Magnon qubit and quantum computing on magnon Bose-Einstein condensates

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Recently, great progress has been made in the creation of a solid-state quantum computer using superconducting qubits on Cooper pairs of charged electrons. However, this approach has met limitations due to decoherence effects caused by the strong Coulomb interaction of the superconducting qubit with the environment. Here, we propose the solution of this problem by switching to another Bose-Einstein condensate (BEC), uncharged long-lived magnons, wherein the magnon BEC qubit can be realized due to the magnon blockade isolating a pair of the magnon condensate energy levels in the mesoscopic and nanoscopic ferromagnetic dielectric sample. We demonstrate the single-qubit gates by operating quantum transition between these states in the external microwave field. We also consider implementation of the two-qubit gates by using the interaction between such magnon BEC qubits due to exchange by virtual photons in a microwave cavity. Finally, we discuss the condition for long-lived magnon BEC qubits, a scalable architecture, and promising advantages of the multiqubit quantum computer based on the magnon qubit.

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I. INTRODUCTION

A quantum computer proposed by Feynman [1] is a tremendous computational resource. Up to now, the most significant progress in its implementation has been achieved on trapped ions where 14 qubits were entangled [2]. However, this approach requires extremely low temperatures of 10^{-6} K; therefore the solid-state prototype on superconducting Josephson qubits working at liquid helium temperatures [3] seems to be very promising. Another critical advantage of the Josephson qubit is the macroscopically enhanced dipole moment of the transition in an effective two-level system that provides the high rate of qubits processing. The main obstacle of superconducting qubits is caused by the strong effect of surrounding electric fields on the charges of superconducting qubits, which limits their decoherence time by hundreds of microseconds [4]. Therefore, it is worthwhile to study the possibility of long-lived qubits in other Bose-Einstein condensates (BECs), such as magnon, polariton, and exciton BECs. Magnons are electrically neutral and, therefore, interact weakly with the surroundings; thus the magnon BEC state can demonstrate the decoherence time within the time scale of several seconds [5,6] that makes the magnon BEC especially interesting for the construction of a long-lived qubit. However, special research needs to be done on the implementation of qubits on magnon condensates.

In the present paper, we propose a scheme for the realization of qubit and quantum gates on BEC magnons. We show that the magnon BEC qubit can be realized due to a type of blockade that we refer to as a magnon blockade, isolating a pair of the magnon condensate energy levels in the mesoscopic ferromagnetic sample. We consider single-qubit gates operating on the transitions between these states in the external microwave field. Also, we show that the two-qubit gates can be built using the interaction between these qubits via exchange by virtual photons in a microwave cavity. Promising advantages of the proposed magnon BEC qubit and logical gates for practical use in quantum information processing are discussed.

II. MAGNON QUBIT

Let us consider the Hamiltonian of interacting spins in a ferromagnetic sample that has the following form:

$$H = -\frac{1}{2}\sum_{i\neq j}\sum_{\alpha\beta}\left[J_{ij}\delta^{\alpha\beta} + D_{ij}^{\alpha\beta}\right]S_i^{\alpha}S_j^{\beta} - \sum_j h_j S_j^z, \tag{1}$$

where $\hat{S}_i^{\alpha},\hat{S}_j^{\beta}$ are spin operators with components α,β of atoms i and j; J_{ij} is the constant of the exchange interaction; $D_{ij}^{\alpha\beta}$ is the constant of the dipole-dipole interaction; $h_j S_j^z$ is the Zeeman energy of the j_mth spin in the external magnetic field oriented along the $\vec{z}$ axis. Applying the Holstein-Primakoff transformation and the operator Fourier transform (see Appendix A), we obtain

$$H = \sum_k A_k \hat{b}_k^{\dagger}\hat{b}_k + \sum_{kk'k''k'''}(B_{k+k'-k''-k'''}\hat{b}_k^{\dagger}\hat{b}_{k'}^{\dagger}\hat{b}_{k''}\hat{b}_{k'''} + \text{H.c.}), \tag{2}$$

where $b_k^{\dagger}$ and b_k are creation and annihilation operators of magnons with the wave vector $\vec{k}$,

$$A_k = S\left(h + J_0 + D_0 - J_k - \frac{D_k^{xx} + D_k^{yy}}{2}\right), \tag{3}$$

$$\begin{aligned} &B_{k+k'-k''-k'''} \\ &= \frac{1}{4N}\left\{\left(J_{k,k'-k''-k'''} + \frac{D^{xx}_{k,k'-k''-k'''} + D^{yy}_{k,k'-k''-k'''}}{2}\right)\right. \\ &\quad + \left(J_{-k,-k'+k''+k'''} + \frac{D^{xx}_{-k,-k'+k''+k'''} + D^{yy}_{-k,-k'+k''+k'''}}{2}\right) \\ &\quad \left. - 2\left(J_{k-k'',k'-k'''} + D^{zz}_{k-k'',k'-k'''}\right)\right\}, \end{aligned} \tag{4}$$

where J_k and $D_k^{\alpha\beta}$ are Fourier images of relevant constants. Using the Hamiltonian (2), the set of main features attributed

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to the magnon Bose condensate can be explained [7–10]. It is worth noting that third-order terms on operators of magnon creation and annihilation are not included in Eq. (2). The main contribution of these terms is not taken into account in Hamiltonian (2) because it determines relatively slow relaxation processes of BEC magnons [8].

