Cascade splitting of two atomic energy levels due to multiphoton absorption

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We have theoretically and experimentally studied the spectroscopic properties of dressed levels in a strong monochromatic field, and propose a model of cascade splitting of two atomic energy levels. In this model two related dressed levels can be split into four levels, and transitions connecting four new levels will constitute spectroscopic structures. Two types of proof-in-principle experiments are performed to verify the model. One experiment measures the probe absorption spectra of a degenerate two-level atomic system with two strong monochromatic coupling fields. The system consists of $5^2S_{1/2}, F = 2$ and $5^2P_{3/2}, F' = 3$ states of ^{87}Rb atoms in a magneto-optical trap (MOT) as well as the cooling beams and an additional coupling field. New spectral features are observed and proven to be due to the transitions of new levels generated by splitting of the dressed levels. The other experiment measures the pump-probe spectra in a degenerate two-level atomic system with one strong monochromatic coupling field. The system consists of $5^2S_{1/2}, F = 2$ and $5^2P_{3/2}, F' = 3$ states of the ^{87}Rb atom in a magneto-optical trap and one coupling field. We have observed spectral features that obviously differ from the prediction that comes from the two-level dressed-atom approach. They cannot be explained by existing theories. The model of cascade splitting of two atomic energy levels is employed to explain the observations in these two types of experiments.

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I. INTRODUCTION

Numerous theoretical and experimental works have focused on the spectroscopic properties of a two-level atomic system driven by a strong monochromatic coupling field. The two-level dressed-atom approach [1,2] developed by Cohen-Tannoudji and collaborators provides a simple and intuitive picture of the energy exchanges in the coupled atom within a coupling field system. Their approach predicts that the probe absorption spectrum has a characteristic triplet structure. More specifically, a very narrow dispersionlike structure is located at the coupling field frequency ω_1. Two sideband spectral features termed the absorption peak and the gain peak are symmetrically distributed at $\pm\Omega_1'$ with respect to ω_1 for the negative detuning case $\Delta_1 < 0$; while for positive detuning $\Delta_1 > 0$, the energy positions of the absorption peak and the gain peak are interchanged [1,2]. Here, $\Omega_1' = \sqrt{\Omega_1^2 + \Delta_1^2}$, where Ω_1 is the Rabi frequency of the coupling field for the corresponding atomic transition, and Δ_1 represents the detuning of the coupling field frequency from the corresponding atomic transition. In addition, the intensities of the three features can be determined by the ratio of Ω_1 to Δ_1, which is represented by the parameter $\theta = \tan^{-1}(-\frac{\Omega_1}{\Delta_1})/2$ [1,2]. In addition, by considering the polarization nature of the electromagnetic field and recognizing that the actual energy levels in atoms are generally degenerate, Lipsich *et al.* used a numerical calculation model to study the coherent interaction of a degenerate two-level atomic system with different coupling and probe field polarizations. This study can be viewed as the extended two-level dressed-atom approach [3]. Based on their calculations, they find that the absorption spectra of driven degenerate two-level atomic systems may not be composed of a characteristic triplet structure. Take the case that the coupling and probe fields are circularly polarized in the $F \rightarrow F+1$ transitions as an example; their calculations show that the probe absorption spectrum is composed of a characteristic triplet structure when the polarizations of coupling and probe fields are circular and equal (cc), while the probe absorption spectrum is composed of four absorption peaks when the polarizations of coupling and probe fields are circular and opposite (coc) [3].

Experimentally, the continuous-wave (cw) pump-probe spectra of cold atoms offer great opportunities for the studies of precise spectroscopic measurements and quantum coherent phenomena due to significantly reduced Doppler broadening and low collision rates. A large number of experimental studies have been devoted to the cycle transitions $F \rightarrow F+1$ for Cs atoms [4–8], ^{85}Rb atoms [9,10], and ^{87}Rb [11,12] confined in a MOT. All these spectra are similar looking in the sense that they are characterized by a very narrow dispersionlike structure and two sideband spectral features termed as the absorption peak and the gain peak, respectively. In addition, the two sideband spectral features are symmetrically distributed with respect to the coupling field frequency ω_1, and have been explicitly interpreted by the two-level dressed-atom approach [1,2].

The response of dressed levels driven by a quasiresonant coupling field is an interesting question in itself. In fact, in the work here, new spectral features are observed when another

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coupling field is added in an operating MOT, and these features cannot be explained by existing theories. Moreover, spectral features that obviously differ from the prediction that comes from the two-level dressed-atom approach [1,2] have been observed for pump-probe spectra in a degenerate two-level atomic system with one strong monochromatic coupling field. We propose a theoretical model of cascade splitting of two atomic energy levels and attribute the above abnormal spectral features to the spectroscopic properties of driven dressed levels.

II. THEORETICAL MODEL AND EXPERIMENTAL METHOD

A. Theoretical model

The response of dressed levels driven by a quasiresonant coupling field can be viewed as a natural generalization of the two-level dressed-atom approach, i.e., the eigenstates (semiclassical dressed states) of a quantum system of the dressed levels in a quasiresonant coupling field are similar to those of a quantum system of coupled atoms in a coupling field. Hence, the dressed levels can be further split. As predicted by the two-level dressed-atom approach [1,2], the frequency of the two transitions associated with the dispersionlike structure is equivalent to the frequency of the coupling field acting on two atomic levels. The frequencies of the two transitions associated with the gain peak and the absorption peak differ from the frequency of the coupling field. Therefore, the coupling field acting on two dressed levels can be the same as or different from the coupling field acting on the two bare atomic levels.

Two simple types of coupled atoms with coupling field(s) systems can demonstrate splitting of the dressed levels. One type is a two-level atomic system with two strong monochromatic coupling fields. Here, the first coupling field acts on the bare atomic levels, and the second coupling field acts on the dressed levels. The frequency of the second coupling field can be around the energy position of the gain peak or the absorption peak. Splitting of the dressed levels is shown in Figs. 1 and 2, respectively. ω_1 represents the frequency of the first coupling field acting on bare atomic levels, and ω_2 represents the frequency of the second coupling field acting on the dressed levels. Δ_1 represents the detuning of the first coupling field frequency from the corresponding atomic transitions, and Δ_2 represents the detuning of the second coupling field frequency from the corresponding transitions connecting two dressed levels. Ω_1 is the Rabi frequency of the first coupling field for the corresponding atomic transition, and Ω_2 is the Rabi frequency of the second coupling field for the transitions connecting two dressed levels and is proportional to the dipole moment that is related to the ratio of Ω_1 to Δ_1 predicted by the two-level dressed-atom approach [1,2]. Similar to the treatment of the two-level dressed-atom approach [1,2], the calculated positions for the levels produced by splitting of the dressed levels are also shown in Figs. 1 and 2. The four transitions with three lines are all allowed, and are also characterized by a very narrow dispersionlike structure near ω_2 with two sideband spectral features symmetrically distributed at $\omega_2 \pm \Omega_2'$, respectively. $\Omega_2' = \sqrt{\Omega_2^2 + \Delta_2^2}$. Actually, the two-level atomic system simultaneously interacts with the two coupling fields, while the above treatment simplifies the issue to a two-step optical process that is similar to a resonant two-photon absorption process. This type of treatment will be a good approximation when one coupling field is quasiresonant with the atomic transition between two bare atomic levels

