Thermal dispersion potential of a diamagnetic atom

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We study the diamagnetic interaction between a ground-state atom, which is located at a distance z from a planar body, e.g., a perfect mirror or a nondispersive and nonabsorbing dielectric substrate, and the body-assisted electromagnetic fields from vacuum, equilibrium, and out of equilibrium thermal fluctuations. We find that the diamagnetic potential at zero temperature is always proportional to z^{-4} in both the retarded and the nonretarded zones, and the Casimir-Polder (CP) force is attractive. The CP potential due to the thermal fluctuations at equilibrium dominates over that due to the zero-point fluctuations in the long-distance or high-temperature limit and behaves like T/z^3, and the corresponding force is attractive. However, in the case of out of thermal equilibrium, the CP potential exhibits a different behavior with slower dependence on the distance and stronger dependence on temperature in the same limit, and it decays like $(T_e^2 - T_s^2)/z^2$, where T_e is the temperature of the environment and T_s is that of the substrate, yielding a CP force that can either be attractive or repulsive. Meanwhile, in the short-distance or low-temperature limit the CP potential is always dominated by the contribution due to the vacuum fluctuations.

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I. INTRODUCTION

Casimir and Polder [1] first found that an electrically polarizable neutral atom near a perfectly conducting plane feels a net attractive force due to the vacuum fluctuations of electromagnetic fields. So, this force is called the Casimir-Polder (CP) force. The CP force behaves like $1/z^4$ (z being the atom-wall separation) in the nonretarded region (short distance), which is like the van der Waals–London interatomic force, while at large distances it exhibits a different behavior and depends on $1/z^5$. Later, Casimir and Polder's work was extended to include the thermal fluctuations by Lifshitz [2]. Assuming the atom and the body in a thermal bath at a temperature T, Lifshitz found that the thermal fluctuation effect also leads to an attractive atom-wall force and the leading term of this force is proportional to T/z^4. In addition, the effects of a medium have been accounted for in [3–7]. Recently, the CP force between an electrically polarizable atom and a dielectric substrate in a stationary configuration out of thermal equilibrium, where the atom and the substrate are assumed to be kept at different local temperatures, has been studied [8–10], and it has been demonstrated that the CP force now shows new asymptotic behaviors, i.e., its leading term has a $\Delta T^2/z^3$ dependence in the retarded limit. Here, $\Delta T^2 \equiv T_s^2 - T_e^2$ with T_e and T_s being the temperatures of the thermal bath in the right and the substrate in the left half spaces, respectively. Apparently, the force can be attractive or repulsive depending on the difference of two temperatures.

The Casimir-Polder force has recently been extended to magnetic atoms, including paramagnetic atoms and diamagnetic ones, and different results have been found. For a paramagnetic atom placed in front of a planar body, such as a perfect mirror, the CP force has the same z dependence as that in the case of an electric atom but it has an opposite force character, i.e., the CP force of paramagnetic atoms is repulsive rather than attractive as opposed to the electric atom [11–13], while for a diamagnetic atom, because the diamagnetic magnetizability is negative and frequency independent, the CP force behaves like $1/z^5$ in both the retarded and the nonretarded limits, and as a result of the Lenz rule its sign is opposite to that of the paramagnetic counterpart [14]. Thus, an attractive force is obtained for a diamagnetic atom.

As mentioned above, for an electric atom, the thermal fluctuations, especially the out of equilibrium thermal fluctuations, can give rise to different characters of the CP force. However, what happens to a magnetic atom remains unclear. In this paper, we plan to investigate this issue using the macroscopic quantum electrodynamics approach [15,16]. At this point, it is worth pointing out that the contribution from diamagnetic coupling is very important when computing highly accurate potentials for alkali-metal atom dimers [17], where it has been found that the diamagnetic-electric contributions are larger than the electric-paramagnetic contributions, and the diamagnetic-paramagnetic interactions are larger than the paramagnetic-paramagnetic contributions.

II. QUANTIZATION

We consider the system with an atom interacting with the electromagnetic field in the presence of the magnetodielectric bodies described by a Kramers-Kronig consistent permittivity $\epsilon(\boldsymbol{r},\omega)$ and permeability $\mu(\boldsymbol{r},\omega)$. Thus, the total Hamiltonian has the form

$$\hat{H} = \hat{H}_A + \hat{H}_F + \hat{H}_{AF}. \tag{1}$$

Here $\hat{H}_A$ is the Hamiltonian of an atom located at position $\boldsymbol{r}_A$:

$$\hat{H}_A = \sum_n E_A^n |n\rangle\langle n|. \tag{2}$$

$\hat{H}_F$ is the Hamiltonian of the body-assisted electromagnetic field. Its electric- and magnetic-field operators can be expressed as