Bose-condensed magnons in the sample can be prepared in two states with wave vectors $\pm\vec{k}_0$ corresponding to the minimal energy in the vicinity of these wave vectors [11]. The BEC stability is provided by the gap in the energy spectrum of the nonideal Bose gas [9,10]. Large values of the energy gap and the high stability of a magnon BEC can be implemented in thin films at the orthogonal orientation of the external magnetic field [12]. In this regard, we have to focus our consideration on the properties of magnons characterized by the wave vectors $\vec{k} = \pm\vec{k}_0$. Herein, we take initially into account the main terms of Hamiltonian $\hat{H}_o$ (2) which are characterized by these wave vectors of magnons and the rest of the terms can be accounted for as the perturbation $\hat{V}$ (Appendix B).

Let us assume for simplicity that the magnons undergo the Bose condensation to the pure quantum state where only one standing magnon mode with the wave vector $k_0 = 0$ is populated. In particular, this can be valid for the case when the magnetic field is perpendicular to the ferromagnetic film [12]. In this state, the main Hamiltonian has the form $\hat{\hat{H}}_0 = E\hat{n} + \lambda\hat{n}^2$, where $\hat{n} = \hat{b}^\dagger\hat{b}$, $\hat{b}^\dagger \equiv \hat{b}_0^\dagger$, $\hat{b} \equiv \hat{b}_0$. Parameters incoming in this Hamiltonian are equal to the following values: $E = [S + 1/(2N)][D_0^{zz} - (D_0^{xx} + D_0^{yy})/2] + Sh$ (main energy of magnon BEC) and $\lambda = [(D_0^{xx} + D_0^{yy})/2 - D_0^{zz}]/(2N)$ (nonlinearity of the magnon BEC spectrum), where N is the total number of atoms in the sample. The following energy levels, $\varepsilon_{-1} = E(n-1) + \lambda(n-1)^2$, $\varepsilon_0 = En + \lambda n^2$, $\varepsilon_1 = E(n+1) + \lambda(n+1)^2$, etc., correspond to this Hamiltonian. The energy of transitions between neighboring levels $\varepsilon_0 \to \varepsilon_{-1}$ and $\varepsilon_1 \to \varepsilon_0$ can be written as $\hbar\omega_{0,-1} = E + \lambda(2n-1)$ and $\hbar\omega_{1,0} = E + \lambda(2n+1)$, etc., and the energy difference for the nearest two transitions is 2λ. If this difference is larger than the transition linewidths, we can limit our consideration by a single pair of the levels obtaining energies ε_0 and ε_{-1} with the transition frequency $\omega_{0,-1}$ and introduce a magnon BEC qubit corresponding to this pair of energy states. This situation can be called a magnon blockade, where excitation of one more magnon blocks transition to excess levels, similar to how it proceeds by excitation of one more atom in the case of a dipole blockade [13]. The difference is that the magnon is the excitation of the whole sample and here we are not limited by the interaction radius as in a dipole blockade. Using numerical data [7], we obtain the following estimate, $\lambda/h \sim 10^{14}/N$ Hz, and our parameters are in the range of 1–100 MHz for $N = 10^8 - 10^6$ which is larger than the experimental resonance linewidths of BEC magnons [5,14]. The implementation of such a large nonlinearity parameter λ/h seems to be practical for nanosize and even for mesoscopic magnon systems and the estimation is in agreement with recent experimental results [15–17] which makes the Bose-Einstein condensation in the mesoscopic systems promising for the implementation of BEC magnon qubits. Below, we consider single- and two-qubit gates on the proposed magnon qubit as well as its decoherence properties.

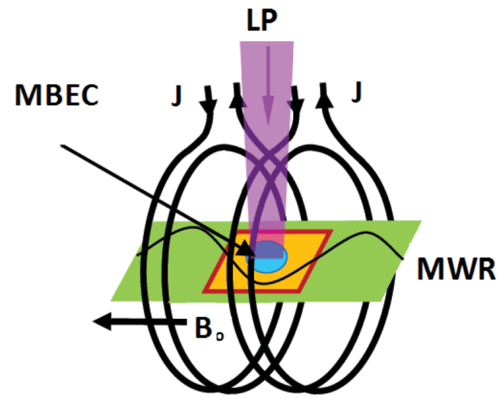

FIG. 1. (Color online) Layout of magnon BEC single-qubit gate (MBEC: magnon Bose-Einstein condensate; LP: laser pumping; MWR: microwave resonator; J: current in magnetic coil; B_0: magnetic field). Here, magnon BEC is excited by the laser pulse and qubit rotation is achieved by the interaction with a standing microwave field in a high-quality resonator.

