rydberg states can be rendered insensitive to small variations of electric fields by microwave (MW) dressing [33]. Previously, radio-frequency dressing of nuclear spins has been proposed to cancel differential Zeeman shifts between optical clock transitions [34]. Microwave fields have further been used to suppress the magnetic-field dependency of qubit states in trapped ions [35] and nitrogen-vacancy centers with both pulsed [36,37] and continuous [38,39] decoupling schemes.

Here, we demonstrate the control and suppression of the differential Zeeman shift between atomic qubit states up to second order by microwave dressing, thereby reducing the magnetic-field sensitivity of the clock transition frequency. The technique does not require the state pair to be close to magic magnetic fields but can be applied for a wide range of chosen magnetic offset fields in the trap. By dressing the state pair $5S_{1/2}$ F = 1, $m_F = -1$ and F = 2, $m_F = 1$ of ⁸⁷Rb, we demonstrate that a variation of <0.1 Hz over a magnetic field range of >100 mG can be achieved, one order of magnitude less than the differential shift of the bare states around the magic offset field. In addition, we demonstrate the existence of points where this dressing becomes insensitive to fluctuations in the dressing Rabi frequency, enabling the generation of noise-protected qubit states. Our model and experimental results show that the frequency of an atomic clock can be engineered by microwave dressing to achieve arbitrary curvatures, e.g., nearly zero differential shift, around a given magnetic offset field.

II. MICROWAVE DRESSING OF ATOMIC TRANSITIONS

In static magnetic fields, the degeneracy of the hyperfine levels of ground-state alkali-metal atoms is lifted according to the Breit-Rabi formula [40]. In small and intermediate fields, the Zeeman effect can be expanded in terms linear and quadratic in the magnetic-field strength. The interaction of the atom with nonresonant ac electromagnetic fields leads to the ac shift of the levels, which depends on the detuning, e.g., $\Delta E \propto \Omega_{dress}^2/\Delta_{dress}$. As this detuning depends on the dc Zeeman shift of the levels, a suitable choice of the microwave field allows for compensation of spatial and temporal variations of the differential Zeeman shift.

While the technique presented here can be applied for all alkali-metal elements, we now discuss this for the specific case of ⁸⁷Rb. The two-photon transition $5S_{1/2}$ $F = 1, m_F = -1 \rightarrow F = 2, m_F = 1$ is commonly used as an atomic-clock transition for magnetically trapped ⁸⁷Rb. Both states exhibit nearly the same first-order Zeeman shift, starkly reducing the sensitivity of the transition to magnetic-field fluctuations and making the two states ideal candidates as atomic qubit states. The energy of the two states $|0\rangle \equiv 5S_{1/2}$ $F = 1, m_F = -1$ and $|1\rangle \equiv 5S_{1/2}$ $F = 2, m_F = 1$ in a magnetic field of magnitude *B* is given by

$$E_0/\hbar = \mu_1 B - 3\beta B^2 \tag{1}$$

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FIG. 1. (Color online) (a) Ground-state hyperfine structure and Zeeman sublevels of ⁸⁷Rb in a magnetic field. A microwave field of frequency ω_{dress} and Rabi frequency Ω_{dress} is used for dressing the clock transition. (b) The atomic transition is probed by means of Ramsey interferometry. A two-photon pulse with $\omega_{MW} \approx 6.833$ GHz and $\omega_{RF} \approx 2$ MHz is used to drive the transition. (c) Schematic of the experimental sequence. A $\pi/2$ pulse is used to prepare the atoms in a superposition state $1/\sqrt{2} \times (|0\rangle + |1\rangle)$. After a variable hold time T_{Ramsey} , the interferometer is closed by the application of a second $\pi/2$ pulse and the population of the two states, oscillating with frequency δ , is measured. The dressing field is left on throughout the interferometer sequence.

and

$$E_1/\hbar = \mu_2 B + 3\beta B^2 + \omega_0,$$
 (2)

where $\mu_1 = 2\pi \times 702.37 \text{ kHz/G}$, $\mu_2 = 2\pi \times 699.58 \text{ kHz/G}$, $\beta = 2\pi \times 71.89 \text{ Hz/G}^2$, and $\omega_0 = 2\pi \times 6.8346826109 \text{ GHz}$ [41] is the frequency difference of the two states in the absence of any fields. The energy difference between the two levels can be expressed by

$$\Delta E_{0,1}/\hbar = 6\beta (B - B_0)^2 + 2\pi \times 6.8346781136 \text{ GHz}, \quad (3)$$

where $B_0 \approx 3.229$ G is the so-called magic offset field [42]. Using microwave dressing of the Zeeman sublevels with an appropriate frequency ω_{dress} and Rabi frequency Ω_{dress} , the second-order Zeeman shift can be compensated for.