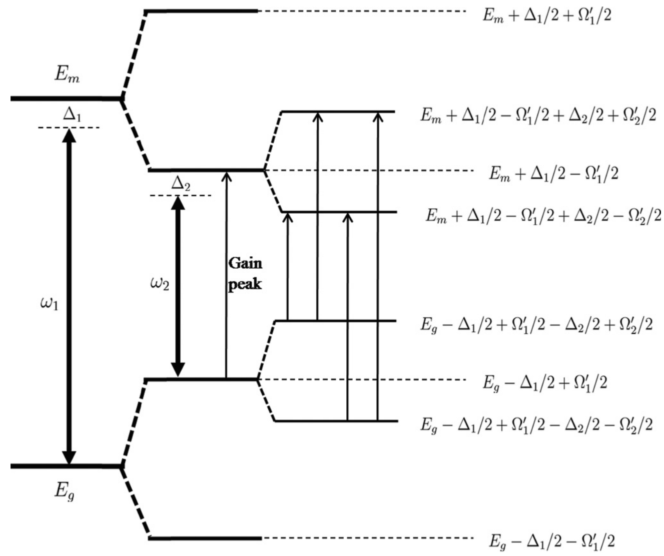

FIG. 1. Cascade splitting of two atomic energy levels in a two-level atomic system with two strong monochromatic coupling fields. The frequency of the second coupling field is near the energy position of the gain peak. Here, ω_1, Δ_1, and Ω_1 represent the frequency, detuning, and Rabi frequency for the first coupling field acting on the bare atomic levels. ω_2, Δ_2, and Ω_2 represent the frequency, detuning, and Rabi frequency for the second coupling field acting on the dressed levels. $\Omega_1' = \sqrt{\Omega_1^2 + \Delta_1^2}$. $\Omega_2' = \sqrt{\Omega_2^2 + \Delta_2^2}$.

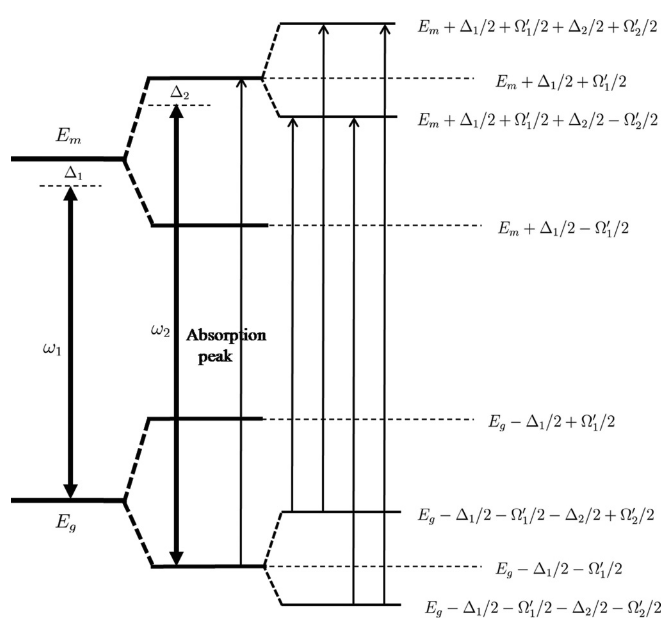

FIG. 2. Cascade splitting of two atomic energy levels in a two-level atomic system with two strong monochromatic coupling fields. The frequency of the second coupling field is near the energy position of the absorption peak. The meanings of physical quantities are the same as those in the caption of Fig. 1.

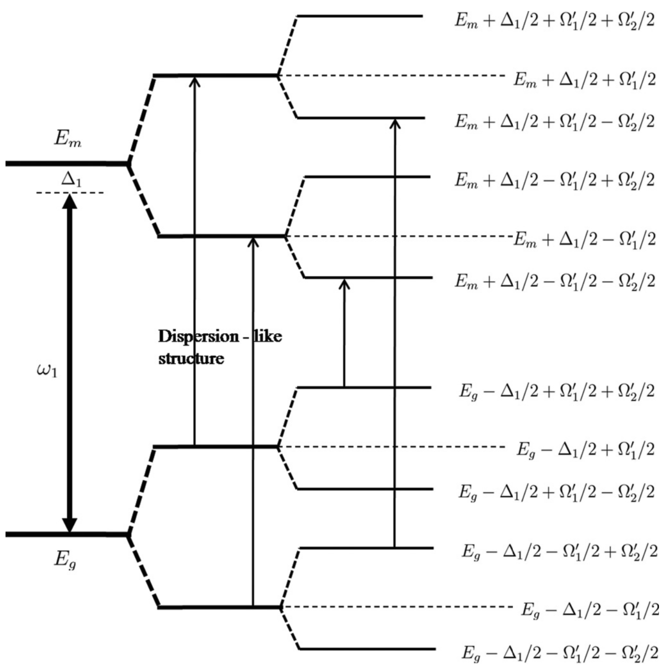

FIG. 3. Cascade splitting of two atomic energy levels in a two-level atomic system with one strong monochromatic coupling field. The meanings of physical quantities are the same as those in the caption of Fig. 1. In this case, $\Delta_2 = 0$ MHz. The transitions connecting the levels formed by splitting of the dressed levels correspond to our observed spectral features under the condition that the pump and probe polarizations are cc shown in Fig. 10.

and the other is quasiresonant with the transition connecting two dressed levels. The other type of system is a two-level atomic system with one strong monochromatic coupling field. As discussed above, the coupling field acting on bare atomic levels is resonant with the two transitions associated with the dispersionlike structure shown in Fig. 3. Therefore, the coupling field splits four dressed levels into eight levels. Following the discussion above, we obtain the calculated positions for the energy levels produced by splitting the dressed levels as shown in Fig. 3. By generalizing such treatment to the case that more than two strong monochromatic coupling fields are introduced, the atomic energy levels can cascade split due to multiphoton absorption. Therefore, we call such theoretical treatment the model of the cascade splitting of two atomic energy levels.

B. Experimental method

We measured the probe absorption spectra of ^{87}Rb atoms in a glass vapor-cell MOT. Two experimental schemes are proposed to demonstrate splitting of the dressed levels shown in Figs. 1, 2, and 3.

Cold ^{87}Rb atoms are produced in a standard MOT. Details of the apparatus were described in our previous work [13]. Briefly, cooling and repumping beams are provided by two frequency-stabilized diode lasers of Toptica TA100 and Toptica DL100, respectively. They overlap and divide into three orthogonal pairs of counterpropagating fields with σ^+ and σ^- circularly polarized configuration. A spherical quadrupole magnetic field with an axial gradient of 30 Gauss/cm is created by a pair of anti-Helmholtz coils. In our measurements, the intensity of the cooling beams varies from 80.0 to 180.0 mW/cm^2 and the intensity of the repumping beams is fixed at about 5.9 mW/cm^2. The intensity of the probe beam is about 0.2 mW/cm^2. Note that the actual atomic energy levels are generally degenerate, e.g., the Zeeman sublevels. The pure two-level systems shown in Figs. 1, 2, and 3 should be replaced by degenerate two-level atomic systems. In addition, even though the spherical quadrupole magnetic field exists in the MOT, splitting of Zeeman sublevels is very small because the cold atomic cloud is very small (diameter ~ 0.6 mm in our case) and located at the zero point of the net magnetic field. Therefore all Zeeman sublevels in the $5^2S_{1/2}, F = 2$ or $5^2P_{3/2}, F' = 3$ state are near degenerate.