III. SINGLE-QUBIT GATES

Let us consider the interaction of the magnon qubit $|\Psi(t)\rangle = \alpha_0|n\rangle + \beta_0|n-1\rangle$ with the external resonant electromagnetic field in a resonator oriented in the $(\vec{x},\vec{y})$ plane whose Hamiltonian can be written as

$$H_{\rm rf}^{(1)} = -\hbar(S^+ + S^-)\Omega(t)\cos(\omega t + \Phi_0), \tag{5}$$

where $\Omega(t)$ and ω are the Rabi frequency and carrier frequency of the variable radio-frequency field, respectively (where $\omega \cong \omega_{0,-1}$), Φ_0 is the initial phase, and S^+ and S^- are raising and lowering operators for effective spin corresponding to the magnon BEC qubit (Fig. 1). The interaction between the BEC magnons and the rectangular radio-frequency pulse leads to the following evolution of the magnon BEC qubit:

$$U\begin{pmatrix}\alpha_0\\ \beta_0\end{pmatrix} = \begin{pmatrix}\cos\,\Omega T/2 & -ie^{-i\Phi_0}\sin\,\Omega T/2\\ -ie^{-i\Phi_0}\sin\,\Omega T/2 & \cos\,\Omega T/2\end{pmatrix} \times \begin{pmatrix}\alpha_0\\ \beta_0\end{pmatrix}, \tag{6}$$

where α_0, β_0 are initial amplitudes of the qubit, and T is the pulse duration. Equation (6) describes the single-qubit rotation in the Hilbert space at the angle $\theta = \Omega T$. It is obvious that the pulse spectral width $\delta\omega$ should satisfy the following relation: $\delta\omega \ll 2\lambda$. Such a qubit rotation can be also realized via two-photon Raman excitation that opens an additional advantage for local driving of the magnon BEC qubits.

Another single-qubit operation is a phase gate. It is implemented here by applying the additional magnetic field for time T in order to change the qubit frequency $\omega'_{0,-1} = \omega_{0,-1} + \Delta\omega_{0,-1}$. As a result, the qubit phase shift is $\Delta\varphi = \Delta\omega_{0,-1}T$. Using this gate it is possible to implement the Hadamard gate and the NOT gate forming a complete set of single-qubit gates.

IV. TWO-QUBIT GATES

Let us consider two films with magnon Bose-Einstein condensates in a common microwave cavity (Fig. 2) among other such films. We equalize the condensate

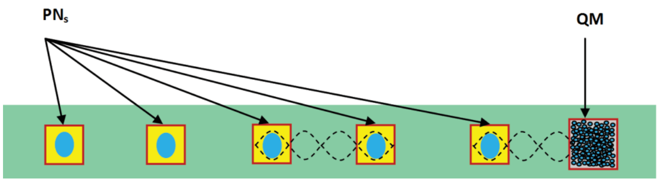

FIG. 2. (Color online) Scheme of quantum computer on magnon BEC qubits and multiqubit atomic quantum memory in waveguide microwave resonator (PN: processor node; QM: quantum memory). Dashed lines show interaction between two qubits and reversible transfer of quantum information between PN and QM via the interaction with virtual photons.

energies in two films $E(n_1 \otimes n_2 + 1) = E(n_1 + 1 \otimes n_2)$ ($|n_1 \otimes n_2 + 1\rangle$ and $|n_1 + 1 \otimes n_2\rangle$ are two resonant states) by the appropriate choice of the magnetic fields for each magnon BEC qubit. Here, we can limit ourselves by four states: $|n_1 - 1 \otimes n_2 - 1\rangle \equiv |0,0\rangle$, $|n_1 - 1 \otimes n_2\rangle \equiv |0,1\rangle$, $|n_1 \otimes n_2 - 1\rangle \equiv |1,0\rangle$, $|n_1 \otimes n_2\rangle \equiv |1,1\rangle$ and their eigenvalues $E(n_1 - 1 \otimes n_2 - 1) \equiv E_{0,0}$, $E(n_1 \otimes n_2 - 1) \equiv E_{1,0} = E(n_1 - 1 \otimes n_2) \equiv E_{0,1}$, and $E(n_1 \otimes n_2) \equiv E_{1,1}$. Taking into account that the magnon-magnon interaction $\hat{W}$ couples only two states, $|n_1 - 1 \otimes n_2\rangle$ and $|n_1 \otimes n_2 - 1\rangle$ (i.e., $|0,1\rangle$ and $|1,0\rangle$), if $n_1\kappa \ll \lambda$ and $n_2\kappa \ll \lambda$, we find the effective Hamiltonian of the long-range interaction $\hat{W} = \sqrt{n_1 n_2}\kappa(S_1^+ S_2^- + S_1^- S_2^+)$ between the two magnon BECs, where we have introduced the effective spin operators of two-level systems [$S_{1,2}^+ = \frac{1}{2}(S_{1,2}^x + iS_{1,2}^y)$ and $S_{1,2}^- = (S_{1,2}^+)^+$ are spin operators of the two-level system] in the films. The interaction arises due to the exchange by virtual photons in the cavity, $\kappa = g^2/\Delta$ is the effective coupling constant, $\Delta = \omega_{0,-1} - \omega$, ω is a resonant frequency of the microwave resonator, and g is a spin-phonon coupling constant in the Jaynes-Cummings model. The effective Hamiltonian $\hat{W}$ is obtained in straightforward manner using Schrieffer-Wolf transformation as in [18,19] for superconducting qubits.