$$\hat{\boldsymbol{E}}(\boldsymbol{r}) = \sum_{\lambda=e,m} \int d^3r' \int_0^\infty d\omega\ \mathbf{G}_\lambda(\boldsymbol{r},\boldsymbol{r}',\omega)\cdot \hat{\boldsymbol{f}}_\lambda(\boldsymbol{r}',\omega) + \text{H.c.}\ , \tag{3}$$

$$\hat{\boldsymbol{B}}(\boldsymbol{r}) = \sum_{\lambda=e,m} \int d^3r' \int_0^\infty \frac{d\omega}{i\omega} \nabla \times \mathbf{G}_\lambda(\boldsymbol{r},\boldsymbol{r}',\omega)\cdot \hat{\boldsymbol{f}}_\lambda(\boldsymbol{r}',\omega) + \text{H.c.}, \tag{4}$$

where $\hat{\boldsymbol{f}}_\lambda^\dagger(\boldsymbol{r},\omega)$ and $\hat{\boldsymbol{f}}_\lambda(\boldsymbol{r},\omega)$ are the creation and annihilation operators of the elementary electric ($\lambda = e$) and magnetic ($\lambda = m$) excitations, respectively. They obey the bosonic commutation relations

$$[\hat{\boldsymbol{f}}_\lambda(\boldsymbol{r},\omega),\hat{\boldsymbol{f}}_{\lambda'}^\dagger(\boldsymbol{r}',\omega')] = \boldsymbol{\delta}(\boldsymbol{r}-\boldsymbol{r}')\delta_{\lambda\lambda'}\delta(\omega-\omega') \tag{5}$$

and

$$[\hat{\boldsymbol{f}}_\lambda(\boldsymbol{r},\omega),\hat{\boldsymbol{f}}_{\lambda'}(\boldsymbol{r}',\omega')] = [\hat{\boldsymbol{f}}_\lambda^\dagger(\boldsymbol{r},\omega),\hat{\boldsymbol{f}}_{\lambda'}^\dagger(\boldsymbol{r}',\omega')] = \mathbf{0}, \tag{6}$$

where $\boldsymbol{\delta}(\boldsymbol{r}-\boldsymbol{r}')$ is a diagonal matrix with the diagonal element given by $\delta(\boldsymbol{r}-\boldsymbol{r}')$, and $\mathbf{0}$ represents a zero matrix. In Eqs. (3) and (4), the quantities $\mathbf{G}_\lambda$ are related to the classical Green's tensor $\mathbf{G}$ by

$$\mathbf{G}_e(\boldsymbol{r},\boldsymbol{r}',\omega) = i\,\frac{\omega^2}{c^2}\sqrt{\frac{\hbar}{\pi\epsilon_0}\,\text{Im}\,\epsilon(\boldsymbol{r}',\omega)}\;\mathbf{G}(\boldsymbol{r},\boldsymbol{r}',\omega), \tag{7}$$

$$\mathbf{G}_m(\boldsymbol{r},\boldsymbol{r}',\omega) = i\,\frac{\omega}{c}\sqrt{\frac{\hbar}{\pi\epsilon_0}\frac{\text{Im}\,\mu(\boldsymbol{r}',\omega)}{|\mu(\boldsymbol{r}',\omega)|^2}}[\nabla'\times\mathbf{G}(\boldsymbol{r}',\boldsymbol{r},\omega)]^{\mathsf{T}}. \tag{8}$$

Here, ϵ_0 and c represent the vacuum permittivity and the light speed, respectively. The Green's function $\mathbf{G}$ satisfies the differential equation

$$\left[\nabla\times\frac{1}{\mu(\boldsymbol{r},\omega)}\nabla\times - \frac{\omega^2}{c^2}\,\epsilon(\boldsymbol{r},\omega)\right]\mathbf{G}(\boldsymbol{r},\boldsymbol{r}',\omega) = \boldsymbol{\delta}(\boldsymbol{r}-\boldsymbol{r}') \tag{9}$$

and the boundary condition

$$\mathbf{G}(\boldsymbol{r},\boldsymbol{r}',\omega)\to\mathbf{0} \quad \text{for} \quad |\boldsymbol{r}-\boldsymbol{r}'|\to\infty. \tag{10}$$

It also fulfills the Schwarz reflection principle and obeys the Onsager-Lorentz reciprocity:

$$\begin{aligned}\mathbf{G}(\boldsymbol{r},\boldsymbol{r}',-\omega^*) &= \mathbf{G}^*(\boldsymbol{r},\boldsymbol{r}',\omega),\\ \mathbf{G}(\boldsymbol{r}',\boldsymbol{r},\omega) &= \mathbf{G}^{\mathsf{T}}(\boldsymbol{r},\boldsymbol{r}',\omega).\end{aligned} \tag{11}$$