We performed two types of proof-in-principle experiments to verify our model. One experiment measures probe absorption spectra in a degenerate two-level atomic system with two strong monochromatic coupling fields, thus splitting of the dressed levels shown in Figs. 1 and 2 can be experimentally demonstrated. Here, the $5^2S_{1/2}, F = 2$ and $5^2P_{3/2}, F' = 3$ states of ^{87}Rb atoms as well as the cooling beams and an additional coupling field constitute the degenerate two-level atomic system in two strong monochromatic coupling fields. The cooling beams play the role of the first coupling field acting on the bare atomic levels, and the second coupling field acting on dressed levels is provided by an independent frequency-stabilized diode laser of Toptica DL100. In this scheme the detuning $\Delta_1/2\pi$ of the first coupling field frequency from the transition $5^2S_{1/2}, F = 2 \rightarrow 5^2P_{3/2}, F' = 3$ is fixed at -12 MHz and the total intensity of the first coupling field is about 180.0 mW/cm^2. The frequency of the second coupling field is controlled by an acousto-optical modulator (AOM) and varies in the energy range from the gain peak to the absorption peak. The intensity of the second coupling field is about 254.8 mW/cm^2. The other experiment measures the pump-probe spectra in the degenerate two-level atomic system with one strong monochromatic coupling field. In this scheme splitting of the dressed levels shown in Fig. 3 can be experimentally demonstrated. Here, the $5^2S_{1/2}, F = 2$ and $5^2P_{3/2}, F' = 3$ states of ^{87}Rb atoms as well as one coupling field constitute the degenerate two-level atomic system with one strong monochromatic coupling field. The coupling field can be the cooling beams or some other coupling field.

As for pump-probe spectra in an operating MOT, the cooling beams simultaneously play the role of the coupling field. It is known that the situation in an operating MOT is extremely complex due to the presence of the spherical quadrupole magnetic field, the flexibility in varying the trap parameters, and the fixed three-dimensional geometry of the cooling beams with σ^+ and σ^- circularly polarized configuration [9,11]. Therefore, the polarizations of the coupling field and the probe field are not well defined, i.e., the configuration of the cc case and the configuration of the coc case coexist. Therefore, the pump-probe spectra in an operating MOT are, in general, essentially different from and more complex than the characteristic triple structure predicted by the two-level dressed-atom approach [1,2].

The complex situation of spectrum measured in an operating MOT motivates us to propose an improved pump-probe spectra of cold ^{87}Rb atoms, in which the polarizations of both the coupling and probe beams are cc. More specifically, the MOT keeps going for 5 s and then is completely switched off in 1 ms. After that 200-ms pulses from the coupling field provided by an independent frequency-stabilized diode laser of Toptica DL100 and a guide magnetic field are turned on for measuring pump-probe spectra. The low guide magnetic field produced by a pair of Helmholtz coils provides a well-defined axis of quantization. The propagation directions of the coupling and probe fields are nearly collinear with the well-defined axis of quantization. In addition, the magnetic field in the interaction region is slightly greater than that of Earth's local magnetic field (∼200 mG) and its influence on atomic energy levels is expected to be negligible for the experimental conditions. In the two experimental schemes we use the corresponding saturated absorption spectrum (SAS) of a rubidium vapor cell for frequency calibration of the probe absorption spectra.

The experimental errors in this work mainly result from the following factors: the statistical uncertainties, the uncertainties in frequency calibration mainly resulting from the frequency jitter ∼1 MHz of the cooling laser and the probe laser, and the errors (less than 1%) from the frequency calibration procedure.

III. RESULTS AND DISCUSSIONS

A. Experimental measurements in a degenerate two-level atomic system with two strong monochromatic coupling fields

Our experiments demonstrate our theoretical prediction shown in Figs. 1 and 2, i.e., the spectroscopic properties in degenerate two-level atomic systems in two strong monochromatic coupling fields. The first coupling field acting on bare atomic levels is provided by the cooling beams. The second coupling field acting on dressed levels is provided by another independent laser. The probe absorption spectra are measured to investigate atom-photon interaction in such a system. Based on Figs. 1 and 2, the frequency of the second coupling field is around the energy position of the gain peak or the absorption peak. Therefore, our analysis is divided into two cases.

1. The frequency of the second coupling field is red-shifted with respect to the energy position of the gain peak

We discuss the case where the frequency of the second coupling field is red-shifted with respect to the energy position of the gain peak, that is, $\Delta_2 < 0$. Splitting of the dressed levels is expected to match that as shown in Fig. 1. The probe absorption spectrum in the presence of two coupling fields is measured. The frequencies of the first coupling field and the second coupling field are −12 and −63 MHz with respect to the transition $5^2S_{1/2}, F = 2 \rightarrow 5^2P_{3/2}, F' = 3$, respectively, as shown in Fig. 4. Clearly, there are many spectral features that emerge around the gain peak in the presence of two coupling fields. To confirm that these spectral features are generated by splitting of the dressed levels, we measure the spectrum with only one coupling field for comparison by controlling the corresponding acousto-optical modulator. The coupling field can be the first coupling field (the cooling beams) or the second coupling field. As shown in Fig. 4,

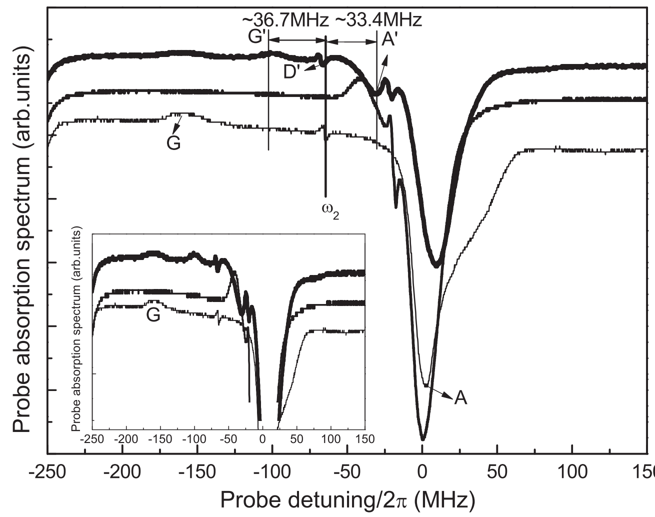

FIG. 4. The probe absorption spectrum measured in degenerate two-level atomic systems with two coupling fields. The frequency of the second coupling field is red-shifted with respect to the energy position of the gain peak. (a) (Thick curve) The probe absorption spectrum in the presence of two coupling fields. The detunings of the two coupling fields are −12 and −63 MHz with respect to the transition $5^2S_{1/2}, F = 2 \rightarrow 5^2P_{3/2}, F' = 3$, respectively. (b) (Medium thickness curve) The pump-probe spectrum measured in an operating MOT. The coupling field is provided by the cooling beams with the detuning of −12 MHz. (c) (Thin curve) The pump-probe spectrum measured in the case that cooling beams are switched off and the coupling field is provided by an independent frequency-stabilized diode laser. The detuning of the coupling field is −63 MHz. (Inset) The detail near the energy position of the gain peak.