The effective Hamiltonian determines the following evolution of the two coupled qubits:

$$\begin{aligned}|\Psi(\tilde{\tau} + t)\rangle &= \exp\{-i(\hat{\hat{H}}_{\text{eff}} + \hat{W})t/\hbar\}|\Psi(\tilde{\tau})\rangle\\ &= \alpha_{0,0}\exp\{-iE_{0,0}t/\hbar\}|0,0\rangle + \alpha_{1,1}\exp\{-iE_{1,1}t/\hbar\}|1,1\rangle + \exp\{-iE_{1,0}t/\hbar\}\{\alpha_{1,0}[\cos(\sqrt{n_1 n_2}\kappa t)|1,0\rangle\\ &\quad - i\sin(\sqrt{n_1 n_2}\kappa t)|0,1\rangle] + \alpha_{0,1}[\cos(\sqrt{n_1 n_2}\kappa t)|0,1\rangle - i\sin(\sqrt{n_1 n_2}\kappa t)|1,0\rangle]\},\end{aligned} \tag{7}$$

where the initial state of two qubits $|\Psi(\tilde{\tau})\rangle = \alpha_{0,0}|0,0\rangle + \alpha_{1,0}|1,0\rangle + \alpha_{0,1}|0,1\rangle + \alpha_{1,1}|1,1\rangle$.

The evolution (7) can be described in a simpler way: $|\Psi(\tilde{\tau} + t)\rangle = \exp\{-i\hat{\hat{H}}_{\text{eff}}t/\hbar\}\hat{S}(t)|\Psi(\tilde{\tau})\rangle$, where $\hat{S}(t)$ is represented by a matrix in the basis of four states $|0,0\rangle$, $|0,1\rangle$, $|1,0\rangle$, $|1,1\rangle$.

It is seen in Table I that the matrix $\hat{S}(t)$ provides the implementation of the gate ISWAP(θ) ($\theta = 2\sqrt{n_1 n_2}\kappa t$). Choosing the corresponding time "t" we can obtain $\sqrt{n_1 n_2}\kappa t = \pi/2$, where the matrix $\hat{S}(t)$ yields the ISWAP gate, and if $\sqrt{n_1 n_2}\kappa t = \pi/4$, we have the $\sqrt{\text{ISWAP}}$ gate. The ISWAP gate gives a possibility of the quantum excitation transfer between the films under the external magnetic fields that can switch on or switch off this transfer. The $\sqrt{\text{ISWAP}}$ gate together with single-qubit gates form a universal set of gates for quantum computations. It is worth noting that the nano- and mesoscopic magnon BEC qubits can be efficiently integrated with multiqubit atomic quantum memory in microwave QED cavity (Fig. 2) similar to the integration of superconducting and other macroscopic qubits [20,21] that is necessary for realization of a multiqubit quantum computer.

TABLE I. The matrix $\hat{S}(t)$ of the two-qubit operation caused by the magnon-magnon interaction of two magnon Bose-Einstein condensates.

	$\|0,0\rangle$	$\|1,0\rangle$	$\|0,1\rangle$	$\|1,1\rangle$
$\|0,0\rangle$	1			
$\|1,0\rangle$		$\cos(\sqrt{n_1 n_2}\kappa t)$	$-i\sin(\sqrt{n_1 n_2}\kappa t)$	
$\|0,1\rangle$		$-i\sin(\sqrt{n_1 n_2}\kappa t)$	$\cos(\sqrt{n_1 n_2}\kappa t)$	
$\|1,1\rangle$				1

V. DECOHERENCE

The number of magnons in BEC should be preserved with accuracy up to one magnon in order to avoid variation of magnon qubit frequency exceeding 2λ. It can be shown that a stable state of BEC magnons can be formed in YIG film at low temperatures during several nanoseconds after excitation [19]. But the magnon qubit still experiences decoherence due to variation of phase in the course of magnon qubit number quantum fluctuations on the background of a stable mean value due to magnon BEC–thermal reservoir interaction. Decoherence of the qubit in a two-level model can be described by the Bloch equation [22]:

$$\dot{\vec{p}} = \omega \times \vec{p} - R\vec{p} + \vec{p}_0, \tag{8}$$

where $\vec{\omega} = (0,0,\omega_{01})$, ω_{01} is the frequency difference between working levels, and $\vec{p}_0$ is the initial value of Bloch vector $\vec{p}$,

$$R = \begin{pmatrix} T_2^{-1} & 0 & 0 \\ 0 & T_2^{-1} & 0 \\ 0 & 0 & T_1^{-1} \end{pmatrix}, \tag{9}$$

$$\frac{1}{T_1} = 2\text{Re}(\Gamma_{0110}^{(+)} + \Gamma_{1001}^{(+)}), \tag{10}$$

is the reverse time of longitudinal relaxation;

$$\frac{1}{T_2} = \frac{1}{2T_1} + \frac{1}{T_\phi} = \frac{1}{2T_1} + \text{Re}(\Gamma_{0000}^{(+)} + \Gamma_{1111}^{(+)} - 2\Gamma_{0011}^{(+)}), \tag{11}$$

is the reverse time of transverse relaxation.

