Improving the cooling performance of a mechanical resonator with two-level-system defects

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We study the cooling performance of a realistic mechanical resonator containing defects. The normal cooling method through an optomechanical system does not work efficiently due to those defects. We show that, by employing periodical σ_z pulses, we can eliminate the interaction between defects and their surrounding heat baths up to the first order of time. Compared with the cooling performance of the no σ_z pulses case, much better cooling results are obtained. Moreover, this pulse sequence has the ability to improve the cooling performance of the resonator with different defect energy gaps and different defect damping rates.

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I. INTRODUCTION

Preparing a ground state of a mechanical resonator is an important topic, and it has applications in testing fundamental quantum theory, exploring the boundary between classical and quantum regions, and studying precision metrology [1–3]. Considering the inevitable interaction between the resonator and its surrounding heat bath, the state of the resonator is far away from the ground state in equilibrium, so cooling the mechanical resonator to the ground state becomes an urgent task. So far, many proposals have been put forward to cool the resonator. Making use of the capacitive coupling, Lorentz force, magnetic field, or strain field induced coupling with a two-level system (Josephson qubit or negatively charged nitrogen-vacancy center, etc.), one can cool the mechanical resonator efficiently [4–13]. Other proposals design an optomechanical system consisting of one cavity mode and a cooled resonator. The radiative pressure from photons is applied to cool the resonator [14–19]. For all of these proposals, we can finally obtain a resonator with a very small phonon number in the long-time limit.

In a realistic experimental optomechanical system, the mechanical resonator is often made by silica. Because of the amorphous nature of silicon, defects reside in the amorphous native oxide of the silicon surface [20–23]. As shown in Ref. [24], due to the coupling between the defect with the resonator and their couplings with heat baths, the thermal noise from the heat bath of the defects can be effectively transferred to the mechanical resonator. Therefore, the normal cooling method used in the optomechanical system does not work well.

In this paper, we show that, by employing the periodical σ_z pulses, we can efficiently remove the detrimental effect of defects and cool the resonator efficiently. The reason is that the periodical σ_z pulses can induce the sign of operators σ_- and σ_+ to be flipped [25–31]. The interaction between defects and the surrounding heat bath can be eliminated up to the first order of time. We display the cooling performance of the resonator with different defect energy gaps and different defect damping rates. We find that with a large number of σ_z pulses ($N = 99$) the phonon occupation of the resonator reduces to a lower value, when compared with the case of no σ_z pulses. We also study the difference between the cooling results through two different calculation approaches; one uses the master equation based on polariton doublets, and the other is a "simple approach" in which the defects and coupling are added into the master equation of the bare resonator directly [24]. In our discussion, we find that these two approaches give a similar qualitative picture in cooling, but the obtained values of phonon occupations are quantitatively different.

The structure of this paper is as follows. Section II introduces the total system we shall study. In Sec. III, we explore the cooling performance with different defect energy gaps and different defect damping rates. The paper is concluded in Sec. IV.

II. MODEL

The total system contains one cavity mode, a mechanical resonator, defects, and their surrounding heat baths. The total Hamiltonian is [23,24]

$$H_{\rm tot} = H_{\rm OM} + H_{\rm JC} + H_{a,e} + H_{b,e} + H_{\sigma,e} + H_B. \qquad (1)$$

Here, H_B denotes the Hamiltonian of the three non-interacting baths. An optomechanical component consists of one cavity mode and the mechanical resonator, that is,

$$H_{\rm OM} = -\hbar\Delta_L a^\dagger a + \hbar g(a + a^\dagger)(b + b^\dagger), \qquad (2)$$

where $\Delta_L = \omega_L - \omega_c$ is the detuning of cavity driving frequency ω_L from cavity mode frequency ω_c. The operators a ($a^\dagger$) and b ($b^\dagger$) stand for the annihilation (creation) operator of the cavity mode and mechanical resonator, respectively.

As discussed above, the defects couple to the mechanical resonator inevitably. Here, we only consider the case of one

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defect, which can be regarded as a two-level system (TLS) [20–24]. The Hamiltonian for the resonator and the defect can be written as a Jaynes-Cummings (JC) form [23,24]:

$$H_{\text{JC}} = \hbar\omega_m b^\dagger b + \tfrac{1}{2}\hbar\omega_z\sigma_z + \hbar\lambda(\sigma_+ b + b^\dagger\sigma_-), \tag{3}$$

where $\omega_z(\omega_m)$ is the frequency of a TLS (mechanical resonator), and λ is the coupling strength.