In addition, there is a useful integral relation for the Green's function:

$$\sum_{\lambda=e,m}\int d^3s\;\mathbf{G}_\lambda(\boldsymbol{r},\boldsymbol{s},\omega)\cdot\mathbf{G}_\lambda^{*\mathsf{T}}(\boldsymbol{r}',\boldsymbol{s},\omega) = \frac{\hbar\mu_0}{\pi}\,\omega^2\,\text{Im}\;\mathbf{G}(\boldsymbol{r},\boldsymbol{r}',\omega), \tag{12}$$

where μ_0 is the vacuum permeability. Thus, using Eqs. (3) and (4), we write $\hat{H}_F$ into the form

$$\hat{H}_F = \sum_{\lambda=e,m}\int d^3r\int_0^\infty d\omega\hbar\omega\hat{\boldsymbol{f}}_\lambda^\dagger(\boldsymbol{r},\omega)\cdot\hat{\boldsymbol{f}}_\lambda(\boldsymbol{r},\omega). \tag{13}$$

It is easy to see that the ground state of $\hat{H}_F$ can be defined as

$$\hat{\boldsymbol{f}}_\lambda(\boldsymbol{r},\omega)|\{0\}\rangle = 0 \quad \forall\lambda,\boldsymbol{r},\omega. \tag{14}$$

Hereafter, we use $|\{\ \}\rangle$ to represent the state of electromagnetic fields, while, for a thermal state, one has

$$\begin{aligned}\langle\{\beta\}|\hat{\boldsymbol{f}}_\lambda(\boldsymbol{r},\omega)\hat{\boldsymbol{f}}_{\lambda'}^\dagger(\boldsymbol{r}',\omega')|\{\beta\}\rangle &= (1+N(\beta))\boldsymbol{\delta}(\boldsymbol{r}-\boldsymbol{r}')\\ &\quad\times\delta_{\lambda\lambda'}\delta(\omega-\omega'),\end{aligned} \tag{15}$$

$$\langle\{\beta\}|\hat{\boldsymbol{f}}_\lambda^\dagger(\boldsymbol{r},\omega)\hat{\boldsymbol{f}}_{\lambda'}(\boldsymbol{r}',\omega')|\{\beta\}\rangle = N(\beta)\boldsymbol{\delta}(\boldsymbol{r}-\boldsymbol{r}')\delta_{\lambda\lambda'}\delta(\omega-\omega'), \tag{16}$$

with $\beta = \hbar c/kT$, k being the Boltzmann constant and

$$N(\beta) = \frac{1}{e^{\beta\omega/c}-1}. \tag{17}$$

In Eq. (1), $\hat{H}_{AF}$ describes the interaction between the atom and the body-assisted electromagnetic field. Within the multipolar coupling scheme, $\hat{H}_{AF}$ contains three different terms [18]:

$$\hat{H}_{AF} = -\hat{\boldsymbol{d}}_A\cdot\hat{\boldsymbol{E}}(\boldsymbol{r}_A) - \hat{\boldsymbol{m}}_A\cdot\hat{\boldsymbol{B}}(\boldsymbol{r}_A) + \sum_{\alpha\in A}\frac{q_\alpha^2}{8m_\alpha}[\hat{\bar{\boldsymbol{r}}}_\alpha\times\hat{\boldsymbol{B}}(\boldsymbol{r}_A)]^2, \tag{18}$$

where the first, second, and third terms in the right-hand side represent the electric, paramagnetic, and diamagnetic interactions, respectively. $\hat{\boldsymbol{d}}_A$ and $\hat{\boldsymbol{m}}_A$ are the respective atomic electric and magnetic dipole operators, and q_α, m_α, and $\hat{\bar{\boldsymbol{r}}}_\alpha$ denote the charges, masses, and positions relative to the center of mass of the particles contained in the atom, respectively.

III. DIAMAGNETIC INTERACTION

The electric interaction between an atom and the fields in the presence of a body, such as a perfect mirror or a dielectric substrate, has been studied extensively including the contributions from the zero-point fluctuations, the equilibrium thermal fluctuations (see [19] for recent reviews), and the out of equilibrium thermal fluctuations [8,10]. In addition, the magnetic interactions, including paramagnetic and diamagnetic ones, arising from the vacuum fluctuations have been investigated in [12,14]. Here, we focus on the diamagnetic interaction between a ground-state atom and the body-assisted electromagnetic fields from the thermal fluctuations. The contributions from the equilibrium and out of equilibrium thermal fluctuations are both considered. We assume that the left half space ($z < 0$) is filled with a nonabsorbing and nondispersive dielectric substrate whose permittivity is real and frequency independent (which we call a real dielectric substrate for short hereafter) at temperature T_s and the right half space ($z > 0$) is filled with a thermal bath at temperature T_e. Apparently, $T_e = T_s$ corresponds to the case of thermal equilibrium. For convenience, we introduce the atomic diamagnetizability operator:

$$\hat{\boldsymbol{m}}_A^d = -\sum_{\alpha\in A}\frac{q_\alpha^2}{4m_\alpha}\left(\hat{\bar{\boldsymbol{r}}}_\alpha^2\,\mathbf{I} - \hat{\bar{\boldsymbol{r}}}_\alpha\hat{\bar{\boldsymbol{r}}}_\alpha\right). \tag{19}$$

