

Dynamics of interacting Dicke model in a coupled-cavity arrayFazal Badshah,^{1,2} Shahid Qamar,¹ and Mauro Paternostro²¹*Department of Physics and Applied Mathematics, Pakistan Institute of Engineering and Applied Sciences, Nilore, Islamabad 45650, Pakistan*²*Centre for Theoretical Atomic, Molecular, and Optical Physics, School of Mathematics and Physics, Queen's University, Belfast BT7 1NN, United Kingdom*

(Received 9 April 2014; published 9 September 2014)

We consider the dynamics of an array of mutually interacting cavities, each containing an ensemble of N two-level atoms. By exploring the possibilities offered by ensembles of various dimensions and a range of atom-light and photon-hopping values, we investigate the generation of multisite entanglement, as well as the performance of excitation transfer across the array, resulting from the competition between on-site nonlinearities of the matter-light interaction and intersite photon hopping. In particular, for a three-cavity interacting system it is observed that the initial excitation in the first cavity completely transfers to the ensemble in the third cavity through the hopping of photons between the adjacent cavities. Probabilities of the transfer of excitation of the cavity modes and ensembles exhibit characteristics of fast and slow oscillations governed by coupling and hopping parameters, respectively. In the large-hopping case, by seeding an initial excitation in the cavity at the center of the array, a tripartite W state, as well as a bipartite maximally entangled state, is obtained, depending on the interaction time. Population of the ensemble in a cavity has a positive impact on the rate of excitation transfer between the ensembles and their local cavity modes. In particular, for ensembles of five to seven atoms, tripartite W states can be produced even when the hopping rate is comparable to the cavity-atom coupling rate. A similar behavior of the transfer of excitation is observed for a four-coupled-cavity system with two initial excitations.

DOI: [10.1103/PhysRevA.90.033813](https://doi.org/10.1103/PhysRevA.90.033813)

PACS number(s): 42.50.Pq, 42.50.Ex, 03.67.-a

I. INTRODUCTION

In cavity quantum electrodynamics (QED), which studies interaction of light and matter, atomic systems are used to embody qubits thanks to the fact that two suitably selected internal electronic states, selected by (quasi)resonant light fields, can be used to coherently store information over considerable time scales. Photons are very effective to transfer information due to their high transport speed and large bandwidth. A high-finesse cavity provides good insulation against the environment. A good cavity can store photons for a long time before they are dissipated. These facts have proved cavity QED to be a very useful platform for processing quantum information [1–7]. An alternative to the cavity-QED scenario is provided by its solid-state analog, termed as a circuit QED, where superconducting qubits are coupled to strip-line resonators [8–10]. When compared to cavity QED, the solid-state platform has the advantages of strong coupling, a well-determined number of standing-still qubits, and greater promise for scalability.

A requirement of distributed quantum information processing (QIP) is the coupling of distant qubits embodying the nodes of a quantum network. Such coupling would be instrumental to the achievement of state transfer among various nodes of the system and the generation of distributed entanglement. Atoms confined in remote cavities may be coupled by connecting the resonators via optical fibers. In this context, one of the early proposals focused on the conversion of an atomic state into a superposition of Fock states of a cavity-field mode [11]. This scheme effectively uses an optical fiber to transfer the state prepared in one cavity to the atom accommodated within a second, remote resonator. The use of a sequence of laser pulses instead of an optical fiber has been suggested to minimize the chance of decoherence due to losses in the fiber [12]. There are various techniques to minimize population of

the cavity modes or fiber while performing a quantum state transfer [13]. Alternatively, a sufficiently strong cavity-fiber coupling effectively eliminates the degrees of freedom of the fiber from the dynamics of the system [14]. Similarly, the use of a highly detuned qubits-cavity mode reduces the chance of cavity-mode population. Thus a careful manipulation of the atom-cavity interaction insures high-fidelity swap and entangling gate generation.

Systems of coupled cavities have received much attention recently [15–21] due to the easy handling of the individual sites (using an optical laser) and the presence of relatively long-lived atomic states suitable for encoding quantum information. Furthermore, such systems provide a number of degrees of freedom to control dynamics of the system in a better way where photons are allowed to hop between neighboring cavities. Coupled-cavity models have potential applications in QIP as well, since a control and measurement of individual lattice sites and an almost lossless guiding and coupling of light pulses at slow group velocities are available in such systems. The coupling among the cavities can be controlled in many ways, which offers great deal of freedom and flexibility to engineer the transfer of quantum states via photonic processes.

An extensive study of the dynamics of two coupled cavities, each containing a single two-level atom, has been carried out in the past [20–22], which has very recently been extended by Zhong *et al.* [23] to the case of three mutually interacting cavities. In the strong-coupling regime ($\Omega \gg J$), cavities embedded with quantum dots [24], atoms [25], and superconducting qubits [26] have been proposed for the implementation of quantum logic gates and the construction of different types of quantum networks [27–30]. Similarly, photon-blockade effects have been studied, thus paving the way to observation of the predicted polaritonic Mott insulator phase [16,18]. In the strong hopping regime, various aspects of the transfer of quantum excitations

between qubits without populating cavity modes have been analyzed [14,15,17,19,20].