the new gain peak and new absorption peak, labeled as G′ and A′, are observed and located at -36.7 ± 2.1 and 33.4 ± 1.6 MHz. They are symmetrically distributed with respect to the second coupling field frequency ω_2 when the two coupling fields act on a cold atomic cloud comparing to the spectrum measured when only one coupling field exists. Meanwhile, a very narrow dispersionlike structure labeled as D′ is located around the second coupling field frequency ω_2. According to our model shown in Fig. 1, the spectroscopic properties of driven dressed levels form a characteristic triplet structure, i.e., the dispersionlike structure is located at the second coupling field frequency ω_2, and two sideband spectral features are symmetrically located at $\omega_2 \pm \Omega_2'$. Therefore, our observations agree well with our theoretical predictions. Note that the spectrum measured in the presence of two coupling fields includes the spectral features obtained from the coupled atom in only one of the two coupling field systems shown in the inset of Fig. 4. This is essentially different from the resonant two-photon spectrum. Furthermore, according to the prediction from our proposed model, the spacing between the two sideband spectral features should become larger as $\Omega_2' = \sqrt{\Omega_2^2 + \Delta_2^2}$ increases. Therefore, we have measured a series of spectra under the condition that Ω_2 is fixed and Δ_2 varies as shown in Fig. 5. Similarly, a new characteristic triplet structure emerges. These new characteristic triplet structures measured at a different frequency of the second coupling field are labeled as (G_1, D_1, and A_1), (G_2, D_2, and A_2), and (G_3,

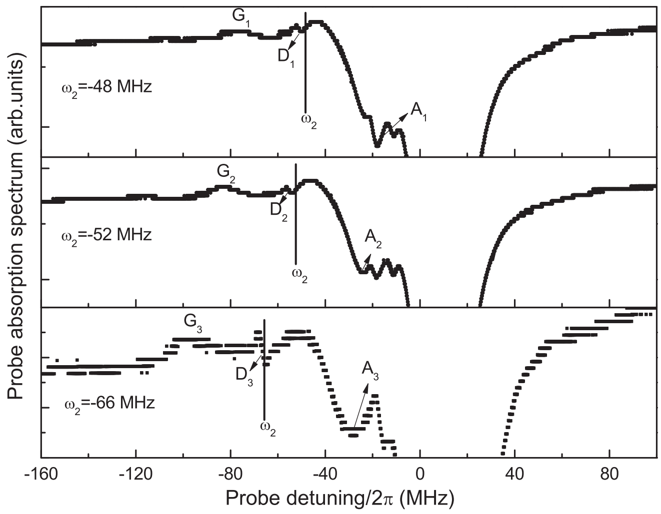

FIG. 5. The probe absorption spectra measured in degenerate two-level atomic systems with two coupling fields. The frequency of the second coupling field is red-shifted with respect to the energy position of the gain peak. The detunings of the second coupling field are −48, −52, and −66 MHz with respect to the transition $5^2S_{1/2}, F = 2 \rightarrow 5^2P_{3/2}, F' = 3$, respectively.

D_3, and A_3), respectively. The energy positions of these new triplet structures are shown in Fig. 6 and Table I. As discussed above, the new dispersionlike structure is located at the second coupling field frequency ω_2 predicted by our model shown in Fig. 1. In our measurement, the energy spacing between the new dispersionlike structure and ω_2 is no more than 2.1 ± 1.3 MHz as shown in Table I, and the observed positions

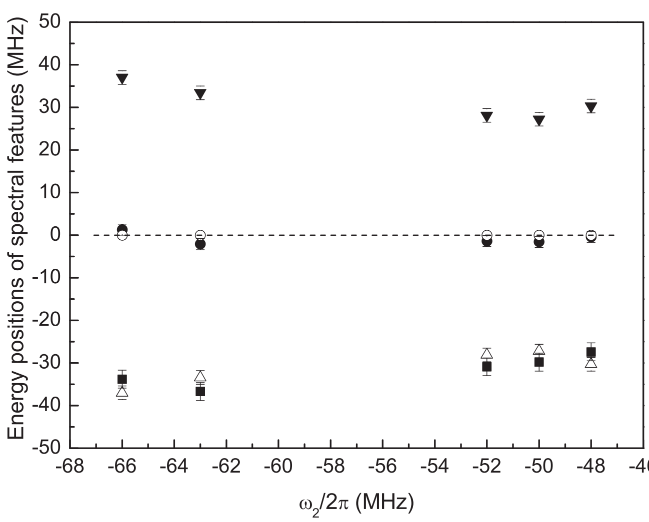

FIG. 6. Energy positions of the new spectral features when the probe absorption spectra are measured in degenerate two-level atomic systems with two coupling fields and also obtained from the proposed model. Solid inverted triangles, circles, and squares represent the energy positions of the new absorption peak, dispersionlike structure, and gain peak, respectively. Open triangles represent the energy positions symmetrical with the new absorption peak and these energy positions should agree well with the energy positions of the new gain peak predicted by our model. Open circles represent the energy positions of the new dispersionlike structure predicted by our model.

TABLE I. Experimental energy levels (present estimated errors in parentheses) for these new spectral features measured in degenerate two-level atomic system with two strong monochromatic coupling fields. The frequency ω_2 of the second coupling field is red-shifted with respect to the energy position of the gain peak or blue-shifted with respect to the energy position of the absorption peak. These data are extracted from the spectrum in Figs. 4 and 5. The detuning of the first coupling field (cooling beams) $\Delta_1/2\pi$ is −12 MHz with respect to the transition frequency $5^2S_{1/2}, F = 2 \rightarrow 5^2P_{3/2}, F' = 3$.

ω_2[a] ($2\pi \times$ MHz)	$\omega_{\text{gain}} - \omega_2$ ($2\pi \times$ MHz)	$\omega_{\text{dis.}} - \omega_2$ ($2\pi \times$ MHz)	$\omega_{\text{abs.}} - \omega_2$ ($2\pi \times$ MHz)
−48	−27.4(2.1)	−0.4(1.3)	+30.3(1.6)
−50	−29.8(2.1)	−1.6(1.3)	+27.2(1.6)
−52	−30.9(2.1)	−1.4(1.3)	+28.1(1.6)
−63	−36.7(2.1)	−2.1(1.3)	+33.4(1.6)
−66	−33.8(2.1)	+1.3(1.3)	+37.0(1.6)

[a]The frequency is with respect to the transition $5^2S_{1/2}, F = 2 \rightarrow 5^2P_{3/2}, F' = 3$.

of new dispersionlike structures coincide to the theoretical values within experimental errors as shown in Fig. 6. These observations show good agreement with our model. From the experimental energy position of the new absorption peak $\omega_{\text{abs.}}$ and our model, we can predict that the new gain peak should be located at $-\omega_{\text{abs.}}$, that is, the symmetrical position of the new absorption peak with respect to the second coupling field frequency ω_2. As shown in Fig. 6, the energy position of the new gain peak ω_{gain} also shows good agreement with $-\omega_{\text{abs.}}$. In addition, the spacing between two sideband spectral features becomes larger as Δ_2 increases as shown in Fig. 6 and Table I. Such observations agree well with our theoretical predictions.

