Polarization control of ultrabroadband supercontinuum generation from midinfrared laser-induced harmonic emission

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We present an efficient scheme for generation of an isolated attosecond (as) pulse based on the combination of two well-known methods: polarization gating method plus a method based on using two colors ($\omega + \omega/2$). The first uses a pulse whose ellipticity is small within a very short time interval (called a polarization gating). By adding a second pulse whose frequency is a half-frequency (midinfrared) of the first pulse, we extend significantly the cutoff frequency, as well as the width of the supercontinuum harmonic spectrum. The high-order harmonic generation is calculated using the strong-field approximation theory. As a result, an isolated 42-as pulse is generated directly by superposing the hundreds of harmonics. To explain the mechanism of the ultrashort attosecond generation, we perform the semiclassical three-step model simulation and wavelet time-frequency transform of the harmonic spectra.

The generation of an ultrashort isolated attosecond (as) pulse has been a subject of much interest in ultrafast science and technology in the past decade, such as controlling electron wave packet [\[1\]](#page-3-0), probing nuclear dynamic [\[2\]](#page-3-0) and photoelectron spectroscopy [\[3\]](#page-3-0), attosecond time-resolved spectroscopy [\[4\]](#page-3-0), tomographic imaging of molecular orbital [\[5\]](#page-3-0), and so on. Currently, the production of an isolated attosecond pulse by means of the superposition of the broadband supercontinuum in high-order harmonic generation (HHG) is one of the most promising routes [\[6–8\]](#page-3-0). The HHG process in single-atom response has been well understood by the semiclassical threestep model [\[9\]](#page-3-0). In this model, the electron first tunnels through the potential barrier formed by the Coulomb potential and the laser field, and it is accelerated by the laser field and acquires additional kinetic energy; finally the electron can recombine with the parent ion and emit radiation. Generally speaking, the phase-locked broadband supercontinuum harmonic spectra are the key to the generation of ultrashort attosecond pulse. Therefore, a series of schemes to generate ultrashort attosecond pulse based on the generation of the broadband supercontinuum harmonic spectra have been proposed, such as two-color laser fields [\[10–12\]](#page-3-0), a few-cycle laser pulse [\[13\]](#page-3-0), the chirped laser pulse $[14–17]$, quantum path control $[18–20]$, and long-wavelength pumping [\[21,22\]](#page-3-0). In experimental works, Goulielmakis *et al.* [\[23\]](#page-3-0) have obtained an isolated 80-as pulse by a single 3.3-fs pump pulse, and Chang *et al.* [\[24\]](#page-3-0) have generated a world-record-shortest 67-as pulse by the double optical gating technique. At present, the main experimental techniques to produce the isolated attosecond pulse from HHG spectrum include spectral selection of half-cycle cutoffs [\[25–27\]](#page-3-0), temporal gating technique with polarization gating [\[28,29\]](#page-3-0), double optical gating [\[30–32\]](#page-3-0), and spatiotemporal gating with the attosecond lighthouse technique [\[33,34\]](#page-3-0). Particularly, the polarization gating is deemed to be an effective method to obtain an isolated attosecond pulse by controlling the ellipticity of the driver field [\[35\]](#page-3-0).

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In this work, we propose an efficient scheme to generate broadband supercontinuum harmonic spectra from He atoms by using the combination of the elliptically polarized pulses and a midinfrared laser. We find that the cutoff of the HHG is extended by the midinfrared laser field, and our scheme allows one to select and control the quantum trajectory by adding a polarization gating, leading to the generation of the broadband supercontinuum harmonic spectra. As a result, an isolated 42-as pulse is obtained directly.

We calculate the HHG spectra from the single-atom response using the strong-field approximation theory (SFA) [\[36\]](#page-3-0), the combination of the elliptically polarized pulses, and a midinfrared laser pulse can be written as

$$
\mathbf{E}(t) = (E_{px} + E_{mi})\hat{\mathbf{x}} + E_{py}\hat{\mathbf{y}},
$$
(1)

where the midinfrared field $E_{mi}(t)$ is expressed by

$$
E_{mi}(t) = E_1 f(t) \cos(\omega_1 t). \tag{2}
$$

In calculation, the elliptically polarized pulse can be constructed by a right circularly polarized pulse and a left circularly polarized pulse [\[37\]](#page-3-0). The left circularly polarized pulse \mathbf{E}_L and right circularly polarized pulse \mathbf{E}_R are expressed as the following form:

$$
\mathbf{E}_L = E_{oL}(t) [\cos(\omega t + \phi_{ce})\hat{\mathbf{x}} + \sin(\omega t + \phi_{ce})\hat{\mathbf{y}}], \quad (3)
$$

$$
\mathbf{E}_R = E_{oR}(t)[\cos(\omega t + \phi_{ce})\hat{\mathbf{x}} - \sin(\omega t + \phi_{ce})\hat{\mathbf{y}}], \quad (4)
$$

where $E_{oL}(t)$ and $E_{oR}(t)$ are the amplitude, respectively. It can be written as

$$
E_{oL}(t) = E_0 \exp\left[-2\ln(2)\left(\frac{t - T_d/2}{T_p}\right)^2\right],
$$
 (5)

$$
E_{oR}(t) = E_0 \exp \left[-2 \ln(2) \left(\frac{t + T_d/2}{T_p} \right)^2 \right].
$$
 (6)