The microwave field leads to a correction of the form

$$\Delta E_{\mathrm{dress},i} = \hbar \sum_{i,\alpha} \frac{\Omega_{i,\alpha}^2}{\Delta_{i,\alpha}} \tag{4}$$

for both of the states *i*, where $\alpha = \sigma_+, \sigma_-, \pi$ denotes all the possible polarizations of the dressing field, for which the relevant detuning Δ_{α} and Rabi frequency Ω_{α} need to be taken into account for each. If we consider a microwave field $B_{\text{dress}} \cos(\omega_{\text{dress}}t)$, which is linearly polarized perpendicular to the quantization axis (given by the magnetic offset field), the situation is simplified, as we only need to take into account σ_+ and σ_- transitions, as sketched in Fig. 1(a). In the rotating-wave approximation, the Hamiltonian relevant for the two states is

$$H_{0} = \hbar \begin{bmatrix} 0 & \sqrt{3}\Omega_{\text{dress}} & \frac{1}{\sqrt{2}}\Omega_{\text{dress}} \\ \sqrt{3}\Omega_{\text{dress}} & -\Delta_{1} & 0 \\ \frac{1}{\sqrt{2}}\Omega_{\text{dress}} & 0 & -\Delta_{2} \end{bmatrix}$$
(5)

and

$$H_1 = \hbar \begin{bmatrix} 0 & \sqrt{\frac{3}{2}} \Omega_{\text{dress}} \\ \sqrt{\frac{3}{2}} \Omega_{\text{dress}} & -\Delta_3 \end{bmatrix},$$
(6)

where we have defined Ω_{dress} as state-independent Rabi frequency

$$\Omega_{\rm dress} = \frac{1}{2\sqrt{2}\hbar} \,\mu_B \,g_F \,|B_{\rm dress}|,\tag{7}$$

and the values of the detunings in Eqs. (5) and (6) and Fig. 1(a) are given by

$$\Delta_1 = \Delta_{\rm dress} + (\mu_1 + 2\mu_2)B - 3\beta B^2, \tag{8}$$

$$\Delta_2 = \Delta_{\rm dress} + \mu_1 B - 7\beta B^2, \tag{9}$$

$$\Delta_3 = \Delta_{\rm dress} - \mu_2 B - 7\beta B^2, \tag{10}$$

where $\Delta_{dress} = \omega_{dress} - \omega_0$. With this notation, we can write the frequency difference between the two states as

$$\Delta E_{0,1}/\hbar = \omega_0 + (\mu_2 - \mu_1)B + 6\beta B^2 + \cdots + \Omega_{dress}^2 \left(\frac{3}{\Delta_1(B)} + \frac{1/2}{\Delta_2(B)} - \frac{3/2}{\Delta_3(B)}\right).$$
(11)

For a given offset field, it is now possible to find numerical solutions ω_{dress} and Ω_{dress} for which the first and second derivatives of Eq. (11) with respect to *B* disappear; i.e., the Zeeman shift of the transitions around that offset field is canceled up to second order. This is illustrated in Fig. 2, where the frequency difference is plotted as a function of the magnetic field for different values of the center of the plateau, B_{center} . For each curve, different optimized values for ω_{dress} and Ω_{dress} were calculated. The choice of B_{center} is completely arbitrary within the limits of Rabi frequencies Ω_{dress} that are achievable in experimental conditions.



FIG. 2. (Color online) Calculated frequency difference of the clock transition as a function of the magnetic field. The Breit-Rabi parabola for the case without the dressing field is plotted in red. The three black curves show the cancellation of the B_{off} dependence around three different central values B_{center} . For an arbitrary value of B_{center} , the optimal detuning and Rabi frequency can be calculated.

III. EXPERIMENTAL PROCEDURE

The measurements are taken with atomic clouds magnetically trapped on a superconducting atom chip. Atoms are loaded into this trap as follows [15]: an ensemble of ultracold ⁸⁷Rb atoms is prepared in a magneto-optical trap and subsequently transferred into an Ioffe-Pritchard-type magnetic trap situated in the room-temperature environment of our setup [43]. The atomic cloud is cooled by forced radiofrequency evaporation and then loaded into an optical dipole trap used to transport the ensemble to a position below the superconducting atom chip at 4.2 K. We load an ensemble of $\sim 1 \times 10^6$ atoms at a temperature of $\sim 1 \ \mu {
m K}$ into the magnetic chip trap, which is based on a Z-wire geometry [23]. The oscillation frequencies in the trap are given by $\omega_x = 2\pi \times$ 30 s^{-1} , $\omega_y = 2\pi \times 158 \text{ s}^{-1}$, and $\omega_z = 2\pi \times 155 \text{ s}^{-1}$, and the offset field $B_{\rm off}$, which defines the quantization axis, is pointing along the x direction. The atomic cloud in the magnetic trap is cooled to a temperature of \sim 250 nK by evaporation. After this sequence, which is repeated every ~ 23 s, we end up with an ensemble of roughly 1×10^5 atoms. After a hold time of 2 s in the magnetic trap, which allows for damping of possible eddy currents in the metallic chip holder, a microwave field for dressing is applied.