A promising alternative to the single-atom scenarios addressed above is embodied by the use of ensembles of atoms, all collectively coupled to the single mode of radiation of a cavity. The collective character of the coupling between N atoms and light enhances the atom-field interaction strength by a factor \sqrt{N} , an effect that has already been experimentally demonstrated in an interesting study [31]. Such enhancement is advantageous, as it allows for the implementation of fast quantum gates at moderate laser intensities. A sample of N excited atoms was shown to form a collective dipole moment, which leads to effects such as a fluorescence intensity proportional to N^2 and a quantum dynamics N times faster than that for a single atom [32]. The collective states of an ensemble can be used to realize quantum memories and store information, thus offering the opportunity of having large quantum registers in a single atomic ensemble [33].

In this paper, we discuss the dynamics of M coupled cavities, each containing an ensemble of N two-level atoms. The transfer of excitations under a large range of operative conditions is demonstrated and explored by tuning the system's control parameters. We find that, for a judicious choice of N , a three-atom W state [34] can be obtained even for coupling and hopping strengths of comparable values. In the case of large hopping strengths, a three-atom W state and two-atom maximally entangled states are obtained for ensembles containing many two-level atoms. We find that a four-cavity system with initial excitations in the second and third ensemble undergoes a similar transfer of excitations among the atoms of different cavities.

II. MODEL AND DYNAMICS

We consider a unidimensional array of M mutually coupled single-mode microcavities (mode frequency ω_c) (see Fig. 1). Adjacent cavities are assumed to be close enough to ensure the mutual transfer of photons via evanescent fields. As the probability for this mechanism to occur drops exponentially with the distance between the cavities, only nearest-neighbor interactions will be considered. The cavities are doped with an ensemble of N two-level atoms each. We call $\{|a\rangle, |b\rangle\}$ the excited and ground state of each atom, respectively, and

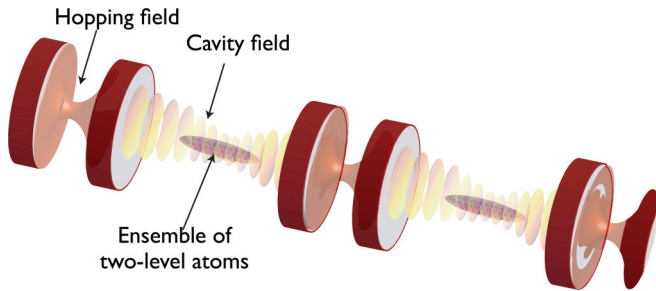


FIG. 1. (Color online) Sketch of the physical configuration studied in the paper. We consider an array of cavities coupled via hopping fields and containing an ensemble of two-level atoms each. The atoms are collectively coupled with the cavity field. We show a two-site subsystem of a longer array.

assume they are separated by the Bohr frequency ω_{ab} . Each atom interacts via electric-dipole coupling with the mode of the respective cavity. The free Hamiltonian for the system is thus (we assume units such that $\hbar = 1$)

$$\hat{H}_0 = \sum_{j=1}^M \left(\omega_c \hat{a}_j^\dagger \hat{a}_j + \omega_{ab} \sum_{k=1}^N \hat{\sigma}_{kj}^z \right), \quad (1)$$

where \hat{a}_j^\dagger (\hat{a}_j) is the creation (annihilation) operator of the j th cavity mode, and $\hat{\sigma}_{kj}^z$ is the z -Pauli matrix of the k th atom of the ensemble in such a cavity. By imposing the resonant condition $\omega_c \simeq \omega_{ab}$ and assuming collective coupling of the atoms in an ensemble to the respective cavity field, the overall coupling Hamiltonian in the interaction picture takes the form

$$\hat{H}_I = - \sum_{j=1}^M J_j \hat{a}_j \hat{a}_{j+1}^\dagger + \sum_{j=1}^M \Omega_j \hat{a}_j^\dagger \hat{S}_j^- + \text{H.c.}, \quad (2)$$

where we have introduced the Dicke lowering operator of the atoms within the j th cavity $\hat{S}_j^\pm = \sum_{k=1}^N \hat{\sigma}_{kj}^\pm$ and the single-atom ladder operator $\hat{\sigma}_{kj}^\pm = (\hat{\sigma}_{kj}^\mp)^\dagger = |a\rangle\langle b|$. Equation (2) contains two different contributions. The first term and its Hermitian conjugate are responsible for the tunneling of photons between adjacent cavities (occurring at rate J_j). The second term and its Hermitian conjugate account for the *in situ* resonant exchange of excitations, at a rate Ω_j , between each of the N elements of an ensemble and the corresponding cavity field mode.

The collective nature of the coupling between the atoms of an ensemble and the field of the respective cavity suggests the use of the Holstein-Primakoff (HP) transformation that maps a collective spin- $N/2$ particle into an effective boson with associated creation and annihilation operators \hat{b}_j^\dagger and \hat{b}_j ($j = 1, \dots, M$) such that [35]

$$\hat{S}_j^+ = (\hat{S}_j^-)^\dagger = \sqrt{N} \hat{b}_j^\dagger \hat{A}_j, \quad \hat{S}_j^z = \hat{b}_j^\dagger \hat{b}_j - N/2. \quad (3)$$

The Hermitian operator $\hat{A}_j = (1 - \hat{b}_j^\dagger \hat{b}_j / N)^{1/2}$ in Eq. (3) allows the Dicke operators to satisfy the SU(2) algebra. Assuming a uniform distribution of coupling rates across the array, the interaction Hamiltonian in Eq. (2) takes the form

$$\hat{H}_{HP} \simeq - \sum_{j=1}^M J \hat{a}_j \hat{a}_{j+1}^\dagger + \sum_{j=1}^M \Omega \sqrt{N} \hat{b}_j^\dagger \hat{A}_j \hat{a}_j + \text{H.c.} \quad (4)$$