Here,

$$\Gamma_{lmnk}^{(+)} = \lim_{\varepsilon \to +0} \int_0^\infty dt e^{-i(\omega_{nk} - i\varepsilon)t} \mathrm{Tr}_B H_{\mathrm{SB}}(t)_{lm} H_{\mathrm{SB}}(0)_{nk} \rho_B, \tag{12}$$

where trace is performed over the modes of bath reservoir with density matrix ρ_B,

$$H_{\mathrm{SB}}(t) = e^{iH_B t} H_{\mathrm{SB}} e^{-iH_B t}, \tag{13}$$

where ω_{nk} is related to the resonant frequencies of the analyzed system, H_B is the Hamiltonian of bath, and H_{SB} is the Hamiltonian of the system-bath interaction.

In the case of magnon BEC $H_S = \hbar\omega_{01} b_0^\dagger b_0$, bath comprises reservoir non-BEC magnons and phonons, $H_B = \sum_{k \neq k_0} \hbar\omega_k b_k^\dagger b_k + \sum_q \hbar\Omega_q c_q^\dagger c_q$ and $H_{\mathrm{SB}} = V_{m-m} + V_{m-ph}$, where

$$V_{m-m}^{(1)} = B {\sum_{k,q}}' [\hat{b}_0^\dagger (\hat{b}_k^\dagger \hat{b}_{k+q} \hat{b}_{-q}) + (\hat{b}_{-q}^\dagger \hat{b}_{k+q}^\dagger \hat{b}_k) \hat{b}_0] \tag{14}$$

is the Hamiltonian of reservoir magnons scattering on BEC magnons ($B \equiv B_0$) and

$$V_{m-ph} = F \sum_q (b_0^\dagger b_q c_q^\dagger + b_q^\dagger b_0 c_q) \tag{15}$$

is the Hamiltonian of phonons scattering on BEC magnons ($F \equiv F_0$). Using these Hamiltonians, we get

$$\begin{aligned} \frac{1}{T_1} = {} & 2\pi B^2 n_0 e^{-\frac{\hbar\Delta_p}{k_B T}} \iint dk dq \rho(\Delta_p + \tilde{\omega}_k)\rho(\Delta_p + \tilde{\omega}_q)\delta(\omega_{-10} + \tilde{\omega}_k - \tilde{\omega}_{k+q} - \tilde{\omega}_{-q} - \Delta_p) \\ & \times \left[e^{-\frac{\hbar\Delta_p}{k_B T}} \left(e^{-\frac{\hbar\Delta_p}{k_B T}} e^{-\frac{\hbar\tilde{\omega}_k}{k_B T}} + 1 \right) e^{-\frac{\hbar\tilde{\omega}_{k+q}}{k_B T}} e^{-\frac{\hbar\tilde{\omega}_{-q}}{k_B T}} + e^{-\frac{\hbar\tilde{\omega}_k}{k_B T}} \left(e^{-\frac{\hbar\Delta}{k_B T}} e^{-\frac{\hbar\tilde{\omega}_{k+q}}{k_B T}} + 1 \right) \left(e^{-\frac{\hbar\tilde{\omega}_{-q}}{k_B T}} + 1 \right) \right] + 2\pi F^2 n_0 e^{-\frac{\hbar\Delta_p}{k_B T}} \\ & \times \int dq \rho(\Delta_p + \tilde{\omega}_q, \Omega_q)\delta(\omega_{-10} - \Delta_p - \tilde{\omega}_q + \Omega_q) \left[e^{-\frac{\hbar\tilde{\omega}_q}{k_B T}} \left(e^{-\frac{\hbar\Delta_p}{k_B T}} e^{-\frac{\hbar(\tilde{\omega}_q - \omega_{10})}{k_B T}} + 1 \right) + \left(e^{-\frac{\hbar\Delta_p}{k_B T}} e^{-\frac{\hbar\tilde{\omega}_q}{k_B T}} + 1 \right) e^{-\frac{\hbar(\tilde{\omega}_q - \omega_{10})}{k_B T}} \right], \end{aligned} \tag{16}$$

$$\frac{1}{T_\phi} = 8\pi B^2 e^{-\frac{\hbar\Delta_p}{k_B T}} \iint dk dk' \rho(\Delta_p + \tilde{\omega}_k)\rho(\Delta_p + \tilde{\omega}_{k'})\delta(\tilde{\omega}_k - \tilde{\omega}_{k'}) e^{-\frac{\hbar\tilde{\omega}_k}{k_B T}} \left(e^{-\frac{\hbar\Delta_p}{k_B T}} e^{-\frac{\hbar\tilde{\omega}_{k'}}{k_B T}} + 1 \right), \tag{17}$$

where we have used $\omega_k = \Delta_p + \tilde{\omega}_k$ and assumed that the temperature of reservoir magnons is equal to the temperature of phonons T; ρ is a density of the magnon modes.

We see from expressions (16) and (17) that $T_1, T_\phi \to \infty$ for sufficiently low temperature $k_B T \ll \hbar\Delta_p$. It means that preferable conditions for magnon qubit quantum computer operation are low temperatures. If $\Delta_p \sim 2$ GHz [12], T should be less than 0.1 K.