The microwave field is irradiated from an antenna outside of the vacuum chamber and is counterpropagating to the quantization axis. We measured the polarization of the microwave by driving resonant σ_+ and σ_- Rabi oscillations. We found a ratio of $\sqrt{6}\Omega_{0,\sigma+}/\Omega_{0,\sigma-} \approx 0.81$, while for a linear (circular) polarization the expected ratio would be 1 (0). The factor $\sqrt{6}$ stems from the different transition strengths, as visible in the Hamiltonian in Eq. (5).

The frequency of the transition is measured by means of Ramsey interferometry. The interferometric sequence is started 100 ms after switching on the dressing field by applying a combined microwave and radio-frequency twophoton pulse with a pulse area of $\pi/2$ ($T_{\pi/2} = 137 \ \mu$ s), which prepares the atomic ensemble in a coherent superposition of states $|0\rangle$ and $|1\rangle$; see Figs. 1(b) and 1(c). The microwave pulses are irradiated from a second external antenna with a wave vector perpendicular to the quantization axis, while the radio-frequency field is generated by an alternating current

IV. MEASUREMENTS AND DISCUSSION

in the trapping wire. Both frequencies are chosen with a

detuning of $\Delta \sim 2\pi \times 310$ kHz with respect to the transition to the intermediate level $5S_{1/2}$ $F = 2, m_F = 0$, so that the

probability of populating this level is negligible. After a

variable hold time T_{Ramsey} , the interferometer is closed by a

second $\pi/2$ pulse and we measure the population of the two

states $|0\rangle$ and $|1\rangle$, which oscillates with the angular frequency

 $\delta = |\omega_{\rm MW} + \omega_{\rm RF} - \Delta E_{0,1}/\hbar|$. We determine this frequency

 δ for different offset fields $B_{\rm off}$ and seek to eliminate the

magnetic-field dependence of the transition.

To demonstrate the control over the differential Zeeman shift, we measure the frequency of the Ramsey interferometer as a function of the magnetic offset field B_{off} for different powers of the dressing field (Fig. 3). For each value of B_{off} , we adjust ω_{RF} and ω_{MW} in order to keep the detuning Δ to the intermediate-state constant, while keeping the sum frequency



FIG. 3. (Color online) Measurement of the differential Zeeman shift between the states $|0\rangle$ and $|1\rangle$ for different Rabi frequencies. The frequency zero point was set to the frequency at the magic offset field without dressing. For a Rabi frequency $\Omega_{dress} = \Omega_0 = 2\pi \times 20.1$ kHz, the frequency is nearly independent of the magnetic offset field in a range of ± 100 mG around the chosen value $B_{center} = 2.65$ G. Inset: Detail of the curve with $\Omega_{dress} = 0.99\Omega_0$. We estimate a measurement error of ± 5 Hz resulting from fluctuations of the MW power. The theory curve (solid red line) is plotted along a polynomial fit (dotted black line), showing the suppression of the first- and second-order Zeeman shift down to a level of -7.3 Hz/G and 5.0 Hz/G².



FIG. 4. (Color online) Frequency difference of the clock transition as a function of the Rabi frequency of the dressing for different magnetic fields. The data was extracted from the measurements in Fig. 3. *Inset*: Frequency difference for different offset fields with respect to the measurements at $B_{\text{off}} = 2.72$ G as a function of the Rabi frequency. At the optimal Rabi frequency Ω_0 , the three curves show nearly identical frequencies, proving the cancellation of the differential Zeeman shift up to second order.

 $\omega_{\rm RF} + \omega_{\rm MW}$ fixed. The measurement without dressing field yields the expected Breit-Rabi parabola which we use to calibrate the magnetic field $B_{\rm off}$.

For the cancellation of the magnetic-field dependence, a magnetic offset field $B_{off} = 2.65$ G was chosen. For this B_{off} , we calculated the optimum detuning Δ_{dress} and Rabi frequency Ω_{dress} for the measured ratio between σ_+ and σ_- transition strengths. We measure δ vs B_{off} in the range 2.1–3.8 G for Rabi frequencies in the range of $2\pi \times 12$ to $2\pi \times 25$ kHz with a calculated optimal Rabi frequency $\Omega_0 = 2\pi \times 20.1$ kHz. The results of these measurements are plotted in Fig. 3 along with the results of the analytical calculations, taking into account the measured imbalance in the Rabi frequency as a free parameter in one of the curves and scaling the other curves according to the MW power applied in the experiment. The data demonstrate the compensation of the differential Zeeman shift around the field value of $B_{center} = 2.65$ G.