The strength of the nonlinear term entering Eq. (4) results from the trade-off between $\langle \hat{b}_j^\dagger \hat{b}_j \rangle$ and N . For a ‘‘mesoscopic’’ number of atoms per ensemble, we can expand \hat{A}_j in the power series of $1/N$, stopping at the first order. Physically, this implies that the number of implanted two-level atoms per cavity should be large enough for the HP transformation to be valid but sufficiently small not to blur any nonlinear effect. By introducing the truncated form of \hat{A}_j in Eq. (4), the Hamiltonian of the system becomes

$$\hat{H}_{HP} \simeq \sum_{j=1}^M \left(\Omega \sqrt{N} \hat{b}_j^\dagger \hat{a}_j - J \hat{a}_j^\dagger \hat{a}_{j+1} - \frac{\Omega}{2\sqrt{N}} \hat{b}_j^{\dagger 2} \hat{b}_j \hat{a}_j + \text{H.c.} \right). \quad (5)$$

A careful look of Eq. (5) reflects its interesting structure: the term $\Omega\sqrt{N}(\hat{b}_j^\dagger\hat{a}_j + \hat{b}_j\hat{a}_j^\dagger)$ embodies a two-mode mixing process, while $(\Omega/2\sqrt{N})(\hat{b}_j^\dagger\hat{n}_j\hat{a}_j + \hat{n}_j\hat{b}_j\hat{a}_j^\dagger)$ [with $\hat{n}_j = \hat{b}_j^\dagger\hat{b}_j$] encompasses the nonlinear character of the interaction, noticeably linked directly to the number of excitations of the HP boson at site j . The Heisenberg equations of motion for the two bosonic species involved in the dynamics thus read

$$\begin{aligned} i\partial_t\hat{a}_j &= -J(\hat{a}_{j+1} + \hat{a}_{j-1}) + \Omega\sqrt{N}(1 - \hat{b}_j^\dagger\hat{b}_j/2N)\hat{b}_j, \\ i\partial_t\hat{b}_j &= \Omega\sqrt{N}\hat{a}_j - \frac{\Omega}{2\sqrt{N}}(2\hat{b}_j^\dagger\hat{b}_j\hat{a}_j + \hat{b}_j^2\hat{a}_j^\dagger). \end{aligned} \quad (6)$$

In what follows we solve the above equations numerically, studying the features of the entangling dynamics arising from the competition between on-site nonlinearity and intersite hopping against the controlling parameters of our model.

III. ANALYSIS OF THE DYNAMICS: MULTIPARTITE ENTANGLEMENT GENERATION

To fix the ideas, we assume homogeneous couplings across the array, the same number of atoms per ensemble, and the same value of Ω per site. Needless to say, this assumption would not be met in an experimental implementation, where fabrication and coupling inaccuracies would induce small differences in the values of such parameters. However, such analysis goes beyond the scope of this paper and will be performed elsewhere [36]. Noticeably, the effects of disorder in arrays of coupled cavities have been studied in Refs. [37] and [38] for both small- and large-scale arrays.

A. Comparable coupling and hopping

As a first limiting case we consider a situation when coupling of the ensembles to their local modes and hopping of photons between the neighboring cavities are of nearly the same strength, i.e., $\Omega \simeq J$. This provides an equal chance of localized excitation transfer in a certain cavity and hopping of the fields between various cavities.

Figure 2 shows the results of our numerical simulations of the dynamics of both fields [cf. panels (a)–(c)] and ensembles [panel (d)], where the probabilities of excitation are plotted vs the time evolution. Obviously, the smaller the time step the more accurate the results of numerical simulations. Here we take $\Delta t = 10^{-4}$, which is sufficiently reasonable for accurate results against $J/\Omega = 1$ and $N = 10$. As initial conditions, here we assume that all the cavities are prepared in vacuum states [$a_{1,2,3}(0) = 0$], and there is a single excitation only in the first cavity's ensemble, i.e., [$b_1(0) = 1, b_2(0) = b_3(0) = 0$]. The plots shown in Figs. 2(a)–2(c) give the evolution of the system in first, second, and third cavity modes, respectively. The corresponding dynamics of the ensembles is given in Fig. 2(d), where the red solid, thick dashed black, and thin dashed blue lines represent excitations in the first, second, and third cavities of the system, respectively.

Evolution of the system in Fig. 2 clearly reflects the transfer of excitations among the cavities. The transformation process of the quantum excitation takes place in the following way. Energy in the form of excitation initially present in the ensemble of cavity 1 first transfers to its corresponding