In our discussion below, we take the interactions between each of these three systems with their surrounding heat bath into account. The interaction Hamiltonian takes the form of

$$H_{a,e} = \sum_k g_{a,k}(a a_k^\dagger + a^\dagger a_k), \tag{4a}$$

$$H_{b,e} = \sum_k g_{b,k}(b b_k^\dagger + b^\dagger b_k), \tag{4b}$$

$$H_{\sigma,e} = \sum_k g_{\sigma,k}(\sigma_- c_k^\dagger + \sigma_+ c_k). \tag{4c}$$

Here, $g_{a,k}$, $g_{b,k}$, and $g_{\sigma,k}$ are the coupling strength between the cavity mode, the mechanical resonator, the defects, and each one's heat bath. The bath modes are labeled by k.

In many cases, the coupling strengths g and λ are comparable, and the cavity damping rate is large. We define polariton states as [24]

$$|n,\alpha\rangle = c_\alpha^n|n\downarrow\rangle + s_\alpha^n|(n-1)\uparrow\rangle, \tag{5}$$

where $n \geqslant 1$, $\alpha = \pm$, $|n\rangle$ is the Fock state of the mechanical resonator, and $|\uparrow\rangle$ and $|\downarrow\rangle$ are the eigenstates of σ_z. Here, $c_+^n = -s_-^n = \cos(\delta_n/2)$, $s_+^n = c_-^n = \sin(\delta_n/2)$, and the expression of δ_n satisfies the relation $\cos(\delta_n/2) = \sqrt{(\omega_n + \delta\omega)/2\omega_n}$, where $\delta\omega = \omega_m - \omega_z$ and $\omega_n = \sqrt{\delta\omega^2 + 4\lambda^2 n}$.

By employing the projection operator technique [32,33], and only keeping terms up to the second order of g, we can obtain the dynamics of the reduced density matrix ρ_s for the composite system of the TLS and resonator [24]:

$$\dot{\rho}_s = -\frac{i}{\hbar}[H_\tau,\rho_s] + \sum_{n,\alpha,\beta}\frac{\Gamma_0^{n\alpha\beta}}{2}\mathcal{L}_0^{n\alpha\beta}\rho_s + \sum_{n,\alpha,\beta}\left|A_{\beta,\alpha}^{(n)}\right|^2\left[\frac{\Gamma_{-,\alpha\beta}^n}{2}\mathcal{L}\left(O_n^{\alpha\beta}\right) + \frac{\Gamma_{+,\alpha\beta}^n}{2}\mathcal{L}\left(O_n^{\alpha\beta\dagger}\right)\right]\rho_s. \tag{6}$$

Here, $\mathcal{L}_0^{n\alpha\beta} = (n_{th}^{n\alpha\beta}+1)\mathcal{L}(O_n^{\alpha\beta}) + n_{th}^{n\alpha\beta}\mathcal{L}(O_n^{\alpha\beta\dagger})$, and $\mathcal{L}(o)\rho = 2o\rho o^\dagger - \rho o^\dagger o - o^\dagger o\rho$. The polariton Hamiltonian $H_\tau = \omega_{n,\alpha}|n,\alpha\rangle\langle n,\alpha|$, where $\omega_{n,\alpha}$ are the eigenenergies of the polariton states. The thermal population $n_{\text{th}}^{n\alpha\beta} = [\exp(\hbar\omega_{n\alpha\beta}/k_B T) - 1]^{-1}$, with $\omega_{n\alpha\beta} = \omega_{n,\alpha} - \omega_{n-1,\beta}$. Here, all heat baths have the same temperature T. The operator $O_n^{\alpha\beta} = |(n-1),\beta\rangle\langle n,\alpha|$. The expressions of $\Gamma_0^{n\alpha\beta}$ and $\Gamma_{\mp,\alpha\beta}^n$ are

$$\Gamma_0^{n\alpha\beta} = \left|A_{\beta,\alpha}^{(n)}\right|^2\gamma_m + \left|\sigma_{\beta,\alpha}^{(n)}\right|^2\gamma_\tau, \tag{7a}$$