There is a delay time between the left circularly polarized pulse and the right circularly polarized pulse. Therefore, the electric field by combining the two pulses produces a gating in the *y*

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direction,

$$
E_{py} = [E_{oL}(t) - E_{oR}(t)]\sin(\omega t + \phi_{ce}),\tag{7}
$$

and the elliptically polarized electric field in the *x* direction is

$$
E_{px} = [E_{oL}(t) + E_{oR}(t)]\cos(\omega t + \phi_{ce}).
$$
 (8)

Figure $1(a)$ shows the superposition of the *x*-direction polarized pulse and a midinfrared laser pulse (green solid line), the *x*-direction polarized pulse alone (red dashed dot line), and the *y*-direction polarized pulse (blue dashed dot line, $E_{py} = 0$, respectively. According to the previous work [\[38\]](#page-3-0), the oscillating electron will be driven away from the parent ion in the circularly polarized pulse, so it is difficult to recombine with the parent ion. However, it is possible to recombine with the parent ion in the linear polarized laser field. Therefore, the polarization gating width plays an important role in the isolated attosecond pulse generation. When the polarization gating width is shorter than one optical cycle, an isolated attosecond pulse can be produced. This method needs a shorter input pulse and a longer delay time [\[39\]](#page-3-0). In the experiment, the duration of the input pulse is larger than the delay time, so this condition suggests that it is possible to have two emissions of the photons when the oscillating electrons recombine twice with the parent ion in the one optical cycle. Therefore, in our calculation, the polarization gating width is 2.6 fs with the wavelength 1064 nm ($\omega = 0.04286$ a.u., $T = 2\pi/\omega$), the intensity is 2.14 \times 10¹⁴ W/cm², and the delay time T_d and the duration of the input pulse T_p are equal to 14 fs. In addition,

for the midinfrared laser fields, the wavelength is 2128 nm $(\omega_1 = \omega/2 = 0.02143 \text{ a.u.})$, the duration time is 20 fs, and the intensity is 2.97×10^{14} W/cm². Figure 1(b) shows the corresponding HHG spectra without the *y*-direction polarized pulse $(E_{py} = 0)$. We find that the cutoff for the *x*-direction polarized pulse alone is located at the 180th harmonic. It is very clear that the plateau of HHG spectrum is not extended. However, by adding a midinfrared field to the *x*-direction polarized pulse, the corresponding cutoff reaches the 720th harmonic. In other words, the plateau of the HHG has been extended by a midinfrared field.

In fact, the ponderomotive energy U_p is proportional to $I\lambda^2$, where *I* and λ are the intensity and wavelength of the driving laser field, respectively. This implies that the cutoff energy of HHG can be extended by increasing the wavelength. Therefore, the cutoff can be extended by the midinfrared laser. In addition, there is a multiplateau structure for the combined laser fields cases due to the coherent superposition of the different emission events [a more detailed discussion will be addressed later in Fig. $2(b)$]. However, the HHG spectra shown in Fig. $1(b)$ are not smooth near the cutoff region, which does not advantage the generation of the isolated attosecond pulse. The reason is that the elliptically polarized pulse in the *y* direction need not be considered. When the atom is exposed to a linear polarized laser field in the whole duration time, the oscillating electrons recombine twice with the parent ion in each optical cycle. Therefore, the multiemission appears in the multicycle linear polarized laser field. Generally, there is

FIG. 1. (Color online) (a) The superposition of the *x*-direction polarized pulse and a midinfrared laser pulse (green solid line), the *x*-direction polarized pulse alone (red dashed dot line), and the *y*-direction polarized pulse alone (blue dashed dot line, *Epy* = 0). (c) Same as (a), for $E_{py} \neq 0$. The corresponding HHG spectra are presented in (b) and (d), respectively.