Here, $\hat{\bar{\boldsymbol{r}}}_\alpha^2 \equiv \hat{\bar{\boldsymbol{r}}}_\alpha\cdot\hat{\bar{\boldsymbol{r}}}_\alpha$ and $\mathbf{I}$ is a unit matrix. In terms of Lagrange's identity $[\boldsymbol{a}\times\boldsymbol{b}]^2 = \boldsymbol{b}\cdot(\boldsymbol{a}^2\,\mathbf{I}-\boldsymbol{a}\boldsymbol{a})\cdot\boldsymbol{b}$, we can re-express the diamagnetic interaction Hamiltonian as

$$\sum_{\alpha\in A}\frac{q_\alpha^2}{8m_\alpha}[\hat{\bar{\boldsymbol{r}}}_\alpha\times\hat{\boldsymbol{B}}(\boldsymbol{r}_A)]^2 = -\frac{1}{2}\hat{\boldsymbol{B}}(\boldsymbol{r}_A)\cdot\hat{\boldsymbol{m}}_A^d\cdot\hat{\boldsymbol{B}}(\boldsymbol{r}_A). \tag{20}$$

The ground-state diamagnetizability of an atom is given by the expectation value:

$$\boldsymbol{m}_A^d \equiv \langle \hat{\boldsymbol{m}}_A^d \rangle = -\sum_{\alpha\in A} \frac{q_\alpha^2}{4m_\alpha} \langle 0_A|\hat{\boldsymbol{r}}_\alpha^2\,\mathbf{I} - \hat{\boldsymbol{r}}_\alpha\hat{\boldsymbol{r}}_\alpha|0_A\rangle. \quad (21)$$

For an isotropic atom, $\boldsymbol{m}_A^d$ can be simplified as

$$\boldsymbol{m}_A^d = -\sum_{\alpha\in A} \frac{q_\alpha^2}{6m_\alpha} \langle \hat{\boldsymbol{r}}_\alpha^2 \rangle\,\mathbf{I} \equiv m_A^d\,\mathbf{I}. \quad (22)$$

Apparently, different from the paramagnetic magnetizability and electric polarizability, the diamagnetic magnetizability is negative and frequency independent.

From the diamagnetic interaction Hamiltonian, one can obtain the CP potential of a purely diamagnetic atom in an out of thermal equilibrium system described by $\langle\{\beta_e,\beta_s\}|$:

$$\begin{aligned} U(r_A) &= \langle\{\beta_e,\beta_s\}|\langle 0_A| -\tfrac{1}{2}\hat{\boldsymbol{B}}(\boldsymbol{r}_A)\cdot\hat{\boldsymbol{m}}_A^d\cdot\hat{\boldsymbol{B}}(\boldsymbol{r}_A)|0_A\rangle|\{\beta_s,\beta_e\}\rangle \\ &= -\tfrac{1}{2}m_A^d\,\mathrm{tr}[\langle\{\beta_e,\beta_s\}|\hat{\boldsymbol{B}}(\boldsymbol{r}_A)\cdot\hat{\boldsymbol{B}}(\boldsymbol{r}_A)|\{\beta_s,\beta_e\}\rangle] \\ &\equiv -\tfrac{1}{2}m_A^d f(r_A). \end{aligned} \quad (23)$$

Using Eq. (4), one can obtain

$$f(r_A) = -\sum_{\lambda=e,m}\int d^3r'\int \frac{d\omega}{\omega^2}[1+2N(r')]\,\mathrm{tr}\Big[\nabla_{\boldsymbol{r}_1} \times \mathbf{G}_\lambda(\boldsymbol{r}_1,\boldsymbol{r}',\omega)\cdot\,\mathbf{G}_\lambda^{*\mathsf{T}}(\boldsymbol{r}_2,\boldsymbol{r}',\omega)\times\overleftarrow{\nabla}_{\boldsymbol{r}_2}\Big]\Big|_{\boldsymbol{r}_1=\boldsymbol{r}_2=\boldsymbol{r}_A}. \quad (24)$$

Since the left and right half spaces have different temperatures, it is useful to reexpress $f(r_A)$ as a summation of three different terms:

$$f(r_A) = f_{\mathrm{vac}}(r_A) + f_{\mathrm{eq}}(r_A) + f_{\mathrm{neq}}(r_A) \quad (25)$$

with

$$f_{\mathrm{vac}}(r_A) = -\frac{\hbar\mu_0}{\pi}\int d\omega\,\mathrm{tr}\Big[\mathrm{Im}\Big[\nabla_{\boldsymbol{r}_1}\times\mathbf{G}(\boldsymbol{r}_1,\boldsymbol{r}_2,\omega)\times\overleftarrow{\nabla}_{\boldsymbol{r}_2}\Big]\Big]\Big|_{\boldsymbol{r}_1=\boldsymbol{r}_2=\boldsymbol{r}_A}, \quad (26)$$