$$\Gamma_{\mp,\alpha\beta}^n = \frac{g^2\kappa}{\kappa^2/4 + (\omega_{n\alpha\beta}\pm\Delta_b)^2}, \tag{7b}$$

where γ_m, γ_τ, and κ stand for the damping rate of the mechanical resonator, the TLS, and the cavity mode, respectively. The coefficients $A_{\beta,\alpha}^{(n)}$ and $\sigma_{\beta,\alpha}^{(n)}$ are from the expressions of $b = \sum A_{\beta,\alpha}^{(n)} O_n^{\alpha\beta}$ and $\sigma_- = \sum \sigma_{\beta,\alpha}^{(n)} O_n^{\alpha\beta}$.

The above system dynamics [Eq. (6)] is the evolution of the density matrix based on polariton doublets; we will also introduce the simple approach to show the different dynamics between the master equation based on polariton doublets and the simple approach. For the "simple approach", the TLS and its interaction with the bath are added into the system dynamics of the bare mechanical resonator directly. The density matrix in the simple approach is

$$\dot{\rho}_s = -\frac{i}{\hbar}[H_{\text{JC}},\rho_s] + \frac{\bar{\gamma}}{2}(\bar{n}+1)\mathcal{L}(b)\rho_s + \frac{\bar{\gamma}}{2}\bar{n}\mathcal{L}(b^\dagger)\rho_s + \frac{\gamma_\tau}{2}(\bar{n}_T+1)\mathcal{L}(\sigma_-)\rho_s + \frac{\gamma_\tau}{2}\bar{n}_T\mathcal{L}(\sigma_+)\rho_s, \tag{8}$$

where the damping coefficient is $\bar{\gamma} = \gamma_m + A^{(-)} - A^{(+)}$ and the population $\bar{n} = \frac{\gamma_m n_m + A^{(+)}}{\gamma_m + A^{(-)} - A^{(+)}}$. The coefficients, $\bar{n}_T$ and n_m are the population of the TLS and mechanical resonator, respectively. The parameter satisfies $A^{(\pm)} = \frac{g^2\kappa}{(\kappa/2)^2 + (\Delta_L \mp \omega_m)^2}$.

σ_z pulses

Many studies have been devoted to designing different pulse sequences, to keep the system away from decoherence induced by the surrounding heat bath [25–31]. Here, we use the periodical σ_z pulses, to eliminate the interaction between the TLS and the bath up to the first order of time. Considering the properties of the operator σ_z,

$$\sigma_z\sigma_-\sigma_z = -\sigma_-, \quad \sigma_z\sigma_+\sigma_z = -\sigma_+, \tag{9}$$

we can obtain

$$\sigma_z e^{-iH_{\text{tot}}t}\sigma_z e^{-iH_{\text{tot}}t} = e^{-it\sigma_z H_{\text{tot}}\sigma_z}e^{-iH_{\text{tot}}t} = e^{-itf_1 + itf_2}e^{-itf_1 - itf_2}, \tag{10}$$

with the coefficients $f_1 = H_{\text{OM}} + \hbar\omega_m b^\dagger b + \frac{1}{2}\hbar\omega_z\sigma_z + H_{a,e} + H_{b,e} + H_B$, $f_2 = \hbar\lambda(\sigma_+ b + b^\dagger\sigma_-) + H_{\sigma,e}$. Clearly, up to the first order of time, we can eliminate the interaction of the TLS and the corresponding heat bath. Combining Eqs. (6) and (10), we obtain the time evolution for the density matrix of $\rho_s(t)$:

$$\rho_s(t) \Rightarrow \cdots e^{\mathcal{H}(t_j - t_{j-1})}\sigma_z \cdots \sigma_z\{e^{\mathcal{H}(t_2-t_1)}\sigma_z\{e^{\mathcal{H}t_1}\rho_s\}\sigma_z\} \times \sigma_z \cdots \sigma_z \cdots. \tag{11}$$

Here, $e^{\mathcal{H}(t_k - t_{k-1})}\rho_s$ denotes the system evolution obeying Eq. (6), and $t_k - t_{k-1}$ is the time duration between two adjacent pulses. With these, we can now investigate the cooling performance of the resonator numerically.