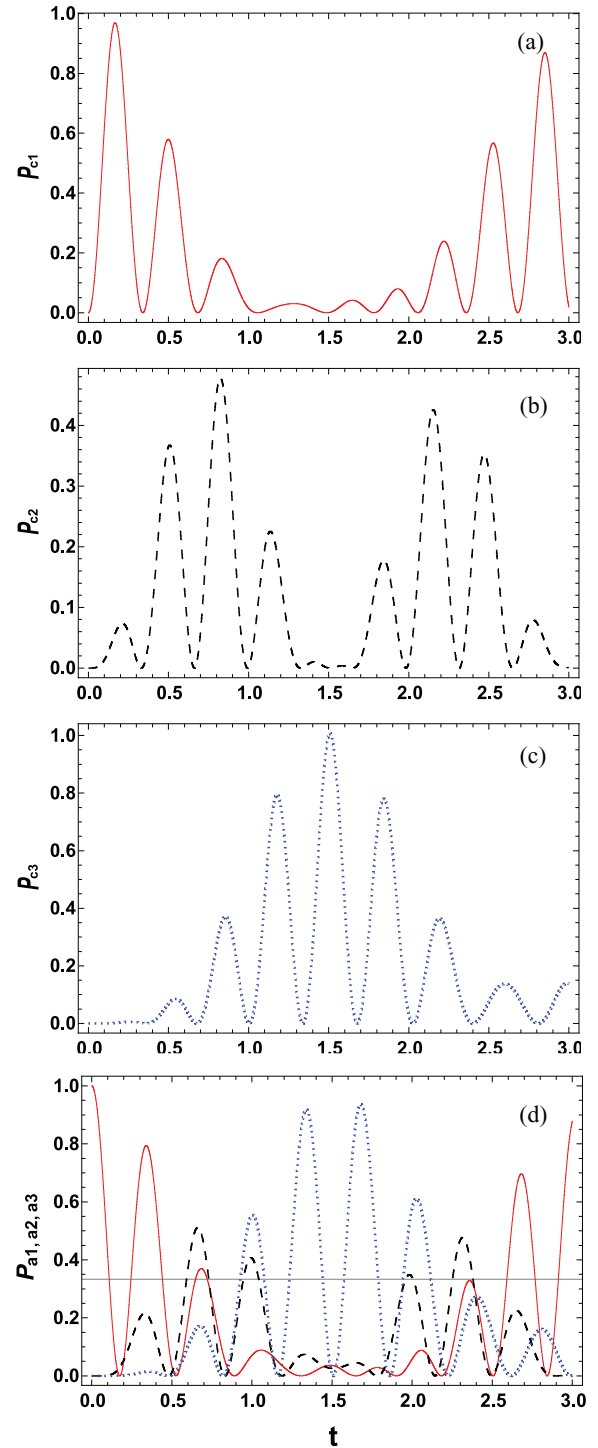


FIG. 2. (Color online) Probabilities of excitation transfer vs time for cavity modes (a)–(c) and ensembles (d), where the solid, thick dashed, and thin dotted lines represent the cases of excitations in the first, second, and third cavity, respectively. Here a single excitation is taken only in the ensemble of the first cavity, i.e., $b_1(0) = 1$, while all the three-cavity modes are initially in their vacuum states. The dynamics of the system corresponds to the resonance case with $J/\Omega = 1$ and $N = 10$ qubits per ensemble.

mode. Then, through cavity-cavity hopping, the energy in cavity mode 1 transfers to cavity mode 2, where it causes excitation of the local ensemble. The phenomenon in cavity

2 is a bit involved because it is coupled with both cavities 1 and 3 through hopping apart from coupling to its local ensemble. In the next step, the excitation transfers to cavity mode 3 from cavity mode 2, which induces excitation in ensemble 3. Then the reverse process takes place, which ultimately leads to the excitation of cavity mode 1 and then the corresponding ensemble, and thus the process repeats again and again.

The probabilities of excitation of cavity modes ($P_{c1} - P_{c3}$) shown in Fig. 2 clearly reveal the presence of some fast and slow characteristic oscillations. Here fast oscillations represent the excitation transfer between an ensemble and the localized cavity mode, while the slow oscillations are due to the transfer of excitation between the adjacent cavities caused by cavity-cavity hopping. The periods of fast and slow oscillations for the current values of parameters are $T_f = 1/3$ and $T_s = 3$, respectively. Here the initial excitation in the ensemble of cavity 1, following the process of excitation transfer, eventually reaches to the mode and ensemble of cavity 3. It is worth mentioning that during half of the slow period of excitation, the mode of cavity 3 becomes fully excited. The probabilities of excitation transfer as high as 100% are observed in the present study. The probability of excitations of cavity 2 is halved and its frequency of excitation is doubled as compared to the cavities at the ends, which is due to simultaneous coupling of the middle cavity with the modes of the first and third cavities. In a recent study, Zhang and Li [21] discussed the resonant interaction of a system of two coupled cavities each having a single two-level atom. It was shown that at certain times, the energy is fully transferred from one quantum subsystem to the other. When the coupling strengths are comparable in magnitude, the role of competition becomes important, which results in a complicated amplitude modulation behavior. The effects of detuning on a system of two coupled cavities each having a single two-level atom have been studied by Jun *et al.* [22]. It was shown that for coupling rates of comparable strengths, the probability of excitation of cavity modes is only 4%. In another interesting study related to three coupled cavities each enclosing a single two-level atom, Zhong *et al.* [23] showed that the probability to transfer excitations to the cavity modes is at most 50%. In the presence of detuning, the probability of finding a photon in each of the three modes further decreases to values that are always less than 0.12% [23].