$$f_{\mathrm{eq}}(r_A) = -\frac{2\hbar\mu_0}{\pi}\int d\omega N(\beta_e)\,\mathrm{tr}\Big[\mathrm{Im}\Big[\nabla_{\boldsymbol{r}_1}\times\mathbf{G}(\boldsymbol{r}_1,\boldsymbol{r}_2,\omega)\times\overleftarrow{\nabla}_{\boldsymbol{r}_2}\Big]\Big]\Big|_{\boldsymbol{r}_1=\boldsymbol{r}_2=\boldsymbol{r}_A}, \quad (27)$$

$$f_{\mathrm{neq}}(r_A) = -2\sum_{\lambda=e,m}\int_{z'<0} d^3r'\int\frac{d\omega}{\omega^2}[N(\beta_s)-N(\beta_e)]\cdot\mathrm{tr}\Big[\nabla_{\boldsymbol{r}_1}\times\mathbf{G}_\lambda(\boldsymbol{r}_1,\boldsymbol{r}',\omega)\cdot\,\mathbf{G}_\lambda^{*\mathsf{T}}(\boldsymbol{r}_2,\boldsymbol{r}',\omega)\times\overleftarrow{\nabla}_{\boldsymbol{r}_2}\Big]\Big|_{\boldsymbol{r}_1=\boldsymbol{r}_2=\boldsymbol{r}_A}, \quad (28)$$

where $f_{\mathrm{vac}}(r_A)$, $f_{\mathrm{eq}}(r_A)$, and $f_{\mathrm{neq}}(r_A)$ correspond to the contributions from zero-point fluctuations, equilibrium, and out of equilibrium thermal fluctuations, respectively. In Eqs. (26) and (27), Eq. (12) has been used. As expected, when $T_e = T_s$ the out of thermal equilibrium term disappears.

Using the Green's functions in the medium configuration given in [20] and assuming that an isotropic atom is located at a distance z from the substrate, we find that, in the case of a real dielectric substrate with a relative permittivity ϵ, $f_{\mathrm{vac}}(r_A)$, $f_{\mathrm{eq}}(r_A)$, and $f_{\mathrm{neq}}(r_A)$ are functions only of z and have the following forms:

$$f_{\mathrm{vac}}(z) = \frac{\hbar\mu_0}{2\pi^2}\int d\omega\int_0^1 dt\,\frac{w^3}{c^3}[\cos(2\omega zt/c)T(t) + e^{-2\omega\sqrt{\epsilon-1}zt/c}A(t)], \quad (29)$$

$$f_{\mathrm{eq}}(z) = \frac{\hbar\mu_0}{\pi^2}\int d\omega\int_0^1 dt\,\frac{w^3}{c^3}\frac{1}{e^{\beta_e\omega/c}-1}[\cos(2\omega zt/c)T(t) + e^{-2\omega\sqrt{\epsilon-1}zt/c}A(t)], \quad (30)$$

$$f_{\mathrm{neq}}(z) = \frac{\hbar\mu_0}{\pi^2}\int d\omega\int_0^1 dt\,\frac{w^3}{c^3}\left(\frac{1}{e^{\beta_s\omega/c}-1} - \frac{1}{e^{\beta_e\omega/c}-1}\right)\times e^{-2\omega\sqrt{\epsilon-1}zt/c}A(t), \quad (31)$$

where

$$T(t) \equiv \frac{1}{2}\frac{\epsilon t - \sqrt{\epsilon-1+t^2}}{\epsilon t + \sqrt{\epsilon-1+t^2}} + \left(\frac{1}{2} - t^2\right)\frac{t-\sqrt{\epsilon-1+t^2}}{t+\sqrt{\epsilon-1+t^2}}, \quad (32)$$

$$A(t) \equiv t\sqrt{\epsilon-1}\sqrt{1-t^2}\left[\frac{\epsilon}{(\epsilon^2-1)t^2+1} + 1 + 2(\epsilon-1)t^2\right]. \quad (33)$$

Notice that here the T and A functions give the contributions from the traveling waves and the evanescent waves, respectively.

Substituting Eqs. (29)–(31) into Eq. (23), one can get the expression of the CP potential $U(z)$ and then the CP force, which is given by differentiating the CP potential with respect to z. For convenience, we divide our discussion into two special cases: a perfectly reflecting planar mirror, which corresponds to $\epsilon \to \infty$, and a real dielectric substrate. Notice that, when the perfect mirror case is considered, we must take first the limit of $\epsilon \to \infty$ before analyzing the retarded or nonretarded limit, otherwise a wrong result may result.

IV. PERFECTLY REFLECTING PLANAR MIRROR

A perfect mirror is characterized by $\epsilon \to \infty$. After taking this limit, we find that $f_{\mathrm{neq}}(z)$ disappears, which means that there is no contribution coming from out of equilibrium thermal fluctuations in this case. For $f_{\mathrm{vac}}(z)$ and $f_{\mathrm{eq}}(z)$, only the term containing the cosine function exists and $A(t) \to t^2$.