2. The frequency of the second coupling field is blue-shifted with respect to the energy position of the absorption peak

Now we discuss the case where the frequency of the second coupling field is blue-shifted with respect to the energy position of the absorption peak, that is, $\Delta_2 > 0$. Splitting of the dressed levels is expected to match that shown in Fig. 2. In this measurement, the frequencies of the first coupling field (the cooling beams) and the second coupling field are −12 and +50 MHz with respect to the transition $5^2S_{1/2}, F = 2 \rightarrow 5^2P_{3/2}, F' = 3$, respectively, as shown in Fig. 7. When two coupling fields act on cold atomic cloud, new spectral features seem to emerge comparing to the pump-probe spectrum as shown in Fig. 7(a). More specifically, a dispersionlike structure is observed located around the second coupling field frequency ω_2, and a broad gain peak labeled as G′ emerges at the blue sideband of ω_2. However the new spectral features are very weak so the smaller scale for the oscilloscope is used to display the spectral features as shown in Fig. 7(b). Similar to the case when the frequency of the second coupling field is red-shifted with respect to the energy position of the gain peak, our model shown in Fig. 2 predicts a characteristic triplet structure. We have observed the dispersionlike structure and gain peak. The absorption peak should be located at the red sideband of ω_2, and the absorption peak and the gain peak are symmetrically distributed with respect to ω_2. Unfortunately, as shown in Fig. 7, the absorption peak related

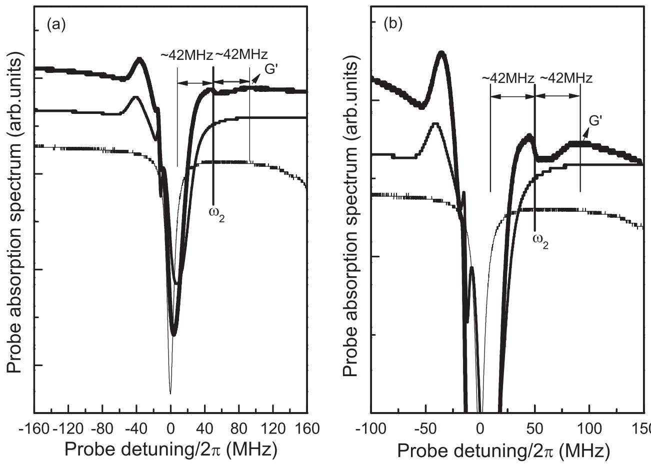

FIG. 7. The probe absorption spectrum measured in degenerate two-level atomic systems with two coupling fields. The frequency of the second coupling field is blue-shifted with respect to the energy position of the absorption peak. (Thick curve) The probe absorption spectrum in the presence of two coupling fields. The detunings of the two coupling fields are -12 and $+50$ MHz with respect to the transition $5^2S_{1/2}, F = 2 \to 5^2P_{3/2}, F' = 3$, respectively. (Medium thickness curve) The pump-probe spectrum measured in an operating MOT. The coupling field is provided by the cooling beams with the detuning of -12 MHz. (Thin curve) The pump-probe spectrum measured in the case that cooling beams are switched off and the coupling field is provided by an independent frequency-stabilized diode laser. The detuning of the coupling field is $+50$ MHz. (a) The whole spectrum around the transition $5^2S_{1/2}, F = 2 \to 5^2P_{3/2}, F' = 3$ when the scale of the digital oscilloscope is big. (b) Amplification of the spectrum around the dispersionlike structure to show the weak gain peak G′.

to the spectroscopic properties of driven dressed levels emerges in a strong and broad absorption peak resulting from the spectroscopic properties of the two-level atomic system driven by the first coupling field, thus, it cannot be distinguished from that peak.

B. Experimental measurements in degenerate two-level atomic system with one strong monochromatic coupling field

Splitting of dressed levels can happen in degenerate two-level atomic systems with one strong monochromatic coupling field as shown in Fig. 3. To demonstrate our theoretical prediction we perform an experiment. In this type of experiment, the coupling field can be cooling beams or not. Therefore, experimental results and discussions are divided into two cases: (1) pump-probe spectra in an operating MOT; (2) pump-probe spectra under the condition that pump and probe polarizations are cc.

1. Pump-probe spectra in an operating MOT

We measure a series of pump-probe spectra over the transitions $5^2S_{1/2}, F = 2 \to 5^2P_{3/2}, F' = 1,2$ and 3 of ^{87}Rb in an operating MOT shown in Figs. 8 and 9. Here, the coupling field is served by the cooling beams. Our analysis procedure is organized as follows: (i) the spectra around the atomic transitions $5^2S_{1/2}, F = 2 \to 5^2P_{3/2}, F' = 1$ and 2; (ii) the spectra around the transition $5^2S_{1/2}, F = 2 \to 5^2P_{3/2}, F' = 3$.

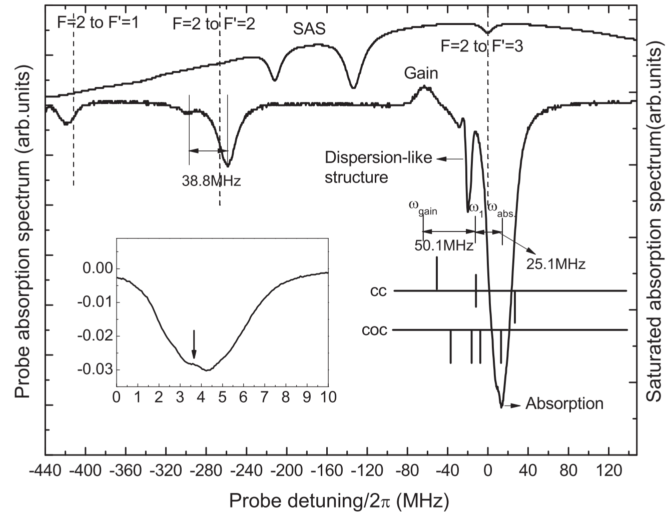

FIG. 8. A typical experimental pump-probe spectrum of cold ^{87}Rb measured in an operating magneto-optical trap with the detuning $\Delta_1/2\pi = -12$ MHz and the cooling laser intensity $I = 170.9$ mW/cm^2. The saturated absorption spectrum of an ^{87}Rb vapor cell is presented for frequency calibration (curve in the top). The energy positions are calculated for these spectral features in cc and coc spectra based on the theoretical model of Lipsich *et al.* [3], in which the Rabi frequency of the coupling field is 36.9 MHz that is obtained from the Autler-Townes doublets for the transition $5^2S_{1/2}, F = 2 \to 5^2P_{3/2}, F' = 2$. (Inset) The detail near the transition $5^2S_{1/2}, F = 2 \to 5^2P_{3/2}, F' = 3$.

As shown in Figs. 8 and 9, there are two peaks around the transition $5^2S_{1/2}, F = 2 \to 5^2P_{3/2}, F' = 2$ and one peak centered at the high-energy side of the atomic transition $5^2S_{1/2}, F = 2 \to 5^2P_{3/2}, F' = 1$. This phenomenon is well known as the Autler-Townes doublet and has been explicitly interpreted by the two-level dressed-atom approach [1,2]. Moreover, as shown in Table II, within experimental errors

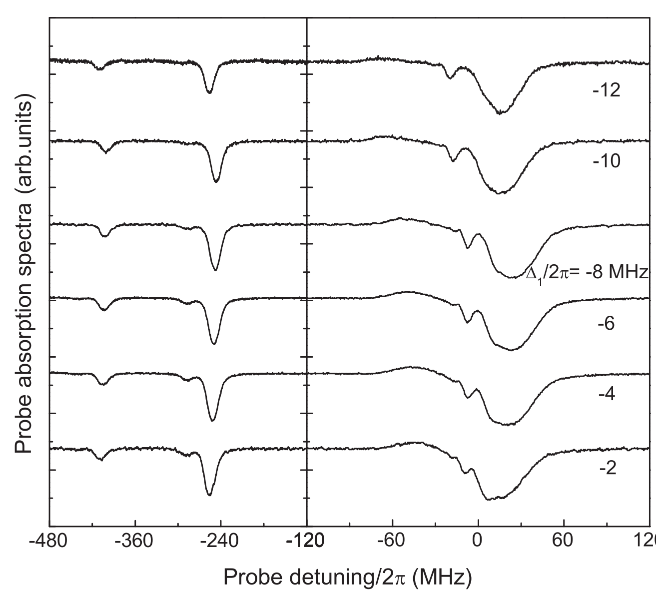

FIG. 9. Experimental pump-probe spectra of cold ^{87}Rb atoms measured in an operating magneto-optical trap with various detunings Δ_1 and the fixed cooling laser intensity $I = 153.4$ mW/cm^2.