III. RESULTS AND DISCUSSIONS

First, by using the master equation based on polariton doublets, we discuss the cooling performance of the mechanical resonator with different numbers of σ_z pulses [see Fig. 1(a)]. The frequency of the mechanical resonator is set as $\omega_m = 200$ MHz. As a result, the more σ_z pulses used,

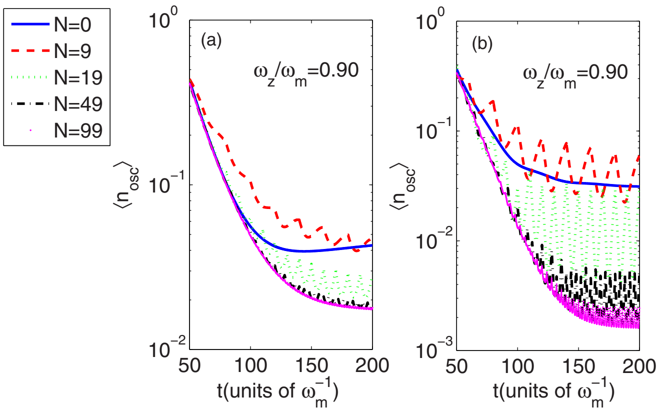

FIG. 1. (Color online) Residual phonon number ($\langle n_{\rm osc}\rangle$) vs different numbers of σ_z pulses. Blue solid, $N = 0$; red dashed, $N = 9$; green dotted, $N = 19$; black dotted-dashed, $N = 49$; purple circle dot, $N = 99$ (a) Master equation based on polariton doublets. (b) Simple approach. Parameters are $\omega_m = 200$ MHz, $\omega_z/\omega_m = 0.9$, $\kappa/\omega_m = 0.15$, $\gamma_m/\omega_m = 10^{-6}$, $\gamma_\tau/\omega_m = 2.5\times10^{-4}$, $g/\omega_m = 0.05$, $\lambda/\omega_m = 0.05$, $\Delta_L/\omega_m = -1$, and $T = 0.1$ K.

the lower the resonator phonon number. If the number of σ_z pulses is too small ($N = 9$), the result is even worse than the case of no σ_z pulses ($N = 0$). Although based on the analysis above the periodical σ_z pulses can eliminate the interaction between the TLS and its surrounding bath up to the first order of time, when the time interval between two adjacent σ_z pulses is too long the consequence of higher-order terms is significant. When there are more σ_z pulses, the time interval between two adjacent incident pulses is smaller, and the effect of those higher-order terms becomes less significant. As shown in Fig. 1(a), we achieve good cooling performance with the pulse numbers $N = 19$, 49, and 99. For the case of no σ_z pulses, at time $\omega_m t = 200$, the residual phonon number of the resonator is 0.04289, while for the case of $N = 99$ the residual phonon number of the resonator is 0.01793 at time $\omega_m t = 200$. There is a decrease of 58.2%. In Fig. 1(b), the cooling results obtained by using the "simple approach" are presented. The same initial states are chosen for both approaches [Figs. 1(a) and 1(b)]. We find that the evolution of the population of the resonator given by the "simple approach" is qualitatively the same as the case shown in Fig. 1(a), but the residual phonon numbers of the resonator in the long-time limit through these two approaches are quantitatively different. As pointed out in Ref. [24], results from the master equation based on polariton doublets [Fig. 1(a)] are more accurate. From Fig. 1(a), it is clearly seen that we can cool the resonator more efficiently with more injecting pulses.