A. Zero-point fluctuations

The vacuum fluctuation corresponds to the case of zero temperature. Thus, only Eq. (29) needs to be considered, and it is simplified as

$$\begin{aligned} f_{\mathrm{vac}}(z) &= \frac{\hbar\mu_0}{2\pi^2}\int d\omega\,\frac{\omega^3}{c^3}\int_0^1 t^2\cos(2\omega zt/c) \\ &= -\frac{3}{16\pi^2}\frac{\hbar\mu_0 c}{z^4}, \end{aligned} \quad (34)$$

which leads to a CP potential:

$$U(z) = -\frac{1}{2} m_A^d f_{\rm vac}(z) = \frac{3}{32\pi^2}\frac{\hbar\mu_0 c}{z^4} m_A^d$$
$$= -\frac{3}{32\pi^2}\frac{\hbar\mu_0 c}{z^4}\sum_{\alpha\in A}\frac{q_\alpha^2}{6m_\alpha}\langle\hat{\boldsymbol{r}}_\alpha^2\rangle. \tag{35}$$

This result is the same as the one found in [14]. It is easy to see that the contribution of zero-point fluctuations depends on z^{-4}, which means that the CP force is attractive and decays like z^{-5} in both the retarded region and the nonretarded one.

B. Equilibrium thermal fluctuations

Here, we assume that the total system is in a thermal equilibrium at temperature T_e. From Eq. (30), one has

$$f_{\rm eq}(z) = \frac{\hbar\mu_0}{\pi^2}\int d\omega\, \frac{\omega^3}{c^3}\frac{1}{e^{\beta_e\omega/c}-1}\int_0^1 t^2\cos(2\omega z t/c)$$
$$= \frac{\hbar\mu_0 c T^4}{\pi^2}\times\frac{3+b\pi\{-\coth(b\pi)-b\pi[1+b\pi\coth(b\pi)]\sinh^{-2}(b\pi)\}}{b^4} \tag{36}$$

with $b\equiv 2z/\beta_e$. For the case of $b\gg 1$, which corresponds to the high-temperature or long-distance limit, we have

$$f_{\rm eq}(z)\simeq\frac{\hbar\mu_0 c}{16\pi^2}\left(\frac{3}{z^4}-\frac{2\pi}{z^3\beta}\right). \tag{37}$$

Since in this case the CP potential is determined by the zero-point fluctuations and the thermal ones, it has the form

$$U(z) = -\frac{1}{2} m_A^d [f_{\rm vac}(z)+f_{\rm eq}(z)]$$
$$\simeq -\frac{\hbar\mu_0 c}{16\pi}\frac{1}{z^3\beta_e}\sum_{\alpha\in A}\frac{q_\alpha^2}{6m_\alpha}\langle\hat{\boldsymbol{r}}_\alpha^2\rangle. \tag{38}$$

Apparently, in the $b\gg 1$ limit the thermal fluctuation is the main source of the CP potential and this potential is proportional to $z^{-3}\beta_e^{-1}$ and gives an attractive force on the atom. These properties are the same as in the case of an electric atom.

Taking the short-distance or low-temperature limit ($b\ll 1$), one has

$$f_{\rm eq}(z)\simeq -\frac{\hbar\mu_0 c}{4\pi^2}\left(\frac{64\pi^6 z^2}{315\beta_e^6}\right), \tag{39}$$

and the CP potential

$$U(z) = -\frac{1}{2} m_A^d [f_{\rm vac}(z)+f_{\rm eq}(z)]$$
$$\simeq -\frac{\hbar\mu_0 c}{8\pi^2}\left(\frac{3}{4z^4}+\frac{64\pi^6 z^2}{315\beta_e^6}\right)\sum_{\alpha\in A}\frac{q_\alpha^2}{6m_\alpha}\langle\hat{\boldsymbol{r}}_\alpha^2\rangle. \tag{40}$$

The thermal fluctuation contribution is proportional to $z^2\beta_e^{-6}$, which is the same as in the case of an electric atom, but it is smaller than the one from zero-point fluctuations. Therefore, in this limit the CP potential is dominated by the z^{-4} term.

V. REAL DIELECTRIC SUBSTRATE

In this section, we calculate the CP potential for the case of a real dielectric substrate.