TABLE II. The comparison between experimental and theoretical (present estimated errors in parentheses) data. The experimental data are extracted from the spectra in Fig. 9. All quantities are in $2\pi \times$ MHz.

Δ_1	Light shift[a]	Light shift[b]	Ω_1[c]	$(-\Delta_1/2 - \Omega_1'/2)$[d] $(-\Delta_1/2 + \Omega_1'/2)$[d]
−12	− 13.9(0.78)	− 11.3(1.27)	35.2(1.60)	− 12.6(1.25)
		26.7(1.27)		24.6(1.26)
−10	− 13.7(0.79)	− 11.3(1.27)	33.5(1.73)	− 12.5(1.25)
		27.7(1.28)		22.5(1.25)
−8	− 14.6(0.78)	− 12.2(1.27)	33.8(1.66)	− 13.4(1.25)
		26.7(1.27)		21.4(1.25)
−6	− 15.5(0.78)	− 12.0(1.27)	32.9(1.64)	− 13.8(1.25)
		24.7(1.27)		20.0(1.25)
−4	− 15.5(0.78)	− 12.0(1.27)	31.2(1.53)	− 13.8(1.25)
		23.0(1.27)		17.8(1.25)
−2	− 15.1(0.79)	− 11.1(1.27)	28.0(1.61)	− 13.1(1.25)
		20.2(1.27)		15.1(1.25)

[a]Obtained from the Autler-Townes doublets for the transition $5^2S_{1/2}, F = 2 \rightarrow 5^2P_{3/2}, F' = 1$.
[b]Obtained from the Autler-Townes doublets for the transition $5^2S_{1/2}, F = 2 \rightarrow 5^2P_{3/2}, F' = 2$.
[c]The averaged value from the Autler-Townes doublets for $5^2S_{1/2}, F = 2 \rightarrow 5^2P_{3/2}, F' = 1$ and 2 transitions.
[d]$\Omega_1' = \sqrt{\Omega_1^2 + \Delta_1^2}$.

the present experimental energy positions for the two Autler-Townes doublets are generally consistent with those predicted by the two-level dressed-atom approach [1,2]. Therefore, the Rabi frequency Ω_1 of the coupling field can be obtained by the averaged value (shown in Table II) from the two Autler-Townes doublets. The observation of only the high-energy side of the Autler-Townes doublet around the transition $5^2S_{1/2}, F = 2 \rightarrow 5^2P_{3/2}, F' = 1$ is due to the fact that the other is too weak to be observed. More specifically, the intensity ratio of the low-energy side to the high-energy side of the Autler-Townes doublet is equivalent to $\tan^6\theta$ [7], both for the transition $5^2S_{1/2}, F = 2 \rightarrow 5^2P_{3/2}, F' = 1$ and for $5^2S_{1/2}, F = 2 \rightarrow 5^2P_{3/2}, F' = 2$. Here, $\theta = \tan^{-1}(-\frac{\Omega_1}{\Delta_1})/2$ represents the ratio of Ω_1 to Δ_1 [1,2], which can be obtained from Table II. Consider the spectrum in Fig. 8 for example, $\tan^6\theta$ is 0.13, consistent with the observation that the intensity ratio of the low-energy side to the high-energy side of the Autler-Townes doublet around the transition $5^2S_{1/2}, F = 2 \rightarrow 5^2P_{3/2}, F' = 2$ is 0.14. In addition, the intensity ratio of the transition $5^2S_{1/2}, F = 2 \rightarrow 5^2P_{3/2}, F' = 1$ to transition $5^2S_{1/2}, F = 2 \rightarrow 5^2P_{3/2}, F' = 2$ is 0.2 according to the Clebsch-Gordan coefficient. Therefore, it is understandable that only the high-energy side of the Autler-Townes doublet around the transition $5^2S_{1/2}, F = 2 \rightarrow 5^2P_{3/2}, F' = 1$ is observed.

Concentrating on the probe absorption spectrum over the cycle transition $5^2S_{1/2}, F = 2 \rightarrow 5^2P_{3/2}, F' = 3$, we observe a characteristic triplet structure over this transition. However, the energy position of the triplet structure differs from the prediction by the two-level dressed-atom approach [1,2] as shown in Fig. 8. Now we discuss it in detail. The energy position of the dispersionlike structure with respect to ω_1 is near −7.8 MHz. Meanwhile, the absorption peak and the gain peak are highly asymmetrical with respect to ω_1; the measured maximum absorption and maximum gain with respect to ω_1 are +25.1 and −50.1 MHz, respectively. In addition, the triplet structure also differs from the prediction by the extended two-level dressed-atom approach [3]. More specifically, considering the polarization configurations of the three pairs of cooling beams, this type of spectrum may be attributed to the mixture of cc and coc spectra according to the extended two-level dressed-atom approach [3]. In fact the absorption peak can be decomposed into two or more peaks as shown in the inset. The calculated energy positions of the spectral features in cc and coc spectra based on the theoretical model of Lipsich *et al.* are also exhibited [3]. In calculations, $\Omega_1 = 36.9$ MHz is estimated using the Autler-Townes doublet for the transition $5^2S_{1/2}, F = 2 \rightarrow 5^2P_{3/2}, F' = 2$. However, our measurements obviously differ from the calculations. The observed triplet structure is red-shifted compared to the prediction by the two-level dressed-atom approach [1,2] or to the extended two-level dressed-atom approach [3].

We see a similar situation when we vary the frequency of the coupling field as shown in Fig. 9 and Table III. More specifically, the detuning $\Delta_1/2\pi$ of the coupling field from the transition $5^2S_{1/2}, F = 2 \rightarrow 5^2P_{3/2}, F' = 3$ varies from −12 to −2 MHz while the fixed intensity $I = 153.4$ mW/cm^2. For all measurements the energy positions of the dispersionlike structures center near −7.0 MHz with respect to the frequency of the pump-coupling field. The energy position of the gain peak differs from the prediction by the two-level dressed-atom approach [1,2]. The asymmetric distribution with respect to the coupling field frequency between the gain peak and the absorption peak becomes apparent as $\Delta_1/2\pi$ varies from −12 to −2 MHz as shown in Table III. However, previous experimental pump-probe spectra measured in an operating MOT for Cs atoms [4–6,8], ^{85}Rb atoms [9,10], and ^{87}Rb atoms [12] seem consistent with the prediction by the two-level dressed-atom approach [1,2]. We will give analysis.

The coupling field acting on the bare atomic levels is resonant with the two transitions associated with the dispersionlike structure. Therefore, the coupling field will split the four dressed levels into eight levels, more transitions will happen, and spectral features beyond the prediction by the two-level dressed-atom approach [1,2] are expected. The reason that the previous experimental observations seem different from ours may be due to the ratio of Ω_1 to Δ_1 [4–6,8]. Based on the two-level dressed-atom approach the intensity of the dispersionlike structure is proportional to $\sin^2\theta\cos^2\theta$ [1,2]. It can be deduced that the effect of splitting of the dressed levels into a degenerate two-level atomic system with one strong monochromatic coupling field becomes evident as the intensity of the dispersionlike structure increases. In fact, previous experimental spectra are measured for the case of $-\Delta_1 \sim \Omega_1$, while in our measurements $-\Delta_1 << \Omega_1$ as shown in Figs. 8 and 9. $|\theta|$ is much bigger than it is in previous experiments [4–6,8]. Therefore, our experimental condition meets the requirement for splitting of the dressed levels connecting two transitions associated with the dispersionlike structure. The transitions connecting two levels formed by splitting of the dressed levels will constitute spectral features not predicted by the two-level dressed-atom approach [1,2] or the extended two-level dressed-atom approach [3]. In fact, as shown in Table III, the pump-probe spectra show that

TABLE III. The comparison between experimental and theoretical (present estimated errors in parentheses) data to demonstrate splitting of the dressed levels in two-level atomic systems with one coupling field shown in Fig. 3. The experimental data are extracted from the spectra in Fig. 9. All the energy positions of spectral features are with respect to ω_1.