FIG. 2. (Color online) (a) Two-cycle combined laser field of the *x*-direction polarized pulse and a midinfrared laser pulse. (b) Return energy as a function of the ionization time t_i and emission time t_r . The parameters used are the same as those in Fig. $1(c)$.

a series of attosecond pulses generated by the linear polarized laser field. Fortunately, we can optimize the HHG spectra by the polarization gating technique. If we add a polarization gating pulse in the *y* direction to control the quantum path of the electrons, the oscillating electrons only recombine with the parent ion in the inside of the gating, in which the laser field approximately is linearly polarized. Once the polarization gating width is close to the half optical cycle of the carrier wavelength, an isolated attosecond pulse can be generated near the center of the polarized pulse. Figure $1(c)$ shows the combined pulse of the *x*-direction polarized pulse and a midinfrared laser pulse, the *x*-direction polarized pulse alone, and the *y*-direction ($E_{py} \neq 0$) polarized pulse alone, respectively, where the intensity of the *y*-direction elliptically polarized pulse is 2.19×10^{14} W/cm², and the time delay T_d is equal to 14 fs, the corresponding polarization gating width is 2.6 fs, and other parameters used are the same as those in Fig. [1\(a\).](#page-1-0) The corresponding HHG spectra are shown in Fig. $1(d)$; there is a smooth plateau after the 460th harmonic order.

To explain the extension of the plateau of the HHG, we calculate the kinetic energy as a function of the ionization time t_i and emission time t_r by using the semiclassical three-step model, as shown in Fig. $2(b)$. The corresponding combined field of the *x*-direction polarized pulse and a midinfrared laser

FIG. 3. (Color online) Wavelet time-frequency profile of the HHG spectra of He atoms driven by the combined laser field with (a) $E_{py} = 0$, (b) $E_{py} \neq 0$. The parameters used are the same as those in Fig. [1\(a\).](#page-1-0)

pulse are presented in Fig. $2(a)$; the parameters used are the same as those in Fig. $1(c)$. There are three major ionization peaks marked by *A*1, *A*2, and *A*3, the corresponding emissions peaks are marked by B_1 , B_2 , and B_3 with the maximum kinetic energies of $26U_p$, $36U_p$, $9U_p$, respectively, in good agreement

FIG. 4. (Color online) Attosecond pulse generation by superposing the harmonics from (a) 460th ∼ 540th harmonic order, (b) 540th ∼ 620th harmonic order, and (c) 620th ∼ 700th harmonic order.

with the multiplateau structures of the HHG as shown in Fig. [1\(b\).](#page-1-0) The multiplateau structures of the HHG are formed due to the coherent superposition of three major emission events marked by *B*1, *B*2, and *B*3. Particularly the major ionization peak marked by *A*² appears at the time −0.67*T* , and the corresponding emission peaks marked by B_2 are located at the time $0.25T$ with the energy $36U_p$, which corresponds to the 719th harmonic order, in good agreement with the cutoff of HHG spectra shown in Fig. $1(d)$. To demonstrate the contributions of the short- and long-quantum path, we perform the wavelet time-frequency profile of the HHG spectra of He atoms driven by the combined laser field shown in Figs. $1(a)$ and $1(c)$, respectively. In Fig. $3(a)$, there are three major emission bursts at the time −0.25*T* , 0.25*T* , and 0.75*T* , which are labeled by B_1 , B_2 , and B_3 . It is very clear that the emission burst marked by B_1 is a lot stronger than the major emission burst B_2 . Although the major emission burst B_2 is extended largely, an isolated attosecond pulse cannot be produced due to the stronger interference between the emission bursts B_1 and B_2 , but the attosecond pulse trains can be obtained by superposing the harmonics near the cutoff regions. However, by adding a *y*-direction polarized pulse, only one emission event appears near the center of the polarized pulse. As shown in Fig. $3(b)$, here there is only one major emission burst B_2 , while the burst B_1 is suppressed, and the short-quantum path (from 460th to 700th) is dominant. That is to say, by adding the

polarization gating in the *y* direction to control the electronic quantum path, only one major emission is chosen near the cutoff. Therefore, we can superpose the regular harmonics from the 460th to 540th order to generate an isolated 42-as pulse, as shown in Fig [4.](#page-2-0) We believe that this proposed metrology can be extended to other systems and produce an isolated ultrashort attosecond pulse.

In summary, we present an efficient scheme for the generation of an ultrashort isolated attosecond pulse by means of using a combined field of the elliptically polarized pulse and a midinfrared laser pulse whose frequency is a half frequency of the elliptically polarized pulse. The results indicate that the cutoff of HHG spectra can be hugely extended by the combined laser field of the midinfrared and *x*-direction polarized laser pulse, and the ultrabroad supercontinuum plateau can be produced by adding a *y*-direction polarized pulse. As a result, an isolated ultrashort 42-as pulse can be generated by superposing the supercontinuum harmonics.

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