The dynamics of the ensembles ($P_{a1} - P_{a3}$) is given in Fig. 2(d), where the solid, thick dashed, and thin dotted lines represent excitations in the ensembles of first, second, and third cavities, respectively. One of the interesting results here is that the excitations of the ensembles follow the pattern of excitations of cavity modes, which was not reported for resonant interaction of two- and three-cavity systems with single two-level atoms [21,22]. In the case of large detuning for two and three coupled cavities, the mode excitation was suppressed and the transfer of excitations from one atom to the other was found to be carried out by virtual photons [22,23]. The number of atoms per ensemble N is an important parameter in the evolution of our system. It is noted that for $N = 30$, while keeping other parameters as in Fig. 2, the period of fast oscillations changes to $T_f = 1/15$ while the period of slow oscillation remains static. This confirms that the local

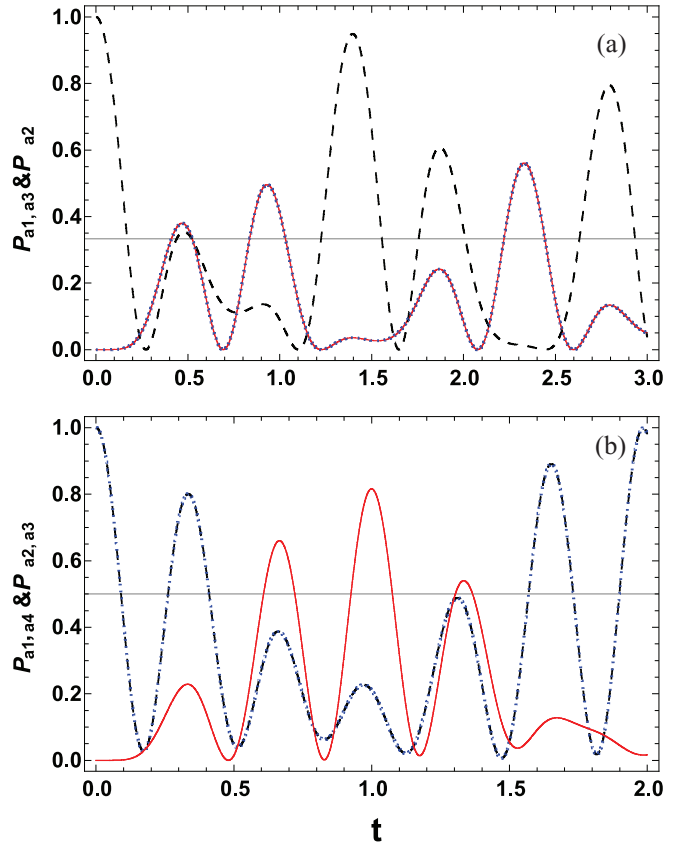


FIG. 3. (Color online) Probabilities of excitation transfer vs time for ensembles in three (a) and four (b) coupled cavities. The dynamics of the system correspond to the resonance case with $J = \Omega = 3$ and the other parameters are (a) $N = 5$, $b_1(0) = b_3(0) = 0$, $b_2(0) = 1$ and (b) $N = 10$, $b_1(0) = b_4(0) = 0$, $b_2(0) = b_3(0) = 1$.

transfer of excitation increases by increasing the number of qubits in the ensemble.

The behavior of the system dramatically changes if instead of ensemble 1, the initial excitation is taken in the ensemble of cavity 2. We reveal evidence of the formation of a W state at $t = 0.55$ for $N = 5$, as shown in Fig. 3(a), unlike the previous study [23] where W states were found only in the presence of detuning ($\Delta = 5\Omega$). It is noted that the position of W states can be controlled by tuning the number of atoms in the ensemble, e.g., it shifts to $t = 2.0$ for $N = 7$. It is clear that tuning of the number of atoms in an ensemble can provide control over the entanglement of the system. The dynamical equation (6) is applicable for any number of coupled cavities. Here we find that for a four-cavity system with two initial excitations in the ensembles of second and third cavities, the transfer of excitation takes place from cavities 2 and 3 to cavities 1 and 4 [cf. Fig. 3(b)].

B. Large hopping

Next we investigate the dynamical behavior of our system for the large-hopping $J \gg \Omega$ limit. In this case, fields of the neighboring cavities are more strongly coupled as compared to the coupling of the ensembles to their local cavity modes. As a result, the hopping terms in Eq. (6) play the dominant role in