VI. CONCLUSION

We have proposed a qubit on magnon BEC using a magnon blockade. We have shown that mesoscopic dimensions of the sample are preferable for realization of such blockade. We have also shown that the decoherence processes of the magnon BEC qubit can be highly suppressed at helium temperatures. Possibility of the proposed mesoscopic magnon BEC qubit realization is supported by recent experimental observations of magnon BECs in mesoscopic systems [15–17] and in some nanostructures [23]. We have also considered a realization of single- and two-qubit gates for the construction of a quantum computer on magnon qubits in QED cavity where small sample dimensions ensure effective energy exchange with electromagnetic waves. The operation rate of the gates are proportional to $\sqrt{n}$ and n, respectively, where n is the BEC magnon number. The observed results open promising possibilities for implementation of a long-lived mesoscopic qubit providing realization of fast operation rate for quantum information processing.

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APPENDIX A: HAMILTONIAN OF MAGNONS

Let us consider Hamiltonian (1):

$$H = -\frac{1}{2} \sum_{i \neq j} \sum_{\alpha\beta} [J_{ij}\delta^{\alpha\beta} + D_{ij}^{\alpha\beta}] S_i^\alpha S_j^\beta - \sum_j h_j S_j^z. \tag{A1}$$

Transformation of Holstein-Primakov is written as

$$\begin{aligned} \hat{S}_i^+ &= \sqrt{2S}\sqrt{1 - \frac{\hat{b}_i^\dagger \hat{b}_i}{2S}}\, b_i \\ &= \sqrt{2S}\left(\hat{b}_i - \frac{\hat{b}_i^\dagger \hat{b}_i \hat{b}_i}{4S} + \cdots \right), \end{aligned} \tag{A2}$$

$$\begin{aligned} \hat{S}_i^- &= \sqrt{2S}\, b_i^\dagger \sqrt{1 - \frac{\hat{b}_i^\dagger \hat{b}_i}{2S}} \\ &= \sqrt{2S}\left(\hat{b}_i^\dagger - \frac{\hat{b}_i^\dagger \hat{b}_i^\dagger \hat{b}_i}{4S} + \cdots \right), \end{aligned} \tag{A3}$$

$$\hat{S}_i^z = S - \hat{b}_i^\dagger \hat{b}_i, \tag{A4}$$

where $b_i^\dagger$ and b_i are operators of excitations' creation and annihilation in the node i, S is a spin. We apply it to

Hamiltonian (1) and get

$$\hat{H} = \hat{H}_0 + \hat{H}_1 + \hat{H}_2 + \hat{H}_3 + \hat{H}_4, \tag{A5}$$

where

$$\hat{H}_0 = -\frac{S^2}{2}\sum_{ij}\left(J_{ij} + D_{ij}^{zz}\right), \tag{A6}$$

$$\hat{H}_1 = -\frac{S}{2}\sqrt{\frac{S}{2}}\sum_{ij}\left[\left(D_{ij}^{zx} - iD_{ij}^{zy}\right)\hat{b}_j + \left(D_{ij}^{zx} + iD_{ij}^{zy}\right)\hat{b}_j^\dagger\right], \tag{A7}$$

$$\hat{H}_2 = \sum_{ij} A(ij)\hat{b}_i^\dagger\hat{b}_j + \frac{1}{2}\sum_{ij}\{B(ij)\hat{b}_i\hat{b}_j + B^*(ij)\hat{b}_i^\dagger\hat{b}_j^\dagger\}, \tag{A8}$$

$$\hat{H}_3 = \frac{1}{2}\sqrt{\frac{S}{2}}\sum_{ij}\left\{\left(D_{ij}^{zx} + iD_{ij}^{zy}\right)\left(\hat{b}_i^\dagger\hat{b}_i\hat{b}_j + \frac{1}{4}\hat{b}_j^\dagger\hat{b}_j^\dagger\hat{b}_j\right) + \text{H.c.}\right\}, \tag{A9}$$

$$\begin{aligned}\hat{H}_4 = &-\frac{1}{2}\sum_{ij} J_{ij}\left\{\hat{n}_i\hat{n}_j - \frac{1}{2}(\hat{b}_i^\dagger\hat{b}_j^\dagger\hat{b}_j\hat{b}_j + \text{H.c.})\right\}\\ &-\frac{1}{2}\sum_{ij}\left\{D_{ij}^{zz}\hat{n}_i\hat{n}_j - \frac{1}{4}\left(D_{ij}^{xx} + D_{ij}^{yy}\right)(\hat{b}_i^\dagger\hat{b}_j^\dagger\hat{b}_j\hat{b}_j + \text{H.c.})\right\}\\ &+\frac{1}{8}\sum_{ij}\left\{\left(D_{ij}^{xx} - iD_{ij}^{xy} - D_{ij}^{yy}\right)\hat{b}_i^\dagger\hat{b}_i\hat{b}_i\hat{b}_j + \text{H.c.}\right\},\end{aligned} \tag{A10}$$

where

$$\begin{aligned}A(ij) = S_1&\left\{\delta_{ij}\left(\sum_{n=1}^{N} J_{\text{in}} + h\right) - J_{ij}\right.\\ &\left.+\delta_{ij}\sum_{n=1}^{N} D_{\text{in}}^{zz} - \frac{D_{ij}^{xx} + D_{ij}^{yy}}{2}\right\},\end{aligned} \tag{A11}$$

$$B(ij) = -\frac{S}{2}\left(D_{ij}^{xx} - iD_{ij}^{xy} - D_{ij}^{yy}\right). \tag{A12}$$