Δ_1 ($2\pi \times$ MHz)	$\tan 2\theta$[a]	$\omega_{\text{disp.,expt}}$[b] ($2\pi \times$ MHz)	$\omega_{\text{abs.,expt}}$[c] ($2\pi \times$ MHz)	$\omega_{\text{gain,expt}}$[d] ($2\pi \times$ MHz)	$\omega_{\text{gain,dressed}}$[e] ($2\pi \times$ MHz)	$\lvert\frac{\omega_{\text{gain,expt}}}{\omega_{\text{abs.,expt}}}\rvert$	FWHM$_{\text{abs.}}$[f] ($2\pi \times$ MHz)
−12	2.9 (0.1)	−7.2(1.3)	27.4(1.2)	−57.8(1.6)	−37.2(1.5)	2.11(0.11)	28.1 (1.3)
−10	3.4 (0.2)	−7.4(1.3)	25.3(1.2)	−56.0(1.6)	−35.0(1.6)	2.21(0.12)	31.7 (1.3)
−8	4.2 (0.2)	−7.2(1.3)	23.5(1.2)	−54.3(1.6)	−34.7(1.6)	2.32(0.14)	33.5 (1.3)
−6	5.5 (0.3)	−7.6(1.3)	22.3(1.2)	−48.9(1.6)	−33.4(1.6)	2.20(0.14)	35.3 (1.3)
−4	7.8 (0.4)	−6.9(1.3)	19.6(1.2)	−47.3(1.6)	−31.5(1.5)	2.41(0.17)	36.5 (1.3)
−2	14.0 (0.8)	−7.1(1.3)	16.8(1.2)	−42.7(1.6)	−28.1(1.6)	2.53(0.20)	38.3 (1.3)

[a] $\tan 2\theta = -\frac{\Omega_1}{\Delta_1}$ and Ω_1 is obtained from the Autler-Townes doublets for $5^2S_{1/2}, F = 2 \rightarrow 5^2P_{3/2}, F' = 1,2$ transitions.
[b] Obtained from the experimental absorption maximum of the dispersionlike structure.
[c] Obtained from the experimental maximum absorption of the absorption peak.
[d] Obtained from the experimental maximum gain of the gain peak.
[e] Obtained from Ω_1 in Table II according to the two-level dressed-atom approach [1,2].
[f] The full width at half maximum (FWHM) of the absorption peak obtained from one peak Gaussian fitting.

the asymmetric distribution between the two sidebands with respect to the coupling field frequency becomes much more apparent as $|\theta|$ increases. In addition, the absorption peak broadens as $|\theta|$ increases as shown in Fig. 9 and Table III.

One question may be raised: Why does the Autler-Townes doublet around the transition $5^2S_{1/2}, F = 2 \rightarrow 5^2P_{3/2}, F' = 2$ and $5^2S_{1/2}, F = 2 \rightarrow 5^2P_{3/2}, F' = 1$ generally agree with those predicted by the two-level dressed-atom approach [1,2] shown in Table II. There are four rather than two transitions around $5^2S_{1/2}, F = 2 \rightarrow 5^2P_{3/2}, F' = 2$ or $5^2S_{1/2}, F = 2 \rightarrow 5^2P_{3/2}, F' = 1$ if splitting of the dressed levels is considered. However, the frequency difference between two levels split from a dressed level is $\Omega_2' = \sqrt{\Omega_2^2 + \Delta_2^2}$, where $\Delta_2 = 0$ MHz. Because Ω_2 is less than Ω_1 in a two-level atomic system with one strong monochromatic coupling field as shown in Fig. 3, the frequency difference is smaller than the frequency difference between the Autler-Townes doublet, i.e., Ω_1'. Therefore, corresponding spectral features overlap and become two broad peaks, which appear to be the Autler-Townes doublet. In addition, the center of gravity of the two levels resulting from splitting of one dressed level is the energy position of the dressed level. Therefore, the two broad peaks resemble the Autler-Townes doublet and can be interpreted by the two-level dressed-atom approach [1,2].

2. Pump-probe spectra under the condition that pump and probe polarizations are circular and equal (cc)

The complex situation in an operating MOT motivates us to study the pump-probe spectra of cold ^{87}Rb atoms under the condition that the polarizations of coupling and probe fields are well defined. This can be accomplished under the condition that the cooling beams and the magnetic field of the MOT are switched off. The coupling field is provided by an independent frequency-stabilized diode laser Toptica DL100. The detailed description is given in Sec. II B. The polarizations of the coupling field and the probe field are cc in the present measurements. We measure a series of spectra with a fixed intensity of the coupling field and various Δ_1. Based on the calculations from the theoretical model of Lipsich *et al.* [3], the characteristic triplet structure around the transition $5^2S_{1/2}, F = 2 \rightarrow 5^2P_{3/2}, F' = 3$ should be observed in this case. In fact, as shown in Figs. 10(a) and 10(e) and Table IV, the gain peak labeled as G_1' and the absorption peak labeled as A_1' can be seen and are approximately symmetrically distributed with respect to the coupling field frequency ω_1, and also are consistent with the calculations from the theoretical model of Lipsich *et al.* [3]. However, as Δ_1 decreases to 0 MHz, the asymmetric distribution with respect to the coupling field frequency between the two sidebands becomes much more apparent as shown in Figs. 10(b)–10(e) and Table IV. More specifically, the extended two-level dressed-atom approach predicts the gain peak should be located at $-\omega_{\text{abs.,expt}}$ [3], while in fact the gain peak is red-shifted with respect to $-\omega_{\text{abs.,expt}}$ as Δ_1 decreases. Therefore we deduce that the observed gain peak and absorption peak are red-shifted compared to the prediction from the extended two-level dressed-atom approach [3]. They should correspond to the two transitions connecting the two levels formed by splitting of the dressed levels as shown in Fig. 3, that is, the gain peak and the absorption peak are located at $-\Omega_1' - \Omega_2'$ and $\Omega_1' - \Omega_2'$, respectively. We now explain in detail.