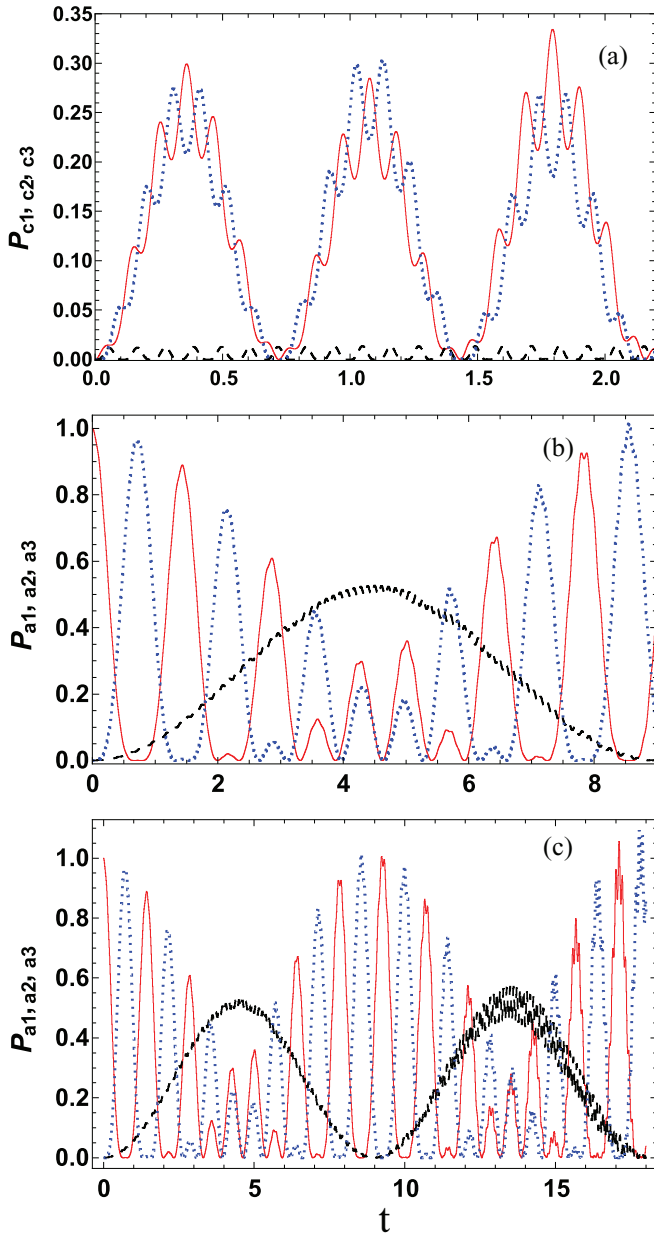


FIG. 4. (Color online) Probabilities of excitation transfer vs time for cavity modes (a) and ensembles (b) and (c), where the solid, thick dashed, and thin dotted lines represent excitations in the first, second, and third cavity, respectively. The dynamics of the system correspond to the large-hopping case with parameters $J = 40, \Omega = 1, N = 20$, and $b_1(0) = 1$.

the evolution process. In Fig. 4 probabilities vs time are plotted where the various parameters are $J = 40, \Omega = 1$, and $N = 20$. Again, all the cavity modes and ensembles 2 and 3 are assumed to be prepared in the initial ground state with $b_1(0) = 1$. Figure 4(a) gives the behavior of various modes of the cavities where the solid, thick dashed, and thin dotted curves correspond to first, second, and third cavities, respectively. It shows that fields of the cavities have small probabilities of excitation as compared to the previous case of comparable coupling-hopping. Here probabilities of various modes do not follow the pattern of oscillations for the corresponding

ensembles. Oscillations of modes 1 and 3 have nearly double frequency as compared to the oscillations of their respective ensembles and also contain sudden jumps. A careful look at the frequency of oscillations of cavity mode 2 indicates that it is 90 times greater than its corresponding ensemble's frequency. This is obviously due to the $J \gg \Omega$, and the simultaneous linking with cavities 1 and 3.

The dynamics of the ensemble of the cavities is shown in Fig. 4(b), which indicates that the initial excitation in ensemble 1 has transferred 100% to ensemble 3, in spite of the small probability of excitation of the modes of different cavities. Here the pattern of the excitation transfer has a regular shape with a continuously varying amplitude. The periods of excitation for the fast oscillations T_f of the ensembles at first and third sites are equal and are 10 times the excitations of the ensemble in the second cavity. Apart from being much slower, the excitation transfer of the second cavity is only 50% of the initial excitation; nevertheless it plays a good mediating role for the transfer of excitation between cavities 1 and cavity 3. In the middle of the plot, the excitation transformation of the second cavity gets the dominant role as compared to the other channels, which are dominant in the beginning and at the end of the plot. The same behavior repeats as time progresses, which is depicted in Fig. 4(c).

For the initial excitation being in ensemble 2, i.e., for $b_2(0) = 1$ [cf. Fig. 5], the probabilities of excitation of cavity modes drops further, as shown in Fig. 5(a), from a bit higher value [see Fig. 4(a)]. A similar behavior was reported in [23] for a system of three coupled cavities containing a single two-level atom. The dynamics of the ensembles is given in Fig. 5(b), which shows the presence of tripartite W states at $t = 2.8$ and $t = 6.3$ and the bipartite maximally entangled state at $t = 4.5$. A four-cavity system having initial excitations in ensembles 2 and 3 also behaves in a similar way, as shown in Fig. 5(c). It may be pointed out that for three coupled cavities each having a single two-level atom, the occurrence of a W state has been previously reported [23]; however, the overall process of excitation transfer was much slower as compared to the case of an atomic ensemble.

C. Strong coupling

The next special case to be discussed is the one with the strong coupling. Here, $\Omega \gg J$, i.e., the ensembles, are more strongly coupled to the local modes of their respective cavities than the hopping of the fields between the two adjacent cavities. We consider the same initial conditions [$a_{1,2,3}(0) = b_{2,3}(0) = 0$ with $b_1(0) = 1$] as were taken in the previous two cases. Evolution of the system for the parameters $\Omega = 10, J = 1.0$, and $N = 10$ is presented in Fig. 6, where panels (a)–(d) give the probabilities of the cavity modes and ensembles. There is a rapid transfer of excitation between the ensemble and the local mode of the cavity due to the high value of coupling, i.e., $\Omega = 10$, while the transfer of excitation between adjacent cavities is much slower than the large-hopping case. During the process of transformation, a slowly varying amplitude phenomenon is observed due to the field-hopping effect. The transfer of excitation of the ensembles follows the pattern of the respective modes similar to the comparable coupling-hopping case. It can be noted that