The operator Fourier transform,

$$\hat{b}_i = \frac{1}{\sqrt{N}}\sum_k e^{i\vec{k}\vec{r}_i}\hat{b}_k, \tag{A13}$$

gives the following expression for terms of the Hamiltonian preserving the number of magnons:

$$H = \sum_k A_k\hat{b}_k^\dagger\hat{b}_k + \sum_{kk'k''k'''}(B_{k+k'-k''-k'''}\hat{b}_k^\dagger\hat{b}_{k'}^\dagger\hat{b}_{k''}\hat{b}_{k'''} + \text{H.c.}), \tag{A14}$$

where

$$A(k) = S\left(h + J_0 + D_0 - J_k - \frac{D_k^{xx} + D_k^{yy}}{2}\right), \tag{A15}$$

$$\begin{aligned}&B_{k+k'-k''-k'''}\\ &= \frac{1}{4N}\left\{\left(J_{k,k'-k''-k'''} + \frac{D_{k,k'-k''-k'''}^{xx} + D_{k,k'-k''-k'''}^{yy}}{2}\right)\right.\\ &\quad+\left(J_{-k,-k'+k''+k'''} + \frac{D_{-k,-k'+k''+k'''}^{xx} + D_{-k,-k'+k''+k'''}^{yy}}{2}\right)\\ &\quad\left.-2\left(J_{k-k'',k'-k'''} + D_{k-k'',k'-k'''}^{zz}\right)\right\},\end{aligned} \tag{A16}$$

$$J_k = \sum_i J_{ij}e^{-i\vec{k}\vec{r}_{ij}}, \tag{A17}$$

$$D_k^{\alpha\beta} = \sum_i D_{ij}^{\alpha\beta}e^{-i\vec{k}\vec{r}_{ij}}, \tag{A18}$$

$$J_{k_1,k_2} = \frac{1}{N}\sum_{ij} J_{ij}e^{-i\vec{k}_1\vec{r}_i}e^{-i\vec{k}_2\vec{r}_j}, \tag{A19}$$

$$D_{k_1,k_2}^{\alpha\beta} = \frac{1}{N}\sum_{ij} D_{ij}^{\alpha\beta}e^{-i\vec{k}_1\vec{r}_i}e^{-i\vec{k}_2\vec{r}_j} \tag{A20}$$

and we used an approximation of spatially homogeneous sample where the interaction constants depend only on the distance between the atoms; we also considered the case of constant magnetic field h inside the sample.

APPENDIX B: MODIFICATION OF MAGNON BEC HAMILTONIAN UNDER THE INFLUENCE OF RESERVOIR MAGNONS

Let us consider the Hamiltonian

$$\begin{aligned}H = &A_0\hat{n}_0 + \sum_k{}' A_k\hat{n}_k + \frac{U}{2N}(\hat{n}_0)^2\\ &+\frac{U}{N}\sum_{k,q}{}'[\hat{b}_0^\dagger(\hat{b}_k^\dagger\hat{b}_{k+q}\hat{b}_{-q}) + (\hat{b}_{-q}^\dagger\hat{b}_{k+q}^\dagger\hat{b}_k)\hat{b}_0]\\ &+\frac{U}{N}\sum_q{}'\{[\hat{b}_0^\dagger\hat{b}_0^\dagger(\hat{b}_q\hat{b}_{-q}) + (\hat{b}_{-q}^\dagger\hat{b}_q^\dagger)\hat{b}_0\hat{b}_0]\\ &+[\hat{n}_0\hat{n}_q + (\hat{n}_q + 1)(\hat{n}_0 + 1)]\},\end{aligned} \tag{B1}$$

that can be obtained from Hamiltonian (A14) neglecting the wave vector dependence of the interaction constant and accounting only terms with participation of magnon BEC. Let us introduce

$$H_0 = A_0\hat{n}_0 + \sum_k{}' A_k\hat{n}_k + \frac{U}{2N}(\hat{n}_0)^2 + \frac{U}{N}\sum_q{}'[\hat{n}_0 + \hat{n}_q + 1]. \tag{B2}$$

Then

$$H = H_0 + V_1 + V_2, \tag{B3}$$

where

$$V_1 = \frac{U}{N}\sum_{k,q}{}'[\hat{b}_0^\dagger(\hat{b}_k^\dagger\hat{b}_{k+q}\hat{b}_{-q}) + (\hat{b}_{-q}^\dagger\hat{b}_{k+q}^\dagger\hat{b}_k)\hat{b}_0], \tag{B4}$$

$$V_2 = \frac{U}{N}\sum_q{}' [\hat{b}_0^\dagger \hat{b}_0^\dagger(\hat{b}_q \hat{b}_{-q}) + (\hat{b}_{-q}^\dagger \hat{b}_q^\dagger)\hat{b}_0\hat{b}_0]. \tag{B5}$$