A. Zero-point fluctuations

Since the result will be divergent when one calculates directly the integrations of Eq. (29), we use the method proposed in [4] to eliminate this divergence. After some tedious calculations, we obtain

$$f_{\rm vac}(z) = -\frac{3}{16\pi^2}\frac{\hbar\mu_0 c}{z^4}g_1(\epsilon) \tag{41}$$

with

$$g_1(\epsilon) = \frac{1}{6}\Bigg(2T(0)+3T'(0)+3T''(1)+3\frac{A'(0)}{(\epsilon-1)^2}+\frac{1}{2}T'''(0)\ln(\epsilon-1)$$
$$-6\int_0^1\frac{dt}{t^4}\left(T(t)-T(0)-T'(0)t-\frac{1}{2}T''(0)t^2+\frac{A(t)-A'(0)t}{(\epsilon-1)^2}\right)\Bigg)$$
$$=\frac{1}{6}\Bigg(-2-6\epsilon+\frac{3(\epsilon+1)}{\sqrt{\epsilon-1}}-\frac{12}{\epsilon-1}+\frac{3(\epsilon+1)}{(\epsilon-1)^{3/2}}+\frac{3(5-5\epsilon+2\epsilon^3)}{2(\epsilon-1)^{3/2}}\ln(\epsilon-1)$$
$$-6\int_0^1\frac{dt}{t^4}\left(T(t)-T(0)-T'(0)t-\frac{1}{2}T''(0)t^2+\frac{A(t)-A'(0)t}{(\epsilon-1)^2}\right)\Bigg). \tag{42}$$

Here, a prime denotes the derivative of functions with respect to their arguments, e.g., $T'(0)=dT(t)/dt|_{t\to 0}$. When $\epsilon\to\infty$, $g_1(\epsilon)\to 1$, and we recover the result of a perfect mirror. As expected, $g_1(\epsilon)\to 0$ for $\epsilon\to 1$. Thus, the CP potential is still proportional to z^{-4} and has the form

$$U(z) = -\frac{3}{32\pi^2}\frac{\hbar\mu_0 c}{z^4} g_1(\epsilon) \sum_{\alpha\in A} \frac{q_\alpha^2}{6m_\alpha}\langle \hat{\boldsymbol{r}}_\alpha^2\rangle. \tag{43}$$

We find that $0 \leqslant g_1(\epsilon) \leqslant 1$, which means that, compared with the perfect mirror, a finite ϵ for the real dielectric decreases the magnitude of the CP force.

B. Thermal equilibrium

Assuming that the system is in a thermal bath at temperature T_e, we find that, when $\min[2z/\beta_e, 2z(\epsilon-1)/\beta_e] \gg 1$,

$$f_{\rm eq}(z) \simeq \frac{\hbar\mu_0 c}{\pi^2}\left[\frac{3}{16z^4} g_1(\epsilon) - \frac{1}{4z^3\beta_e}\frac{\epsilon+1}{\epsilon-1}\right]. \tag{44}$$

Using Eqs. (41) and (44), one obtains the CP potential

$$\begin{aligned} U(z) &= -\frac{1}{2} m_A^d [f_{\rm vac}(z) + f_{\rm eq}(z)] \\ &\simeq -\frac{\hbar\mu_0 c}{8\pi^2}\frac{1}{z^3\beta_e}\frac{\epsilon+1}{\epsilon-1}\sum_{\alpha\in A}\frac{q_\alpha^2}{6m_\alpha}\langle\hat{\boldsymbol{r}}_\alpha^2\rangle. \end{aligned} \tag{45}$$

As in the case of a perfect mirror, the CP potential depends on $z^{-3}\beta_e^{-1}$. When $\epsilon \to \infty$, the result reduces to the one obtained in the perfect mirror. Different from the case of zero-point fluctuations, the real dielectric increases rather than decreases the CP potential due to the thermal fluctuations since $\frac{\epsilon+1}{\epsilon-1} \geqslant 1$. Notice that, when $\epsilon \to 1$, $U(z)$ diverges, but this limit is not allowed since we have already assumed that $\min[2z/\beta_e, 2z(\epsilon-1)/\beta_e] \gg 1$.

For $\max[2z/\beta_e, 2z(\epsilon-1)/\beta_e] \ll 1$, one can obtain

$$f_{\rm eq}(z) \simeq \frac{\hbar\mu_0 c}{2\pi^2}\left[-\frac{96\,\xi(5)\,z}{\beta_e^5} g_2(\epsilon) - \frac{32\pi^6 z^2}{63\beta_e^6} g_3(\epsilon)\right], \tag{46}$$

where $\xi(x)$ is the Riemann zeta function:

$$g_2(\epsilon) = \frac{(5+2\epsilon+\epsilon^2)(\epsilon^2-\epsilon)\pi}{16(\epsilon+1)^2}, \tag{47}$$

$$g_3(\epsilon) = \int_0^1 dt t^2[T(t) - (\epsilon-1)A(t)]. \tag{48}$$

Comparing Eqs. (39) and (46), one can see that $f_{\rm eq}(z)$ in the real dielectric case has a different character. From Eqs. (41) and (46), the CP potential is

$$\begin{aligned} U(r) \simeq &-\frac{\hbar\mu_0 c}{4\pi^2}\left[\frac{3}{8z^4} g_1(\epsilon) + \frac{96\,\xi(5)\,z}{\beta_e^5} g_2(\epsilon) + \frac{32\pi^6 z^2}{63\beta_e^6} g_3(\epsilon)\right] \\ &\times \sum_{\alpha\in A}\frac{q_\alpha^2}{6m_\alpha}\langle\hat{\boldsymbol{r}}_\alpha^2\rangle. \end{aligned} \tag{49}$$

In the limit of $\max[2z/\beta_e, 2z(\epsilon-1)/\beta_e] \ll 1$, the contribution of zero-point fluctuations is dominated over that of thermal fluctuations.