The reduced sensitivity of the clock transition to magneticfield variations is shown in Fig. 4. Here we plot the measured frequencies and the theory curves for three different offset fields as a function of the Rabi frequency, as extracted from the values in Fig. 3. For the optimum Rabi frequency Ω_0 , all three curves show the same ac Zeeman shift. The inset in Fig. 4 shows the frequency difference between the curves measured for the three offset fields with respect to the value $B_{\rm off} = 2.72$ G. The three curves cross nearly at the same point, showing the strong suppression of the differential Zeeman shift over a field range larger than 0.2 G. The analysis of the theory curves in Fig. 3 shows that it is possible to generate plateaus where the frequency differs by less than 0.1 Hz over a magnetic-field range of more than 100 mG. As is visible in the inset of Fig. 3, the measurement does not reach this accuracy. We estimate a frequency uncertainty of ± 5 Hz, based on the



FIG. 5. (Color online) *Double magic* dressing of the atomicclock transition, for which the dependence of the frequency on both the magnetic field and the Rabi frequency disappears around a field value of $B_{\text{center}} = 2.59$ G. The calculation assumes a Rabi-frequency imbalance of $\sqrt{6}\Omega_{0,\sigma+}/\Omega_{0,\sigma-} = 1.25$, and the obtained optimal parameters are $\Omega_0 = 2\pi \times 86.7$ kHz, $\Delta_{\text{dress}} = -2\pi \times 309$ kHz.

limited time between the Ramsey pulses and the uncertainty of the unstabilized microwave power.

The stability of the microwave Rabi frequency is expected to be the strongest limitation on the frequency stability. In order to reach the 0.1-Hz range at the field point of 2.65 G, a power stability on the order of $\Delta\Omega_{dress}/\Omega_{dress} \sim 1 \times 10^{-4}$ would be required. For certain offset fields, however, it is possible to find solutions for Eq. (11) where both the *B*-field dependency as well as the dependency on the Rabi frequency Ω_{dress} disappear. An example for such a solution can be seen in Fig. 5: Here, we calculate that the transition frequency varies by less than ± 0.1 Hz over a range of 100 mG around $B_{\text{center}} = 2.59 \text{ G}$. At the center of the plateau, the frequency δ becomes independent of the Rabi frequency for a detuning of $\Delta_{dress} = -2\pi \times 309$ kHz. In a range of ± 10 mG around B_{center}, a Rabi-frequency stabilization on the order of 1% would be sufficient to reach a level of 0.1-Hz stability. Such double magic dressing enables the employment of this technique with on-chip microwave devices, where Rabi frequencies are inversely proportional to the distance to the chip.

Manipulation of the differential Zeeman shift can be used to decrease the frequency spread over the size of the cloud. For a cloud of $N = 5 \times 10^4$ atoms at T = 250 nK and $B_{\text{off}} =$ 2.65 G, the standard deviation of the frequency distribution due to the inhomogeneity of the magnetic field without dressing is on the order of $\sigma_{\text{inh}} \approx 4$ Hz, about an order of magnitude larger than the spread σ_{dens} caused by the inhomogeneous mean-field interaction due to the density distribution in the trap [12]. Microwave dressing can be employed to decrease σ_{inh} to a level on the order of σ_{dens} , thereby balancing the two effects and leading to a nearly homogeneous frequency over the size of the cloud. For the parameters above and our trap, we calculate that the differential Zeeman shift can be engineered to cancel the collisional frequency shift down to a level of $\sigma \approx 2\pi \times 0.25$ Hz. In addition, our scheme can be used to prepare states with nearly arbitrary δ vs B_{off} curvatures around the desired field point, enabling one to suppress, enhance, or spatially structure the differential Zeeman shifts. Enhancing the *B*-field dependence could, for example, be used to counteract the strong mean-field shifts in a Bose-Einstein condensate. Further engineering of differential clock frequencies can be achieved by using multifrequency microwave fields. This opens up new possibilities for microwave and radio-frequency dressing of atomic transitions, which has previously been used for trapping and manipulation of cold atoms [44–48] and the generation of state-dependent potentials [49,50].

V. CONCLUSION

In summary, we have shown both experimentally and theoretically that dressing of Zeeman sublevels in magnetically

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trapped atoms can render hyperfine transitions insensitive to magnetic-field fluctuations around an arbitrary field value. We have furthermore identified double magic points, where the clock frequency becomes independent of the Rabi frequency. Microwave dressing can be used to enhance the coherence time of quantum superposition states in arbitrary magnetic fields and for the creation of noise-protected quantum memories. The scheme is further applicable in atomic-clock schemes in magnetically noisy environments or portable setups.

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