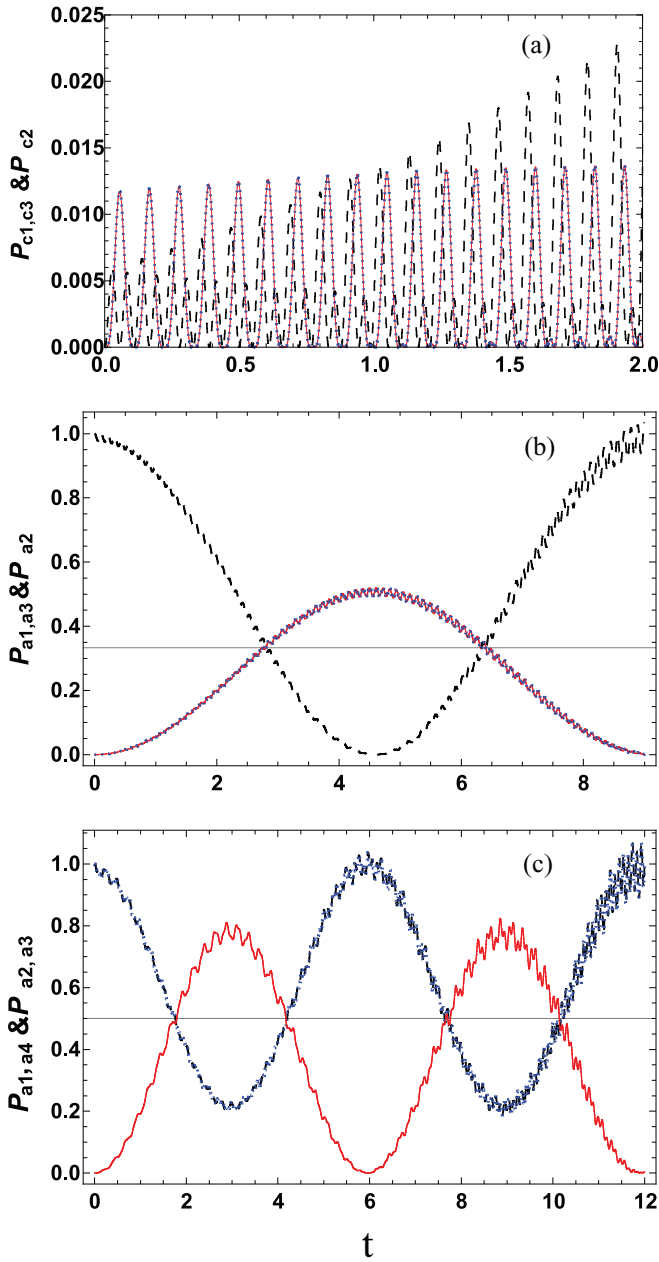


FIG. 5. (Color online) Probabilities of excitation transfer vs time for cavity modes (a) and ensembles (b) and (c). The dynamics of the system correspond to the large-hopping case with $J = 40$, $\Omega = 1$, $N = 20$, and other parameters are (a) and (b): three cavities with $b_1(0) = b_3(0) = 0, b_2(0) = 1$ and (c) four cavities, $b_1(0) = b_4(0) = 0, b_2(0) = b_3(0) = 1$.

the excitation is transferred completely to the ensemble of cavity 3 through the field of the adjacent cavity 2. Here the behavior of the middle cavity is similar to that shown for a comparable coupling-hopping case.

The two extreme situations are when one of the coupling constants and hopping constants is negligibly small as compared to the other. For $\Omega = 1$ and $J = 0.01$ with vacuum cavity modes, while only $b(0) = 1$, the system reduces to the Jaynes-Cummings model. Here fast Rabi oscillations