Similar to the discussion for pump-probe spectra in an operating MOT, the angle $|\theta|$ increases as $|\Delta_1|$ decreases as shown in Table IV. Therefore, the secular limit in which the coupling field is much stronger than the decay rates of the related dressed states can be satisfied. Under this condition, splitting of the dressed levels connecting the transitions corresponding to the dispersionlike structure happens. The transitions connecting the two new levels constitute spectroscopic structures that are different from the prediction by the extended two-level dressed-atom approach [3]. As shown in Fig. 10(e) and Table IV, the asymmetric distribution between the two sidebands with respect to the coupling field frequency becomes much more apparent as $|\theta|$ increases. Figure 10(f) shows the comparison between Ω_1 and $\Omega_{1,\text{fit}}$. Ω_1 is obtained from the Autler-Townes doublet for the transition $5^2S_{1/2}, F = 2 \rightarrow 5^2P_{3/2}, F' = 2$, while $\Omega_{1,\text{fit}}$ is calculated according to energy level formulas predicted by our model shown in Fig. 3. Although the two data sets are consistent within experimental

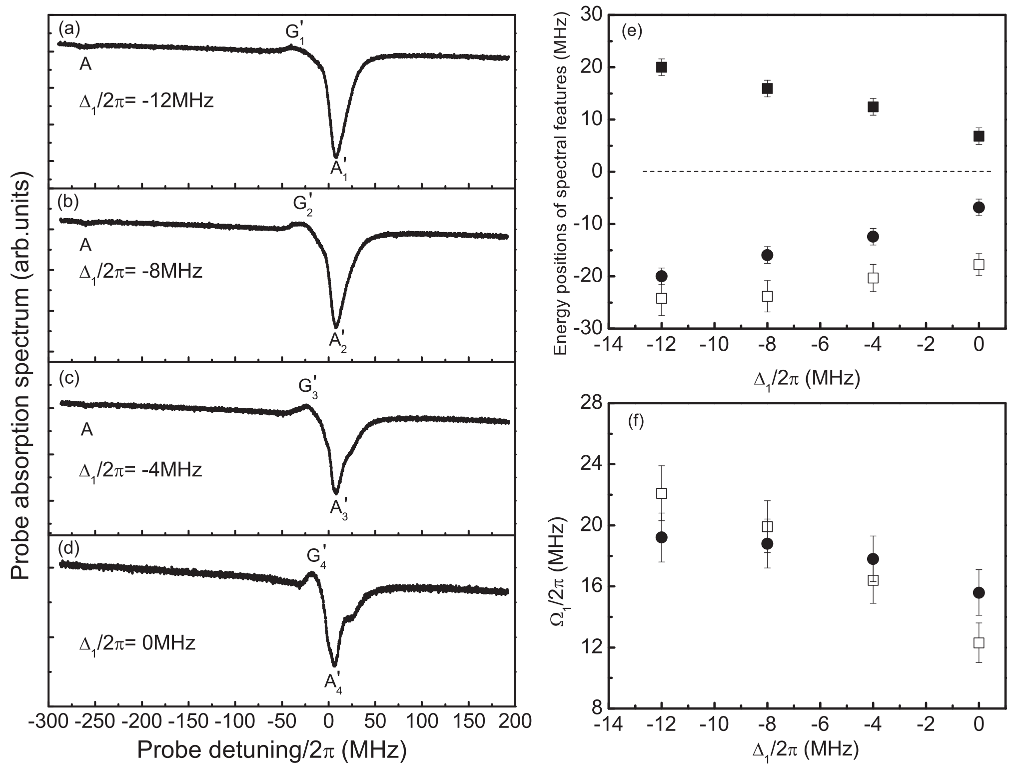

FIG. 10. Experimental pump-probe spectra of cold ^{87}Rb atoms measured in the well-defined polarization configurations of the coupling and probe pump fields, i.e., pump and probe polarizations are circular and equal (cc). (a) $\Delta_1/2\pi = -12$ MHz; (b) $\Delta_1/2\pi = -8$ MHz; (c)$\Delta_1/2\pi = -4$ MHz; (d) $\Delta_1/2\pi = 0$ MHz. The Rabi frequency of the coupling field $\Omega_1 \sim 19$ MHz is estimated from the Autler-Townes doublet (labeled as A) for the transition $5^2S_{1/2}, F = 2 \rightarrow 5^2P_{3/2}, F' = 2$. (e) Energy positions of spectral features. Solid squares and circles represent the energy positions of the absorption peak and the gain peak, respectively. Open squares represent the energy positions symmetrical with the absorption peak and these energy positions should agree well with the energy positions of the gain peak predicted by our model. (f) Comparison between Ω_1 obtained from the Autler-Townes doublet for the transition $5^2S_{1/2}, F = 2 \rightarrow 5^2P_{3/2}, F' = 2$, and $\Omega_{1,\text{fit}}$ calculated using the energy level formulas predicted by our model shown in Fig. 3. Solid circles and open squares represent Ω_1 and $\Omega_{1,\text{fit}}$, respectively.

errors, the slopes of the two data sets look different; this indirectly supports our model, i.e., splitting of dressed levels as shown in Fig. 3. Further work is underway.

IV. CONCLUSIONS

In conclusion, we have experimentally and theoretically studied the response of dressed levels driven by a

TABLE IV. The comparison between experimental and theoretical (present estimated errors in parentheses) data to demonstrate splitting of the dressed levels in two-level atomic systems with one coupling field shown in Fig. 3. The experimental data are extracted from the spectra in Fig. 10. All the energy positions of spectral features are with respect to ω_1.

Δ_1 ($2\pi \times$ MHz)	Ω_1[a] ($2\pi \times$ MHz)	$\tan 2\theta$[b]	$\omega_{\text{abs.,expt}}$[c] ($2\pi \times$ MHz)	$\omega_{\text{gain,expt}}$[d] ($2\pi \times$ MHz)	$\lvert\frac{\omega_{\text{gain,expt}}}{\omega_{\text{abs.,expt}}}\rvert$	$\Omega_{1,\text{fit}}$[e] ($2\pi \times$ MHz)
−12	19.2(1.6)	1.6(0.1)	20.0(1.6)	−24.2(3.3)	1.2 (0.2)	22.1(1.8)
−8	18.8(1.6)	2.4(0.2)	15.9(1.6)	−23.8(3.0)	1.5 (0.2)	19.9(1.7)
−4	17.8(1.5)	4.5(0.4)	12.4(1.6)	−20.3(2.6)	1.6 (0.3)	16.4(1.5)
0	15.6(1.5)	∞	6.8 (1.6)	−17.8(2.1)	2.6 (0.7)	12.3(1.3)

[a]Obtained from the Autler-Townes doublet for the transition $5^2S_{1/2}, F = 2 \rightarrow 5^2P_{3/2}, F' = 2$.
[b]$\tan 2\theta = -\frac{\Omega_1}{\Delta_1}$.
[c]Obtained from the experimental maximum absorption of the absorption peak. It is equal to $\Omega_1' - \Omega_2'$ as shown in Fig. 3.
[d]Obtained from the experimental maximum gain of the gain peak. It is equal to $-\Omega_1' - \Omega_2'$ as shown in Fig. 3.
[e]Obtained from comparison between energy positions of two experimental features (the gain peak and the absorption peak) and corresponding energy level formulas shown in Fig. 3.

quasiresonant coupling field. Theoretically, we propose a model for cascade splitting of two atomic energy levels shown in Figs. 1, 2, and 3, i.e., the atomic energy levels would cascade split due to multiphoton absorption. Two types of experiments are performed to test the model. The first experiment measures probe absorption spectra over the atomic transitions $5^2S_{1/2}, F = 2 \rightarrow 5^2P_{3/2}, F' = 3$ of cold ^{87}Rb atoms with two strong monochromatic coupling fields. The second experiment measures a series of the pump-probe spectra over the transitions $5^2S_{1/2}, F = 2 \rightarrow 5^2P_{3/2}, F' = 1,2$ and 3 of cold ^{87}Rb atoms. New spectral features are observed as shown in Figs. 4, 5, and 7. Some spectral features obviously differ from the prediction by the two-level dressed-atom approach [1,2] or the extended two-level dressed-atom approach [3] as shown in Figs. 8, 9, and 10. The model of cascade splitting of two atomic energy levels can explain the above observations in these two types of experiments.

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