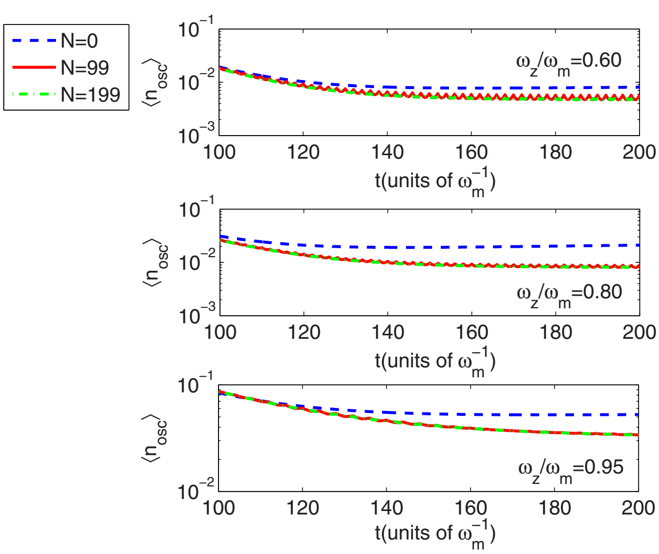

FIG. 2. (Color online) Residual phonon number ($\langle n_{\rm osc}\rangle$) vs different numbers of σ_z pulses (N=0, 99, and 199). Parameters are $\omega_m = 200$ MHz, $\kappa/\omega_m = 0.15$, $\gamma_m/\omega_m = 10^{-6}$, $\gamma_\tau/\omega_m = 2.5\times10^{-4}$, $g/\omega_m = 0.05$, $\lambda/\omega_m = 0.05$, $\Delta_L/\omega_m = -1$, and $T = 0.1$ K. From top to bottom, ω_z/ω_m is 0.6, 0.8, and 0.95.

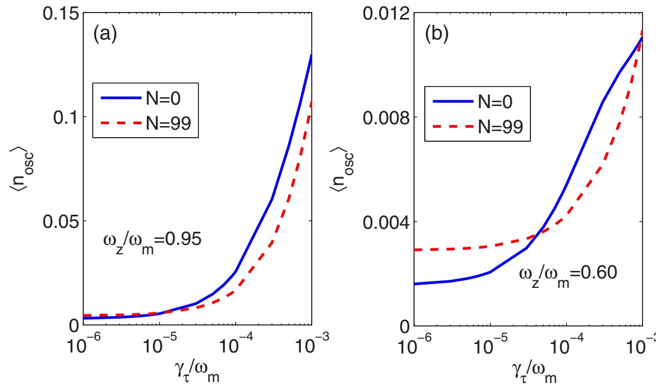

FIG. 3. (Color online) Residual phonon number ($\langle n_{\rm osc}\rangle$) vs γ_τ. Different numbers of σ_z pulses ($N = 0$ and 99) are applied. The value of $\langle n_{\rm osc}\rangle$ is chosen at time $t = 200\omega_m^{-1}$, where the residual phonon number has been stable already. Parameters are $\omega_m = 200$ MHz, $\kappa/\omega_m = 0.15$, $\gamma_m/\omega_m = 10^{-6}$, $g/\omega_m = 0.05$, $\lambda/\omega_m = 0.05$, $\Delta_L/\omega_m = -1$, and $T = 0.1$ K.

Second, we study the cooling performance of the resonator with different defect energy gaps. Figure 2 presents the results of three different cases ($\omega_z/\omega_m = 0.60$, 0.80, and 0.95). Cooling results from different pulse numbers, $N = 99$ and 199, are compared. These results show that, by employing the periodical σ_z pulses, we can cool the resonator efficiently within a wide range of defect energy gaps.

Finally, we compare the cooling performance of the resonator with different TLS damping rates γ_τ in Fig. 3. The near resonance condition is chosen ($\delta\omega/\omega_m = 0.05$), and the parameters satisfy $\delta\omega \leqslant \lambda < \kappa$. From Fig. 3(a), we find that when the TLS damping rate is small enough (between 10^{-6} and 10^{-5}) the cooling performance of the resonator is insensitive to the pulse number applied and the residual phonon number of the resonator is very small ($\simeq$0.004). In Fig. 3(b), we change the defect energy gap to a new value, $\omega_z/\omega_m = 0.60$, and the resonance condition is not satisfied, $\delta\omega > \lambda = 0.05\omega_m$. When the TLS damping rate is larger than 5×10^{-5}, the resonator is cooled efficiently with the pulse number $N = 99$, while the result is not effective with no σ_z pulses applied.

IV. CONCLUSION

In summary, we introduce periodical σ_z pulses to eliminate the bad effect from the defects in cooling the mechanical resonator. The periodical σ_z pulses can remove the interaction between the TLS and the heat bath up to the first order of time. By applying σ_z pulses, we can cool the resonator efficiently with different defect energy gaps and different TLS damping rates. Other designed pulse sequences eliminating the interaction in more than the first order of time [26,28–31] might be more efficient to cool the resonator. This deserves further study in the future.

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