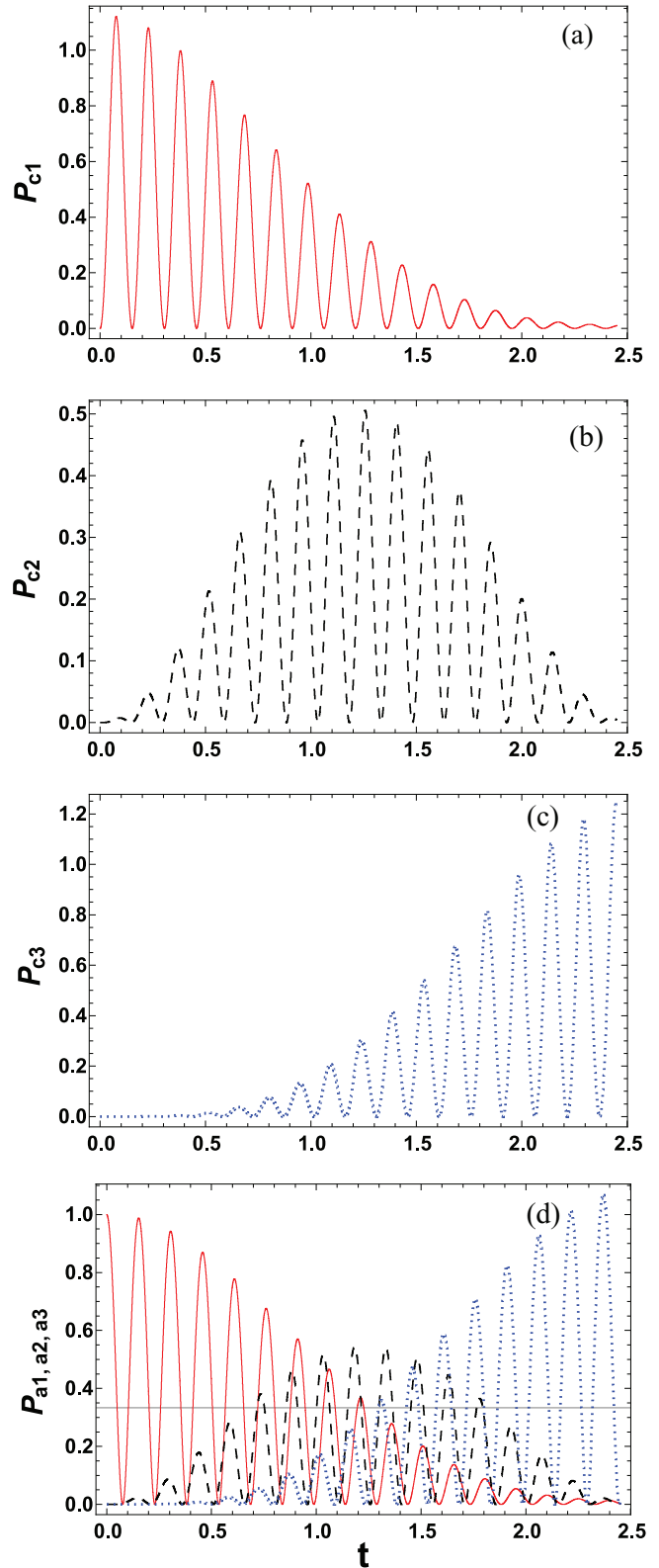


FIG. 6. (Color online) Probabilities of excitation transfer vs time for cavity modes (a)–(c) and ensembles (d), where the solid, thick dashed, and thin dotted lines represent excitations in the first, second, and third cavity, respectively. The dynamics of the system corresponds to the strong-coupling case with parameters $\Omega = 10$, $J = 2$, $N = 5$, and $b_1(0) = 1$.

for the reduced single-cavity system is observed in cavity 1. On the other hand, for strong hopping ($J = 1$) and weak coupling ($\Omega = 0.01$), for a three-coupled-cavity model the initial atomic excitation transfers to the third cavity without populating the middle cavity.

The initial excitation can be taken arbitrarily in any mode or ensemble alone or simultaneously in various modes and ensembles. If a single excitation is considered in cavity 3 instead of cavity 1, the dynamics of the system gets a slight change only, i.e., the pattern of oscillation in the middle cavity remains the same, while there is a flipping of transformation behavior between the other two cavities. On the other hand, if a single excitation is taken initially in the middle cavity, both cavities at the end start behaving in the same way, as shown in Fig. 5(b).

IV. CONCLUSIONS

We have considered an array of M mutually coupled cavities each containing an ensemble of N two-level atoms. It is assumed that all the cavities are prepared in the initial vacuum states, while a single excitation is seeded mostly in the first site. Our system offers a great degree of freedom to control its evolution by modulating the parameters, e.g., coupling of the ensembles to their respective cavities, cavity-cavity hopping, number of atoms in the ensembles, and position of the initial excitation. Analytically, the dynamics of the system is given by coupled differential equations obtained using the Holstein-Primakoff transformation [35,39]. Here we have examined evolution of the system generally for three coupled cavities. A number of interesting cases of comparable coupling-hopping ($\Omega \simeq J$), large hopping ($J \gg \Omega$), and strong coupling ($\Omega \gg J$) are discussed in detail.

For the comparable coupling-hopping case, with an initial excitation taken in the ensemble of cavity 1, we obtained a 100% transfer of excitation to the modes of cavities 1 and 3. The middle cavity gets a 50% excitation; however, it attains a frequency of excitation which is almost twice that of the frequencies in the other two cavities. In addition, it is noted that dynamics of the transfer of excitation of the ensembles follow resonances in the probability curves for the

respective cavity modes. The probabilities of excitation were found to exhibit both fast and slow oscillations. A single fast oscillation represents a complete transfer of excitation between the ensemble and its local cavity mode, which is governed by the coupling constant Ω . The increase in population of the ensembles has a positive impact on the rate of these fast oscillations (see Fig. 3). The slow oscillations are related to the process of the intersite excitation transfer initiated by the cavity-cavity hopping constant J .

In the large-hopping regime, excitation of the cavity modes is much smaller as compared to the equal coupling-hopping. Nevertheless, these weak excitations of cavity modes are equally effective to cause the transfer of excitation among the ensembles of various cavities. Here the ensemble of cavity 2 has a 50% probability of excitation, while the ensemble of cavity 3 becomes fully excited, as was the case for comparable coupling-hopping. For an initial excitation seeded in the middle cavity, we obtained tripartite W states at $t = 2.8$, $t = 6.3$, and a bipartite maximally entangled state at $t = 4.5$. It is worth mentioning that a control on the size of the ensemble provides us an opportunity to obtain a W state even for comparable coupling-hopping, as shown in Fig. 3. For the strong-coupling regime, the fast oscillations have a continuously varying amplitude due to the field-hopping effect. Here the probability of excitation transfer in the middle cavity is half as compared to the probabilities in the other two cavities. The excitation transformation curves of the ensembles follow resonances of the corresponding mode excitation, just like the equal coupling-hopping regime.

ACKNOWLEDGMENTS

F.B. thanks T. J. G. Apollaro, C. Di Franco, and J. Wragg for invaluable help. He is grateful to the Higher Education Commission of Pakistan for financial support and the Centre for Theoretical Atomic, Molecular, and Optical Physics, School of Mathematics and Physics, Queen's University Belfast for kind hospitality during the completion of this work. M.P. acknowledges financial support from the UK EPSRC for a Career Acceleration Fellowship and a grant awarded under the "New Directions for Research Leaders" initiative (EP/G004579/1).

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