C. Out of thermal equilibrium

Now we study the contribution of out of equilibrium thermal fluctuations. We first consider the limit of $\min[2z, 2z(\epsilon-1) \gg \max[\beta_e, \beta_s]$ and obtain

$$f_{\rm neq}(z) \simeq \frac{\hbar\mu_0 c}{24z^2}\frac{\epsilon+1}{\sqrt{\epsilon-1}}\left(\frac{1}{\beta_e^2} - \frac{1}{\beta_s^2}\right). \tag{50}$$

Combining Eqs. (41), (44), and (50) gives the CP potential of total contributions including zero-point and thermal fluctuations:

$$\begin{aligned} U(z) &= -\frac{1}{2} m_A^d [f_{\rm vac}(z) + f_{\rm eq}(z) + f_{\rm neq}(z)] \\ &\simeq -\frac{\hbar\mu_0 c}{8\pi^2}\left[\frac{\pi^2}{6z^2}\frac{\epsilon+1}{\sqrt{\epsilon-1}}\left(\frac{1}{\beta_e^2} - \frac{1}{\beta_s^2}\right) + \frac{1}{z^3\beta_e}\frac{\epsilon+1}{\epsilon-1}\right] \\ &\quad\times \sum_{\alpha\in A}\frac{q_\alpha^2}{6m_\alpha}\langle\hat{\boldsymbol{r}}_\alpha^2\rangle. \end{aligned} \tag{51}$$

If the temperature difference is not too small, one can see that the contribution of out of equilibrium thermal fluctuation plays a dominant role and the CP force can be attractive or repulsive, depending on this difference. This character is the same as what was obtained in the electric atom case [8,10].

When the limit $\max[2z, 2z(\epsilon-1)] \ll \min[\beta_e, \beta_s]$ is taken, one can obtain

$$f_{\rm neq}(z) \simeq \frac{\hbar\mu_0 c}{\pi}\frac{48\,\xi(5)\,z}{\pi} g_2(\epsilon)\left(\frac{1}{\beta_e^5} - \frac{1}{\beta_s^5}\right), \tag{52}$$

and the total CP potential

$$\begin{aligned} U(r) \simeq &-\frac{\hbar\mu_0 c}{4\pi^2}\left[\frac{3}{8z^4} g_1(\epsilon) + \frac{96\,\xi(5)\,z}{\beta_s^5} g_2(\epsilon) + \frac{32\pi^6 z^2}{63\beta_e^6} g_3(\epsilon)\right] \\ &\times \sum_{\alpha\in A}\frac{q_\alpha^2}{6m_\alpha}\langle\hat{\boldsymbol{r}}_\alpha^2\rangle. \end{aligned} \tag{53}$$

In this limit, the CP potential is still dominated by the zero-point fluctuations.

VI. CONCLUSION

It has been found that the thermal fluctuations, especially out of equilibrium thermal fluctuations, play a very important role in the retarded CP force of an electric atom. In this paper we extend the study to the case of a magnetic atom. We consider the diamagnetic interaction between a ground-state atom and the body-assisted electromagnetic field from the thermal fluctuations. The contributions from both the equilibrium and the out of equilibrium thermal fluctuations are analyzed. We examine two special cases: a perfectly reflecting planar mirror and a real dielectric substrate. For the case of a perfect mirror, we find that the diamagnetic CP potential due to the zero-point fluctuations carries the same sign as the well-known electric potential but different from the paramagnetic one. The diamagnetic potential at zero temperature is always proportional to z^{-4} in both the retarded and the nonretarded zones. When the atom-mirror system is in a thermal bath, the CP potential is dominated by the contribution of the thermal fluctuations and behaves like T_e/z^3 in the long-distance or high-temperature limit and the corresponding CP force is attractive, while in the short-distance or low-temperature limit the main contribution of the CP potential is from the vacuum fluctuations.

When a real dielectric substrate is considered, we find that the CP potential has the same sign as in the case of a perfect mirror for both cases of zero temperature and a thermal bath at finite temperature. However, the dielectric substrate decreases the CP force due to the vacuum fluctuations while it increases that due to the thermal fluctuations at equilibrium in the long-distance or high-temperature limit. For the CP potential in the case of out of thermal equilibrium, we find in the long-distance or high-temperature limit the CP force is proportional to $(T_e^2 - T_s^2)/z^2$. So, the force can be attractive or repulsive, depending on the difference of two temperatures. In the short-distance or low-temperature limit, the CP potential is still dominated by the vacuum fluctuations. These properties are the same as those of an electric atom [8,10].

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