We denote wave functions of BEC giving us

$$H_0|\varphi_{n,m_k}\rangle = E_{n,\{m_k\}}|\varphi_{n,\{m_k\}}\rangle, \tag{B6}$$

where energy E_{n,m_k} includes the impact from magnon BEC and from other magnons,

$$E_{n,\{m_k\}} = A_0 n + \frac{U}{2N}(n)^2 + \frac{U}{N}\sum_k{}' [n + m_k + 1] + \sum_k{}' A_k m_k, \tag{B7}$$

$$|\varphi_{n,m_k}\rangle = |n\rangle_0 \prod_k |m\rangle_k. \tag{B8}$$

We are interested in the ground state of BEC that in the approximation of absent interaction with magnons outside BEC is written as

$$|\varphi_{n,m_k=0}\rangle \equiv |\varphi_{n,0}\rangle = |n\rangle_0 \prod_k |0\rangle_k. \tag{B9}$$

Using perturbation theory, we find the BEC wave function in the first order of perturbation theory,

$$|\psi_{n,0}\rangle \equiv |\varphi_{n,0}\rangle + \sum_{n'\neq n}\sum_k{}' |n\rangle_0 \prod_k |0\rangle_{k_0}\langle n| \prod_k {}_k\langle 0| \times \frac{(V_1+V_2)(V_1+V_2)}{(E_{n,0} - E_{n',\{m_k\}})}|n'\rangle_0 \prod_k |m\rangle_{k'}, \tag{B10}$$

and BEC energy in the perturbation theory second order by interaction with other magnons,

$$\tilde{E}_{n,0} \equiv E_{n,0} + \sum_{n'\neq n}\sum_{q,m_q}{}' \frac{\Omega_{n,n'}^{0_k,m_q}\left(\Omega_{n,n'}^{0_k,m_q}\right)^*}{(E_{n,0} - E_{n',\{m_q\}})}, \tag{B11}$$

where

$$\Omega_{n,n'}^{0_k,m_q} = \langle n| \prod_k {}_k\langle 0|(V_1+V_2)\prod_q |m\rangle_q |n'\rangle_0. \tag{B12}$$

Finally, we get $\tilde{E}_{n,0} \equiv E_{n,0} + \delta E_{n,0}$, where energy correction

$$\delta E_{n,0} = \left(\frac{U}{N}\right)^2 n(n-1)\sum_q \frac{1}{2(A_0 - A_q) - \frac{|U|}{N}(n-2)} \tag{B13}$$

is calculated at the negative value of constant $U = -|U|$, when there is no resonance between frequencies of BEC magnons and other magnons since $A_q > A_0$, so that the denominator in (B13) never becomes zero when summing over various magnon frequencies. Such a sign of constant $U = -|U|$ indicates an attractive character of magnon interaction that provides condensate stability. Let us consider the case of sufficiently large gap $A_q - A_0 \gg \frac{|U|}{N}(n-2) \cong \frac{|U|}{N}n$, when the influence of the spectral region where this condition does not fulfill is negligibly small, which gives the following expression for energy correction:

$$\delta E_{n,0} \cong \delta\varepsilon n - \delta\lambda n^2 + \delta\mu n^3, \tag{B14}$$

where at $U > 0$ constants

$$\delta\varepsilon = -\frac{1}{2}\left(\frac{U}{N}\right)^2 \sum_q \frac{1}{(A_0 - A_q)}\left\{1 - \frac{1}{(A_0 - A_q)}\frac{U}{N}\right\}, \tag{B15}$$

$$\delta\lambda = -\frac{1}{2}\left(\frac{U}{N}\right)^2 \sum_q \frac{1}{(A_0 - A_q)}\left\{1 - \frac{3}{2(A_0 - A_q)}\frac{U}{N}\right\}, \tag{B16}$$

$$\delta\mu = \frac{1}{4}\left(\frac{U}{N}\right)^3 \sum_q \frac{1}{(A_0 - A_q)^2}, \tag{B17}$$

are positively definite. The term $\delta\varepsilon\hat{n}$ leads to linear magnon frequency shift that can be compensated by the external constant magnetic field. The second term, $-\delta\lambda\hat{n}^2$, indicates the presence of attractive interaction of condensed magnons via magnon modes of other frequencies that again indicate the stability of magnon condensate. The third term, $\delta\mu\hat{n}^3$, is significantly less than the first two terms and we neglect its influence on the nonlinear character of magnon BEC spectrum.

Returning in (B13) back to the magnon operators we get

$$\hat{\tilde{H}} \cong \tilde{E}\hat{n} + \tilde{\lambda}\hat{n}^2, \tag{B18}$$

where

$$\tilde{E} = A_0 - \frac{M}{N}|U| + \delta\varepsilon, \tag{B19}$$

$$\tilde{\lambda} = -\frac{|U|}{2N} - \delta\lambda. \tag{B20}$$

M is the number of magnon modes.

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